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Taylor problem and onset of plasmoid instability in the Hall-magnetohydrodynamics

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A well-known Taylor problem in the theory of forced magnetic reconnection is investigated in the framework of the Hall-Magnetohydrodynamics. In the first part of the paper, we deal with the linear theory of the Hall-mediated forced reconnection. Then, in the second part, these results are used for demonstrating how the secondary tearing (plasmoid) instability can develop in the course of this process. *Published by AIP Publishing.* https://doi.org/10.1063/1.4996982

I. INTRODUCTION

Magnetic reconnection, which is a change in the connectivity of magnetic field lines in a highly conducting fluid, plays a crucial role in various phenomena occurring in space and laboratory plasmas (solar flares, magnetospheric substorms, tokamak disruptions, etc.). This paper investigates a particular model problem of forced magnetic reconnection suggested by J. B. Taylor and known as the "Taylor problem." Being by itself of considerable intrinsic interest in the plasma physics theory, this model has also many important implications for laboratory and astrophysical plasmas. Therefore, since a pioneer paper by Hahm and Kulsrud[1] (hereafter HK), it has been studied in quite a large number of publications (see, e.g., the list of references in Ref. 2).

More recently, this model attracted a renewed interest because of its relevance to study of the secondary tearing (plasmoid) instability[3,4] as a mechanism leading to fast magnetic reconnection. The latter is a primary focus in the research field of magnetic reconnection, which is to explain why the observed rate of reconnection is usually much faster than predicted by conventional magnetohydrodynamics (MHD) models with a large Lundquist number. It is realized now that highly elongated current sheets (CSs), which are typically formed in a course of magnetic reconnection under a large Lundquist number, cannot persist. The secondary tearing instability breaks the initially long current sheet into a chain of magnetic islands (plasmoids), whose subsequent nonlinear evolution paves the way to fast reconnection.[5–7] Therefore, the issue of the plasmoid-mediated fast reconnection is presently a hot topic with a significant number of related publications. Most of them are numerical simulations, because self-consistent description of this process is not a simple task: it requires following an entire current sheet evolution, which at some point brings about the onset of the secondary tearing instability.[8]

However, in order to get deeper understanding of the issue, in particular, on how the process scales with the plasma and magnetic field parameters, one needs some tractable analytical models. Here is where the Taylor model comes to help. Thus, development of the plasmoid instability in the process of forced reconnection was first observed in the numerical simulation.[9] Then, the standard single-fluid MHD analytical theory of nonlinear forced reconnection and the onset of plasmoid instability was presented in Ref. 10. However, such a scheme is not applicable when, as it is often the case, the Lundquist number is so large that the current sheet thickness becomes comparable or smaller than the ion inertial skin-depth $d_i = c/\omega_{pi}$. In this situation, the flow of electrons and ions inside the current sheet is strongly decoupled, which manifests itself as the Hall-effect. Therefore, in this paper, our goal is to explore the Taylor model in order to find out how the Hall-effect changes the process of forced magnetic reconnection and the associated onset of the plasmoid instability. The first step is to derive analytical theory of the Hall-MHD Taylor problem. Quite surprisingly, such an analogue to the standard MHD analysis of HK still remained outstanding. A few relevant results available so far are either numerical simulations[11,12] or some heuristic theoretical estimates.[13] Then, this theory is used as a foundation for a self-consistent analytical study of the plasmoid instability in an evolving current sheet. In this respect, our approach is profoundly different from the previous publications[14,15] concerned with the Hall-mediated plasmoid instability. Unlike our case, at the starting point, there is the secondary tearing instability of a pre-existing magnetic configuration with the parameters chosen more or less arbitrarily. Moreover, in Ref. 14, it was the Sweet-Parker current sheet, which, as is now well understood (see, e.g., Ref. 8), cannot be realized in any real system.

In what follows, we use the force-free modification[16] of the Taylor's model, when a uniform plasma with the initial magnetic field

$$\vec{B}^{(0)} = (0, B_0 \sin\alpha x, B_0 \cos\alpha x) \tag{1}$$

is confined between the two perfectly conducting boundaries located at $x = x_b^{(\pm)} = \pm a$. Here, α is a constant which determines the local current density $\vec{j}^{(0)} = \frac{c}{4\pi}(\nabla \times \vec{B}^{(0)}) = \frac{c}{4\pi}\alpha\vec{B}^{(0)}$, while the degree of the "non-potentiality" of the magnetic configuration as a whole is characterised by a non-dimensional parameter $\mu \equiv \alpha a$. This equilibrium is subjected to a boundary deformation as

$$x_b^{(\pm)} = \mp(a + \delta\cos ky), \tag{2}$$

with $\frac{\delta}{a} \ll 1$. In the linear approximation with respect to this small parameter, there are two new equilibria consistent with the deformed boundaries. In terms of the flux-function $\Psi(x, y,)$, they can be written as

$$\vec{B}^{(i,r)} = (\vec{\nabla}\Psi^{(i,r)} \times \hat{z}) + \alpha\Psi^{(i,r)}\hat{z},$$
$$\Psi^{(i,r)} = \Psi^{(0)}(x) + \psi_1^{(i,r)}(x)\cos ky. \quad (3)$$

Here, $\Psi^{(0)}(x) = \frac{B_0}{\alpha}\cos\alpha x$ corresponds to the initial magnetic field (1), while two functions related to the perturbation are equal to

$$\psi^{(i)}(x) = B_0\delta\frac{\sin\alpha a}{\sin\kappa a}|\sin\kappa x|,\ \psi^{(r)}(x) = B_0\delta\frac{\sin\alpha a}{\cos\kappa a}\cos\kappa x. \quad (4)$$

It is assumed that $\kappa^2 = \alpha^2 - k^2 > 0$ (as explained below, long-wave perturbations with $k \ll \alpha$ are most interesting). The first of the above-given solutions, $\psi^{(i)}(x)$, represents the ideal MHD perturbed equilibrium, which preserves the topology of the initial field (1) but acquires discontinuity at $x = 0$: the magnetic field component $B_y = -\partial\Psi/\partial x$ has there a finite jump

$$\{B_y\} \equiv B_y|_{0+} - B_y|_{0\_} = -2B_0\kappa\delta\frac{\sin\alpha a}{\sin\kappa a}. \quad (5)$$

On the other hand, the solution $\psi^{(r)}$ is a regular one, but topology of the respective equilibrium differs from that of the initial filed: magnetic field lines reconnect and form magnetic islands located at the plane $x = 0$. The magnetic flux confined inside a single island is equal to

$$\Delta\psi^{(r)} = 2\psi^{(r)}(x = 0) = 2B_0\delta\frac{\sin\alpha a}{\cos\kappa a}. \quad (6)$$

Of course, such a model of forced reconnection makes physical sense only if the initial magnetic configuration (1) is MHD stable. As shown in Ref. 16, this is the case when the non-potentiality parameter $\mu \equiv \alpha a < \pi/2$, which is, therefore, assumed in what follows. Note also that since the value of μ determines the ratio of the magnetic field components $B_y^{(0)}$ and $B_z^{(0)}$, it is sometimes called the guide-field parameter (with $\mu \ll 1$ corresponding to the limit of a strong guide-field $B_z^{(0)}$).

As demonstrated by HK, forced reconnection is a process of the transition from the ideal MHD equilibrium to the reconnected one, which takes place when a small but finite plasma resistivity is present. The dynamics of forced reconnection is determined by the evolution of the central current sheet located at $x = 0$. If plasma thermal pressure is not too small (namely, the parameter $\beta \equiv 8\pi P/B_0^2 > S^{-2/5}$, where $S \equiv aV_A/\eta \gg 1$ is the relevant Lundquist number), in the resistive Hall-MHD magnetic reconnection, the plasma flow inside the current sheet may be considered as incompressible.[17] Then, by representing the magnetic field and the plasma velocity as

$$\vec{B}(x, y, t) = \vec{\nabla}\Psi(x, y, t) \times \hat{z} + B_z(x, y, t)\hat{z},$$
$$\vec{V}(x, y, t) = \vec{\nabla}\Phi(x, y, t) \times \hat{z} + V_z(x, y, t)\hat{z},$$

where Φ is a stream-function of the flow in the $(x - y)$ plane, equations of motion for Φ and V_z take the form

$$\rho\frac{d}{dt}(\nabla^2\Phi) = \frac{1}{4\pi}\left[\vec{\nabla}\Psi \times \vec{\nabla}(\nabla^2\Psi)\right]\cdot\hat{z}, \quad (7)$$
$$\rho\frac{dV_z}{dt} = \frac{1}{4\pi}(\vec{\nabla}B_z \times \vec{\nabla}\Psi)\cdot\hat{z}. \quad (8)$$

These should be complemented with the Maxwell's equations

$$\frac{\partial\vec{B}}{\partial t} = -c(\vec{\nabla} \times \vec{E}), \quad \vec{j} = \frac{c}{4\pi}(\vec{\nabla} \times \vec{B}), \quad (9)$$

where the electric field $\vec{E}$ is equal to

$$\vec{E} = -\frac{1}{c}(\vec{V}_e \times \vec{B}) + \frac{1}{\sigma}\vec{j} = -\frac{1}{c}(\vec{V} \times \vec{B}) - \frac{1}{4\pi ne}\left[\vec{B} \times (\vec{\nabla} \times \vec{B})\right] + \frac{c}{4\pi\sigma}(\vec{\nabla} \times \vec{B}), \quad (10)$$

[note that the bulk velocity of electrons, $\vec{V}_e = \vec{V} - \frac{\vec{j}}{ne} = \vec{V} - \frac{c}{4\pi ne}(\vec{\nabla} \times \vec{B}))$].

It follows then from (9) and (10) that

$$\frac{\partial\Psi}{\partial t} = (\vec{\nabla}\Psi \times \vec{\nabla}\Phi)\cdot\hat{z} + \eta\nabla^2\Psi + \frac{c}{4\pi ne}(\vec{\nabla}\Psi \times \vec{\nabla}B_z)\cdot\hat{z}, \quad (11)$$

$$\frac{\partial B_z}{\partial t} = (\vec{\nabla}B_z \times \vec{\nabla}\Phi)\cdot\hat{z} - (\vec{\nabla}V_z \times \vec{\nabla}\Psi)\cdot\hat{z} + \eta\nabla^2 B_z + \frac{c}{4\pi ne}(\vec{\nabla}\nabla^2\Psi \times \vec{\nabla}\Psi)\cdot\hat{z}, \quad (12)$$

with $\eta \equiv c^2/4\pi\sigma$ being the plasma magnetic viscosity.

Thus, our analysis of the Hall-MHD forced magnetic reconnection is based on Eqs. (7) and (8) and (11) and (12), and the paper is organized as follows. Section II is devoted to the linear theory, results of which are then used in Sec. III for demonstrating how the onset of the plasmoid instability is affected by inclusion of the Hall effect. A brief summary of the results and discussion is presented in Sec. IV.

II. LINEAR REGIME OF THE HALL-MHD FORCED RECONNECTION

In the linear approximation, the governing Eqs. (7)–(12) take the form

$$\frac{\partial}{\partial t}(\nabla^2\Phi) = -\frac{1}{4\pi\rho}\left[\frac{d\Psi^{(0)}}{dx}\cdot\frac{\partial}{\partial y}(\nabla^2\Psi_1) - \frac{d^3\Psi^{(0)}}{d^3x}\cdot\frac{\partial\Psi_1}{\partial y}\right], \quad (13a)$$

$$\frac{\partial V_z}{\partial t} = \frac{1}{4\pi\rho}\left(\frac{dB_z^{(0)}}{dx}\cdot\frac{\partial\Psi_1}{\partial y} - \frac{d\Psi^{(0)}}{dx}\cdot\frac{\partial B_z^{(1)}}{\partial y}\right), \quad (13b)$$

$$\frac{\partial\Psi_1}{\partial t} = -\frac{d\Psi^{(0)}}{dx}\cdot\frac{\partial\Phi}{\partial y} + \eta\nabla^2\Psi_1 + \frac{c}{4\pi ne}\left(\frac{d\Psi^{(0)}}{dx}\cdot\frac{\partial B_z^{(1)}}{\partial y} - \frac{dB_z^{(0)}}{dx}\cdot\frac{\partial\Psi_1}{\partial y}\right), \quad (13c)$$

$$\frac{\partial B_z^{(1)}}{\partial t} = -\frac{dB_z^{(0)}}{dx}\cdot\frac{\partial\Phi}{\partial y} - \frac{d\Psi^{(0)}}{dx}\cdot\frac{\partial V_z}{\partial y} + \eta\nabla^2 B_z^{(1)} + \frac{c}{4\pi ne}\left(\frac{d^3\Psi^{(0)}}{d^3x}\cdot\frac{\partial\Psi_1}{\partial y} - \frac{d\Psi^{(0)}}{dx}\cdot\frac{\partial}{\partial y}\nabla^2\Psi_1\right), \tag{13d}$$

where $\Phi, V_z, \Psi_1, B_z^{(1)}$ are perturbations proportional to the first power of a small parameter (δ/a). Since reconnection takes place inside a narrow central current sheet with a thickness $\Delta x \ll a$ (as well as $\Delta x \ll k^{-1}$—a wavelength of the external boundary perturbation), in what follows one can simplify $\frac{dB_z^{(0)}}{dx}, \frac{d\Psi^{(0)}}{dx} \propto \sin\alpha x \approx \alpha x$, and assume that $\nabla^2(\Psi_1, \Phi, B_z^{(1)}) \approx \partial^2(\Psi_1, \Phi, B_z^{(1)})/\partial^2 x$. Furthermore, it is useful to introduce non-dimensional variables by scaling all lengths with a, time with the Alfven time-scale $\tau_A = a/V_A = a\sqrt{4\pi\rho}/B_0$, and, for the perturbations: velocity V_z with the Alfven speed as $V_A(\delta/a)$, stream-function Φ with $aV_A(\delta/a) = V_A\delta$, flux-function Ψ_1 with $aB_0(\delta/a) = B_0\delta$, and $B_z^{(1)}$ with $B_0(\delta/a)$. These transform Eq. (13) into the following:

$$\frac{\partial}{\partial t}\left(\frac{\partial^2\Phi}{\partial^2 x}\right) \approx \mu x\frac{\partial}{\partial y}\left(\frac{\partial^2\Psi_1}{\partial^2 x}\right), \tag{14a}$$

$$\frac{\partial V_z}{\partial t} \approx \mu x\frac{\partial}{\partial y}\left(B_z^{(1)} - \mu\Psi_1\right), \tag{14b}$$

$$\frac{\partial\Psi_1}{\partial t} \approx \mu x\frac{\partial\Phi}{\partial y} + \frac{1}{S}\frac{\partial^2\Psi_1}{\partial^2 x} - \mu d_i x\frac{\partial}{\partial y}\left(B_z^{(1)} - \mu\Psi_1\right), \tag{14c}$$

$$\frac{\partial B_z^{(1)}}{\partial t} \approx \mu^2 x\frac{\partial\Phi}{\partial y} + \frac{1}{S}\frac{\partial^2 B_z^{(1)}}{\partial^2 x} + \mu x\frac{\partial V_z}{\partial y} + \mu d_i x\frac{\partial}{\partial y}\left(\frac{\partial^2\Psi_1}{\partial^2 x}\right), \tag{14d}$$

where $S \equiv \frac{aV_A}{\eta}$ is the Lundquist number, and $d \equiv d_i/a$ is the scaled ion inertial length. In what follows, it is assumed that $S \gg 1$ and $d \ll 1$, which is the case for a vast majority of applications.

Consider now symmetry properties of the perturbations. As far as the flux-function Ψ_1 is concerned, that is imposed by the boundary deformation (2): $\Psi_1(x, y, t) = \psi(x, t)\cos ky$, with $\psi(x, t)$ being an even function of x. Then, according to (14a), $\Phi(x, y, t) = \phi(x, t)\sin ky$, where ϕ is an odd function of x. The magnetic field component $B_z^{(1)}$ is, according to (14d), a superposition of both modes

$$B_z^{(1)}(x, y, t) = b_1(x, t)\cos ky + b_2(x, t)\sin ky, \tag{15}$$

where b_1 and b_2 are, respectively, even and odd functions of x. The appearance of the latter is entirely due to the Hall-effect: the second term on the r.h.s. of Eq. (15) represents a quadrupole magnetic structure which is a signature of the Hall-mediated magnetic reconnection.[18] Then, a straightforward inspection of Eqs. (14c) and (14d) reveals that $b_1(x, t) = \mu\psi(x, t)$; so, according to (14c), this part of $B_z^{(1)}$ does not affect evolution of the flux-function Ψ_1. Finally, Eq. (14b) yields $V_z(x, y, t) = v(x, t)\cos ky$, where v is an even function of x, and one gets the following set of evolution equations for the above-introduced functions ψ, ϕ, b_2, v:

$$\frac{\partial}{\partial t}\phi'' = -\mu kx\psi'', \tag{16a}$$

$$\frac{\partial v}{\partial t} = \mu kxb_2, \tag{16b}$$

$$\frac{\partial\psi}{\partial t} = \mu kx\phi + \frac{1}{S}\psi'' - \mu kdxb_2, \tag{16c}$$

$$\frac{\partial b_2}{\partial t} = -\mu kxv + \frac{1}{S}b_2'' - \mu kdx\psi''. \tag{16d}$$

The last terms on the r.h.s. of Eqs. (16c) and (16d), which are proportional to the parameter d, are due to the Hall effect. The limit $d = 0$ corresponds to the standard single-fluid MHD, when $b_2 = v = 0$. Thus, we aim to derive a threshold value of d, above which the Hall effect makes a difference, and to investigate the resulting process of the Hall-mediated forced magnetic reconnection.

As demonstrated by HK, the boundary deformation (2) leads to the formation of the current sheet (CS) located around the plane $x = 0$, the thickness of which is decreasing with time. Under the conditions $d \ll 1, S \gg 1$, the initial stage of this process can be described in terms of the ideal single-fluid MHD as follows. Let $\Delta(t) \equiv (\Delta x)/a$ be the scaled thickness of this CS, so that at $x \le \Delta$ one gets

$$B_y^{(1)} \sim \frac{x}{\Delta} \Rightarrow \psi_i \sim \frac{x^2}{\Delta} \sim \Delta \Rightarrow \frac{\partial\psi_i}{\partial t} \sim \frac{d\Delta}{dt}, \quad \psi_i'' \sim \frac{1}{\Delta}, \tag{17}$$

(a symbol ψ_i here indicates the ideal MHD flux-function). It follows then from (16a) that

$$\frac{\partial}{\partial t}\phi'' \sim \frac{\partial}{\partial t}\left(\frac{\phi}{\Delta^2}\right) \sim \mu k \Rightarrow \phi \sim \mu kt\Delta^2. \tag{18}$$

On the other hand, Eq. (16c) yields

$$\frac{d\Delta}{dt} \sim \mu k\Delta\cdot\mu kt\Delta^2 \Rightarrow \Delta(t) \sim (\mu kt)^{-1}, \tag{19}$$

in accordance with HK. Thus, such shrinking of the CS would bring about (though only asymptotically in time) the singular ideal MHD equilibrium given by Eqs. (3) and (4). However, this process comes to the end when, eventually, a finite plasma resistivity or the Hall effect intervenes. Consider first a role of the resistivity. The respective term in Eq. (16c) can be estimated to be $S^{-1}\psi_i'' \sim S^{-1}\Delta^{-1} \sim S^{-1}\mu kt$, and it becomes comparable with $\frac{\partial\psi_i}{\partial t} \sim \frac{d\Delta}{dt} \sim (\mu k)^{-1}t^{-2}$ at

$$t \sim t_S \sim (\mu k)^{-2/3}S^{1/3}. \tag{20}$$

A similar derivation for the Hall effect is as follows. One can use Eqs. (16d) and (17) to estimate generation of the quadrupole field b_2 in the CS: $\frac{\partial b_2}{\partial t} \sim kd \Rightarrow b_2 \sim \mu kdt$, so the Hall term in Eq. (16c) is of order of $\mu kdxb_2 \sim \mu kd^2$. By comparing it with the l.h.s. term $\frac{\partial\psi_i}{\partial t} \sim \frac{d\Delta}{dt} \sim (\mu k)^{-1}t^{-2}$, one concludes that the Hall effect comes into play at

$$t \sim t_H \sim (\mu kd)^{-1}, \tag{21}$$

when, according to (19), the CS thickness $\Delta(t \sim t_H) \sim d$.

Therefore, an interplay between the two effects depends on the relation between these two time-scales, t_S and t_H. Consider first the case when the resistivity comes first, i.e., $t_S \ll t_H$, which implies that

$$d < d_1 \sim (\mu k)^{-1/3} S^{-1/3}. \tag{22}$$

It turns out that in this case the Hall effect does not play any role at all in the process of forced reconnection.

In order to demonstrate this, consider now what happens at $t > t_S$, when, according to HK, the system evolves in the so-called "constant-Ψ" regime.[19] Indeed, the amount of reconnected magnetic flux, ψ_r, is equal to $\psi(x=0)$, hence, as it follows from Eqs. (16c), (17), and (19), at $t \leq t_S$

$$\frac{d\psi_r}{dt} = \frac{1}{S}\psi_i'' \sim S^{-1}\Delta^{-1} \sim S^{-1}\mu k t \Rightarrow \psi_r \sim S^{-1}\mu k t^2.$$

Thus, at $t \sim (\mu k)^{-2/3} S^{1/3} \sim t_S$ the reconnected flux becomes comparable to the total variation of the magnetic flux function inside the CS, which is given by $\Delta\psi = \psi_i(x \sim \Delta) - \psi_i(0) = \psi_i(\Delta) \sim \Delta \sim (\mu k t)^{-1}$. Therefore, at $t > t_S$, the "constant-Ψ" approximation holds, and temporal evolution of the reconnected flux, $\psi_r(t)$, and the CS thickness, $\Delta(t)$, can be obtained in the following way. First, as long as the reconnected flux is still small compared to its terminal value, i.e., $\psi_r \ll 1$, the discontinuity of $B_y^{(1)}$ across the CS persists, so $\psi'' \cdot \Delta \sim 1$. Second, in this regime, the convective and resistive terms in Eq. (16c) should be comparable, which yields $S^{-1}\psi'' \sim S^{-1}\Delta^{-1} \sim \mu k x \phi \sim \mu k \Delta \phi \Rightarrow \phi \sim S^{-1}(\mu k)^{-1} \Delta^{-2}$. By inserting this expression for ϕ into Eq. (16a), one gets

$$\frac{\partial}{\partial t}\phi'' \sim \frac{\partial}{\partial t}\left(\frac{\phi}{\Delta^2}\right) \sim \mu k x \psi'' \sim \mu k \Delta \psi'' \sim \mu k \Rightarrow \Delta \sim S^{-1/4}(\mu k)^{-1/2} t^{-1/4}. \tag{23}$$

As seen from (23), the CS shrinking continues, and, according to (16c), it yields the reconnection rate

$$\frac{d\psi_r}{dt} \sim \frac{1}{S}\psi'' \sim \frac{1}{S\Delta} \sim S^{-3/4}(\mu k)^{1/2} t^{1/4} \Rightarrow \psi_r \sim S^{-3/4}(\mu k)^{1/2} t^{5/4}. \tag{24}$$

Thus, it follows then from (24) that the reconnected flux becomes of order of unity at

$$t \approx \tau_r \sim S^{3/5}(\mu k)^{-2/5}, \tag{25}$$

which, in accordance with HK, is the standard MHD reconnection time.

Now one can estimate the magnitude and, hence, significance of the ignored so far Hall term in Eq. (16c). In order to do so, it is necessary first to evaluate the quadrupole field, b_2, generated in the CS by the Hall effect [see Eq. (16d)]. It turns out that at $t > t_S$, the respective last term on the r.h.s. of (16d) is balanced by the resistive diffusion of b_2, hence

$$\mu k d x \psi'' \sim \mu k d \sim \frac{b_2''}{S} \sim \frac{b_2}{S\Delta^2} \Rightarrow b_2 \sim \mu k d S \Delta^2 \sim d(S/t)^{1/2}.$$

Inserting this expression for b_2 into the Hall term in Eq. (16c), one gets $\mu k d x b_2 \sim \mu k d \Delta b_2 \sim (\mu k)^{1/2} d^2 S^{1/4}/t^{3/4}$, which at $t > t_S$ is small compared to other terms in this equation. Indeed, its ratio to $d\psi_r/dt$ [see Eq. (24)] reads

$$\frac{d^2 S}{t} \sim \left(\frac{d}{(\mu k)^{-1/3} S^{-1/3}}\right)^2 \cdot \frac{t_S}{t} \ll 1 \text{ under the condition (22).}$$

In this context, it is worth noting that, contrary to a common wisdom, the Hall effect could remain insignificant even when in a course of reconnection the CS thickness, Δ, gets smaller than d. Indeed, according to (23) and (25), $\Delta(t \sim \tau_r) \sim (\mu k)^{-2/5} S^{-2/5}$, which could be smaller than d even under the constraint (22).

Thus, for realization of the Hall-mediated regime of forced reconnection, it is necessary (but, as shown below, not sufficient) that the inequality opposite to (22) holds

$$d > d_1 \sim (\mu k)^{-1/3} S^{-1/3}, \tag{26}$$

which implies that $t_H < t_S$, so the Hall effect comes into play before a finite plasma resistivity intervenes. Therefore, in this case what initially follows at $t > t_H \sim (\mu k d)^{-1}$ is a phase of the ideal Hall-MHD, when the evolution of ψ and b_2 is governed entirely by the Hall terms in Eqs. (16c) and (16d). It results in a further shrinking of the CS, which can be derived in the same way as explored in Eqs. (17)–(19). Thus, Eq. (16d) now yields

$$\frac{\partial b_2}{\partial t} \sim \mu k d \Delta \psi'' \sim \frac{1}{t_H} \Rightarrow b_2 \sim t/t_H, \tag{27}$$

and, by inserting it into Eq. (16c), one gets

$$\frac{\partial \psi}{\partial t} \sim \frac{d\Delta}{dt} \sim \mu k d x b_2 \sim \Delta \frac{t}{t_H^2} \Rightarrow \Delta(t) \sim d \exp\left(-t^2/t_H^2\right). \tag{28}$$

Such exponential shrinking of the CS [which is much faster than that in the standard MHD, see Eq. (19)] is caused by the dispersive character of the Hall-MHD waves (whistlers). This ideal phase of evolution holds until the resistivity intervenes at some time $t \sim t_*$, when the CS thickness becomes sufficiently small: $\Delta(t_*) \ll d$. This instant can be obtained by equating the resistive and Hall terms in Eq. (16c)

$$\frac{1}{S}\psi'' \sim \frac{1}{S\Delta(t_*)} \sim \mu k d \Delta(t_*) b_2(t_*). \tag{29}$$

Then, since temporal variation of Δ is, according to (28), much stronger than that of b_2 in (27), with a logarithmic accuracy the sought after time is $t_* \sim t_H$. Therefore, $b_2(t_*) \sim 1$, and it follows than from (29) that

$$\Delta(t_*) \equiv \Delta_H \sim S^{-1/2}(\mu k d)^{-1/2} \sim S^{-1/2} t_H^{1/2}, \tag{30}$$

[note that the anticipated inequality, $\Delta_H \ll d$, is satisfied indeed due to the condition (26)].

The subsequent resistive Hall-MHD reconnection is quite similar to the standard MHD case discussed earlier, albeit advection of the magnetic field into the CS is now provided by the Hall effect rather than by the plasma inflow. First, one can verify that, as before, reconnection proceeds now in the "constant-Ψ" regime. Indeed, in the course of the CS shrinking, its internal magnetic flux is decreasing with time [see Eq. (17)] as $\Delta\psi = \psi_i(x\sim\Delta) - \psi_i(0) = \psi_i(x\sim\Delta) \sim \Delta$; hence, $\Delta\psi(t_*) \sim \Delta_H$. On the other hand, the reconnected flux ψ_r is growing with time as

$$\frac{d\psi_r}{dt} = \frac{1}{S}\psi'' \sim \frac{1}{S\Delta_H} \Rightarrow \psi_r(t_*) \sim \frac{t_*}{S\Delta_H} \sim \frac{t_H}{S\Delta_H};$$

hence, it becomes comparable to $\Delta\psi(t_*)$ with Δ_H given by Eq. (30). Thus, the set of relations governing the subsequent temporal evolution of Δ, b_2, and ψ_r is as follows:

$\Delta \cdot \psi'' \sim 1$—the required discontinuity of $B_y^{(1)}$ across the CS;

$\frac{1}{S}\psi'' \sim \mu k d x b_2 \Rightarrow \frac{1}{S\Delta} \sim \mu k d \Delta b_2$—balance of the resistive and Hall terms in Eq. (16c);

$\frac{1}{S}b_2'' \sim \mu k d x \psi'' \Rightarrow \frac{b_2}{S\Delta^2} \sim \mu k d \Delta \psi'' \sim \mu k d$—balance of the resistive and Hall terms in Eq. (16d). These yield $\Delta \sim \Delta_H$, $b_2 \sim 1$ and

$$\frac{d\psi_r}{dt} = \frac{1}{S}\psi'' \sim \frac{1}{S\Delta} \sim S^{-1/2}(\mu k d)^{1/2} \Rightarrow \psi_r(t) \sim S^{-1/2}(\mu k d)^{1/2} t. \tag{31}$$

Therefore, if this regime proceeded until full completion of the process of forced reconnection, i.e., when $\psi_r \approx 1$, the respective reconnection time, according to (31), would be equal to

$$\tau_r^{(H)} \sim S^{1/2}(\mu k d)^{-1/2}. \tag{32}$$

Note that the scaling (32) yields the reconnection time that does not involve the ion mass m_i. Indeed, S is proportional to $m_i^{-1/2}$, $d \propto m_i^{1/2}$, and the unit of time imposed in (32), $\tau_A \propto m_i^{1/2}$. Therefore, it corresponds to the electron-MHD limit in the theory of forced magnetic reconnection.[20]

It turns out, however, that this is the case only when the Hall parameter d exceeds a certain second threshold, d_2 (see below), which is much higher than d_1 given in Eq. (26). Otherwise, at some time, $\tilde{t} \ll \tau_r^{(H)}$, the Hall regime (31) gives way to the standard MHD reconnection, and the overall reconnection time becomes equal to τ_r defined in Eq. (25). The reason lies in a double-layer structure of the internal solution during the resistive phase of the Hall-MHD reconnection.[21,22] Thus, the resistive region, $x \le \Delta_H$, is surrounded by a much wider layer, $\Delta_H < x < x_H$, where the plasma resistivity plays no role, but the poloidal magnetic field described by the flux function ψ is still advected towards the reconnection site by the Hall effect [the last term on the r.h.s. of Eq. (16c)]. Therefore, by using Eqs. (16c) and (31), one can evaluate the required quadrupole field component as

$$b_2 = -\frac{1}{\mu k d x}\frac{\partial\psi}{\partial t} \sim \frac{S^{-1/2}(\mu k d)^{-1/2}}{x} \sim \frac{\Delta_H}{x}. \tag{33}$$

On the other hand, the very same field (33) also generates, according to Eq. (16b), the z-component of the plasma velocity, v

$$\frac{\partial v}{\partial t} = \mu k x b_2 \sim \mu k \Delta_H \Rightarrow v \sim \mu k \Delta_H t.$$

Furthermore, in Eq. (16d), the Hall term is balanced by the first term on the r.h.s. of this equation, which is due to this velocity component. Therefore, $\mu k d x \psi'' \sim \mu k x v$; hence, the electric current in this layer, ψ'', can be estimated as

$$\psi'' \sim (\mu k) d^{-1} \Delta_H t \sim d^{-2}\Delta_H (t/t_H). \tag{34}$$

This current accelerates the poloidal plasma flow at the rate given by Eq. (16a)

$$\frac{\partial}{\partial t}(\phi'') \sim \mu k x d^{-2} \Delta_H \frac{t}{t_H} \Rightarrow \phi'' \sim d^{-3}\Delta_H x \left(\frac{t}{t_H}\right)^2 \Rightarrow \phi \sim \Delta_H \left(\frac{x}{d}\right)^3 \left(\frac{t}{t_H}\right)^2.$$

Finally, by inserting this expression into Eq. (16c), one can estimate the width x_H of the ideal Hall-MHD sublayer by requiring that at $x \sim x_H$ the advection of the magnetic field by the plasma flow becomes comparable to that by the Hall term. Hence,

$$\mu k x_H \phi(x_H) \sim \mu k d x_H b_2(x_H) \sim \mu k d \Rightarrow x_H \sim d\left(\frac{t}{t_H}\right)^{-1/2}, \tag{35}$$

so at $x > x_H$ the Hall effect is not important, and the standard MHD description applies.

Therefore, the Hall-MHD regime of reconnection described by Eq. (31) holds as long as the width of the resistive sublayer, Δ_H, is smaller than x_H, i.e., according to (30) and (35), $t < \tilde{t} \sim S d^2$. This leaves one with the following two possibilities. If

$$d > d_2 \sim S^{-1/5}(\mu k)^{-1/5}, \tag{36}$$

[note that $d_2 \gg d_1 \sim S^{-1/3}(\mu k)^{-1/3}$], $\tilde{t} \gg \tau_r^{(H)} \sim S^{1/2} t_H^{1/2}$; hence, the Hall-MHD regime has enough time to complete the reconnection process. If otherwise, i.e., when $d_1 < d < d_2$, at $t \approx \tilde{t}$ a transition from the Hall-MHD regime (31) to the standard MHD reconnection (24) occurs. At this point, the amount of already reconnected magnetic flux is still small: indeed, according to (31), $\psi_r(\tilde{t}) \sim (d/d_2)^{5/2} \ll 1$; hence, the main part of reconnection is completed in the standard MHD regime. It is worth emphasizing here that this transition from the Hall- to the standard MHD occurs when the thickness of the resistive CS is much smaller than the ion inertial length. Moreover, the former reduces even further in the course of the subsequent standard MHD reconnection [see Eq. (23)]. Finally, note that these results confirm simple preliminary estimates given in Ref. 13, but disagree with the respective linear Hall-MHD scaling presented in Ref. 11.

III. ONSET OF THE PLASMOID INSTABILITY

According to Ref. 10, in the framework of the standard single-fluid MHD, the onset of plasmoid instability during forced magnetic reconnection is possible only when the amplitude of the external perturbation is large enough

$$\delta/a > S^{-1/3}. \quad (37)$$

In this case, a central role is played by the nonlinear equilibrium with the CS of thickness $\Delta x \sim \delta$,, which is formed at the time $t = t_1 \sim \tau_A(\delta/a)^{-1} \ll t_S$. In this section, we revert to dimensional units and assume, for the sake of simplicity, that the guide-field strength parameter $\mu \sim 1$, and the perturbation wave-length is comparable to the spatial scale of the system, i.e., $ka \sim 1$. Thus, consider first what difference, if any, is caused by the Hall effect in the plasmoid instability of this CS. First of all, note that its very formation is due to the nonlinear torque in the vorticity Eq. (7) [see Ref. 10]; therefore, it can happen only before the Hall effect comes into play, i.e., when $t_1 < t_H \sim \tau_A(d_i/a)^{-1}$ [see Eq. (21)]; hence, it requires $d_i < \delta$. It turns out, however, that even under this restriction, the Hall effect could be significant. In order to demonstrate this, one may find helpful a brief summary of the resistive tearing instability theory in the standard MHD,[19,23] and in the Hall-MHD[17] frameworks, applied to the CS of thickness l, length $L \gg l$, and magnetic field B. These parameters define the respective Alfven velocity $V_A^{(l)}$, the Alfven transit time $\tau_A^{(l)} = l/V_A^{(l)}$, and the Lundquist number $S_l = lV_A^{(l)}/\eta$. Then, the standard MHD yields the instability growth rate

$$\gamma\tau_A^{(l)} \sim [S^{(l)}]^{-3/5}(ql)^{-2/5}, \quad (38)$$

where q is a wave-number of the unstable tearing mode. Expression (38), which assumes the "constant-ψ" approximation, is valid only for a wave-length $\lambda = 2\pi/q$ in the interval $l < \lambda < \lambda_* \sim lS_l^{1/4}$ (it is assumed that $S_l \gg 1$). In the non-constant-ψ case, when $\lambda > \lambda_*$, the growth rate falls sharply,[23] which makes such modes of no interest. Therefore, as seen from (38), the most unstable mode (the one with a maximum growth rate γ) corresponds to a wave-length $\lambda = \min\{\lambda_*, L\}$[8] (clearly, the CS of a finite length L cannot accommodate perturbations with $\lambda > L$). Thus, it has the following implication to the nonlinear CS under consideration, for which $l \approx \delta, L \approx a, B \approx B_0(\delta/a)$, hence $V_A^{(l)} \approx V_A(\delta/a)$, $\tau_A^{(l)} \approx \tau_A, S_l \approx S(\delta/a)^2$ [note that this $S_l \gg 1$ due to condition (37)]. Therefore, the ratio $\frac{\lambda_*}{a} \sim S^{1/4}(\frac{\delta}{a})^{3/2}$, so the most unstable appropriate mode is the one with $\lambda \sim \lambda_*$, if $(\delta/a) < S^{-1/6}$, and with $\lambda \sim a$ if otherwise. As pointed out in Ref. 10, whatever the case, their growth rate is sufficient for the plasmoid instability development during the CS life-time $(\Delta t) \sim \delta^2/\eta \sim \tau_A S(\delta/a)^2$.

In the Hall-MHD case, the situation is even more favourable to the plasmoid instability development. Indeed, the Hall effect makes the secondary tearing instability faster by providing additional inflow of magnetic flux into the reconnection site, but leaves intact the CS resistive life-time (Δt). Therefore, now the question to answer is how a finite value of d_i affects the most unstable tearing mode, in particular, its wave-length. The latter is important parameter which determines a number of plasmoids initially generated during the linear phase of the plasmoid instability. Thus, for the Hall-mediated tearing mode, a summary, analogous to the one given above for the standard MHD case, reads as follows.[17] For a mode with a wave-number q, transition to the Hall regime of instability occurs when

$$d_i/l > S_l^{-1/5}(ql)^{1/5}, \quad (39)$$

which in the constant-ψ approximation brings about the growth rate

$$\gamma\tau_A^{(l)} \sim S_l^{-1/2}(d_i/l)^{1/2}(ql)^{-1/2}. \quad (40)$$

This expression holds for

$$l < \lambda < \lambda_*^{(H)} \sim lS_l^{1/3}(d_i/l)^{1/3}, \quad (41)$$

and the growth rate falls sharply for the non-constant-ψ modes with $\lambda > \lambda_*^{(H)}$. By applying these results to the particular CS under consideration $[l \sim \delta, L \sim a, B \sim B_0(\delta/a)]$, one should also recall two constraints that are necessary for the formation of this CS: $(\delta/a) > S^{-1/3}, d_i < \delta$. Thus, consider first the case when $S^{-1/3} < (\delta/a) < S^{-1/6}$, for which in the above discussed standard MHD framework a large number of plasmoids is initially formed: $N_p \sim (a/\lambda_*) \sim S^{-1/4}(\delta/a)^{-3/2} > 1$. If a similar multiple-plasmoids regime takes place in the Hall-MHD, the following two conditions must be met. First, the optimal wave-length $\lambda_*^{(H)}$, given by Eq. (44), must be shorter than L, which in our case translates into

$$\lambda_*^{(H)} \sim \delta S^{1/3}\left(\frac{\delta}{a}\right)^{2/3}\left(\frac{d_i}{\delta}\right)^{1/3} < a \Rightarrow \frac{d_i}{\delta} < S^{-1}\left(\frac{\delta}{a}\right)^{-5}. \quad (42)$$

Second, d_i should be large enough to bring about the Hall-mediated reconnection [see Eq. (39)]; hence, $\frac{d_i}{\delta} > S^{-1/5}(\frac{\delta}{a})^{-2/5}(\frac{\delta}{\lambda_*^{(H)}})^{1/5} \Rightarrow \frac{d_i}{\delta} > S^{-1/4}(\frac{\delta}{a})^{-1/2}$. These two inequalities are compatible if $(\delta/a) < S^{-1/6}$, while for $(\delta/a) < S^{-1/5}$, the validity of (42) is guaranteed by the requirement $d_i < \delta$. Therefore, in the Hall-MHD scenario, the multiple-plasmoids regime survives when $S^{-1/3} < (\delta/a) < S^{-1/5}$. Within the interval $S^{-1/5} < (\delta/a) < S^{-1/6}$, there are two possibilities. The multiple-plasmoids case realizes if the inequality (42) still holds; otherwise, the most unstable mode is the one with $\lambda \sim a$, i.e., the number of the initially generated plasmoids is just a few. The latter is also the case when $(\delta/a) > S^{-1/6}$. Indeed, the Hall-reconnection condition (39) then takes the form

$$\frac{d_i}{\delta} > S^{-1/5}\left(\frac{\delta}{a}\right)^{-2/5}\left(\frac{\delta}{a}\right)^{1/5} \Rightarrow \frac{d_i}{\delta} > S^{-1/5}\left(\frac{\delta}{a}\right)^{-1/5},$$

[it guarantees that the inequality opposite to (42) holds], which is also compatible with the requirement $d_i < \delta$.

Thus, the impact of the Hall effect on the plasmoid instability of the CS formed at the nonlinear stage of the ideal MHD evolution is two-fold. The instability develops faster, and a wave-length of the most unstable mode becomes longer, which means a reduced number of the initially generated plasmoids. These changes, however, are not dramatic, as the overall scenario is basically the same as in the standard MHD case.

A very different situation is possible when the ion inertial length is large enough, so that the Hall-effect becomes instrumental during the entire resistive phase of the forced reconnection process. According to Sec. II, this is the case when

$$d_i > aS^{-1/5}, \quad \delta < d_i. \tag{43}$$

The point is that in the standard MHD framework, the plasmoid instability cannot develop at this stage: the system slips into the Rutherford regime of slow magnetic reconnection[9,10] even under quite a small perturbation amplitude. However, this effect is irrelevant in the Hall-MHD, where magnetic field is advected to the reconnection site by the Hall-generated electric current rather than by the plasma flow. Therefore, the situation becomes more favourable to the plasmoid instability development.

Thus, consider tearing stability of the CS formed at the major phase of the Hall-MHD forced reconnection [see Eqs. (30)–(32)] under the conditions (43). In this case, the CS parameters are as follows: $L \sim a, l \sim a\Delta_H \sim aS^{-1/2}(d_i/a)^{-1/2}, B \sim B_0(\delta/a)$, which yield

$$V_A^{(l)} \sim V_A\frac{\delta}{a}, \quad \tau_A^{(l)} \sim \frac{l}{V_A^{(l)}} \sim \tau_A S^{-1/2}\left(\frac{d_i}{a}\right)^{-1/2}\left(\frac{\delta}{a}\right)^{-1},$$
$$S_l = \frac{lV_A^{(l)}}{\eta} \sim S^{1/2}\left(\frac{d_i}{a}\right)^{-1/2}\left(\frac{\delta}{a}\right).$$

Then, according to (41), the wave-length of the most unstable tearing mode is equal to

$$\lambda_*^{(H)} \sim lS_l^{(1/3}\left(\frac{d_i}{l}\right)^{1/3} \sim aS^{-1/6}\left(\frac{d_i}{a}\right)^{-1/6}\left(\frac{\delta}{a}\right)^{1/3}; \tag{44}$$

hence, $\lambda_*^{(H)} < a$ under the conditions (43). Therefore, this mode can develop in the CS of length $L \sim a$ and, hence, lead to a multiple-plasmoids $(N_p \sim a/\lambda_*^{(H)} > 1)$ initial phase of the instability, provided that its growth rate is sufficiently high. In order to verify that the latter is the case, one should compare the respective growth rate, $\gamma_*^{(H)}$, with the life-time (Δt) of this CS, which in this case is the Hall reconnection time (33): $(\Delta t) \sim \tau_r^{(H)} \sim \tau_A S^{1/2}(d_i/a)^{-1/2}$. Thus, according to (40) and (44), $\gamma_*^{(H)} \sim [\tau_A^{(l)}]^{-1}S_l^{-1/3}(\frac{d_i}{l})^{2/3} \sim \tau_A^{-1}S^{2/3}(\frac{d_i}{a})^{5/3}(\frac{\delta}{a})^{2/3}$, which yields $\gamma_*^{(H)} \cdot (\Delta t) \sim S^{7/6}(\frac{d_i}{a})^{7/6}(\frac{\delta}{a})^{2/3}$. Therefore, the plasmoid instability requirement, $\gamma_*^{(H)} \cdot (\Delta t) > 1$, reads $\frac{\delta}{a} > S^{-7/4}(\frac{d_i}{a})^{-7/4}$, which can be readily satisfied under the conditions (43).

IV. SUMMARY AND DISCUSSION

The first part of the paper (Sec. II) presents the analytical theory of the Hall-MHD forced magnetic reconnection. The role of the Hall effect in this process is determined by the parameter $d \equiv d_i/a$. Thus, it is shown that there are two threshold values, $d_1 \sim S^{-1/3}$ and $d_2 \sim S^{-1/5}$, which separate different regimes of reconnection. If $d < d_1$, the Hall-effect plays no role at all, so the reconnection time follows the standard MHD scaling:[1] $\tau_r \sim \tau_A S^{3/5}$. In the intermediate case, when $d_1 < d < d_2$, initially the reconnection proceeds in the Hall-MHD regime. However, it quickly gives way to the standard MHD phase, and the overall reconnection time still does not depend on the Hall parameter d. Only when the latter exceeds the second threshold, i.e., $d > d_2$, the Hall effect becomes dominant, and the reconnection time scales as $\tau_r = \tau_r^{(H)} \sim \tau_A S^{1/2}d^{-1/2}$.

Two relevant points are due here. The first one is about a widely accepted paradigm[24] that transition from the standard-to Hall-MHD occurs when the ion-inertial length d_i exceeds the CS thickness Δx. Our results clearly demonstrate that, generally speaking, this is not correct. Thus, consider the intermediate case of Sec. II, namely, $d_1 < d < d_2$. It implies that in the course of the CS shrinking during the initial ideal MHD phase of the system evolution, the transition to the Hall-MHD does take place at the point when $\Delta x \approx d_i$ [see Eqs. (17)–(19)]. However, later on, the Hall effect becomes insignificant, and the system reverses back to the standard MHD evolution despite the fact that at this stage the CS thickness $\Delta x \ll d_i$ (see the last part of Sec. II). Note also that in this case the CS thickness is, by definition, equal to a length scale associated with resistive effects. Therefore, we conclude that the paradigm under discussion holds only in the ideal MHD limit [see Eq. (21)], but it may cause confusion when the resistive effects become important. In the latter case, an interplay between now the two small parameters, $d \equiv \frac{d_i}{a} \ll 1$ and $S \equiv \frac{\eta}{aV_A} \ll 1$, makes the situation much more complicated.

The second point concerns the perturbation of the magnetic field component perpendicular to the reconnection plane, $B_z^{(1)}$. A part of it, b_2 [see Eq. (15)], has a quadrupole symmetry, and is commonly considered as a signature of the Hall-mediated magnetic reconnection.[18] However, it has been already pointed out[25] that the overall structure of $B_z^{(1)}$ could be more complicated. It is shown here that although this effect is weak in the case of a strong guide field, $(\mu \equiv \alpha a \ll 1 \Rightarrow b_1 \ll b_2)$, it could be significant when $\mu \sim 1$, making then $b_1 \sim b_2$.

The second part (Sec. III) deals with the onset of plasmoid instability in the framework of the Hall-MHD. As shown in Ref. 10, in the standard MHD case, the plasmoid instability becomes involved in the process of forced magnetic reconnection via the nonlinear CS forming at the ideal MHD stage of the system evolution. The main difference made in this case by the Hall-effect is a reduction in the number of initially generated plasmoids. This is because in the Hall-MHD framework, the most unstable secondary tearing mode has a longer wave-length: according to Eq. (39), the Hall effect has stronger impact on a tearing perturbation with a smaller wave-number q.

There is, however, another more significant change: the onset of plasmoid instability in the course of the resistive phase of the CS evolution. This is not possible in the standard MHD, because the system slips into the Rutherford regime due to halting of the plasma flow. On the contrary, in the respective Hall-MHD phase, this effect becomes irrelevant because in this case advection of the poloidal magnetic field is provided by the Hall-generated electric current rather than by the bulk flow of the plasma. This enables a multiple-plasmoids regime of the secondary tearing instability.

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[1]T. S. Hahm and R. M. Kulsrud, Phys. Fluids **28**, 2412 (1985).
[2]L. Comisso, D. Grasso, and F. L. Waelbroeck, Phys. Plasmas **22**, 042109 (2015).
[3]D. Biskamp, Phys. Fluids **29**, 1520 (1986).
[4]N. Loureiro, A. Schekochihin, and S. C. Cowley, Phys. Plasmas **14**, 100703 (2007).
[5]A. Bhattacharjee, Y. M. Huang, and B. Rogers, Phys. Plasmas **16**, 112102 (2009).
[6]D. Uzdensky, N. Loureiro, and A. Schekochihin, Phys. Rev. Lett. **105**, 235002 (2010).
[7]T. Shibayama, K. Kusano, T. Miyoshi, T. Nakabou, and G. Vekstein, Phys. Plasmas **22**, 100706 (2015).
[8]D. Uzdensky and N. Loureiro, Phys. Rev. Lett. **116**, 105003 (2016).
[9]L. Comisso, D. Grasso, and F. L. Waelbroeck, J. Phys.: Conf. Ser. **561**, 012004 (2014).
[10]G. Vekstein and K. Kusano, Phys. Plasmas **22**, 090707 (2015).
[11]R. Fitzpatrick, Phys. Plasmas **11**, 937 (2004).
[12]R. Fitzpatrick, Phys. Plasmas **11**, 3961 (2004).
[13]G. Vekstein and N. Bian, Phys. Plasmas **13**, 122105 (2006).
[14]S. D. Baalrud, A. Bhattacharjee, Y. M. Huang, and K. Germaschewski, Phys. Plasmas **18**, 092108 (2011).
[15]Z. W. Ma, L. C. Wang, and L. J. Li, Phys. Plasmas **22**, 062104 (2015).
[16]G. Vekstein and R. Jain, Phys. Plasmas **5**, 1506 (1998).
[17]N. Bian and G. Vekstein, Phys. Plasmas **14**, 072107 (2007).
[18]Y. Ren, Y. Yamada, S. Gerhardt, H. Ji, R. Kulsrud, and A. Kuritsin, Phys. Rev. Lett. **95**, 055003 (2005).
[19]H. P. Furth, J. Killeen, and M. N. Rosenbluth, Phys. Fluids **6**, 459 (1963).
[20]S. V. Bulanov, F. Pegoraro, and A. S. Sakharov, Phys. Fluids B **4**, 2499 (1992).
[21]T. Terasawa, Geophys. Res. Lett. **10**, 475, doi:10.1029/GL010i006p00475 (1983).
[22]A. B. Hassam, Phys. Fluids **27**, 2877 (1984).
[23]B. Coppi, R. Galvao, R. Pellat, M. Rosenbluth, and P. Rutherford, Sov. J. Plasma Phys. **2**, 533 (1976) [Fiz. Plazmy **2**, 961 (1976) (in Russian)].
[24]M. Yamada, R. Kulsrud, and H. Ji, Rev. Mod. Phys. **82**, 603 (2010).
[25]N. Bian and G. Vekstein, Phys. Plasmas **14**, 120702 (2007).