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BRS structure of simple model of cosmological constant and cosmology

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In Mod. Phys. Lett. A **31**, 1650213 (2016), Nojiri proposed a simple model in order to solve one of the problems related to the cosmological constant. The model is induced from a topological field theory, and the model has an infinite number of BRS symmetries. The BRS symmetries are, in general, spontaneously broken, however. We investigate the BRS symmetry in detail and show that there is one and only one BRS symmetry which is not broken, and the unitarity can be guaranteed. In the model, the quantum problem of the vacuum energy, which may be identified with the cosmological constant, reduces to the classical problem of the initial condition. We investigate the cosmology given by the model and specify the region of the initial conditions, which could be consistent with the evolution of the Universe. We also show that there is a stable solution describing the de Sitter space-time, which may explain the accelerating expansion in the current Universe.

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Recent observations indicate that the expansion of the Universe is accelerating. The energy density generating the accelerating expansion is called dark energy. The simplest model of dark energy may be a cosmological term with a small cosmological constant, $\Lambda^{1/4} \sim 10^{-3}$ eV. The cosmological term can be regarded with the energy density of the vacuum but, as is well known, the corrections from the matter in quantum field theory to the vacuum energy ρ_{vacuum} diverges and it is necessary to introduce the cutoff scale $\Lambda_{\mathrm{cutoff}}$, which might be the Planck scale, to regularize the divergence. Then the obtained value of the vacuum energy, $\sim\Lambda_{\mathrm{cutoff}}^4$, is much larger than the observed value of the energy density in the Universe, $(10^{-3}\ \mathrm{eV})^4$. Even if we impose the supersymmetry in the high energy, the vacuum energy by the quantum corrections is evaluated as $\sim\Lambda_{\mathrm{cutoff}}^2\Lambda_{\not{\mathrm{SUSY}}}^2$. Here, we denote the scale of the supersymmetry breaking by $\Lambda_{\not{\mathrm{SUSY}}}$. The vacuum energy coming from the quantum corrections can then be very large. We may use the counterterm in order to obtain the observed very small vacuum energy $(10^{-3}\ \mathrm{eV})^4$, but very significant fine-tuning is necessary, and it looks extremely unnatural. For a discussion of why the vaccum energy is so small but does not vanish, see, for example, [1]. Unimodular gravity theories [2–28] were proposed to solve this problem. For other scenarios to solve the cosmological constant problems, see, for example, [29–35].

In [36], motivated by the unimodular gravity theories, a new model has been proposed. The action of this model is given by

$$S' = \int d^4x\sqrt{-g}\{\mathcal{L}_{\mathrm{gravity}} - \lambda + \partial_\mu\lambda\partial^\mu\varphi - \partial_\mu b\partial^\mu c\} + S_{\mathrm{matter}}. \tag{1}$$

Here, λ and φ are scalar fields and b and c are also scalar fields, but they are fermionic (Grassmann odd), and b is later identified with the antighost and c with the ghost. The action without the ghost c and antighost b appeared in [37] for other purposes. Recently, the cosmological perturbation based on the model in (1) was investigated in [38]. In (1), we express the action of matters by S_{matter} and the Lagrangian density of the gravity $\mathcal{L}_{\mathrm{gravity}}$ can be that of an arbitrary model. We should note that there is not any parameter except the parts coming from S_{matter} and $\mathcal{L}_{\mathrm{gravity}}$.

We divide the gravity Lagrangian density $\mathcal{L}_{\mathrm{gravity}}$ into the sum of some constant Λ, which may include the large quantum corrections, and another part $\mathcal{L}^{(0)}_{\mathrm{gravity}}$, where $\mathcal{L}_{\mathrm{gravity}} = \mathcal{L}^{(0)}_{\mathrm{gravity}} - \Lambda$. We also redefine the scalar field λ by $\lambda \to \lambda - \Lambda$. Then the action (1) is rewritten as

$$S' = \int d^4x\sqrt{-g}\{\mathcal{L}^{(0)}_{\mathrm{gravity}} - \lambda + \partial_\mu\lambda\partial^\mu\varphi - \partial_\mu b\partial^\mu c\} + S_{\mathrm{matter}}. \tag{2}$$

The obtained action (2) does not include the constant Λ, which tells us that the constant Λ does not affect the dynamics. Although the constant Λ may include the large quantum corrections from matter to the vacuum energy, the large quantum corrections can be tuned to vanish.

The model in (1) includes ghosts [36], which generate the negative norm states in the quantum theory—and therefore the model is inconsistent—but the negative norm

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states can be excluded by defining the physical states through the use of BRS symmetry [39]. In fact, the action is invariant under the infinite numbers of the BRS transformation,

$$\delta\lambda = \delta c = 0, \qquad \delta\varphi = \epsilon c, \qquad \delta b = \epsilon(\lambda - \lambda_0). \quad (3)$$

Here, ϵ is a fermionic parameter and λ_0 is a solution of the equation

$$0 = \nabla^\mu \partial_\mu \lambda, \quad (4)$$

which can be obtained by the variation of the action (1) with respect to φ.[1] If we define the physical states as the states invariant under the BRS transformation in (3), we can consistently exclude the negative norm states as in gauge theory [40,41]. By assigning the ghost number 1 for c and -1 for b and ϵ, we find that the ghost number is also conserved. The four kinds of fields λ, φ, b, and c can be identified with a quartet in Kugo-Ojima's quartet mechanism in gauge theory [40,41].

We should note that the Lagrangian density in the action (1),

$$\mathcal{L} = -\lambda + \partial_\mu \lambda \partial^\mu \varphi - \partial_\mu b \partial^\mu c, \quad (5)$$

can be regarded as the Lagrangian density of a topological field theory [42], where the Lagrangian density is BRS exact—that is, it is given by the BRS transformation of a certain quantity. We may start with a field theory including only the scalar field φ, but the Lagrangian density vanishes ($\mathcal{L}_\varphi = 0$). Because the Lagrangian density vanishes, under any transformation of φ, the action is trivially invariant. In this sense, we may regard this theory as a gauge theory. We now impose the following gauge condition in order to fix the gauge symmetry:

$$1 + \nabla_\mu \partial^\mu \varphi = 0. \quad (6)$$

Then the gauge-fixing Lagrangian [43] is given by the BRS transformation (3) of $-b(1 + \nabla_\mu \partial^\mu \varphi)$. In fact, we find

$$\begin{aligned} &\delta(-b(1 + \nabla_\mu \partial^\mu \varphi)) \\ &\quad = \epsilon(-(\lambda - \lambda_0)(1 + \nabla_\mu \partial^\mu \varphi) + b \nabla_\mu \partial^\mu c) \\ &\quad = \epsilon(\mathcal{L} + \lambda_0 + (\text{total derivative terms})). \end{aligned} \quad (7)$$

Therefore, the Lagrangian density (5) is surely BRS exact up to the total derivative terms if $\lambda_0 = 0$, and we find that the theory in (5) could be regarded with a topological field theory. We should note that for the unbroken BRS symmetry—where, in general, $\lambda_0 \neq 0$—the Lagrangian density (5) is not BRS exact. In this sense, the Lagrangian density (5) is not that of the exact topological field theory, which may be a reason why the Lagrangian density (5) gives nontrivial and physically relevant contributions.

We should note that the gauge condition (6) does not fix the gauge symmetry completely, and there remains residual gauge symmetry. In fact, the gauge condition (6) is invariant under the residual gauge transformation

$$\varphi \to \varphi + \delta\varphi. \quad (8)$$

Here, $\delta\varphi$ satisfies the equation $\nabla_\mu \partial^\mu \delta\varphi = 0$. Then, by using the residual gauge symmetry, we can choose (be restricted to) the initial condition where φ is a constant—or even zero.[2]

We should also note that Eq. (3) tells that λ is nothing but the Nakanishi-Lautrup field [44–46]. Then, by using Eq. (3), $\lambda - \lambda_0$ is BRS exact, which indicates that the vacuum expectation value of $\lambda - \lambda_0$ must vanish. If the vacuum expectation value of $\lambda - \lambda_0$ does not vanish, the BRS symmetry is spontaneously broken, and we may not be able to consistently impose the physical state condition. We should note that there is only one unbroken BRS symmetry in the infinite numbers of the BRS symmetry in (3). Because Eq. (4) is the field equation for λ, the real world should be realized by one and only one solution of (4) for λ. Therefore, in the real world, only one λ_0 is chosen so that $\lambda = \lambda_0$ and the corresponding BRS symmetry is not broken. Therefore, by using the unbroken BRS symmetry, we can exclude the negative norm state (the ghost states) and the unitarity is guaranteed. We should also note that λ_0 can include the classical fluctuation as long as λ_0 satisfies the classical equation (4). Therefore, although the quantum fluctuations are prohibited by the BRS symmetry, there could appear the classical fluctuations.

The above arguments suggest that the quantum problem of the cosmological constant or vacuum energy might be solved. There is not, however, any principle to determine the value of λ or $\Lambda + \lambda$ in the quantum theory. The value could be determined by the initial conditions in the classical theory. In other words, the quantum problem of the vacuum energy is replaced with the classical problem of the initial conditions. In the following, we investigate the cosmology given by the model (1) and specify the region of the initial conditions, which could be consistent with the evolution of the observed Universe. We may assume the Friedmann-Robertson-Walker (FRW) metric with a flat spacial part,

$$ds^2 = -dt^2 + a(t)^2 \sum_{i=1}^{3} (dx^i)^2, \quad (9)$$

and λ and φ are assumed to depend only on the time coordinate t. In (9), $a(t)$ is called the scale factor. By a variation of λ in the action (1), we obtain Eq. (6), which has the following form in the FRW metric (9):

[1]The existence of the BRS transformation where λ_0 satisfies Eq. (4) was pointed out by Saitou.

[2]This argument comes from discussions with S. Akagi.

$$0 = 1 - \left(\frac{d^2\varphi}{dt^2} + 3H\frac{d\varphi}{dt}\right). \tag{10}$$

Here, H is the Hubble rate H defined by $H \equiv \frac{1}{a}\frac{da}{dt}$. The general solution of (10) is given by

$$\varphi(t) = \int^t \frac{dt_1}{a(t_1)^3}\int^{t_1} dt_2 a(t_2)^3 + \varphi_1 \int^t \frac{dt_1}{a(t_1)^3} + \varphi_2. \tag{11}$$

Here, φ_1 and φ_2 are constants. On the other hand, the equation given by a variation of φ is given by (4), which has the following form:

$$0 = \frac{d^2\lambda}{dt^2} + 3H\frac{d\lambda}{dt}, \tag{12}$$

whose general solution is given by

$$\lambda = \lambda_1 + \lambda_2 \int^t \frac{dt_1}{a(t_1)^3}. \tag{13}$$

As a gravity theory, we simply consider the Einstein gravity whose Lagrangian density is given by

$$\mathcal{L}_{\text{gravity}} = \frac{R}{2\kappa^2} - \Lambda. \tag{14}$$

Here, R is the scalar curvature and κ is the gravitational coupling constant. Λ is a cosmological constant, but it may include the large quantum correction from matter.

First, by neglecting the contributions from matter, we consider the FRW cosmology. Then the first and second FRW equations have the following forms:

$$\frac{3}{\kappa^2}H^2 = \Lambda + \lambda - \frac{d\lambda}{dt}\frac{d\varphi}{dt}, \tag{15}$$

$$-\frac{1}{\kappa^2}\left(3H^2 + 2\frac{dH}{dt}\right) = -\Lambda - \lambda - \frac{d\lambda}{dt}\frac{d\varphi}{dt}. \tag{16}$$

We can delete Λ from Eqs. (15) and (16), and we find

$$\frac{1}{\kappa^2}\frac{dH}{dt} = \frac{d\lambda}{dt}\frac{d\varphi}{dt}. \tag{17}$$

Then we find that there is a solution, where λ is a constant $\lambda = \lambda_1$. In fact, $\lambda = \lambda_1$ is a solution of (12) or the solution in (13) with $\lambda_2 = 0$. Then Eq. (17) indicates that H is a constant, $H = H_0$, and the space-time is therefore the de Sitter space-time. By using (15) or (16), we obtain the explicit value of $\lambda = \lambda_1$ as follows:

$$\lambda_1 = -\Lambda + \frac{3H_0^2}{\kappa^2}. \tag{18}$$

A solution of Eq. (10) is given by $\varphi = \frac{t}{3H_0}$, which is a special case in (11). We should note that the value of H_0 does not depend on the value of the cosmological constant Λ. Because H_0 is given by the constant of the integration in (17), the value of H_0 could be determined by the initial condition or something else. In any case, the value of the cosmological constant Λ is irrelevant for the cosmology. The above result also tells that the problem in the quantum theory for the vacuum energy reduces to the initial condition problem in the classical theory in our model.

We now investigate the stability of the solution in (18) expressing the de Sitter space-time. For this purpose, we consider the perturbation from the solution

$$H = H_0 + \delta H, \qquad \lambda = -\Lambda + \frac{3H_0^2}{\kappa^2} + \delta\lambda,$$
$$\varphi = \frac{t}{3H_0} + \delta\varphi. \tag{19}$$

Then, by using (10), (12), and (15), we obtain the following equations, respectively:

$$0 = \delta\ddot{\varphi} + 3H_0\delta\dot{\varphi} - \frac{1}{3H_0}\delta H, \tag{20}$$

$$0 = \delta\ddot{\lambda} + 3H_0\delta\dot{\lambda}, \tag{21}$$

$$\frac{6}{\kappa^2}H_0\delta H = \delta\lambda + \frac{1}{3H_0}\delta\dot{\lambda}. \tag{22}$$

By deleting δH from (20) and (22), we obtain

$$0 = \delta\ddot{\varphi} + 3H_0\delta\dot{\varphi} - \frac{\kappa^2}{18H_0^2}\left(\delta\lambda + \frac{1}{3H_0}\delta\eta\right). \tag{23}$$

Here, we have defined a new variable $\delta\eta$ by

$$\delta\eta \equiv \delta\dot{\lambda}. \tag{24}$$

Then we can rewrite (21) as follows:

$$0 = \delta\dot{\eta} + 3H_0\delta\eta. \tag{25}$$

By summarizing Eqs. (23), (24), and (25), we can write the equations in the matrix form,

$$\begin{pmatrix} \delta\dot{\lambda} \\ \delta\dot{\eta} \\ \delta\ddot{\varphi} \end{pmatrix} = A \begin{pmatrix} \delta\lambda \\ \delta\eta \\ \delta\dot{\varphi} \end{pmatrix}, \qquad A \equiv \begin{pmatrix} 0 & 1 & 0 \\ 0 & -3H_0 & 0 \\ \frac{\kappa^2}{18H_0^2} & -\frac{\kappa^2}{54H_0^3} & -3H_0 \end{pmatrix}. \tag{26}$$

The eigenvalues of the matrix A are given by $-3H_0$ and two 0s. Because there are not positive eigenvalues, the solution is stable—or at least quasistable. Then the solution (18) describing the de Sitter space-time might correspond to the accelerating expansion in the current Universe.

We now investigate what could be the initial condition corresponding to the value of the vacuum energy in the Universe. After inflation, the Universe passed through the radiation-dominated and matter-dominated eras and entered into the dark energy–dominated era. In the radiation-dominated and matter-dominated eras, the contributions from λ and φ can be neglected, and these scalar fields are expected to evolve by following (11) and (13). In the future dark energy–dominated era, the Universe is expected to be described by the asymptotically de Sitter space-time in (18).

In the radiation-dominated era, the scale factor is given by

$$a(t) = a_{\rm rad} t^{1/2}, \tag{27}$$

in the matter-dominated era,

$$a(t) = a_{\rm mat} t^{2/3}, \tag{28}$$

and in the dark energy–dominated era,

$$a(t) = a_\Lambda \mathrm{e}^{H_0\sqrt{\Omega_\Lambda}t}. \tag{29}$$

Here, $a_{\rm rad}$, $a_{\rm rad}$, and a_Λ are constants depending on the energy density of the radiation, the matter density, and the dark energy density, respectively. We express the value of the Hubble rate H in the current Universe by H_0 and the dark energy density parameter by Ω_Λ.

Then, by using (11) and (13), the scalar fields $\lambda(t)$ and $\varphi(t)$ in the radiation-dominated era are given by

$$\begin{aligned} \varphi(t) &= \varphi_{\rm rad}(t) \equiv \varphi_{\rm rad2} - \frac{2\varphi_{\rm rad1}}{a_{\rm rad}^3} t^{-1/2} + \frac{1}{5} t^2, \\ \lambda(t) &= \lambda_{\rm rad}(t) \equiv \lambda_{\rm rad1} - \frac{2\lambda_{\rm rad2}}{a_{\rm rad}^3} t^{-1/2}. \end{aligned} \tag{30}$$

On the other hand, in the matter-dominated and dark energy–dominated eras, the scalar fields are given by

$$\begin{aligned} \varphi(t) &= \varphi_{\rm mat}(t) \equiv \varphi_{\rm mat2} - \frac{\varphi_{\rm mat1}}{a_{\rm mat}^3} t^{-1} + \frac{1}{6} t^2, \\ \lambda(t) &= \lambda_{\rm mat}(t) \equiv \lambda_{\rm mat1} - \frac{\lambda_{\rm mat2}}{a_{\rm mat}^3} t^{-1}, \end{aligned} \tag{31}$$

$$\begin{aligned} \varphi(t) &= \varphi_\Lambda(t) \equiv \varphi_{\Lambda 2} - \frac{\varphi_{\Lambda 1}}{3H_0\sqrt{\Omega_\Lambda} a_\Lambda^3} \mathrm{e}^{-3H_0\sqrt{\Omega_\Lambda}t} + \frac{t}{3H_0\sqrt{\Omega_\Lambda}}, \\ \lambda(t) &= \lambda_\Lambda(t) \equiv \lambda_{\Lambda 1} - \frac{\lambda_{\Lambda 2}}{3H_0\sqrt{\Omega_\Lambda} a_\Lambda^3} \mathrm{e}^{-3H_0\sqrt{\Omega_\Lambda}t}. \end{aligned} \tag{32}$$

Here, $\varphi_{\rm rad1}$, $\varphi_{\rm rad2}$, $\lambda_{\rm rad1}$, $\lambda_{\rm rad2}$, $\varphi_{\rm mat1}$, $\varphi_{\rm mat2}$, $\lambda_{\rm mat1}$, $\lambda_{\rm mat2}$, $\varphi_{\Lambda 1}$, $\varphi_{\Lambda 2}$, $\lambda_{\Lambda 1}$, and $\lambda_{\Lambda 2}$ are constants.

We now use approximations where the radiation-dominated era transitioned to the matter-dominated era at the time $t = t_1$, and the matter-dominated era to the dark energy–dominated era at $t = t_2$. We connect the solutions in (30), (31), and (32) by imposing the continuities of the values of φ, λ, $\dot{\varphi}$, and $\dot{\lambda}$ at the transit points. Then, at the point $t = t_1$, we require that

$$\begin{aligned} \varphi_{\rm rad2} - \frac{2\varphi_{\rm rad1}}{a_{\rm rad}^3} t_1^{-1/2} + \frac{1}{5} t_1^2 &= \varphi_{\rm mat2} - \frac{\varphi_{\rm mat1}}{a_{\rm mat}^3} t_1^{-1} + \frac{1}{6} t_1^2, \\ \lambda_{\rm rad1} - \frac{2\lambda_{\rm rad2}}{a_{\rm rad}^3} t_1^{-1/2} &= \lambda_{\rm mat1} - \frac{\lambda_{\rm mat2}}{a_{\rm mat}^3} t_1^{-1}, \end{aligned} \tag{33}$$

and

$$\begin{aligned} \frac{\varphi_{\rm rad1}}{a_{\rm rad}^3} t_1^{-3/2} + \frac{2}{5} t_1 &= \frac{\varphi_{\rm mat1}}{a_{\rm mat}^3} t_1^{-2} + \frac{1}{3} t_1, \\ \frac{\lambda_{\rm rad2}}{a_{\rm rad}^3} t_1^{-3/2} &= \frac{\lambda_{\rm mat2}}{a_{\rm mat}^3} t_1^{-2}. \end{aligned} \tag{34}$$

Then we find

$$\begin{aligned} \varphi_{\rm mat1} &= \left(\frac{a_{\rm mat}}{a_{\rm rad}}\right)^3 t_1^{1/2} \varphi_{\rm rad1} + \frac{1}{15} a_{\rm mat}^3 t_1^3, \\ \varphi_{\rm mat2} &= \varphi_{\rm rad2} - \frac{t_1^{-1/2}\varphi_{\rm rad1}}{a_{\rm rad}^3} + \frac{1}{10} t_1^2, \\ \lambda_{\rm mat2} &= \left(\frac{a_{\rm mat}}{a_{\rm rad}}\right)^3 t_1^{1/2} \lambda_{\rm rad2}, \\ \lambda_{\rm mat1} &= \lambda_{\rm rad1} - \frac{\lambda_{\rm rad2}}{a_{\rm rad}^3} t_1^{-1/2}. \end{aligned} \tag{35}$$

On the other hand, at the point $t = t_2$, we require

$$\begin{aligned} \varphi_{\rm mat2} - \frac{\varphi_{\rm mat1}}{a_{\rm mat}^3} t_2^{-1} + \frac{1}{6} t_2^2 &= \varphi_{\Lambda 2} - \frac{\varphi_{\Lambda 1}}{3H_0\sqrt{\Omega_\Lambda} a_\Lambda^3} \mathrm{e}^{-3H_0\sqrt{\Omega_\Lambda}t_2} \\ &\quad + \frac{t_2}{3H_0\sqrt{\Omega_\Lambda}}, \\ \lambda_{\rm mat1} - \frac{\lambda_{\rm mat2}}{a_{\rm mat}^3} t_2^{-1} &= \lambda_{\Lambda 1} - \frac{\lambda_{\Lambda 2}}{3H_0\sqrt{\Omega_\Lambda} a_\Lambda^3} \mathrm{e}^{-3H_0\sqrt{\Omega_\Lambda}t_2}, \end{aligned} \tag{36}$$

and

$$\begin{aligned} \frac{\varphi_{\rm mat1}}{a_{\rm mat}^3} t_2^{-2} + \frac{1}{3} t_2 &= \frac{\varphi_{\Lambda 1}}{a_\Lambda^3} \mathrm{e}^{-3H_0\sqrt{\Omega_\Lambda}t_2} + \frac{1}{3H_0\sqrt{\Omega_\Lambda}}, \\ \frac{\lambda_{\rm mat2}}{a_{\rm mat}^3} t_2^{-2} &= \frac{\lambda_{\Lambda 2}}{a_\Lambda^3} \mathrm{e}^{-3H_0\sqrt{\Omega_\Lambda}t_2}, \end{aligned} \tag{37}$$

and we obtain

$$\begin{aligned}
\varphi_{\Lambda 1} &= \frac{a_\Lambda^3}{a_{\rm mat}^3} t_2^{-2} \mathrm{e}^{3H_0\sqrt{\Omega_\Lambda}t_2}\varphi_{\rm mat1} - \frac{a_\Lambda^3 \mathrm{e}^{3H_0\sqrt{\Omega_\Lambda}t_2}}{3H_0\sqrt{\Omega_\Lambda}} + \frac{1}{3} t_2 a_\Lambda^3 \mathrm{e}^{3H_0\sqrt{\Omega_\Lambda}t_2}, \\
\varphi_{\Lambda 2} &= \varphi_{\rm mat2} - \left(1 - \frac{1}{3H_0 t_2\sqrt{\Omega_\Lambda}}\right)\frac{\varphi_{\rm mat1}}{t_2 a_{\rm mat}^3} + \frac{t_2}{9H_0\sqrt{\Omega_\Lambda}} + \frac{1}{6}t_2^2 - \frac{1}{9H_0^2\Omega_\Lambda}, \\
\lambda_{\Lambda 2} &= \left(\frac{a_\Lambda}{a_{\rm mat}}\right)^3 t_2^{-2}\mathrm{e}^{3H_0\sqrt{\Omega_\Lambda}t_2}\lambda_{\rm mat2}, \\
\lambda_{\Lambda 1} &= \lambda_{\rm mat1} - \left(1 - \frac{1}{3H_0 t_2\sqrt{\Omega_\Lambda}}\right)\frac{\lambda_{\rm mat2}}{t_2 a_{\rm mat}^3}.
\end{aligned} \tag{38}$$

By combining the above equations, we find

$$\begin{aligned}
\lambda_0 + \Lambda = \frac{3H_c^2}{\kappa^2} &= \Lambda + \lambda_{\Lambda 1} - \frac{\lambda_{\Lambda 2}}{3H_0\sqrt{\Omega_\Lambda}a_\Lambda^3}\mathrm{e}^{-3H_0\sqrt{\Omega_\Lambda}t_2}, \\
\lambda_{\Lambda 1} &= \lambda_{\rm rad1} - \frac{\lambda_{\rm rad2}}{a_{\rm rad}^3} t_1^{-1/2}\left[1 + t_1 t_2^{-1}\left(1 - \frac{t_2^{-1}}{3H_0\sqrt{\Omega_\Lambda}}\right)\right], \\
\lambda_{\Lambda 2} &= \lambda_{\rm rad2}\left(\frac{a_\Lambda}{a_{\rm rad}}\right)^3 \mathrm{e}^{3H_0\sqrt{\Omega_\Lambda}t_2} t_2^{-2} t_1^{1/2}, \\
\varphi_{\Lambda 1} &= \varphi_{\rm rad1}\left(\frac{a_\Lambda}{a_{\rm rad}}\right)^3 t_2^{-2} t_1^{1/2}\mathrm{e}^{3H_0\sqrt{\Omega_\Lambda}t_2} + \frac{1}{15}a_\Lambda^3 t_1^3 t_2^{-2}\mathrm{e}^{3H_0\sqrt{\Omega_\Lambda}t_2} - \frac{a_\Lambda^3\mathrm{e}^{3H_0\sqrt{\Omega_\Lambda}t_2}}{3H_0\sqrt{\Omega_\Lambda}} + \frac{1}{3}t_2 a_\Lambda^3\mathrm{e}^{3H_0\sqrt{\Omega_\Lambda}t_2}, \\
\varphi_{\Lambda 2} &= \varphi_{\rm rad2} - \left\{\frac{t_1^{-1/2}}{a_{\rm rad}^3} + \left(1 - \frac{1}{3H_0 t_2\sqrt{\Omega_\Lambda}}\right)\frac{t_1^{1/2}}{t_2 a_{\rm rad}^3}\right\}\varphi_{\rm rad1} + \frac{t_2}{9H_0\sqrt{\Omega_\Lambda}} - \frac{1}{9H_0^2\Omega_\Lambda} \\
&\quad - \frac{1}{15}\left(1 - \frac{1}{3H_0 t_2\sqrt{\Omega_\Lambda}}\right)\frac{t_1^3}{t_2} + \frac{1}{10}t_1^2 + \frac{1}{6}t_2^2.
\end{aligned} \tag{39}$$

Now we consider the constraints on the scalar fields coming from the observations. For this purpose, we use the values of the cosmological parameters in [47].

(a) The scale factor and the cosmological time when the density of the radiation was equal to the density of matter: $a_{\rm rm} = 2.8\times 10^{-4}$, $t_1 = 4.7\times 10^4$ yr $\sim 1.5\times 10^{12}$ s $= 2.3\times 10^{27}$ [eV^{-1}].
(b) The scale factor and the cosmological time when the density of the matter was equal to the density of the dark energy: $a_{\rm m\Lambda} = 0.75$, $t_2 = 9.8\times 10^9$ yr $\sim 3.1\times 10^{17}$ s $= 4.7\times 10^{32}$ [eV^{-1}].
(c) The cosmological time when the radiation-dominated era began: $t_3 = 10^{-32}$ s $= 10^{-17}$ [eV^{-1}].
(d) The scale factor and the cosmological time in the current Universe: $a_0 = 1$, $t_0 = 13.5\times 10^9$ yr $\sim 4.3\times 10^{17}$ s $= 6.5\times 10^{34}$ [eV^{-1}].
(e) The Hubble constant in the current Universe: $H_0 = 70\,{\rm km\,s^{-1}\,Mpc^{-1}} \sim 2.2\times 10^{-18}\,{\rm s^{-1}} = 1.5\times 10^{-33}$ [eV].
(f) The density parameters of the radiation, the matter, and the dark energy: $\Omega_{\rm r} = 8.4\times 10^{-5}$, $\Omega_{\rm m} = 0.30$, $\Omega_\Lambda \sim 0.70$.

Then we obtain

(a) $a_{\rm rad} \sim (2H_0\sqrt{\Omega_{\rm r}})^{1/2} = 2.0\times 10^{-10}$ s$^{-1/2}$ $= 5.1\times 10^{-18}$ [eV$^{1/2}$]
(b) $a_{\rm mat} \sim (\frac{3}{2}H_0\sqrt{\Omega_{\rm m}})^{2/3} \sim 5.7\times 10^{-13}$ s$^{-2/3}$ $= 4.3\times 10^{-24}$ [eV$^{2/3}$]
(c) The critical density: $\rho_0 = \frac{3H_0^2}{8\pi G} = 5\times 10^{-24}$ kg m^{-3} $= 4.2\times 10^{-11}$ [eV^4].
(d) Newton's gravitational constant: $G \sim 6.6\times 10^{-11}$ m^3 kg^{-1} s^{-2} $= 6.7\times 10^{-57}$ [eV^{-2}].

First constraints can be obtained by requiring that $\Lambda + \lambda$ become a constant corresponding to the cosmological constant $\Lambda_0 \sim 10^{-11}$ [eV^4],

$$\Lambda_0 \sim \Lambda + \lambda_{\Lambda 1} \gg \left|\frac{\lambda_{\Lambda 2}}{3H_0\sqrt{\Omega_\Lambda}a_\Lambda^3}\mathrm{e}^{-3H_0\sqrt{\Omega_\Lambda}t_0}\right|. \tag{40}$$

By using (39), we can rewrite the constraints in (40) as follows:

$$10^{-11}\ [\mathrm{eV}^4] \sim \Lambda + \lambda_{\mathrm{rad1}} - \frac{\lambda_{\mathrm{rad2}}}{a_{\mathrm{rad}}^3} t_1^{-1/2}\left[1 + t_1 t_2^{-1}\left(1 - \frac{t_2^{-1}}{3H_0\sqrt{\Omega_\Lambda}}\right)\right] \sim \Lambda + \lambda_{\mathrm{rad1}} - \lambda_{\mathrm{rad2}} \times (3.1\times 10^{65}\ [\mathrm{eV}^{\frac{1}{2}}]), \tag{41}$$

$$10^{-11}\ [\mathrm{eV}^4] \gg |\lambda_{\mathrm{rad2}}| \times (1.8\times 10^{36}\ [\mathrm{eV}^{\frac{1}{2}}]). \tag{42}$$

Then Eq. (42) gives the following constraint:

$$|\lambda_{\mathrm{rad2}}| \ll 10^{-47}\ [\mathrm{eV}^5]. \tag{43}$$

The next constraint requires that the matter should surely be dominant compared to the contributions from λ and φ in the matter-dominated era $t_1 \ll t \ll t_2$,

$$\Lambda + \lambda_{\mathrm{rad1}} - \frac{\lambda_{\mathrm{rad2}}}{a_{\mathrm{rad}}^3} t_1^{-1/2}(1 + t_1 t^{-1}) - \frac{\lambda_{\mathrm{rad2}}}{a_{\mathrm{rad}}^3} t_1^{1/2}\left\{\left(\frac{t_1^{1/2}}{a_{\mathrm{rad}}^3}\varphi_{\mathrm{rad1}} + \frac{1}{15}t_1^3\right)t^{-4} + \frac{1}{3}t^{-1}\right\} \ll \rho = \Omega_{\mathrm{m}}\rho_0 a_{\mathrm{mat}}^{-3} t^{-2}. \tag{44}$$

We should require that the radiation be dominant in the radiation-dominated era $t \ll t_1$,

$$\Lambda + \lambda_{\mathrm{rad1}} - \frac{\lambda_{\mathrm{rad2}}}{a_{\mathrm{rad}}^3}\left(\frac{\varphi_{\mathrm{rad1}}}{a_{\mathrm{rad}}^3}t^{-3} + \frac{12}{5}t^{-1/2}\right) \ll \rho = \Omega_{\mathrm{r}}\rho_0 a_{\mathrm{rad}}^{-4} t^{-2}. \tag{45}$$

It is, in general, not so straightforward to solve the constraints (44) and (45). We may, however, evaluate the constraints as follows. When the matter-dominated era transitioned to the dark energy–dominated era at $t = t_2$, the lhs is almost equal to the rhs, by definition of the transition. Each of the terms, except for the first constant terms, on the lhs becomes larger when $t \to t_1$ and the most dominant term is the t^{-4} one. Then we may have the following constraint:

$$\left|\lambda_{\mathrm{rad2}}\left(\varphi_{\mathrm{rad1}} + \frac{2}{5}a_{\mathrm{rad}}^3 t_1^{5/2}\right)\right| \ll \Omega_{\mathrm{m}}\rho_0 \frac{a_{\mathrm{rad}}^6}{a_{\mathrm{mat}}^3} t_1, \tag{46}$$

that is,

$$|\lambda_{\mathrm{rad2}}(\varphi_{\mathrm{rad1}} + 1.4\times 10^{16}\ [\mathrm{eV}^{-1}])| \ll 10^{-23}\ [\mathrm{eV}^4]. \tag{47}$$

At the beginning of the radiation-dominated era $t = t_3$, the t^{-3} term dominates on the lhs of Eq. (45) and we obtain the following constraint:

$$|\lambda_{\mathrm{rad2}}\varphi_{\mathrm{rad1}}| \ll \Omega_{\mathrm{r}}\rho_0 a_{\mathrm{rad}}^2 t_3, \tag{48}$$

that is,

$$|\lambda_{\mathrm{rad2}}\varphi_{\mathrm{rad1}}| \ll 10^{-62}\ [\mathrm{eV}^4]. \tag{49}$$

We may summarize the obtained constraints as

$$\begin{aligned} \Lambda + \lambda_{\mathrm{rad1}} - \lambda_{\mathrm{rad2}} \times (3.1\times 10^{65}\ [\mathrm{eV}^{-1}]) &\sim 10^{-11}\ [\mathrm{eV}^4], \\ |\lambda_{\mathrm{rad2}}| &\ll 10^{-47}\ [\mathrm{eV}^5], \\ |\lambda_{\mathrm{rad2}}(\varphi_{\mathrm{rad1}} + 1.4\times 10^{16}\ [\mathrm{eV}^{-1}])| &\ll 10^{-23}\ [\mathrm{eV}^4], \\ |\lambda_{\mathrm{rad2}}\varphi_{\mathrm{rad1}}| &\ll 10^{-62}\ [\mathrm{eV}^4]. \end{aligned} \tag{50}$$

The first constraint in (50) or (41) seems to indicate that we require fine-tuning of the initial conditions.

We now consider more about the initial condition for λ. By choosing t as a present time, λ can be expressed as

$$\lambda \sim 10^{-11}\ [\mathrm{eV}^4] \sim \lambda_{\mathrm{rad1}} - \frac{\lambda_{\mathrm{rad2}}}{6.4\times 10^{-39}\ [\mathrm{eV}]}. \tag{51}$$

This may suggest that

$$\lambda_{\mathrm{rad1}} \sim (10^{-3}\ [\mathrm{eV}])^4, \qquad \lambda_{\mathrm{rad2}} \sim (10^{-10}\ [\mathrm{eV}])^5. \tag{52}$$

The obtained value then seems to be very small. If we assume $\lambda_{\mathrm{rad1}} = 0$, which might be unnatural, then by using (30), we find the value of λ at the beginning of the radiation-dominated era $t \sim t_3$,

$$\lambda = \lambda_{\mathrm{rad}}(t_3) \sim (0.1\ [\mathrm{keV}])^4. \tag{53}$$

The obtained value might be a little bit more reasonable. Then, even if $\lambda \sim 10^{-12}\ [\mathrm{eV}^4]$ in the present Universe, $\lambda \sim [(0.1\ [\mathrm{keV}])^4]$ at the beginning of the radiation-dominated era. The converse is not true because, in general, $\lambda_{\mathrm{rad1}} \neq 0$: If we only require $\lambda \sim (0.1\ [\mathrm{keV}])^4$ at the beginning of the radiation-dominated era, we may find $\lambda \sim (0.1\ [\mathrm{keV}])^4 \gg 10^{-12}\ [\mathrm{eV}^4]$ even in the present Universe.

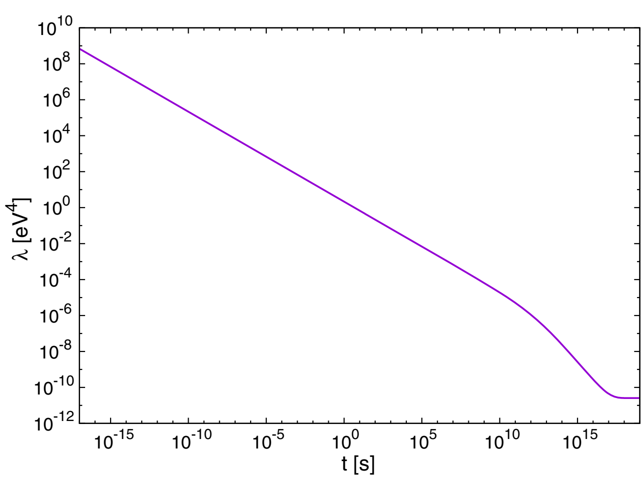

FIG. 1. The development of λ.

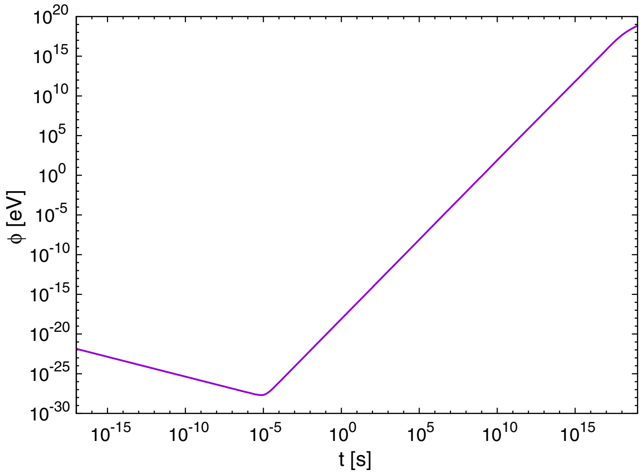

FIG. 2. The development of φ.

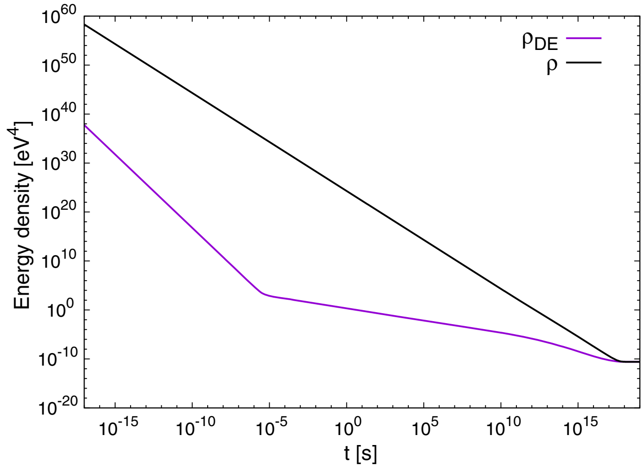

FIG. 3. The development of energy density.

We now solve Eqs. (10), (12), and (15) numerically. In Fig. 1, the time development of λ is given. The obtained value of λ at the beginning of the radiation-dominated era is consistent with the analytic result in (53). In Fig. 2, the time development of $\phi \equiv M_{\rm Pl}^3\varphi$ is given. Figure 3 shows the development of the energy density. The parameters $\lambda_{\rm rad1}$ and $\lambda_{\rm rad2}$ are chosen to reproduce the value of the dark energy density in the current Universe. The dark energy density in the matter-dominated era or the radiation-dominated era is surely negligible.

In summary, we have clarified the structure of the model in [36] and investigated the cosmology given by the model. Although the model has an infinite number of BRS symmetries, most of the symmetry is broken, and there remains one and only one BRS symmetry which guarantees the unitarity of the model. We have also shown that, by using the residual gauge symmetry, the initial condition where φ is a constant can be chosen. Because the quantum problem of the vacuum energy reduces to the classical problem of the initial condition in the model, we have investigated the region of the initial conditions which could be consistent with the evolution of the Universe. It seems difficult to solve the fine-tuning problem in the initial condition in this model. It has been also shown that a stable solution describing the de Sitter space-time exists in this model.

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