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主 論 文 の 要 旨

論文題目　**New approaches to quantum correlations: Majorana braiding dynamics and information indistinguishability**

– A quantum beasts' monomyth –

（量子相関への新たなアプローチ：マヨラナ・ブレイディング・ダイナミックスおよび情報不可弁別性

–量子獣モノミス–）

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論 文 内 容 の 要 旨

Modern quantum theory studies various microscopic phenomena of intrinsic uniqueness. As a form of probabilistic theory that relies on complex linear spaces, many physical quantities are manifested under the form of different correlations. Here, we shall particularly focus on the idiosyncrasies of non-locality from two different perspectives: topological superconducting wires where fermion-like edge-mode pairs displays exotic non-Abelian statistics [1,2] and general non-local correlations between binary degrees of freedom [3]. The first is recent object of interest for the realization of fault-tolerant quantum computation [4,5], owing to the reliability of topological states that do not couple to disturbances from the environment. The second stumbles on older questions around quantum mechanics like EPR paradox [6-8] brought under new questionings about the physical limits of nonlocality in recent years [9].

Topological quantum computation & Majorana braiding dynamics

Quantum information processing has been lurking around since 80s as an idea of using quantum mechanics to handle information like computers do. It has attracted more and more attention in recent years as efforts towards its realization grew stronger and more visible results have piled up, with major companies also taking part in this quest. Needless to say, the challenge of manipulating information on quantum level is far from trivial and has always been bathed in skepticism even among leaders in the field. One obvious problem arises from the fragility of quantum states to various sources of noise from the environment, leading to parasitic decoherence that severely limits any attempt of computation. Similarly, the difficulties to achieve precise control even on an isolated noiseless system pose another critical obstacle that must be overcome. Different approaches to solve such matters exist, among them the idealization of topological quantum computation.

Topological quantum computation relies on topology to give stability to a system and achieve fault-tolerance [10]. It can be realized by using *anyons*, topological excitations on low-dimensional systems, to encode and process information. In three space dimensions and above, particle exchange can be represented with a permutation group, whose representation becomes simply ±1, corresponding to bosons and fermions. However, in one or two space dimensions, particles must go around each other if they are exchanged, and their path can no longer be ignored. This exchange is called *braiding*, which makes up the *braiding group*. In general, the representation of the braiding group can assume any form. If the braising group is Abelian, its representation reduces to a phase factor $e^{i\theta}$, which is not physically relevant alone. Nonetheless, if the braiding group is non-Abelian, we have *non-Abelian anyons* whose braiding representation becomes higher dimensional, i.e. non-commuting matrices. This allows for state transitions upon particle exchange, which can be a reliable and robust method for controlling quantum states and performing computations, making this property the heart of topological information processing.

The simplest example of such non-Abelian anyons is the so-called Ising anyon. Essentially, two Ising anyons taken together can add up to either a fermion or a boson (vacuum). It has been noticed that Majorana fermions in condensed matter hetero-structures can behave exactly as Ising anyons, allowing their application in topological quantum computers. Majorana fermions are fermions that are their own antiparticle, following anti-commutation relations but squaring to 1. This algebraic structure reproduces the same properties as an Ising anyon, and braiding Majorana fermions should have non-trivial results. Such particles appear in condensed matter as zero-energy edge-modes in topological superconductors, making them more often called Majorana bound states (MBS) or Majorana zero modes (MZM). Theoretically, electrons and holes coalesce on edges of semiconducting nanowires with strong spin-orbit interaction under Zeeman field, made superconducting by proximity with a standard s-wave superconductor [11,12].

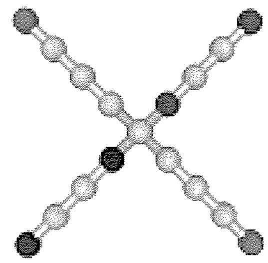

Fig. 1: Cruciform junction for braiding of Majorana fermions. Orange links represent control gates.

We follow Kitaev model [13] for quantum wires with free Majorana fermions on the edges, under Hamiltonian

$$H = \mu_0 \sum_j c_j^\dagger c_j + \sum_j \lambda c_{j+1}^\dagger c_j + \Delta_s c_{j+1} c_j + \text{h.c.}$$

to simulate the dynamics of the system, where the first term gives on-site interaction with chemical potential μ_0, and the second summation has a hopping term and a superconducting pairing term, respectively. We can implement the exchange of two edge-mode by making a cross junction of two such wires and switching central gates on/off to connect/disconnect edges (fig. 1). To realize a NOT-gate, for example, one only has to exchange two edge modes belonging to different wires twice, and the overall state should change to an orthogonal state. The procedure for such exchange can be seen explicitly in fig. 2, with T standing for each gate operation time.

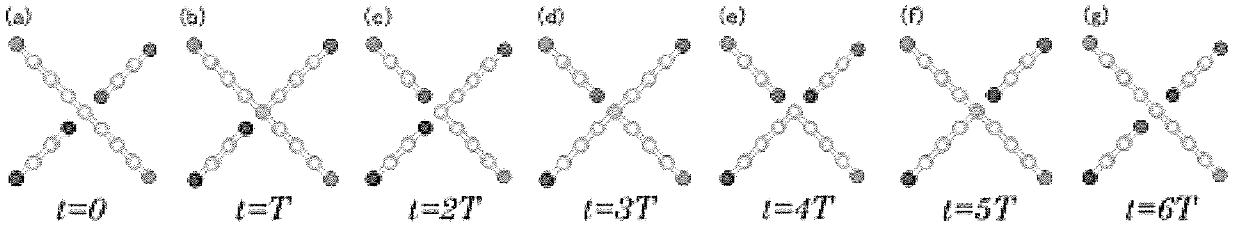

Fig. 2: Exchange process of two Majorana bound states. Each central gate is shut/open with operation time T. Central links are shown present or cut according to the state of the gates in each given instant. (a) and (g) have the same gate state, but inn er MBSs have been exchanged.

Starting from a pair of MBSs and numerically calculating the time evolution steps of fig. 2 (i.e., by solving the time-dependent Bogoliubov-de Gennes equation), we can assess the braiding conditions and its success ratio. As a NOT-gate is expected to have orthogonal initial and final states, one only has to observe the projection of the wavefunction on different bases. We set our chemical potential to $\mu_0 = 0.7\lambda$ and $\Delta_s = 0.1\lambda$, and examine various wire length conditions (defined as number of sites per half-wire). Figure 3 shows the success rate under such conditions.

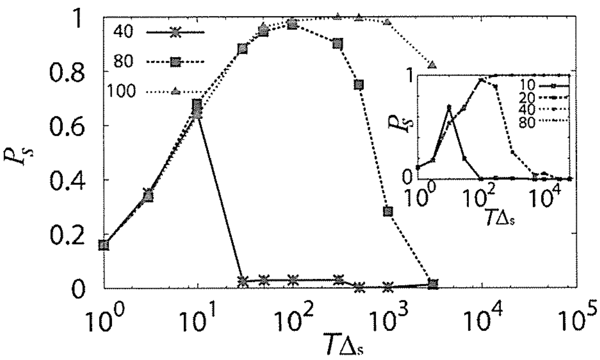

Fig. 3: Success rate of Majorana braiding. Each line accounts for different wire lengths, with $\mu_0 = 0.7\lambda$ and $\Delta_s = 0.1\lambda$ (inset, $\Delta_s = \lambda$).

From this analysis, one can infer operation time windows and possible wire lengths. For a superconducting gap around 250 μeV, gate operations are limited to tens of nanoseconds at fastest. The slowest possible operation depends on the coupling energy of Majorana edge-modes, which is governed by wires length and depends on material. We estimate a length of micrometer order to be necessary for common icosagen pnictogenide [14] semiconducting nanowires.

Information indistinguishability & quantum nonlocality

When we deal with quantum information, the underlying physical foundation for any discussion is the information unit called quantum bit or *qubit* for short. As the name should make clear, qubits represent any two-level system or two-value quantum degrees of freedom. As such, it does not have a single clear physical character. For instance, electron spins (fermionic) may serve as qubits as well as light polarization (bosonic), or even non-local Majorana zero modes (anyonic).

Although the concept may sound relatively new, many physical aspects that have been investigated in the past can be rephrased in information theory formalism without much change. As an example, the study of quantum properties of a spin singlet state dates the dawn of quantum mechanics, and its non-triviality has been pointed as a possible flaw in quantum theory early on, since their anti-correlation could be a margin for superluminal information transmission [6], what is nowadays called "signaling." The alternative advocated by Einstein *et al.* was to consider some sort of hidden variable that would account for each spin state even if we have no possible access to such variable, hence defining each spin state by itself. But however far-fetched a spin singlet may seem, it was later shown that such states, if complying with quantum theory, cannot accept a hidden variable to determine their parts [7,8]. In other words, either quantum theory correctly describes such states without accepting any other parameters, or quantum theory is wrong and other parameters may be adopted in a different theory. This prohibition of hidden variables defines "non-locality," indicating the presence of correlations that can be arbitrarily space-separated without a common source. It has also been shown that the quantum limits for the correlation calculated by Clauser-Horne-Shimony-Holt (CHSH) inequality [8] is given by a $\sqrt{2}$ factor on the classical limit, what is now known as Tsirelson bound [15].

But in 1994, Popescu and Rohrlich showed that Tsirelson bound cannot be reproduced only by non-locality and no-signaling axioms alone [9]. This has far reaching physical implications: is there any other fundamental principle in nature that limits quantum correlations? Or could quantum theory be short of complete and even "more non-locality" be actually possible? We address this problem by taking a new perspective: *information indistinguishability*.

Usually in quantum mechanics, "indistinguishability" denotes the lack os possible discernment of identical particles, which are generally fermions or bosons. For anyons, indistinguishability is not usually considered, since they have a somewhat hard core and can be identified by their position in real space. Still, this traditional notion of indistinguishability can have non-trivial consequences [16,17]. On the top of it, information indistinguishability is postulated as the indistinguishability of information building block, i.e. qubits. By treating qubits as "identical particles," like electrons and photons they can be identified and rendered distinguishable when out-of-bounds regarding one another, i.e., when they are not interacting. On the other hand, qubit coupling may leave once distinguishable qubits indistinguishable. It can be understood in terms of lack of which-way information [17] (see fig. 4).

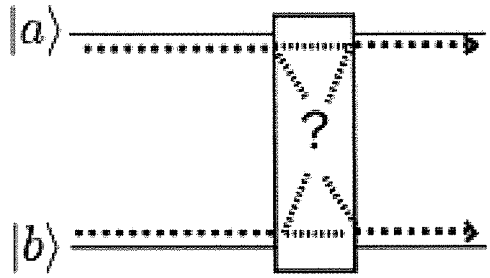

Fig. 4: Lack of which-way information [17] depicted on an arbitrary entangling gate.

It is worth noticing that the words "distinguishable" and "distinct" hold different meaning in information indistinguishability context. The later is used to indicate when two qubits are physically unambiguously identifiable, like left/right quantum dots, superconducting qubit and NV-center qubit pair, carbon and hydrogen nuclear spins in a chloroform molecule, or even an electron and a proton spin in a hydrogen atom. All these are *distinct* qubits, for there is at least one physical property that uniquely define them. On the other hand, their information is said *distinguishable* only if we can unambiguously assign this information to a distinct qubit. For instance, one may not be able to access an electron's qubit of a hydrogen atom in its ground state until a magnetic field or equivalent influence overcomes spin-spin coupling and produces different responses (nuclear magnetic resonance and electron spin resonance).

In order to examine entanglement between qubits we consider indistinguishable, we can learn from particle indistinguishability. Sciara *et al.* have discussed the universality of Schmidt decomposition for such systems [18] by using a symmetric inner product of quantum states [17]. This product, if written for a projection of two-particle states onto a single-particle, becomes

$$\langle \xi | \psi, \phi \rangle = \langle \xi | \psi \rangle | \phi \rangle + \eta \langle \xi | \phi \rangle | \psi \rangle$$

with $\eta = \pm 1$ according to statistics. Looking at particle exchange as a particular case of symmetry (exchange symmetry), we can write it in more general terms as

$$\langle \xi | \psi, \phi \rangle = \langle \xi | \psi \rangle | \phi \rangle + \langle \xi | \psi' \rangle | \phi' \rangle$$

where the prime indicates a symmetry transformation (e.g., $\phi' = \phi$ and $\phi' = \eta\,\phi$ for exchange symmetry). Indistinguishability shall be regarded as a sort of symmetry to be dealt with this kind of inner product.

To handle indistinguishability, we shall rethink how to represent qubits first. To define distinguishable qubits, we need to grasp the physical background to which we can assign information. Hence, it is natural to rewrite kets as

$$|q\rangle \longrightarrow |r, q\rangle$$

with the physical background reference r explicitly indicated. We can now further expand the generalized symmetric product to exclude the background information, writing

$$\langle r_1, r_2 | r_1, \psi; r_2 \phi \rangle = \langle r_1 | r_1, \psi \rangle \langle r_2 | r_2, \phi \rangle + \langle r_1 | r_1, \psi' \rangle \langle r_2 | r_2, \phi' \rangle$$

for the general symmetric product of two (distinguishable) qubits defined on each ones reference. Writing each qubit bases as $|r, q\rangle$ $\{|r, 0\rangle, |r, 1\rangle\}$, we consider symmetries following $q' = q + 1 \bmod 2$, owing to its relative ubiquity and possibility to represent time reversal as well as exchange symmetries. Also, we assume that applying a symmetry transformation may contribute with a general phase factor to the transformed state.

It is possible to prove that, for indistinguishable bases spanning a subspace V, with an operator Π being the projector onto this indistinguishable subspace, there exist entanglement for states living in such space. For this, we compute $\| \Pi - S \|$, where S is the projector onto Schmidt bases, and notice that $\| \Pi - S \| < 1$, what guarantees the two projectors to have the same rank [19]. Since Schmidt rank is a well known mark of entanglement, as the rank equals to the number of indistinguishable bases, we may notice that non-separability can be assured by indistinguishability. Tsirelson's bound can be inferred by calculating the maximum overlap of a projector on the indistinguishable subspace and an arbitrary observable, computing the Hermitian inner product of operators Π and joint observables A and B, each acting on a different qubit.

We can further generalize this result by taking together the "exclusivity principle" approach proposed by Cabello [20]. On general probabilistic theories not restricted to quantum mechanics, one can define general bits (gbits). By considering two copies of a system and a joint sharp measurement on both copies, and supposing such measurement to exist in compatible pairs of different bases, Cabello demonstrates that a √2 factor appears as a tight bound multiplier (compared with the classical case) for gbits correlations. The justification of such assumption is nonetheless unclear. Using an information indistinguishability argument, the measurement in question has its existence assured by the way which it is defined. Cabello's measurement compares results of measurements on both copies of the system, establishing a relative reference relation between the observed gbits and rendering them indistinguishable. In such case, there ought to be at least two (indistinguishable) bases on which these measurements are compatible, supporting Cabellos's proof.

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