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Output and Welfare Implications of Oligopolistic Third-Degree Price Discrimination*

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Abstract

Using estimable concepts, we provide sufficient conditions for price discrimination to lower or raise aggregate output and social welfare under symmetrically differentiated oligopoly with general demand functions and cost differences across separated markets. Assuming that all markets are open under uniform pricing, we show that if the *markup* ratio in the strong market (where the discriminatory price is higher than the uniform price) relative to the weak market (where it is lower) is sufficiently large under uniform pricing, then social welfare will be *lower* if price discrimination is allowed. It is also shown that if either the *conduct* ratio, the *pass-through* ratio, or the *markup* ratio is sufficiently small in the strong market under price discrimination, then it *raises* social welfare.

Keywords: Third-Degree Price Discrimination; Differentiated Oligopoly; Social Welfare; Pass-through.

JEL classification: D21; D43; D60; L11; L13.

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1 Introduction

In this paper, we extend Aguirre, Cowan, and Vickers' (2010) arguments of the welfare effects of monopolistic third-degree price discrimination to the case of (symmetric) oligopoly. In particular, we emphasize the role of *pass-through* as in Weyl and Fabinger (2013) and Adachi and Fabinger (2017). Furthermore, we allow *cost differentials* across discriminatory markets as in Chen and Schwartz (2015). Assuming that marginal costs are constant and that all markets are open under uniform pricing, we show that if the *markup* ratio in the strong market (where the discriminatory price is higher than the uniform price) relative to the weak market (where it is lower) is sufficiently large under uniform pricing, then social welfare will be *lower* if price discrimination is allowed. It is also shown that if either the *conduct* ratio, the *pass-through* ratio or the *markup* ratio is sufficiently small in the strong market under price discrimination, then it *raises* social welfare.

In almost all of the theoretical studies on price discrimination, researchers (manually) assume that there are no cost differentials across discriminatory markets to focus on the demand side. However, in many real-world cases of price discrimination, cost differentials are quite often observed, not to mention the typical example of a first-class seat and an economy-class seat (see Phlips 1983, pp. 5-7). In the narrow definition of price discrimination, this is not price discrimination because they are considered different products. However, airlines are arguably motivated to offer different types of seats because they aim to exploit consumer surplus by making use of heterogeneity among consumers. Thus, ideally, a theoretical analysis of price discrimination should also allow cost differentials across discriminatory markets.

Even if costs differ across markets, sellers, in reality, may have to be engaged in uniform pricing due to the universal service requirement, fairness concerns from consumers, and so on. Following Robinson (1933), we call one market s (strong), where the equilibrium discriminatory price, p_s, will be higher, if price discrimination is allowed, than the equilibrium uniform price, $\overline{p}$, and the other w (weak), where the opposite is true, and the equilibrium discriminatory price is denoted by p_w.[1] In the

[1] In this paper, price discrimination is present when $p_s > p_w$. As Clerides (2004, p. 402) argues,

case of monopoly with constant marginal costs, Chen and Schwartz (2015) derive sufficient conditions for consumer surplus to be higher under differential pricing. To ensure that the strong market is indeed strong when cost differentials are allowed, it is sufficient to assume that the marginal cost in the strong market is higher than in the weak market: $c_s > c_w$ (though c_s should not be too much higher than c_w). Then, under uniform pricing, the markup in the strong market $\overline{p} - c_s$ is smaller than the markup in the weak market $\overline{p} - c_w$. Differential pricing allows the monopolist to sell more products in the weak market which is more efficient than the other market. Chen and Schwartz (2015) find that while differential pricing with no cost differentials (third-degree price discrimination in a traditional manner) tends to increase the average price after differential pricing is allowed, differential pricing with cost differentials does not. As in Chen and Schwartz (2015), this paper does not have to make an explicit assumption on c_s and c_w as long as the second-order conditions are satisfied and a large discrepancy between c_w and c_w does not change the order of the discriminatory prices from the one with no cost differentials.[2]

We also emphasize that our sufficient conditions are related to estimable concepts. Interestingly, own- and cross-price elasticities *per se* do not play an important role in welfare evaluation. Our theoretical predictions, equipped with an estimable framework, would be utilized to understand the mechanism behind an empirical result. For example, in their empirical analysis of within-store brand competition, Hendel and Nevo (2013) show that social welfare is higher under third-degree price discrimination than with the case with no discrimination. However, it is not clear what mechanism makes this empirical result. Although welfare evaluation is ultimately an empirical matter, one still wishes to know more about which force derives

once cost differentials are allowed, "there is no single, widely accepted definition of price discrimination." To understand this issue, let mc_s and mc_w be equilibrium marginal costs in markets s and w, respectively. Then, there are two alternative definitions. One is the *margin* definition: price discrimination occurs when $p_s - mc_s > p_w - mc_w$. The other one is the markup definition: price discrimination occurs when $p_s/mc_s > p_w/mc_w$. In this paper, we employ the simplest definition. As long as cost differentials are sufficiently small, these differences will not alter the results significantly because these three definitions are equivalent if $mc_s = mc_w$.

[2] In the context of reduced-fare parking as a form of third-degree price discrimination, Flores and Kalashnikov (2017) characterize a sufficient condition for free parking (drivers receive a price discount in the form of complentary parking while pedestrians do not) to be welfare improving.

the result. Thus, this paper also aims to fill the gap between theoretical predictions and the empirical literature of price discrimination, where researchers often have to remain agnostic about the mechanism behind the result.

The rest of the paper is organized as follows. Section 2 presents our basic model with symmetric firms and constant marginal costs. Then, we derive output and welfare implications in Section 3. Section 4 concludes the paper.

2 Symmetric Firms with Constant Marginal Costs

For ease of exposition, we, following Holmes (1989) and Aguirre, Cowan, and Vickers (2010), consider the case of two symmetric firms and two separate markets or consumer groups (simply called markets hereafter). Extending the following analysis to the case of J (≥ 3) symmetric firms and M (≥ 3) separate markets is straightforward. As explained above, we call one market s (strong), where the equilibrium discriminatory price will be higher than the equilibrium uniform price, and the other w (weak), where the opposite is true. Two firms, A and B, have an identical cost structure in each market. We assume that they have a constant marginal cost in each market m, $c_m \geq 0$. In the spirit of Chen and Schwartz (2015), c_s and c_w can be different.

In market $m = s, w$, given firms A and B's prices $p_{A,m}$ and $p_{B,m}$, the representative consumer consumes $q_{A,m} > 0$ and $q_{B,m} > 0$, and her (net) utility is written as $U_m(q_{A,m}, q_{B,m}) - p_{A,m}q_{A,m} - p_{B,m}q_{B,m}$, where U_m is twice continuously differentiable, $\partial U_m/\partial q_{jm} > 0$, $\partial^2 U_m/\partial q_{jm}^2 > 0$, $j = A$, B, and $\partial^2 U_m/(\partial q_{A,m}\partial q_{B,m}) < 0$. The *direct demands* in market m are derived from the representative consumer's utility maximization: $\partial U_m(q_{jm}, q_{-j,m})/\partial q_{jm} - p_{jm} = 0$, which leads to firm j's demand in market m, $q_{jm} = x_{jm}(p_{jm}, p_{-j,m})$. We assume that x_{jm} is twice continuously differentiable. The corresponding *inverse demand* can be written as $p_{jm} = p_{jm}(q_{jm}, q_{-j,m})$. Because of the assumptions on the utility, firm j's demand in market m falls as its own price increases ($\partial x_{jm}/\partial q_{jm} < 0$), and it rises as the rival's price increases ($\partial x_{jm}/\partial q_{-j,m} > 0$; the firms' products are substitutes). We assume that for a consumer's perspective firms are symmetric: $U_m(q', q'') = U_m(q'', q')$ for any $q' > 0$ and $q'' > 0$. Then, the firms' demands in market m are also symmetric: $x_{A,m}(p', p'') = x_{B,m}(p'', p')$ for any $p' > 0$ and $p'' > 0$. Because the firms' technologies are also identical, we, throughout this paper, focus on symmetric Nash equilibrium. Under the regime of uniform pricing, the equilibrium uniform price for both markets is $\overline{p}$. If price discrimination is allowed, the equilibrium discriminatory prices are p_s^* in the strong market and p_w^* in the weak market, and functional and parametric restrictions are imposed to assure that $p_s^* > \overline{p} > p_w^*$.[3]

We define the demand in symmetric pricing by $q_m(p) \equiv x_{A,m}(p, p)$. Another interpretation of $q_m(p)$ is: both firms take $2q_m(p)$ as the joint demand, 'cooperatively' choose the same price (behaving as an 'industry'), and divide the joint demand equally to obtain $q_m(p)$. Note that:

$$q'_m(p) = \underbrace{\frac{\partial x_{A,m}}{\partial p_A}(p_A, p)\bigg|_{p_A=p}}_{<0} + \underbrace{\frac{\partial x_{A,m}}{\partial p_B}(p, p_B)\bigg|_{p_B=p}}_{>0 \text{ if substitutes}}.$$

Thus, for $q'_m(p)$ to be negative, we assume that $|\partial x_{A,m}(p, p)/\partial p_A| > \partial x_{A,m}(p, p)/\partial p_B$. Note also that by symmetry the following relationship also holds:

$$\underbrace{\frac{\partial x_{A,m}}{\partial p_A}(p, p)}_{\text{own}} = \underbrace{q'_m(p)}_{\text{industry}} - \underbrace{\frac{\partial x_{B,m}}{\partial p_A}(p, p)}_{\text{strategic effects}},$$

which corresponds to Holmes' (1989) equation (4). This exchangeability is to key in Holmes' (1989) derivation below. Intuitively, under symmetry, each firm treats the industry demand $q_m(p)$ as if it is its own demand. Thus, how a firm's pricing behavior affects its own demand as an industry demand has the following two effects: a small decrease in p^A by firm A by deviating from the 'coordinated' price p (i) not only raises its own demand by $\partial x_{A,m}/\partial p_A$ as the *residual* monopolist (taking the rival's pricing as fixed; *intrinsic effects*), (ii) firm A can now also obtain some of the

[3]However, see Nahata, Ostaszewski, and Sahoo (1990) for an example of all discriminatory prices being lower than the uniform price with a plausible demand structure under monopoly. In the case of oligopoly, Corts (1998) show that best-response asymmetry, by which firms differ in ranking strong and weak markets, is necessary to all discriminatory prices to be lower than the uniform price ("all-out price competition").

consumers originally attached to firm B, and this amount is $\partial x_{B,m}/\partial p_A$ (*strategic effects*).

Under symmetric pricing, we define, following Holmes (1989, p.245), the *price elasticity of the industry's demand* by $\varepsilon_m^I(p) \equiv -pq_m'(p)/q_m(p)$. This corresponds to ϵ_D in Weyl and Fabinger (2013, p.542), and it should not "be confused with the elasticity of the residual demand that any of the firms faces." Similarly, the *own-price and the cross-price elasticities of the firm's demand* are defined by $\varepsilon_m^F(p) \equiv -(p/q_m(p))(\partial x_{A,m}(p,p)/\partial p_A)$ and by $\varepsilon_m^C(p) \equiv (p/q_m(p))(\partial x_{B,m}(p,p)/\partial p_A)$, respectively. Then, Holmes (1989) shows that under symmetric pricing, $\varepsilon_m^F(p) = \varepsilon_m^I(p) + \varepsilon_m^C(p)$ holds. This implies that the own-price elasticity must be greater than the cross-price elasticity ($\varepsilon_m^F(p) > \varepsilon_m^C(p)$). Here, $\partial^2 x_{A,m}(p,p)/\partial(p_A)^2$ can be positive, zero or negative. Following Dastidar's (2006, p.234) Assumption 2 (iv), we assume that $\partial^2 x_{jm}(p,p)/\partial p_j^2 + \partial^2 x_{jm}(p,p)/\partial p_A \partial p_B \leq 0$.

Firm j's profit in market m is written as $\pi_{jm}(p_{jm}, p_{-j,m}; c_m) = (p_{jm} - c_m)x_{jm}(p_{jm}, p_{-j,m})$. As in Dastidar's (2006, pp.235-6) Assumptions 3 and 4, for the existence and the global uniqueness of pricing equilibrium under either uniform pricing or price discrimination, we assume that for each firm $j = A, B$, $\partial^2 \pi_{jm}/\partial p_{jm}^2 < 0$, $\partial^2 \pi_{jm}/(\partial p_{jm} \partial p_{-j,m}) > 0$, and $-[\partial^2 \pi_{jm}/(\partial p_{jm} \partial p_{-j,m})]/[\partial^2 \pi_{jm}/\partial p_{jm}^2] < 1$ (see Dastidar's (2006) Lemmas 1 and 2 for the existence and the uniqueness). We then define the first-order partial derivative of the profit in market m, evaluated at a symmetric price p, by

$$\begin{aligned} \partial_p \pi_m(p; c_m) &\equiv \left. \frac{\partial \pi_{jm}(p_{jm}, p_{-j,m}; c_m)}{\partial p_{jm}} \right|_{p_{jm}=p_{-j,m}=p} \\ &= q_m(p) + (p - c_m)\frac{\partial x_{A,m}}{\partial p_A}(p,p). \end{aligned}$$

Then, under symmetric discriminatory pricing, $p_m^* = p_m^*(c_m)$ satisfies $\partial_p \pi_m(p_m^*; c_m) = 0$ for $m = s, w$. Under symmetric uniform pricing, $\overline{p} = \overline{p}(c_s, c_w)$ is a (unique) solution of $\partial_p \pi_s(\overline{p}; c_s) + \partial_p \pi_w(\overline{p}; c_w) = 0$.[4] Throughout this paper, we consider the

[4] If one considers quantity-setting, rather than price-setting, firms, as in Agurre (2017), then firm j's profit in market m is defined by $\pi_{jm} = p_{jm}(q_{jm}, q_{-j,m})q_{jm} - c_m(q_{jm})$, and thus the first-order partial derivative in symmetric equilibrium is $\partial \pi_{jm}/\partial q_{jm}|_{q_{jm}=q_{-j,m}=q}$, which is equivalent to $p_m(q) - mc_m(q) + q(\partial p_{A,m}(q,q))/\partial q_A) = 0$. Under uniform pricing, firm j's quantity-setting problem is formulated as $\max_{q_s,q_w} \sum_{m=s,w}[p_{jm}(q_{jm}, q_{-j,m})q_{jm} - c_m(q_{jm})$ subject to $p_{j,s}(q_{j,s}, q_{-j,s}) = p_{j,w}(q_{j,w}, q_{-j,w})$.

situation where the weak market is open under uniform pricing: $q_w(\overline{p}) > 0$.[5]

Let the equilibrium profit in market m in symmetric equilibrium under uniform pricing and under price discrimination be denoted by π_m^* and $\overline{\pi}_m$, respectively. Accordingly, the aggregate profits are defined by $\Pi^* \equiv \pi_s^* + \pi_w^*$ and $\overline{\Pi} \equiv \overline{\pi}_s + \overline{\pi}_w$, respectively. If price discrimination lowers firms' profit in symmetric pricing equilibrium (i.e., $\Pi^* < \overline{\Pi}$), the firms may want to agree not to price discriminate even if they are allowed to do so. This is a situation of Prisoners' Dilemma: one firm's deviation is profitable.[6] We assume that firms cannot commit to not engaging in price discrimination even if $\overline{\Pi} > \Pi^*$: we consider Nash equilibrium under each regime of pricing (uniform pricing or price discrimination).

The equilibrium discriminatory price in market $m = s, w$, $p_m^* \equiv p_m^*(c_m)$, satisfies the following Lerner formula: $\varepsilon_m^F(p_m^*)(p_m^* - c_m)/p_m^* = 1$. This shows that the discriminatory price in market m approaches to the marginal cost as the own-price elasticity for the firm, $\varepsilon_m^F(p_m^*)$, becomes large. Because of Holmes' (1989) elasticity formula explained above, $\varepsilon_m^F(p_m^*)$ can be large (i) when $\varepsilon_m^I(p_m^*)$ is very large even if $\varepsilon_m^C(p_m^*)$ is close to zero, or (ii) when $\varepsilon_m^C(p_m^*)$ is very large even if $\varepsilon_m^I(p_m^*)$ is close to zero. These are two polar cases of a large $\varepsilon_m^F(p_m^*)$: of course if both $\varepsilon_m^I(p_m^*)$ and $\varepsilon_m^C(p_m^*)$ are very large, then $\varepsilon_m^F(p_m^*)$ is also very large. Case (i) is where the industry is under strong pressure from other substitutable industries[7] so that a small price increase in symmetric pricing causes a large number of consumers switching to purchasing a product in other industries instead, although any consumers are very loyal to either firm so that a small price increase by one firm causes a very small number

[5]Note that $q_w(\overline{p}) > q_w(p_s^*)$ because $q_w(\cdot)$ is strictly decreasing and $p_s^* > \overline{p}$. By Assumption, $q_w(p_s^*) > 0$. Thus, the weak market is open under uniform pricing, i.e., $q_w(\overline{p}) > 0$. Alternatively, we would be able to show that there exist $\underline{c}_s$ and $\overline{c}_s$, $\underline{c}_s < \overline{c}_s$, such that $p_s^* > p_w^*$ and $q_w(\overline{p}) > 0$ for $c_s \in (\underline{c}_s, \overline{c}_s)$ in a similar spirit of Adachi and Matsushima (2014).

[6]Dastidar (2006) provides a sufficient condition for firms' equilibrium profit to be higher under price discrimination. More specifically, define the price difference between the discriminatory price and the uniform price in market m by $\Delta p_m^* \equiv p_m^* - \overline{p}$. Then, Dastidar's Proposition 2 (2006, p. 241) shows that if $|\Delta p_s^*| \geq |\Delta p_w^*|$ and $\partial x_{B,s}(p,p)/\partial p_A \geq \partial x_{B,w}(p,p)/\partial p_A$ for all $p \in (p_w^*, p_s^*)$, then the per-firm profit difference, $\Delta\Pi \equiv \Pi^* - \overline{\Pi}$, is positive.

[7]In the demand construction based on random utility and discrete choice, this would be interpreted as an outside option. Note also that if the industry is defined by the SSNIP ("Small but Significant and Non-transitory Increase in Price") test, then by definition, $\varepsilon_m^I(p_m^*)$ is close to zero, and thus, $\varepsilon_m^F(p_m^*)$ is closely approximated by $\varepsilon_m^C(p_m^*)$.

of consumers switching to other rivals' product in the same industry (most of them leave the industry to purchase something outside the industry). For example, in a residential area, consumers (especially young consumers) would have a strong taste for their favorite soda (thus, $\varepsilon_m^C(p_m^*)$ is close to zero), although soda is highly substitutable by mineral water (thus, $\varepsilon_m^I(p_m^*)$ is very large) if consumers just want to quench their thirst (whether soda or water does not matter).

On the other hand, case (ii) is where the competitive pressure from other industries is weak, although inside the industry, firms are fiercely competing for consumers. For example, in a resort, consumers may not care much about the difference between Coke and Pepsi (thus, $\varepsilon_m^C(p_m^*)$ is very large), though soda would not be easily substitutable by mineral water (thus, $\varepsilon_m^I(p_m^*)$ is close to zero) because consumers want to get perfectly refreshed: having water instead of soda does not relieve their throat. From a firm's perspective, these two polar cases are equivalent with respect to pricing in the sense that if it raises its price by even a small amount, it loses a large number of consumers: whether they leave the industry or switch to rivals' products does not matter to that firm. Thus, in the examples above, the firms' (discriminatory) prices are close to marginal cost both in a residential area and in a resort due to different reasons. Recall again that these are two polar cases: in reality, $\varepsilon_m^F(p_m^*)$ may be large because both $\varepsilon_m^I(p_m^*)$ and $\varepsilon_m^C(p_m^*)$ are large. We can also think of the following alternative possibility: in a resort filled by young visitors, Coke's and Pepsi's prices are close to the marginal cost because consumers do not care about water or soda as long as they can relieve their throat (i.e., $\varepsilon_m^I(p_m^*)$ is very large), although they are very loyal to either brand once they choose soda ($\varepsilon_m^C(p_m^*)$ is close to zero).

Next, let y_m be per-firm (symmetric) share output in market m,[8] that is, $y_m(p_s, p_w) \equiv q_m(p_m)/[q_s(p_s) + q_w(p_w)]$. Then, the equilibrium uniform price, $\overline{p} \equiv \overline{p}(c_s, c_w)$, satisfies: $\sum_{m=s,w} \overline{y}_m \varepsilon_m^F(\overline{p})(\overline{p} - c_m)/\overline{p} = 1$, where $\overline{y}_m \equiv y_m(\overline{p}(c_s, c_w), \overline{p}(c_s, c_w))$ for $m = s, w$.[9]

[8] Of course, the total output in market m is $2q_m(p_m)$, aggregated across symmetric firms.

[9] If there are no cost differentials, i.e., $c_s = c_w$ ($\equiv c$), then the formula is simpler: $\sum_{m=s,w} \overline{x}_m \varepsilon_m^F(\overline{p})(\overline{p} - c)/\overline{p} = 1$ as shown by Holmes (1989, p. 247): the markup rate (common

3 Output and Welfare

In the analysis below, we, following Schmalensee (1981), Holmes (1989), and Aguirre, Cowan, and Vickers (2010), add the constraint $p_s - p_w \leq r$, where $r > 0$, to the firms' profit maximization problem (under symmetric pricing). Then, $r = 0$ corresponds to uniform pricing, and $r = r^* \equiv p_s^* - p_w^*$ to price discrimination. We express social welfare (and aggregate output) as a function of r in $[0, r^*]$. Note that under this constrained problem of profit maximization, p_w satisfies $\partial_p \pi_s(p_w + r) + \partial_p \pi_w(p_w) = 0$. Thus, we write the solution by $p_w(r)$. Then, we define $p_s(r) \equiv p_w(r) + r$. Applying the implicit function theorem to this equation yields to $p_w'(r) = -\pi_s''/(\pi_s'' + \pi_w'') < 0$ and $p_s'(r) = \pi_w''/(\pi_s'' + \pi_w'') > 0$.

Here, note that $\pi_m''(p, c_m)$ and $\partial_p^2 \pi_m(p, c_m)$ are different:

$$\begin{aligned}
\pi_m''(p, c_m) &= q_m'(p) + \frac{\partial x_{A,m}}{\partial p_A}(p,p) + (p - c_m)\frac{d}{dp}\left(\frac{\partial x_{A,m}}{\partial p_A}(p,p)\right) \\
&= \partial_p^2 \pi_m(p) + \frac{\partial x_{A,m}}{\partial p_B}(p,p) + (p - c_m)\frac{\partial^2 x_{A,m}}{\partial p_B \partial p_A}(p,p),
\end{aligned}$$

where $\partial_p^2 \pi_m(p, c_m)$ is defined by

$$\partial_p^2 \pi_m(p) \equiv \left[2 + (p - c_m)\frac{\partial^2 x_{A,m}(p,p)/\partial p_A^2}{\partial x_{A,m}(p,p)/\partial p_A}\right]\frac{\partial x_{A,m}}{\partial p_A}(p,p),$$

which corresponds to Aguirre, Cowan, and Vickers' (2010) $\pi_m''(p)$. We assume that $\pi_m''(p, c_m) < 0$ for all $p \geq 0$.[10]

We define the representative consumer's utility in symmetric pricing by $\widetilde{U}_m(q) = U_m(q,q)$. Aggregate output under symmetric pricing is given by $Q(r) = Q_s(r) + Q_w(r) = 2\left(q_s(p_s(r)) + q_w(p_w(r)\right)$. Social welfare under symmetric pricing as a function of r is written as $W(r; c_s, c_w) \equiv \widetilde{U}_s(q_s(p_s(r))) + \widetilde{U}_w(q_w(p_w(r))) - 2c_s \cdot q_s(p_s(r)) - 2c_w \cdot q_w(p_w(r))$, which implies $W'(r) \equiv (\widetilde{U}_s' - 2c_s) \cdot q_s' \cdot p_s'(r) + (\widetilde{U}_w' - 2c_w) \cdot q_w' \cdot p_w'(r)$. Now, note that $\widetilde{U}_m' = \partial U_m/\partial q_A + \partial U_m/\partial q_B = 2\partial U_m/\partial q_A$ (by symmetry). Thus, $W'(r) = 2\left(p_s(r) - c_s\right) \cdot q_s' \cdot p_s'(r) + 2\left(p_w(r) - c_w\right) \cdot q_w' \cdot p_w'(r)$.

to all markets) is equal to the inverse of the average of own-price elasticities weighted by the output shares.

[10] Appendix A of Aguirre, Cowan, and Vickers (2010) discusses the concavity of the profit function.

3.1 Output

Now, we can further proceed:

$$\begin{aligned}
\frac{W'(r)}{2} &= (p_s(r) - \overline{p} + \overline{p} - c_s)\, q_s'(p_s(r))p_s'(r) \\
&\quad + (p_w(r) - \overline{p} + \overline{p} - c_w)\, q_w'(p_w(r))p_w'(r) \\
&= \underbrace{(p_s(r) - \overline{p})\, q_s'(p_s(r))p_s'(r)}_{<0} + \underbrace{(p_w(r) - \overline{p})\, q_w'(p_w(r))p_w'(r)}_{<0} \\
&\quad + \sum_{m=s,w} (\overline{p} - c_m)\, q_m'(p_m(r))p_m'(r).
\end{aligned}$$

This derivation coincides with the case of monopoly as shown in Aguirre, Cowan, and Vickers' (2010, p. 1604) equality (3) if there are no cost differentials (i.e., $c_s = c_w \equiv c$), with two minor modifications: (i) the left hand side is $W'(r)/2$ rather than $W'(r)$ itself, and (ii) the last term of Aguirre, Cowan, and Vickers' (2010) equality (3) is replaced by $Q'(r)/2$ rather than $Q'(r)$ because $(1/2)\sum_{m=s,w}(\overline{p} - c_m)\, q_m'(p_m(r))p_m'(r) = (\overline{p} - c)\,(Q'(r)/2)$. If cost differentials are allowed, it is observed that an increase in the *weighted* aggregate output, $\sum_{m=s,w}(\overline{p} - c_m)\, q_m'(p_m(r))p_m'(r)$, is *necessary* for price discrimination to raise social welfare, as in the case of monopoly.[11]

To proceed further, we define the *curvature of the firm's (direct) demand* in market m by

$$\alpha_m^F(p) \equiv -\frac{p}{\partial x_{A,m}(p,p)/\partial p_A}\frac{\partial^2 x_{A,m}}{\partial p_A^2}(p,p)$$

(which measures the concavity/convexity of the firm's direct demand, and corresponds to $\alpha_m(p)$ in Aguirre, Cowan, and Vickers, 2010, p. 1603), and the *elasticity of the cross-price effect* of the firm's direct demand in market m by

$$\begin{aligned}
\alpha_m^C(p) &\equiv -\frac{p}{\partial x_{A,m}(p,p)/\partial p_B}\frac{\partial^2 x_{A,m}}{\partial p_B \partial p_A}(p,p) \\
&= -\underbrace{\frac{p}{\partial x_{B,m}(p,p)/\partial p_A}}_{>0}\underbrace{\frac{\partial^2 x_{B,m}}{\partial p_A^2}(p,p)}_{<0},
\end{aligned}$$

[11]However, if externalities across consumers, such as network externalities and congestion, exist, then an increase in aggregate output would be no longer a necessary condition, as implied by Adachi (2002, 2005), who studies monopoly with linear demands. See also Czerny and Zhang (2015) as a recent study of price discrimination and congestion.

which is new to oligopoly.[12] Here, $\alpha_m^F(p)$ is positive (resp. negative) if and only if $\partial^2 x_{A,m}(p,p)/\partial p_A^2$ is negative (resp. positive), while $\alpha_m^C(p)$ is always positive (because of our assumption, $\partial^2 x_{jm}(p,p)/(\partial p_A \partial p_B) < 0$). Note that the sign of $\alpha_m^F(p)$ indicates whether the firm's own part of the demand slope under symmetric pricing given the rival's price being p, $\partial x_{A,m}(\cdot,p)/\partial p_A$, is *convex* ($\alpha_m^F(p)$ is positive) or *concave* ($\alpha_m^F(p)$ is negative). On the other hand, $\alpha_m^C(p)$ measures how the rival's price level matters to how many of the firm's customers switch to the rival's product when the firm raises its own price ($\partial x_{B,m}/\partial p_A$). Thus, a large $\alpha_m^C(p)$ implies that $\partial x_{B,m}/\partial p_A$ is very responsive to a change in p_B, and vice versa.

Next, we define the *conduct index* (see, e.g., Bresnahan 1989; Genesove and Mullin 1998; and Corts 1999) in market m by $\theta_m(p) \equiv 1 - A_m(p)$,[13] where $A_m(p)$ is the *aggregate diversion ratio* (Shapiro 1996) in market m, which is is defined by $A_m(p) \equiv -(\partial x_{B,m}(p,p)/\partial p_A)/(\partial x_{A,m}(p,p)/\partial p_A) = \varepsilon_m^C(p)/\varepsilon_m^F(p)$. Here, $A_m(p)$ measures the degree of *rivalness*: if $A_m(p)$ is close to one, consumers who leave a firm as a response to an increase in its price are nearly all switching to its rival's product. In this way, Aguirre, Cowan, and Vickers' (2010) derivation in the case of monopoly (where the method by Schmalensee (1981) is utilized) is connected to Weyl and Fabinger's (2013) condition in the case of symmetric oligopoly. In particular, Aguirre, Cowan, and Vickers' (2010, p.1606) Proposition 2 (a sufficient condition for price discrimination to raise social welfare) is extended to the case of

[12] This is because $\partial\left(\partial x_{A,m}(p,p)/\partial p_B\right)/\partial p_A = \partial\left(\partial x_{B,m}(p,p)/\partial p_A\right)/\partial p_A$.

[13] Alternatively, Weyl and Fabinger (2013, p. 531) define the conduct index in a market (which, in our interest in price discrimination, can be indexed by m) by $\theta_m \equiv [(p-c_m)/p]\varepsilon_m^I$ (their mc and ε_D are replaced by our c_m and ε_m^I, respectively) as the Lerner index adjusted by the elasticity of the *industry's* demand. If the first-order condition is given for each market (that is, if full price discrimination is allowed), then $\theta_m(p)$ defined as in Weyl and Fabinger (2013) coincides with $1 - A_m(p)$ because $\varepsilon_m^F[(p_m - c_m)/p_m] = 1$ and thus

$$\begin{aligned}
\varepsilon_m^I(p)\frac{p-c_m}{p} &= \frac{1}{\varepsilon_m^F(p)}\left(-\frac{p}{q_m(p)}\right)q_m'(p) \\
&= -\frac{q_m(p)}{p}\frac{1}{\partial x_{A,m}(p,p)/\partial p_A}\left(-\frac{p}{q_m(p)}\right)\left(\frac{\partial x_{A,m}}{\partial p_A}(p,p)+\frac{\partial x_{A,m}}{\partial p_B}(p,p)\right) \\
&= \frac{\partial x_{A,m}(p,p)/\partial p_A + \partial x_{B,m}(p,p)/\partial p_A}{\partial x_{A,m}(p,p)/\partial p_A} \text{ (by symmetry).}
\end{aligned}$$

See also Adachi and Fabinger (2017) for a generalized definition of the conduct index that allows for the possibly of non-zero specific and ad valorem taxes.

oligopoly in a simpler manner, using the concept of *pass-through* introduced in the next subsection.

As Weyl and Fabinger (2013, p. 544) argue, $\theta_m(p)$ captures the degree of *industry-level brand loyalty or stickiness*[14] in market m: if $\theta_m(p)$ is zero (close to one), market m is captured by perfect competition (almost monopoly): firms' products are perfect substitutes (nearly non-substitutable products).[15] *The markup rate* (the Lerner index), $L_m(p_m, c_m) \equiv (p_m - c_m)/p_m$, *alone is not appropriate to measure the rivalness within market* m because it can be the case that p_m is close to c_m (the markup rate is close to zero) simply because the price elasticity of the industry's demand $\varepsilon_m^I(p_m)$ is very large while the brand rivalness is so weak that the cross-price elasticity, $\varepsilon_m^C(p_m)$, remains very small (as a result, in total, $\varepsilon_m^F(p_m)$ is very large, which is actually reason for the low markup rate). However, if $\varepsilon_m^C(p_m)$ is close to $\varepsilon_m^F(p_m)$ (i.e., almost of all consumers who leave a firm as a response to its price increase are switching to other rivals' products), then θ_m becomes close to zero *irrespective of the value of the markup rate.* Thus, $\theta_m(p)$, which ranges between 0 and 1, better captures the brand stickiness than $L_m(p, c_m)$ does.

Now, we consider the effects of price discrimination on aggregate output. First, note that

$$\begin{aligned} \frac{Q'(r)}{2} &= q_w' \cdot p_w' + q_s' \cdot p_s' \\ &= \underbrace{\left(-\frac{\pi_s''\pi_w''}{\pi_s'' + \pi_w''}\right)}_{>0} \\ &\quad \times \left(\frac{1 - L_s(p_s(r))[\alpha_s^F(p_s(r)) - (1 - \theta_s(p_s(r))\alpha_s^C(p_s(r))]}{\theta_s(p_s(r))}\right. \\ &\qquad \left. - \frac{1 - L_w(p_w(r))[\alpha_w^F(p_w(r)) - (1 - \theta_w(p_w(r))\alpha_w^C(p_w(r))]}{\theta_w(p_w(r))}\right). \end{aligned}$$

Note also that $\varepsilon_m^C(p_m)/\varepsilon_m^I(p_m) = [1 - \theta_m(p_m)]/\theta_m(p_m)$ measures the *substitutability*

[14] Even if the firms' products have the same characteristics across different markets (with no product differentiation), the degree of brand loyalty may differ across markets, reflecting differences in market characteristics (summarized in demand functions).

[15] Because $(p_m - c_m)\varepsilon_m^F(p_m)/p_m = 1$ and $\varepsilon_m^F(p_m) = \varepsilon_m^I(p_m) + \varepsilon_m^C(p_m)$, it is verified that $\theta_m(p_m, c_m) + \varepsilon_m^C(p_m)(p_m - c_m)/p_m = 1$. Thus, as long as the products are substitutes ($\varepsilon_m^C(p_m) > 0$), $\theta_m(p_m, c_m)$ is less than one.

between brands (adjusted by the elasticity of the industry's demand): if the brand stickiness is very strong (i.e., $\theta_m(p)$ is close to one), $\varepsilon_m^C(p_m)/\varepsilon_m^I(p_m)$ is close to one, while if the brand stickiness is very weak (i.e., $\theta_m(p)$ is close to zero), then $\varepsilon_m^C(p_m)/\varepsilon_m^I(p_m)$ becomes infinitely large. Then, the following lemma holds with cost differentials being allowed.

Lemma 1. *$Q'(r) > 0$ if and only if (suppressing the dependence on $p_m(r)$ and c_m)*

$$L_w \cdot \frac{\alpha_w^F - (1-\theta_w)\alpha_w^C}{\theta_w} - L_s \cdot \frac{\alpha_s^F - (1-\theta_s)\alpha_s^C}{\theta_s} + \frac{1}{\theta_s} - \frac{1}{\theta_w} > 0. \qquad (1)$$

Now, suppose that the brand stickiness in the weak market is so weak that $\theta_w(p_w)$ is close to zero ($\theta_w(p_w) \approx 0$), while the brand stickiness in the strong market is moderate or strong ($\theta_s(p_s) \gg 0$). Then, the left hand side of the inequality above is approximated by

$$\frac{1 - L_s\left[\alpha_s^F - (1-\theta_s)\alpha_s^C\right]}{\theta_s} - \left\{1 - L_w\left[\alpha_w^F - \alpha_w^C\right]\right\}\frac{1}{\theta_w}.$$

Thus, as long as $1 > L_w[\alpha_w^F - \alpha_w^C]$, the left hand side becomes infinitely negative as $\theta_w(p_w)$ approaches to zero (assuming the first term is bounded). Counterintuitively, in the weak market, where price discrimination lowers the price, the brand rivalness has a *negative* effect on an increase in aggregate output by price discrimination. This is because the uniform price is already very low due to the fierce level of competition and thus there is little room for a price reduction by price discrimination to increase the output in the weak market. The opposite argument holds if the strong market is characterized by a low brand stickiness (i.e., $\theta_s(p_s, c_s) \approx 0$). This implies that, as is expected, a fierce level of competition in the strong market has a positive effect on an increase in aggregate output by price discrimination. The rivalness in the strong market keeps the price increase by price discrimination small, and thus a reduction in output in the strong market is also kept small.

Following Holmes (1989), we call the first and the second terms in the left hand side of (1) the *adjusted-concavity part*, and the third and the fourth terms the *elasticity-ratio part*:

$$\underbrace{L_w \cdot \frac{\alpha_w^F - (1-\theta_w)\alpha_w^C}{\theta_w} - L_s \cdot \frac{\alpha_s^F - (1-\theta_s)\alpha_s^C}{\theta_s}}_{\text{adjusted-concavity}} + \underbrace{\frac{1}{\theta_s} - \frac{1}{\theta_w}}_{\text{elasticity-ratio}}. \tag{1'}$$

First, look at the elasticity-ratio part. If the difference $\theta_w - \theta_s$ is greater, it is more likely that $Q'(r) > 0$. Thus, competitiveness in the strong market, rather than in the weak market, is important. Next, look at the adjusted-concavity part. A larger α_w^F and/or a smaller α_w^C make a positive $Q'(r)$ more likely. A larger α_w^F means that the firm's own part of the demand in the weak market ($\partial x_{A,w}/\partial p_A$) is more convex. On the other hand, a smaller α_w^C means that how many of the firm's customers switch to the rival's product as response to the firm's price increase is not so affected by the current price level. In this sense, the strategic concerns in the firm's pricing are small. Thus, both a larger α_w^F and a smaller α_w^C indicate that the weak market is competitive. Even if $\partial x_{A,w}/\partial p_A$ is not so convex, a smaller α_w^C (i.e., $\partial x_{B,m}/\partial p_A$ is not responsive to the level of p_B) can substitute it. A similar argument also holds for α_s^F and α_s^C. In the Appendix, we show that Holmes' (1989) expression for $Q'(r)$ (expression (9) in Holmes (1989, p. 247)) is equivalent to (1').

Now, define $h_m(p, c_m) \equiv 1/(q_m'(p)/\pi_m''(p, c_m)) > 0$ so that

$$\frac{Q'(r)}{2} = \underbrace{\left(-\frac{q_s' q_w'}{\pi_s'' + \pi_w''}\right)}_{>0} [h_s(p_s(r), c_s) - h_w(p_w(r), c_w)].$$

We assume that $h_m(\cdot, c_m)$ is *decreasing* (and call it the *Decreasing Inverse Ratio Condition*: DIRC).[16] It is also shown that

$$\frac{Q''(r)}{2} = \left(-\frac{q_s' q_w'}{\pi_s'' + \pi_w''}\right) [h_s' p_s' - h_w' p_w'] + [h_s - h_w] \frac{d}{dr}\left(-\frac{q_s' q_w'}{\pi_s'' + \pi_w''}\right).$$

[16]It is verified that $h_m' < 0$ is equivalent to $q_m'' > (q_m'/\pi_m'')\pi_m'''$ because

$$h_m'(p, c_m) = \left(\frac{\pi_m''(p, c_m)}{q_m'(p)}\right)' = \frac{\pi_m''' q_m' - \pi_m'' q_m''}{[q_m']^2}.$$

Thus, DIRC states that the profit function decreases quickly enough as p increases. To see this, if $q_m'' > 0$, then it is sufficient to assume $\pi_m''' < 0$. This means that π_m'', which is negative, should be smaller, that is, the the negative slope of π_m'' should be steeper, as p increases. If $q_m'' \leq 0$, then π_m'' be should be not only negative but sufficiently small that $\pi_m''' < q_m''/(q_m'/\pi_m'')$. In both cases, $\pi_m$should decrease quickly as p increases.

Then, there exists $\widehat{r}$ such that $Q'(\widehat{r}) = 0$ and $Q''(\widehat{r})/2 = [-q_s' q_w' / (\pi_s'' + \pi_w'')] \, [h_s' p_s' - h_w' p_w'] < 0$ because $h_s' p_s' < 0$ and $h_w' p_w' > 0$. Then, $(1/2)Q(r)$ behaves on $[0, r^*]$ in either manner:[17]

1. If $Q'(0) \leq 0$, then $(1/2)Q(r)$ is monotonically decreasing in r, and a result $\Delta Q/2 = [Q(r^*; c_s, c_w) - Q(0; c_s, c_w)]/2 < 0$; *price discrimination lowers aggregate output.*

2. If $Q'(0) > 0$, then $(1/2)Q(r)$ either

   (a) is monotonically increasing (if $Q'(r^*) > 0$, this is true), and as a result, $\Delta Q/2 > 0$; *price discrimination raises aggregate output.*

   (b) first increases, and then after the reaching the maximum (where $Q'(r) = 0$), decreases until $r = r^*$. In this case, *price discrimination may raise or lower aggregate output*: it cannot be determined whether $\Delta Q/2 < 0$ or $\Delta Q/2 > 0$ without further functional and/or parametric restrictions.

Now, we determine the sign of $Q'(0)$. It follows that $\text{sign}[Q'(0)] = \text{sign}[\pi_s''(\overline{p}, c_s)/q_s'(p) - \pi_w''(\overline{p}, c_w)/q_w'(p)]$, which implies that $Q'(0) \leq 0 \Leftrightarrow \pi_w''(\overline{p}, c_w)/q_w'(\overline{p}) \geq \pi_s''(\overline{p}, c_s)/q_s'(\overline{p})$. Note also that $\text{sign}[Q'(r^*)] = \text{sign}[h_s(p_s^*, c_s) - h_w(p_w^*, c_w)]$, which implies that $Q'(r^*) > 0 \Leftrightarrow \pi_s''(p_s^*, c_w)/q_s'(p_s^*) > \pi_w''(p_w^*, c_w)/q_w'(p_w^*)$. Because

$$\frac{\pi_m''(\overline{p}, c_m)}{q_m'(\overline{p})} = \frac{\left[2 - L_m(\overline{p}, c_m)\alpha_m^F(\overline{p})\right] - [1 - \theta_m(\overline{p})] \left[1 - L_m(\overline{p}, c_m)\alpha_m^C(\overline{p})\right]}{\theta_m(\overline{p})}$$

holds, the following proposition obtains.

Proposition 1. *Given the DIRC, if $\theta_s(\overline{p}) \geq \theta_w(\overline{p})$ and*

$$\frac{\alpha_s^F(\overline{p}) - [1 - \theta_s(\overline{p})]\alpha_s^C(\overline{p})}{\theta_s(\overline{p})} \geq \frac{\alpha_w^F(\overline{p}) - [1 - \theta_w(\overline{p})]\alpha_w^C(\overline{p})}{\theta_w(\overline{p})}$$

then price discrimination lowers aggregate output. If $\theta_w(p_w^*) > \theta_s(p_s^*)$ and

$$\frac{\alpha_w^F(p_w^*) - [1 - \theta_w(p_w^*)]\alpha_w^C(p_w^*)}{\theta_w(p_w^*)} \geq \frac{\alpha_s^F(p_s^*) - [1 - \theta_s(p_s^*)]\alpha_s^C(p_s^*)}{\theta_s(p_s^*)},$$

then price discrimination raises aggregate output.

[17] This is because the modified version of Aguirre, Cowan, and Vickers' (2010, p. 1605) Lemma also holds in our oligopoly setting.

Proof. First, note that

$$\frac{\left[2 - L_w(\overline{p}, c_w)\alpha_w^F(\overline{p})\right] - [1 - \theta_w(\overline{p})]\left[1 - L_w(\overline{p}, c_w)\alpha_w^C(\overline{p})\right]}{\theta_w(\overline{p})} \geq \frac{\left[2 - L_s(\overline{p}, c_s)\alpha_s^F(\overline{p})\right] - [1 - \theta_s(\overline{p})]\left[1 - L_s(\overline{p}, c_s)\alpha_s^C(\overline{p})\right]}{\theta_s(\overline{p})},$$

then price discrimination lowers aggregate output. The first part is a sufficient condition for this inequality to hold. Next, note that

$$\frac{\left[2 - L_s(p_s^*, c_s)\alpha_s^F(p_s^*)\right] - [1 - \theta_s(p_s^*)]\left[1 - L_s(p_s^*, c_s)\alpha_s^C(p_s^*)\right]}{\theta_s(p_s^*)} > \frac{\left[2 - L_w(p_w^*, c_w)\alpha_w^F(p_w^*)\right] - [1 - \theta_w(p_w^*)]\left[1 - L_w(p_w^*, c_w)\alpha_w^C(p_w^*)\right]}{\theta_w(p_w^*)},$$

then price discrimination raises aggregate output. Thus, $\theta_w(p_w^*) > \theta_s(p_s^*)$ and

$$L_w(p_w^*, c_w)\frac{\alpha_w^F(p_w^*) - [1 - \theta_w(p_w^*)]\alpha_w^C(p_w^*)}{\theta_w(p_w^*)} > L_s(p_s^*, c_s)\frac{\alpha_s^F(p_s^*) - [1 - \theta_s(p_s^*)]\alpha_s^C(p_s^*)}{\theta_s(p_s^*)}.$$

□

3.2 Social Welfare

Now, we study the effects of allowing third-degree price discrimination on social welfare. To proceed further, note that

$$\frac{W'(r)}{2} = \underbrace{\left(-\frac{\pi_s''\pi_w''}{\pi_s'' + \pi_w''}\right)}_{>0} \times \left(\frac{(p_w(r) - c_w)q_w'(p_w(r))}{\pi_w''} - \frac{(p_s(r) - c_s)q_s'(p_s(r))}{\pi_s''}\right).$$

We follow Aguirre, Cowan, and Vickers (2010, p. 1605), who define $z_m(p, c_m) \equiv (p - c_m)q_m'(p)/\pi_m''(p, c_m)$, which is "the ratio of the marginal effect of a price increase on social welfare to the second derivative of the profit function." However, our q_m' and π_m'' have *strategic effects*. More specifically, our q_m' and π_m'' are written as

$$q_m'(p_m) = \underbrace{\frac{\partial x_{A,m}}{\partial p_A}(p_m, p_m)}_{<0 \text{ (ACV's } q_m')} + \underbrace{\frac{\partial x_{B,m}}{\partial p_B}(p_m, p_m)}_{>0 \text{ (strategic)}}$$

and

$$\pi_m''(p_m, c_m) = \underbrace{D^2\pi_m(p_m, c_m)}_{\text{ACV's } \pi_m''} + \underbrace{G_m(p_m, c_m)}_{\text{strategic}}$$

where

$$G_m(p, c_m) = \frac{\partial x_{A,m}}{\partial p_B}(p, p) + (p - c_m)\left[\frac{d}{dp}\left(\frac{\partial x_{A,m}}{\partial p_A}(p, p)\right) - \frac{\partial^2 x_{A,m}}{\partial p_A^2}(p, p)\right].$$

As in Aguirre, Cowan, and Vickers (2010, p.1605), we can write

$$\frac{W'(r)}{2} = \underbrace{\left(-\frac{\pi_s''\pi_w''}{\pi_s'' + \pi_w''}\right)}_{>0}\left[z_w(p_w(r), c_w) - z_s(p_s(r), c_s)\right],$$

and their lemma also holds in our case of oligopoly if we assume $z_m(\cdot, c_m)$ is *increasing* (the *Increasing Ratio Condition*; IRC).[18] Then, $(1/2)W(r)$ behaves on $[0, r^*]$ in either manner:[19]

1. If $W'(0) \leq 0$, then $(1/2)W(r)$ is monotonically decreasing in r, and a result $\Delta W/2 = [W(r^*; c_s, c_w) - W(0; c_s, c_w)]/2 < 0$; *price discrimination lowers social welfare.*

2. If $W'(0) > 0$, then $(1/2)W(r)$ either

   (a) is monotonically increasing (if $W'(r^*) > 0$, this is true), and as a result, $\Delta W/2 > 0$; *price discrimination raises social welfare.*

   (b) first increases, and then after the reaching the maximum (where $W'(r) = 0$), decreases until $r = r^*$. In this case, *price discrimination may raise or lower social welfare*: it cannot be determined whether $\Delta W/2 < 0$ or $\Delta W/2 > 0$ without further functional and/or parametric restrictions.

[18]Note that $z_m'(p; c_m) = \{[(p - c_m)q_m''(p) + q_m'(p)]\pi_m''(p) - (p - c_m)q_m'(p)\pi_m'''(p)\}/[\pi_m''(p)]^2$ and thus, IRC is equivalent to $[(p - c_m)q_m''(p) + q_m'(p)]\pi_m''(p) > (p - c_m)q_m'(p)\pi_m'''(p)$. Appendix B of Agguire, Cowan, and Vickers (2010) discusses sufficient conditions for the IRC in the case of monopoly. If $h_m(\cdot, c_m)$ is decreasing, as we assume throughout, then $z_m(\cdot, c_m)$ is increasing because $z_m'(p; c_m) = [1 - z_m(p; c_m)h_m'(p; c_m)]/h_m(p; c_m)$ so that z_m' is positive if h_m' is negative. That is, DIRC is a sufficient condition for IRC to hold.

[19]This is because the modified version of Aguirre, Cowan, and Vickers' (2010, p. 1605) Lemma also holds in our oligopoly setting.

Now, we determine the sign of $W'(0)$. First, define the *markup* in market m by $\mu_m(p, c_m) \equiv p - c_m$. Then, it follows that $\mathrm{sign}[W'(0)] = \mathrm{sign}[\mu_w(\overline{p}, c_w) q_w'(\overline{p}) / \pi_w''(\overline{p}, c_w) - \mu_s(\overline{p}, c_s) q_s'(\overline{p}) / \pi_s''(\overline{p}, c_s)]$, and thus, the following proposition is obtained.

Proposition 2. *Given the IRC, if the markup in strong market relative to the weak market at the uniform price $\overline{p}$ is sufficiently large, i.e.,*

$$\mu_s(\overline{p}, c_s) \frac{q_s'(\overline{p})}{\pi_s''(\overline{p})} \geq \mu_w(\overline{p}, c_w) \frac{q_w'(\overline{p})}{\pi_w''(\overline{p})}, \tag{2}$$

then price discrimination lowers social welfare.

If there are no strategic effects (i.e., $\partial x_{B,m} / \partial p_A = 0$ or $\theta_m(p, c_m) = 1$), then $\pi_m''(\overline{p}, c_m) / q_m'(\overline{p}) = 2 - L_m(\overline{p}, c_m) \alpha_m^F(\overline{p})$, and inequality (2) above reduces to $\mu_s(\overline{p}, c_s) / \mu_w(\overline{p}, c_w) \geq [2 - L_s(\overline{p}, c_s) \alpha_s^F(\overline{p})] / 2 - L_w(\overline{p}, c_w) \alpha_w^F(\overline{p})$. On the other hand, if there are no cost differentials (i.e., $c_s = c_w \equiv c$), then inequality (2) above reduces to $\pi_w''(\overline{p}, c) / q_w'(\overline{p}) \geq \pi_s''(\overline{p}, c) / q_s'(\overline{p})$ because the markups are the same in the two markets. Thus, if there are no strategic effects and no cost differentials, then inequality (2) coincides with Aguirre, Cowan, and Vickers' (2010, p. 1605) Proposition 1 ($\alpha_s^F(\overline{p}) \geq \alpha_w^F(\overline{p})$ in our notation; in their notation, $\alpha_s(\overline{p}) \geq \alpha_w(\overline{p})$) because $L_s(\overline{p}, c_s) = L_w(\overline{p}, c_w)$. That is, the firm's "direct demand function in the strong market is at least as convex as that in the weak market at the nondiscriminatory price" (Aguirre, Cowan, and Vickers, 2010, p. 1602).

Recall that in our case of oligopoly,

$$\frac{\pi_m''(\overline{p}, c_m)}{q_m'(\overline{p})} = \frac{\left[2 - L_m(\overline{p}, c_m) \alpha_m^F(\overline{p})\right] - \left[1 - \theta_m(\overline{p})\right] \left[1 - L_m(\overline{p}, c_m) \alpha_m^C(\overline{p})\right]}{\theta_m(\overline{p})}$$

holds, which leads to the following corollary, another expression for the sufficient condition for price discrimination to lower social welfare in the case of no cost differentials.

Corollary 1. *Suppose there are no cost differentials across markets ($c_s = c_w$). Given the IRC, if $\theta_w \geq \theta_s$ and*

$$\frac{\alpha_s^F - (1 - \theta_s) \alpha_s^C}{\varepsilon_s^I(\overline{p})} \geq \frac{\alpha_w^F - (1 - \theta_w) \alpha_w^C}{\varepsilon_w^I(\overline{p})}$$

at $\overline{p}$, then price discrimination lowers social welfare.

This is because

$$\begin{aligned}
&\frac{\pi_s''(\overline{p}, c_s)}{q_s'(\overline{p})} - \frac{\pi_w''(\overline{p}, c_w)}{q_w'(\overline{p})} \leq 0 \\
\Leftrightarrow \quad &\overline{L} \cdot \left(\frac{\alpha_w^F - (1-\theta_w)\alpha_w^C}{\theta_w} - \frac{\alpha_s^F - (1-\theta_s)\alpha_s^C}{\theta_s} \right) + \frac{1}{\theta_s} - \frac{1}{\theta_w} \leq 0,
\end{aligned}$$

where $\overline{L} \equiv L_s(\overline{p}, c) = L_w(\overline{p}, c)$.

We also define the *pass-through rate* (under symmetric equilibrium discriminatory pricing) in market m by $\rho_m \equiv (p_m^*)'(c_m)$. Then, we obtain the following sufficient condition on welfare improvement by allowing price discrimination.

Proposition 3. *Given the IRC, if the markup in strong market relative to the weak market under price discrimination is sufficiently small, i.e.,*

$$\theta_s(p_s^*)\rho_s(p_s^*)\mu_s(p_s^*, c_s) \leq \theta_w(p_w^*)\rho_w(p_w^*)\mu_w(p_w^*, c_w),$$

then price discrimination raises social welfare.

Proof. To prove this proposition, note first that

$$\begin{aligned}
z_m(p_m, c_m) &= -\frac{(p_m - c_m)\varepsilon_m^I(p_m)q_m(p_m)}{p_m \pi_m''(p_m, c_m)} \\
&= -\theta_m(p_m)\frac{q_m(p_m)}{\pi_m''(p_m, c_m)}
\end{aligned}$$

holds. Now, define

$$F(p_m, c_m) = \frac{q_m(p_m)}{\frac{\partial x_{A,m}}{\partial p_A}(p_m, p_m)} + p_m - c_m$$

so that $F(p_m^*, c_m) = 0$. Then, by applying the implicit function theorem, we have

$$\begin{aligned}
\rho_m &= \frac{1}{1 + \dfrac{q_m'}{\partial x_{A,m}/\partial p_A} - q_m \dfrac{d\left(\partial x_{A,m}/\partial p_A\right)/dp_m}{\left(\partial x_{A,m}/\partial p_A\right)^2}} \\
&= \frac{\partial x_{A,m}/\partial p_A}{\dfrac{\partial x_{A,m}}{\partial p_A} + q_m' - \dfrac{q_m}{\partial x_{A,m}/\partial p_A}\dfrac{d}{dp_m}\left(\dfrac{\partial x_{A,m}}{\partial p^A}\right)},
\end{aligned}$$

and under the equilibrium discriminatory prices,

$$\begin{aligned}\rho_m &= \frac{\partial x_{A,m}(p_m^*, p_m^*)/\partial p_A}{q_m'(p_m^*) + \dfrac{\partial x_{A,m}}{\partial p_A}(p_m^*, p_m^*) + (p_m^* - c_m)\dfrac{d}{dp_m}\left(\dfrac{\partial x_{A,m}}{\partial p_A}(p_m^*, p_m^*)\right)} \\ &= \frac{\partial x_{A,m}(p_m^*, p_m^*)/\partial p_A}{\pi_m''(p_m^*, c_m)},\end{aligned}$$

which implies that

$$\begin{aligned}z_m(p_m^*, c_m) &= -\theta_m(p_m^*)\rho_m(p_m^*)\frac{q_m(p_m^*)}{\dfrac{\partial x_{A,m}}{\partial p_A}(p_m^*, p_m^*)} \\ &= \theta_m(p_m^*)\rho_m(p_m^*)\mu_m(p_m^*, c_m)\end{aligned}$$

and thus

$$\begin{aligned}\frac{W'(r^*)}{2} &= \left(-\frac{\pi_s''\pi_w''}{\pi_s'' + \pi_w''}\right) \\ &\quad \times \left[\theta_w(p_w^*)\rho_w(p_w^*)\mu_w(p_w^*, c_w) - \theta_s(p_s^*)\rho_s(p_s^*)\mu_s(p_s^*, c_s)\right].\end{aligned}$$

This completes the proof of Proposition 3. □

Here, Aguirre, Cowan, and Vickers' (2010) IRC for $z_m(p, c_m)$ is equivalent to that condition on $\theta_m(p)\rho_m(p)\mu_m(p, c_m)$. Roughly speaking, if (in symmetric equilibrium) (i) the *brand loyalty* (θ), (ii) the *pass-through* (ρ), or (iii) the *markup* (μ) is sufficiently *small* in the *strong* market, then social welfare is likely to be higher under price discrimination. In particular, if these three measures are calculated (or estimated) in each separate market (and symmetry is not so far away from the reality), then it would assist one to judge whether price discrimination is desirable from a society's viewpoint. To the best of our knowledge, Propositions 2 and 3 are the most general statements on when allowing (symmetric) oligopolisitc firms to price discriminate lowers or raises social welfare, allowing cost differentials that Chen and Schwartz (2015) study in the case of monopoly, although Weyl and Fabinger (2013, p. 565) also briefly mention the importance of $\theta_m\rho_m\mu_m$.

Even if there are no cost differentials (i.e., $c_s = c_w$), this expression cannot be further simplified. In other words, this expression is already robust to the inclusion of cost differentials. Now, if we further assume that there are no strategic effects (i.e., $\theta_m = 1$), then the above condition becomes $(p_s^* - c)/(p_w^* - c) \leq (1/\rho_s(p_s^*))/(1/\rho_w(p_w^*))$, which coincides with $(p_w^* - c)/(2 - \sigma_w^I(q_w(p_w^*))) \geq (p_s^* - c)/(2 - \sigma_s^I(q_s(p_s^*)))$ in Proposition 2 of Aguirre, Cowan, and Vickers (2010, p. 1606), where $\sigma_m^I(q) \equiv -qp''/p'$ is the *curvature of the industry's inverse demand function* (in symmetric pricing), because it is shown that $\rho_m(p_m^*) = 1/[2 - \sigma_m^*(q_m(p_m^*))]$ in our oligopoly setting as well. Thus, price discrimination raises social welfare "if the discriminatory prices are not far apart and the inverse demand function in the weak market is locally more convex than that in the strong market" (Aguirre, Cowan, and Vickers 2010, p. 1602).

Suppose that price discrimination is being conducted. Then, to evaluate it from a viewpoint of social welfare, we first compute θ_m, ρ_m and μ_m for each $m = s, w$, then if the sufficient condition above is satisfied, then the ongoing price discrimination is justified. Notably, to compute θ_m, ρ_m and μ_m in equilibrium, the cost information is not necessary: once a specific form of demand function in market m for firm j, $q_{jm} = x_{jm}(p_{jm}, p_{-j,m})$ is provided (and if IRC is satisfied), then the three variables are computed in the following manner: $\theta_m = 1 - \varepsilon_m^C(p_m^*)/\varepsilon_m^F(p_m^*)$, $\rho_m = 1/[2 - \sigma_m^*(p_m^*)] = [q_m'(p_m^*)]^2/(2[q_m'(p_m^*)]^2 - q_m(p_m^*)q_m''(p_m^*))$, and $\mu_m = p_m^*/\varepsilon_m^F(p_m^*)$. Thus, if the firm's demand for each market m is estimated and the discriminatory price p_m^* is observed, then one can easily compute θ_m, ρ_m, and ρ_m.

It is also possible to provide coherent sufficient conditions for an increase and a decrease in social welfare by price discrimination, using $\theta_m \rho_m \mu_m$ that appears in Proposition 3 above. To do so, consider the case where the prevailing uniform price is not a result from banning price discrimination: the uniform price results from a quantity transfer from the strong market to the weak market. Let the amount of the transfer be denoted by $\widetilde{q} > 0$. Then, if per-firm quantity transfer is made from the strong market to the weak market, the first-order condition in the strong market is

$$q_s(p) - \widetilde{q} + (p - c_s)\frac{\partial x_{A,s}}{\partial p_A}(p, p) = 0,$$

while that in the weak market is

$$q_w(p) + \widetilde{q} + (p - c_w)\frac{\partial x_{A,w}}{\partial p_A}(p, p) = 0.$$

It is possible to find a unique $\widetilde{q} = \overline{q}$ such that $p_s = p_w = \overline{p}$ because summing these equations yields the first-order condition under uniform pricing. As $\widetilde{q}$, starting at $\overline{q}$, approaches to zero, $p_m(\widetilde{q})$ moves from the uniform price $\overline{p}$ to the discriminatory price p^*_m. In this sense, the role of $\widetilde{q}$ is similar to considering the constraint $p_s - p_w = r$ as above. Now, let the corresponding equilibrium (per-firm) output be defined by (with an abuse of notation) $q_m(\widetilde{q}) \equiv q_m(p_m(\widetilde{q}))$. Then, dead-weight loss from oligopoly in market m can be written as a function of $\widetilde{q}$, $DWL_m(\widetilde{q})$, and as Weyl and Fabinger (2013, p.538) show,

$$\frac{dDWL_m}{d\widetilde{q}} = -\theta_m(\widetilde{q})\rho_m(\widetilde{q})\mu_m(\widetilde{q}).$$

Thus price discrimination raises social welfare ($\Delta W > 0$) if and only if

$$|\Delta DWL_w| = \int_{-\overline{q}}^{0} \theta_w(\widetilde{q})\rho_w(\widetilde{q})\mu_w(\widetilde{q})d\widetilde{q}$$

is greater than

$$|\Delta DWL_s| = \int_{0}^{\overline{q}} \theta_s(\widetilde{q})\rho_s(\widetilde{q})\mu_s(\widetilde{q})d\widetilde{q},$$

and vice versa. Notice that the IRC for $z_m(p, c_m)$ is equivalent to $\theta_m(\widetilde{q})\rho_m(\widetilde{q})\mu_m(\widetilde{q})$ decreasing. Then, as Figure 1 shows, the following proposition holds.

Proposition 4. *Given the IRC, if* $\theta_w(\widetilde{q})\rho_w(\widetilde{q})\mu_w(\widetilde{q}) > \theta_s(\widetilde{q})\rho_s(\widetilde{q})\mu_s(\widetilde{q})$ *for all* $\widetilde{q} \in [0, \overline{q}]$, *then price discrimination raises social welfare.*

Note that Proposition 4 is stronger than Proposition 3. Similarly, as Figure 2 shows, the following proposition also holds.

Proposition 5. *Given the IRC, if* $\overline{\theta}_s\overline{\rho}_s\overline{\mu}_s > \overline{\theta}_w\overline{\rho}_w\overline{\mu}_w$, *then price discrimination lowers social welfare.*

Again, Proposition 5 is stronger than Proposition 2. To see this, note that

$$\overline{\theta}_s\overline{\rho}_s \;=\; \frac{\overline{p} - c_s}{\overline{p}}\left(-\frac{\overline{p}q_s'(\overline{p})}{q_s(\overline{p})}\right)\overline{\rho}_s$$

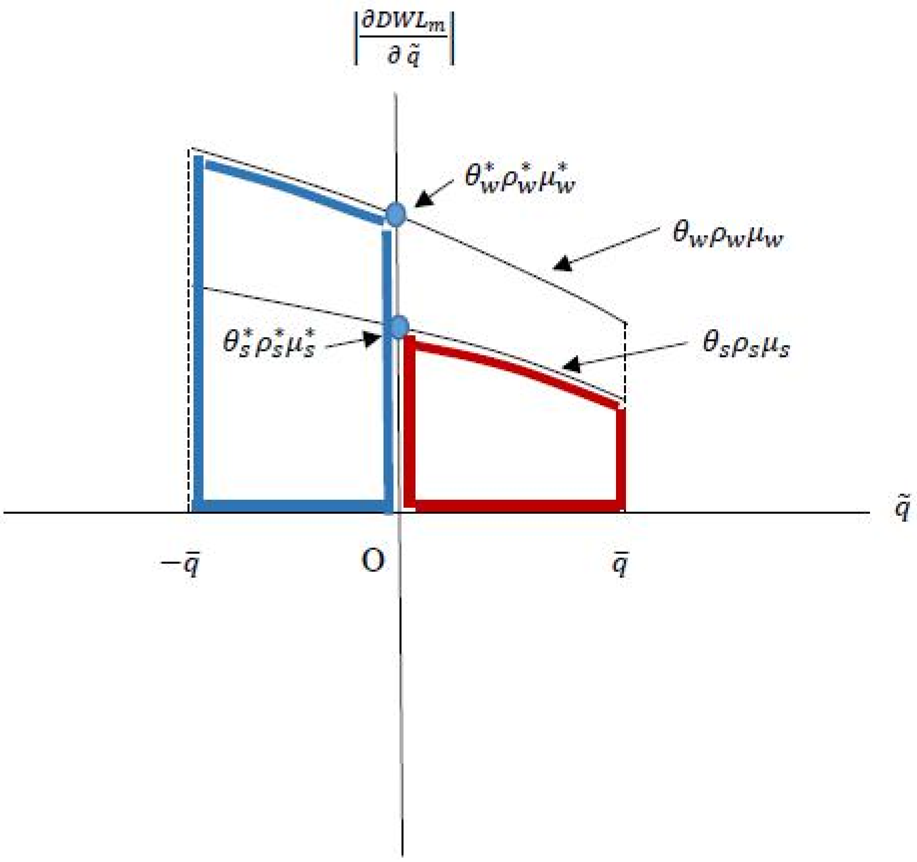

Figure 1: Sufficient condition for price discrimination to raise social welfare: $\theta_w \rho_w \mu_w > \theta_s \rho_s \mu_s$ for all $\widetilde{q} \in [0, \overline{q}]$. (Weaker sufficient condition (Prop 3): $\theta_w^* \rho_w^* \mu_w^* > \theta_s^* \rho_s^* \mu_s^*$)

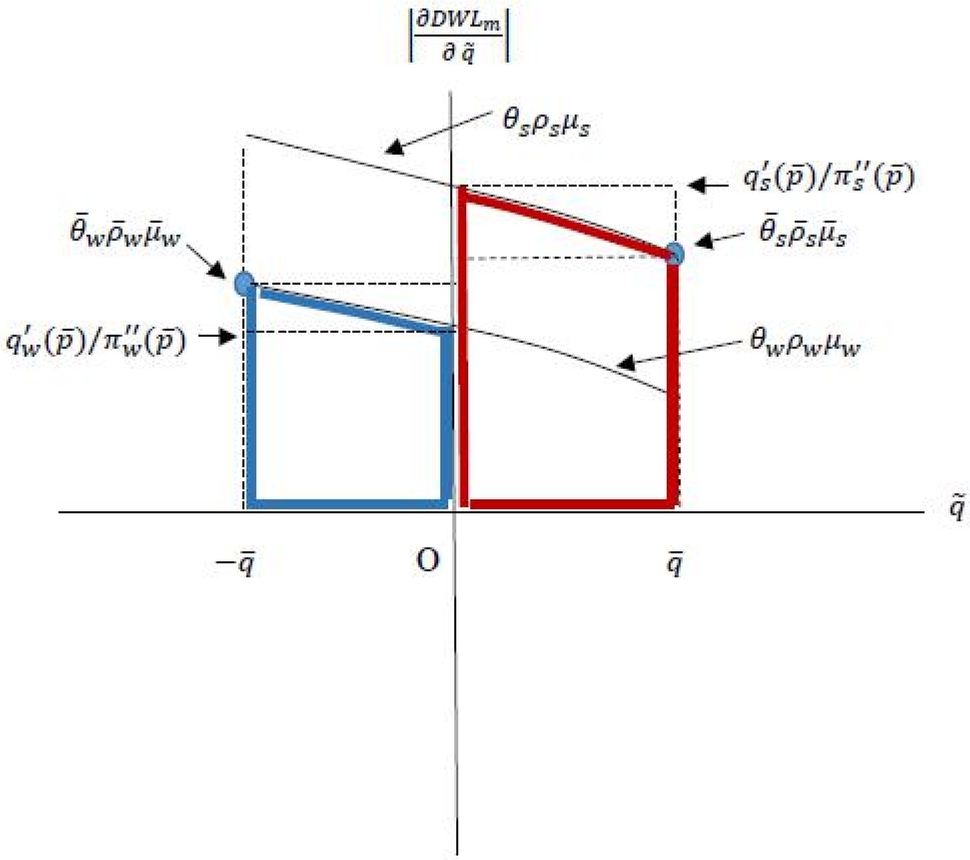

Figure 2: Sufficient condition for price discrimination to lower social welfare: $\overline{\theta}_s\overline{\rho}_s\overline{\mu}_s > \overline{\theta}_w\overline{\rho}_w\overline{\mu}_w$

$$= (\overline{p} - c_s)\left(-\frac{q_s'(\overline{p})}{q_s(\overline{p})}\right)\overline{\rho}_s.$$

Now, by applying the implicit function theorem to $F(p; \overline{q}, c_s) \equiv q_s(p) - \overline{q} + (p - c_s)\partial q_s^A(p,p)/\partial p^s$, one obtains

$$\begin{aligned}
\overline{\rho}_s &= \frac{\partial p}{\partial c_s}(\overline{q}, c_s) \\
&= -\frac{\partial F/\partial c_s}{\partial F/\partial p} \\
&= \frac{\partial x_{A,s}(\overline{p},\overline{p})/\partial p_s}{\pi_s''(\overline{p})} \\
&= -\frac{1}{\pi_s''(\overline{p})}\frac{q_s(\overline{p}) - \overline{q}}{\overline{p} - c_s},
\end{aligned}$$

which leads to

$$\begin{aligned}
\overline{\theta}_s\overline{\rho}_s &= -\frac{1}{\pi_s''(\overline{p})}\frac{q_s(\overline{p}) - \overline{q}}{\overline{p} - c_s}(\overline{p} - c_s)\left(-\frac{q_s'(\overline{p})}{q_s(\overline{p})}\right) \\
&= \frac{q_s'(\overline{p})}{\pi_s''(\overline{p})}\frac{q_s(\overline{p}) - \overline{q}}{q_s(\overline{p})}.
\end{aligned}$$

Thus, it is shown that $\overline{\mu}_s(q_s'(\overline{p})/\pi_s''(\overline{p})) > \overline{\theta}_s\overline{\rho}_s\overline{\mu}_s$. Similarly, it is verified that $\overline{\theta}_w\overline{\rho}_w > q_w'(\overline{p})/\pi_w''(\overline{p})$ because

$$\overline{\theta}_w\overline{\rho}_w = \frac{q_w'(\overline{p})}{\pi_w''(\overline{p})}\frac{q_w(\overline{p}) + \overline{q}}{q_w(\overline{p})}.$$

In summary, if $\overline{\theta}_s\overline{\rho}_s\overline{\mu}_s > \overline{\theta}_w\overline{\rho}_w\overline{\mu}_w$ holds, then $\overline{\mu}_s \cdot (q_s'(\overline{p})/\pi_s''(\overline{p})) > \overline{\mu}_w \cdot (q_w'(\overline{p})/\pi_w''(\overline{p}))$.

3.3 Consumer Surplus

One can extend the analysis above to consumer surplus. First, consumer surplus is defined by replacing c_m in $W(r)$ by $p_m(r)$ to define

$$CS(r; c_s, c_w) = U_s(q_s(p_s(r))) + U_w(q_w(p_w(r))) - 2p_s(r) \cdot q_s(p_s(r)) - 2p_w(r) \cdot q_w(p_w(r)),$$

which implies that

$$\begin{aligned}
\frac{CS'(r)}{2} &= p_s(r) \cdot q_s' \cdot p_s'(r) + p_w(r) \cdot q_w' \cdot p_w'(r) \\
&\quad -p_s'(r)\left[p_s(r) \cdot q_s' + q_s\right] - p_w'(r)\left[p_w(r) \cdot q_w' + q_w\right]
\end{aligned}$$

$$
\begin{aligned}
&= -\left[p_s'(r)q_s + p_w'(r)q_w\right] \\
&= \underbrace{\left(-\frac{\pi_s''\pi_w''}{\pi_s''+\pi_w''}\right)}_{>0}\left(\frac{q_s}{\pi_s''}-\frac{q_w}{\pi_w''}\right). \\
&= \underbrace{\left(-\frac{\pi_s''\pi_w''}{\pi_s''+\pi_w''}\right)}_{>0}\left[g_s(p_s(r),c_s)-g_w(p_w(r),c_w)\right]
\end{aligned}
$$

where $g_m(p,c_m) \equiv q_m(p)/\pi_m''(p,c_m)$. If $g_m(\cdot,c_m)$ is assumed to be decreasing, then one can use a similar argument.

4 Concluding Remarks

This paper provides theoretical implications of oligopolistic third-degree price discrimination with general nonlinear demands, allowing cost differentials across separate markets. In this sense, this paper, with the help of Weyl and Fabinger (2013), synthesizes Aguirre, Cowan, and Vickers' (2010) analysis of monopolistic third-degree price discrimination with general demands and Chen and Schwartz' (2015) analysis of monopolistic differential pricing, to extend them to the case of symmetrically differentiated oligopoly.

If some of price-discriminating oligopolists merge into a single firm, what happens? Price discrimination is often neglected in a merger analysis. Traditionally, in merger analyses, it has been considered as important to estimate own- and cross-price elasticities. However, our theoretical analysis suggests that own- and cross-price elasticities *per se* may not be so important in welfare evaluation. We conjecture that our main thrust obtained under symmetric oligipoly, namely the fundamental importance of the conduct index and the pass-through rate in welfare evaluation, remains valid if asymmetric firms are allowed.

Appendix: Equivalence of Holmes' (1989) and Our Expressions for $Q'(r)$

Holmes (1989, p. 247), who assumes no cost differentials ($c \equiv c_s = c_w$) as in most of the papers on third-degree price discrimination, also derives a necessary and sufficient condition for $Q'(r) > 0$ under symmetric oligopoly. It is (using our notation) written as:

$$\underbrace{\frac{p_s - c}{q_s'(p_s)} \cdot \frac{d}{dp_s}\left(\frac{\partial x_{A,s}(p_s, p_s)}{\partial p_A}\right) - \frac{p_w - c}{q_w'(p_w)} \cdot \frac{d}{dp_w}\left(\frac{\partial x_{A,w}(p_w, p_w)}{\partial p_A}\right)}_{\text{adjusted-concavity condition (Robinson 1933)}}$$
$$+ \quad \underbrace{\frac{\varepsilon_s^C(p_s)}{\varepsilon_s^I(p_s)} - \frac{\varepsilon_w^C(p_w)}{\varepsilon_w^I(p_w)}}_{\text{elasticity-ratio condition (Holmes 1989)}} \quad > 0.$$

Recall that $1/\theta_s - 1/\theta_w = \varepsilon_s^C/\varepsilon_s^I - \varepsilon_w^C/\varepsilon_w^I$. The first and the second terms in the left hand side of Holmes' (1989) inequality is rewritten as:

$$\begin{aligned}
& \frac{p_s - c}{q_s'(p_s)} \cdot \frac{d}{dp_s}\left(\frac{\partial x_{A,s}(p_s, p_s)}{\partial p_A}\right) - \frac{p_w - c}{q_w'(p_w)} \cdot \frac{d}{dp_w}\left(\frac{\partial x_{A,w}(p_w, p_w)}{\partial p_A}\right) \\
= \ & L_w(p_w) \cdot \left[\left(-\frac{p_w}{q_w'(p_w)}\right)\frac{d}{dp_w}\left(\frac{\partial x_{A,w}(p_w, p_w)}{\partial p_A}\right)\right] \\
& -L_s(p_s) \cdot \left[\left(-\frac{p_s}{q_s'(p_s)}\right)\frac{d}{dp_s}\left(\frac{\partial x_{A,s}(p_s, p_s)}{\partial p_A}\right)\right].
\end{aligned}$$

Now, it is also observed that

$$\begin{aligned}
& \frac{\alpha_m^F - (1-\theta_m)\alpha_m^C}{\theta_m} \\
= \ & \frac{\alpha_m^F}{\theta_m} - \frac{\partial x_{B,m}/\partial p_A}{q_m'} \frac{p_m}{\partial q_{B,m}/\partial p_A} \frac{\partial^2 x_{A,m}}{\partial p_A \partial p_B} \\
= \ & -\frac{p_m}{q_m'}\left(-\frac{q_m'}{p_m}\frac{\alpha_m^F}{\theta_m} + \frac{\partial^2 x_{A,m}}{\partial p_A \partial p_B}\right).
\end{aligned}$$

This shows that inequality (1) is another expression for Holmes' (1989, p. 247) inequality (9). To see this, note that

$$\frac{d}{dp_m}\left(\frac{\partial x_{A,m}(p_m, p_m)}{\partial p_A}\right) = \frac{\partial^2 x_{A,m}}{\partial p_A^2}(p_m, p_m) + \frac{\partial^2 x_{A,m}}{\partial p_A \partial p_B}(p_m, p_m)$$

in Holmes' (1989) expression is equivalent to $-(q'_m/p_m)(\alpha^F_m/\theta_m)+\partial^2 x_{A,m}/(\partial p_A \partial p_B)$ because

$$
\begin{aligned}
-\frac{q'_m}{p_m}\frac{\alpha^F_m}{\theta_m} &= -\frac{q'_m}{p_m\theta_m}\left(-\frac{p_m}{\partial x_{A,m}/\partial p_A}\right)\frac{\partial^2 x_{A,m}}{\partial p_A^2} \\
&= \frac{1}{\theta_m}\frac{\partial x_{A,m}/\partial p_A + \partial x_{B,m}/\partial p_A}{\partial x_{A,m}/\partial p_A}\frac{\partial^2 x_{A,m}}{\partial p_A^2} \\
&= \frac{1}{\theta_m}\left(1 - A_m\right)\frac{\partial^2 x_{A,m}}{\partial p_A^2} \\
&= \frac{\partial^2 x_{A,m}}{\partial p_A^2}.
\end{aligned}
$$

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