Equity Bargaining with Common Value

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July 20, 2016

Abstract

We study a common-value bilateral bargaining model with equity offer. In particular, we consider a model in which players bargain over an equity share of a common-value stochastic pie (i.i.d. over time) and players receive private signals on the size of the pie each period. Efficient agreement is a stochastic rule: delay is efficient if the expected size of today's pie is small and the discount factor is high. Hence, information aggregation is crucial for efficiency. We derive the conditions under which an equilibrium that attains the efficient agreement exists. The key idea is that the proposer makes an offer in such a way that the responder will use her signal if the responder's signal is crucial for an efficient agreement.

Keywords: asymmetric information bargaining, information aggregation, common value.
JEL Classification Code: C78, D82.

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1 Introduction

In many bargaining situations, players bargain over the equity share of a surplus, especially in a business environment such as merger negotiations, franchise agreements, and partnership formations. If information on the size of the pie is complete, whether the players bargain over the equity share or cash payment does not affect the analysis. However, bargaining with an equity share differs significantly from bargaining with cash payment if the size of the pie changes across periods and information on the size of the pie is incomplete.[1] Both sides of the negotiation have some private signals about the size of the pie at the time of negotiation, which changes over time. Then the offer not only describes how much the players would receive in an agreement but also contains information about the proposer's signal. Consider, for example, two firms bargaining over a potential merger contract with a share payment. Neither firm knows exactly how large the synergies are, but each has private signals about the size of the synergies, which changes over time. Because the realized value of the negotiated equity share depends on the true state, the firms will benefit from aggregating information across players.

In such cases, information aggregation may allow players to collectively make an efficient decision, i.e., to reach an agreement today if and only if the size of today's pie evaluated with the help of the information is larger than the sum of continuation values. However, information aggregation is not straightforward because players' interests may be conflicting. If a player can enjoy better terms of agreement by manipulating information, the player might like to distort the efficient agreement decision for rent. In other words, a player might have an incentive to misrepresent the private signal in order to seek a larger share of the pie at the cost of an inefficient agreement decision.

In this paper, we ask to what degree players can aggregate information in bilateral equity bargaining and whether an efficient outcome can be attained. While information aggregation is studied extensively in the auction and the voting literature, it has attracted less attention in the bargaining literature in spite of its importance. In many actual bargaining situations, players bargain over a pie while having different information about its size. Consider, for example, two venture capitalists (VCs) trying to syndicate a new venture investment that can only be realized if the two VCs agree to fund it. The common-value profitability of the investment is uncertain, and each VC has a private signal about the profitability of the investment.[2] Using the private signals they have, VCs bargain over whether to fund the

[1] In the auction literature, DeMarzo, Kremer, and Skrzypacz (2005) study auctions with contingent payments in which payment can take forms other than cash, including equity. Examples of such auctions include an oil-lease auction, where the government sells a certain percentage of the oil revenue from an oil field if successfully explored. See Skrzypacz (2014) for a survey of the auction literature on contingent payment.

[2] See, e.g., Lerner (1994) for the importance of information aggregation in the syndication of VCs. He

investment—and how to divide the profit—or wait for another investment opportunity in the next period.[3]

To address this issue, we consider a random proposer stochastic bargaining game (Merlo and Wilson, 1995, 1998; Eraslan and Merlo, 2002) into which we introduce a common-value environment, i.e., each player has a private signal about the common-value size of the pie. After Nature chooses a proposer randomly and determines the true size of the pie (either high or low), each player receives a conditionally independent private signal about the size of the pie. Then the proposer makes an equity offer, which is a function of his private signal, to the responder. The responder chooses whether to accept or reject the offer by inferring the size of the pie using her own signal together with the proposer's offer. The game ends if the responder agrees; otherwise, it continues to the next period with a new draw of the identity of the proposer and a new i.i.d. draw of the size of the pie each period.[4,5]

We begin our analysis by characterizing the efficient agreement schedules that maximize the ex ante social value as a mapping from aggregated private signals $\{h,\ell\}\times\{h,\ell\}$ to agreement decisions in a stage $\{Agreement,\ No\ Agreement\}$.[6] This generalizes the efficient agreement schedules in Merlo and Wilson (1995, 1998) to an incomplete-information environment. Efficiency in this context does not imply immediate agreement because of the stochastic nature of the pie. Waiting for a better realization in the next period would be more efficient than agreeing with the expectation of a small pie size today if the discount factor is high enough. Therefore, information aggregation is necessary for efficient delay.

Indeed, there are three possible efficient agreement schedules (Proposition 1). For a high discount factor, the players should agree only if they both receive good signals (High Delay Schedule). For an intermediate discount factor, they should agree unless they both receive bad signals (Medium Delay Schedule). For a low discount factor, they should agree

reports that experienced VCs only syndicate with other VCs with similar experience in the first-round (i.e., new) investment due to the importance of private signals.

[3] Our environment differs from that of Myerson and Satterthwaite (1983) with respect to studying a bilateral trade with *independent* private values: our model can be interpreted as bargaining between a buyer and a seller with (perfectly) *correlated* values. Evans (1989),Vincent (1989), and Deneckere and Liang (2006) study bargaining with correlated values, assuming that the size of the pie is *persistent*, the incomplete information is one-sided, and only the uninformed party makes offers.

[4] The i.i.d. assumption helps us focus on the issue of information aggregation across players while avoiding complexity due to the intertemporal transmission of information.

[5] The model can also be interpreted as bargaining between a randomly matched buyer and seller. Both the buyer and the seller have only a noisy signal about the size of the surplus from the trade and will be rematched with another player if they cannot agree. The size of the pie is i.i.d. for each matching, and the players have different information about the true size of the surplus. They will benefit from information aggregation because they will gain if they trade when the surplus is large (and wait for another random match if the surplus is small).

[6] The i.i.d. assumption implies the irrelevance of history for efficient agreement. We assume transferable utility and thus the terms of agreement for players are also irrelevant for efficiency. Note that the efficient agreement schedule is based on the information profile at the interim stage but not on the unknown, true size of the pie. An actual efficient agreement could end up with a low social value ex post.

immediately regardless of the signals (No Delay Schedule). The degree to which efficient information aggregation requires a high discount factor depends on the precision of the signals: the more accurate the signals, the lower the required level of the discount factor for efficient information aggregation. This is because the more accurate signal lowers the likelihood of agreeing when the pie is small. We then study a stationary perfect Bayesian equilibrium of the bargaining game and ask whether an equilibrium exists that attains each efficient agreement schedule on the equilibrium path.

We present the result that there exists an equilibrium that implements the efficient outcome if the discount factor and precision of signals satisfy certain conditions (Proposition 2, Parts 1–3). We show this proposition for each case corresponding to High, Medium, or No Delay Schedule.

First, we show that there exists an equilibrium that attains High Delay Schedule when it is efficient (Proposition 2, Part 1). The intuition is somewhat similar to that of the swing voters' curse (Feddersen and Pesendorfer, 1996). Under High Delay Schedule, efficiency requires both players to agree only when the signals of both players are good. A proposer with a bad signal can simply make an unacceptable offer, because the responder's signal is not important in such a case. The key is how the responder reacts to the offer by a proposer who has a good signal.

If the proposer with a good signal makes an offer such that the responder will not be signal-responsive (i.e., the responder believes that the proposer has a bad signal, and thus she thinks she has no need to make a decision dependent on her signal), the responder has no incentive to use her signal. To avoid such an event, the proposer wants to make an offer so that the responder is signal-responsive. To do so, the proposer cannot be greedy—a greedy offer will be interpreted as a sign that the proposer's signal was bad. This results in disagreement despite the fact that they both have good signals (and the expected size of the pie is large). Accordingly, the proposer demands less of a share to avoid the proposer's curse that the players cannot agree when both signals are good due to the responder's not making use of her signal. This is similar to avoiding the swing voters' curse in the sense that the proposer transfers final decision power in order to make the responder signal-responsive (by making a relevant and "fair"[7] offer). Thus, in this equilibrium, the proposer has to give up some share of the pie so the responder will believe that the proposer's signal is good. Note that the proposer with a bad signal also has no reason to imitate the good type, because doing so would make the players agree with a bad signal that yields a lower payoff than the continuation.

[7] If the share upon agreement is expected to be around half, both parties have incentive to act according to the efficient agreement schedule; note first that their continuation payoff is equal due to the random proposer setup. Thus by making the share around half, their payoffs upon agreement increase proportionally to that of the social surplus. Therefore efficiency and individual interests align for a "fair" share.

Next, we study the attainability of an efficient equilibrium under Medium Delay Schedule (Proposition 2, Part 2). In this case, efficiency requires at least one of the signals to be good, thus a proposer with both bad and good signals can reach an efficient agreement. The difference from the case of the High Delay Schedule equilibrium is that it is now the proposer with a bad signal who needs to make the responder signal-responsive, while a proposer with a good signal has no need to make the responder signal-responsive. The difficulty of sustaining an efficient equilibrium is due to the fact that the offer that makes the responder signal-responsive must be fairly greedy, to the level that the responder is indifferent between making use of her signal or not. The proposer with a bad signal requires an "unfair" share, much larger than the proposer in the case of High Delay Schedule. This makes various incentive constraints binding, depending on the discount factor and the accuracy of the signals. For example, the discount factor cannot be too low, because a low discount factor gives an incentive for the proposer with a bad signal to lie about his signal and agree today (by acting as if he had a good signal) even if both signals are low.

When No Delay Schedule is efficient, agreement should occur immediately regardless of the signals, and therefore, information aggregation is unnecessary for efficiency. We show that an equilibrium that attains the schedule fails to exist as the accuracy of information improves (Proposition 2, Part 3). As the accuracy improves, the proposer with a good signal considers it more likely that the responder received the same signal as the proposer. The proposer would then find it optimal to demand a higher share that is only acceptable to the responder with a good signal, contradicting the pooling equilibrium that is necessary for No Delay Schedule.

The remainder of the paper proceeds as follows. The literature review is provided in the next subsection. Section 2 presents the model and the characterization of efficient agreement schedules. In Section 3, we study efficient equilibria. In addition to the analysis for fully efficient equilibria, we discuss approximately efficient ones when the information is almost complete. Section 4 offers concluding remarks.

1.1 Related Literature

The paper is related to three strands of literature. The closest is the literature on bargaining with contingent payment. To the best of our knowledge, Eraslan et al. (2014) is the first study that considers a common value bargaining model with contingent payment. They consider bargaining between a lender and a shareholder in the context of bankruptcy reorganization, where efficiency in operational restructuring requires the aggregation of each player's private information and proposals in negotiation take the form of contingent payments (financial restructuring). They highlight the importance of reorganization procedure to induce efficient operational restructuring; in particular, efficiency arises in an equilibrium if players negotiate financial restructuring prior to operational restructuring or if they negotiate operational and financial restructuring simultaneously. While their paper allows for a wider class of contingent payment considering bankruptcy reorganization, we focus on the share offer, given that writing a certain class of contingent contract can be costly in many situations.[8] Another study addressing contingent payment is a companion paper, Hanazono and Watanabe (2014), in which we consider a one-period bargaining model with common value. While the continuation value is endogenous in the current paper, in Hanazono and Watanabe (2014) and in Bond and Eraslan (2010), the players receive an exogenous status quo outside option value if they do not agree. This simplification allows Hanazono and Watanabe (2014) to investigate issues such as the asymmetric precision of private signals, while the dynamic setup of the current paper allows it to address the issue of dynamic patterns of an (in)efficient agreement, such as the expected time to agreement.

The second strand is the literature on information aggregation in committees. Bond and Eraslan (2010) consider bargaining between a proposer and a set of voters who collectively decide whether to accept the offer in a common-value environment. Their bargaining model is a static game, and status quo prevails in the case of no agreement. Their focus is on information aggregation among voters rather than information aggregation across voters and proposers. Additionally, we consider different forms of offers in an environment where the value of disagreement is endogenously determined by the continuation value of the game. Damiano et al. (2010) consider information aggregation in a collective decision-making problem between two players where players may disagree (i) because they have different information and/or (ii) because they have a conflicting interest. While their paper focuses on the issue of information aggregation by simplifying the space of available decisions, our paper incorporates the information aggregation problem into a bargaining environment in which players endogenously choose terms of agreement.

The third strand is the literature on stochastic bargaining. Merlo and Wilson (1995, 1998) consider a complete information bargaining game in which the size of the pie is stochastic. Players know the size of the pie at the beginning of each period. The authors show that an efficient delay occurs in such a model when the realization of the pie is small. Our model adds two-sided asymmetric information in stochastic bargaining by introducing private signals about the stochastic size of the pie. Our equilibrium converges to their equilibrium as the accuracy of the signal converges to one.

Our paper is also related to the literature on bargaining with incomplete information. A bargaining model with incomplete information has been studied extensively.[9] In this

[8] For example, debt contracts need to specify maturity, which may not be easy to set, since it is uncertain when the return will materialize. In general, writing contingent payments involves transaction costs to find and agree to these sorts of contrat terms. Equity contracts, however, are immune to maturity.

[9] See, e.g., the survey by Kennan and Wilson (1982) and Ausubel et al. (2002).

literature, bargaining with one-sided private information and interdependent valuations is analyzed in a series of papers.[10] The uninformed party makes an offer to the informed party in these studies. By taking a mechanism design approach to the sequential bargaining model, Ausubel and Deneckere (1989) show that every individually rational and incentive-compatible bargaining mechanism is implementable if the uninformed player makes all of the offers and that the set of implementable equilibria shrinks as the frequency of offers by the informed player increases. There is also a series of papers in which the informed party makes an offer: Reinganum and Wilde (1986), Schweinzer (2010a), and Brooks et al. (2010) consider signaling in a common-value bargaining environment. Two papers close to ours are Schweizer (1989) and Schweinzer (2010b), which are the only studies with two-sided asymmetric information with common-value bargaining.[11] In Schweizer (1989) each of the two parties has one perfect signal about one of the two-dimensional states in a one-period pretrial bargaining model. Thus, players have perfect information on one dimension but not on the other dimension. He shows that the outcome cannot be efficient in the refined equilibrium. Schweinzer (2010b) considers an alternating-offer sequential bargaining game in which the proposal cannot decrease over time and the offer space is discrete. Our paper differs from others in this group of literature because it considers the issue of information aggregation.

2 Model

Primitives We consider an infinite horizon two-person random-proposer bargaining model. Denoting the time by $t \in \{0, 1, ..., \infty\}$, the value of the pie to be divided in time t is a random variable x_t, which is an i.i.d. draw in each period t. Specifically, x_t takes either value L or H, $0 < L < H$, with probability $1/2$ for each realization. Each party, $i \in \{1, 2\}$, receives a conditionally independent, private signal $s_{it} \in \{h, \ell\} = S$, with probability $\Pr(s_{it} = h | x_t = H) = \Pr(s_{it} = \ell | x_t = L) = q > 1/2$.

When the offer is accepted, the value of the pie materializes. The payoff is the share of the realized value of the pie. Players discount future payoffs with the common discount factor $\delta \in [0, 1)$.

[10] See, e.g., Evans (1989), Vincent (1989), Spier (1992), Deneckere and Liang (2006), and Fuchs and Skrzypacz (2010). These papers study bargaining with correlated values, assuming that the size of the pie is *persistent*, the incomplete information is one-sided, and the uninformed party makes offers so that the game is a screening game. Our paper differs from these papers in that we allow two-sided asymmetric information, and signaling is considered. On the other hand, the size of the pie is i.i.d. over time in this paper, and dynamic screening is not considered.

[11] There are also papers that study two-sided asymmetric information with private values, such as Fudenberg and Tirole (1983), Cramton (1984, 1992), and Chatterjee and Samuelson (1987), where information aggregation is not the focus of the study.

Timing The bargaining proceeds as follows: at the beginning of a stage in time t, Nature assigns one party as the proposer, with probability $1/2$, and determines the value of the pie, x_t. Each party then receives a private signal, s_{it} for $i = 1, 2$ conditional on x_t. The proposer makes a share offer $(\alpha_t, 1-\alpha_t)$, $\alpha_t \in [0,1]$, and if the responder accepts it, the bargaining is terminated with proposer receiving α_t of the pie, and the responder receiving $1-\alpha_t$ of the pie. Otherwise the bargaining proceeds to the next stage.

Symmetric Pure Strategy Equilibrium in Stationary Strategies In our analysis we focus on a symmetric pure strategy equilibrium in stationary strategies $((\alpha(h), \alpha(\ell)), \sigma : [0,1] \times \{h, \ell\} \to \{0,1\})$ and a (stationary) belief system $\beta(\alpha, s_r)$ where $\alpha(s_p) \in [0,1]$ is the share the player offers to keep to himself when he is the proposer and his signal is s_p, $\sigma(\alpha, s_r) = 1$ if the responder accepts offer α when her signal is s_r, and $\sigma(\alpha, s_r) = 0$ if the responder rejects offer α when her signal is s_r, and $\beta(\alpha, s_r)$ is the probability that the responder attaches to the proposer having h signal when the proposer offers α and the responder's own signal is s_r, such that (note the recursive structure and the one shot deviation principle)

1. given $\sigma(\cdot)$ and the continuation payoff V, offer $\alpha(s_p)$ maximizes proposer's expected payoff when his signal is s_p, i.e.,

$$\alpha(s_p) \in \arg\max_{\alpha} E[\sigma(\alpha, s_r)\alpha x + \delta(1-\sigma(\alpha, s_r))V|s_p],$$

   where V is recursively determined by

$$\begin{aligned} V &= \frac{1}{2}E[\sigma(\alpha(s_p), s_r)x + \delta(1-\sigma(\alpha(s_p), s_r))2V] \\ &= \frac{E[\sigma(\alpha(s_p), s_r)x]}{2[1-\delta(1-E[\sigma(\alpha(s_p), s_r)])]}; \end{aligned}$$

2. given offer α, signal s_r, belief β, and the continuation payoff V, $\sigma(\alpha, s_r)$ maximizes responder's expected payoff,

$$\sigma(\alpha, s_r) = 1 \text{ iff } (1-\alpha)[E[x|s_p = h, s_r]\beta(\alpha, s_r) + E[x|s_p = \ell, s_r](1-\beta(\alpha, s_r))] \geq \delta V;$$

3. the belief system is consistent with strategy $(\alpha(h), \alpha(\ell))$ and the prior distribution, i.e., if $\alpha(h) \neq \alpha(\ell)$,

$$\beta(\alpha, s_r) \begin{cases} = 1 & \text{if } \alpha = \alpha(h) \\ = 0 & \text{if } \alpha = \alpha(\ell) \\ \in [0,1] & \text{otherwise} \end{cases}$$

and if $\alpha(h) = \alpha(\ell) = \alpha^{pool}$

$$\beta(\alpha, s_r) \begin{cases} = \Pr(s_p = h | s_r = h) = q^2 + (1-q)^2 & \text{if } \alpha = \alpha^{pool}, s_r = h \\ = \Pr(s_p = h | s_r = \ell) = 2q(1-q) & \text{if } \alpha = \alpha^{pool}, s_r = \ell \\ \in [0, 1] & \text{otherwise} \end{cases}$$

Note that the beliefs about past signals are irrelevant and thus not included in the above definition since the pie size is i.i.d. Also, we suppress the time index t and player index i for the rest of the paper for all variables as we focus on the stationary symmetric strategy profile.

2.1 Efficient Agreement Schedule

We first characterize the efficient agreement schedule with respect to signal realization before moving on to analyzing the bargaining game. Because of the stochastic nature of the size of the pie, forgoing agreement may be preferred to immediate agreement if signal realizations are not good. Thus, the efficient agreement schedule depends on the discount factor, the signal accuracy, and the relative size of L and H. Our characterization is a natural extension of the idea of efficient delay in stochastic bargaining by Merlo and Wilson (1995, 1998) to signal realization. Denoting the size of the pie by x, we first define the efficient agreement schedule.

Definition 1 *An efficient agreement schedule is a mapping from the signal space to agreement or no agreement, i.e.* $f : S \times S \to \{Agreement, No\ Agreement\}$*, such that* $f(s_1, s_2) = \{Agreement\}$ *iff* $E[x|(s_1, s_2)] \geq 2\delta W(\delta, q, L, H)$*, where* $W(\delta, q, L, H)$ *is recursively defined by*

$$W(\delta, q, L, H) = \frac{1}{2}[\Pr(Agreement|f)E[x|Agreement, f] + \Pr(No\ Agreement|f)2\delta W(\delta, q, L, H)].$$

An efficient agreement schedule means that players agree if and only if the expected size of the pie given the signals is larger than the continuation value. The continuation value is multiplied by two because $W(\delta, q, L, H)$ is the continuation value for each player. Note that we do not need an index for each player or period because we assume that the recognition probability is equal across players and that the size of the pie is i.i.d. across periods. To characterize an efficient agreement schedule, consider the following three cases.

High Delay Schedule: agreement only for $(s_1, s_2) = (h, h)$. This case corresponds to the situation where waiting is not costly (δ is sufficiently high), and thus the players should reach an agreement only if both signals are h. In this case, the continuation value at the beginning of each period, $W(\delta, q, L, H)$, is written as

$$W(\delta, q, L, H) = \frac{1}{2}(\Pr(h,h)E[x|h,h]) + \delta W(\delta, q, L, H)(1 - \Pr(h,h))$$
$$\Leftrightarrow W(\delta, q, L, H) = \frac{\Pr(h,h)E[x|h,h]}{2(1-(1-\Pr(h,h))\delta)}.$$

If the two signals are different (one is h and the other is ℓ), the expected size of the pie is $E[x] = E[x|h,\ell]$. Hence, the condition that the discount factor should be sufficiently high can be obtained by comparing the expected size of the pie given two different signals, $E[x]$, and the continuation value where an agreement is reached only if the signal is (h,h), i.e.,

$$E[x] \leq 2\delta W(\delta, q, L, H) = \frac{\delta \Pr(h,h)E[x|h,h]}{1-(1-\Pr(h,h))\delta}$$
$$\Leftrightarrow \delta \geq \frac{E[x]}{\Pr(h,h)E[x|h,h] + (1-\Pr(h,h))E[x]} \equiv \bar{\delta}(q, L/H).$$

Medium Delay Schedule: agreement for $(s_1, s_2) = (h,h), (h,\ell), (\ell,h)$. This case corresponds to the situation where waiting is relatively costly (δ is not too high) and an agreement is reached if at least one signal is h. In this case, the continuation value is written as

$$W(\delta, q, L, H) = \frac{1}{2}(E[x] - \Pr(\ell,\ell)E[x|\ell,\ell]) + \delta W(\delta, q, L, H)\Pr(\ell,\ell)$$
$$\Leftrightarrow W(\delta, q, L, H) = \frac{E[x] - \Pr(\ell,\ell)E[x|\ell,\ell]}{2(1-\Pr(\ell,\ell)\delta)}.$$

Note that efficiency implies that the expected size of the pie given signals (ℓ,ℓ) should be lower than the continuation value of forgoing agreement, i.e.,

$$E[x|\ell,\ell] \leq 2\delta W(\delta, q, L, H) = \frac{\delta(E[x] - \Pr(\ell,\ell)E[x|\ell,\ell])}{1-\Pr(\ell,\ell)\delta}$$
$$\Leftrightarrow \delta \geq \frac{E[x|\ell,\ell]}{E[x]} \equiv \underline{\delta}(q, L/H).$$

Combining this with the High Delay Case, we obtain the result that an agreement in the case of $(s_1, s_2) = (h,h), (h,\ell), (\ell,h)$ is efficient if

$$\underline{\delta}(q, L/H) \leq \delta < \bar{\delta}(q, L/H).$$

No Delay Schedule: agreement for any profile. If the discount factor is very low, immediate agreement is always efficient. The associated continuation value, W, is $E[x]/2$. The threshold discount factor is obtained by the condition in Medium Delay Case with an opposite sign, since $E[x|\ell,\ell] \geq 2\delta W(\delta,q,L,H) \Leftrightarrow \delta \leq \underline{\delta}(q,L/H)$. This proves the following proposition.

Proposition 1 *The efficient schedule is High Delay Schedule if $\delta \in [\overline{\delta}(q,L/H),1)$, Medium Delay Schedule if $\delta \in [\underline{\delta}(q,L/H),\overline{\delta}(q,L/H)]$, and No Delay Schedule if $\delta \in [0,\underline{\delta}(q,L/H)]$. The efficient schedule is unique except for $\delta = \overline{\delta}(q,L/H)$ and $\delta = \underline{\delta}(q,L/H)$.*

Note that thresholds $\overline{\delta}(q,L/H)$ and $\underline{\delta}(q,L/H)$ depend on q and L/H. For instance, if q is approaching 1 and/or L/H is approaching zero, both $\overline{\delta}(q,L/H)$ and $\underline{\delta}(q,L/H)$ take lower values as illustrated in Figure 1. In Figure 1 we plot the thresholds $\overline{\delta}(q,L/H)$ (solid) and $\underline{\delta}(q,L/H)$ (dashed) for the case where $L/H = 0.6$ and $L/H = 0.1$.

An important point here is the extent of information aggregation required to achieve an efficient agreement schedule. For a low δ (that is $\delta < \underline{\delta}(q,L/H)$), we do not need information aggregation across the players in order to achieve efficiency, because immediate agreement is always preferred. For the first two cases (a high and middle δ), information aggregation is necessary to achieve efficiency. The question is whether and how the player can aggregate information in equilibrium, which we discuss in Section 3.

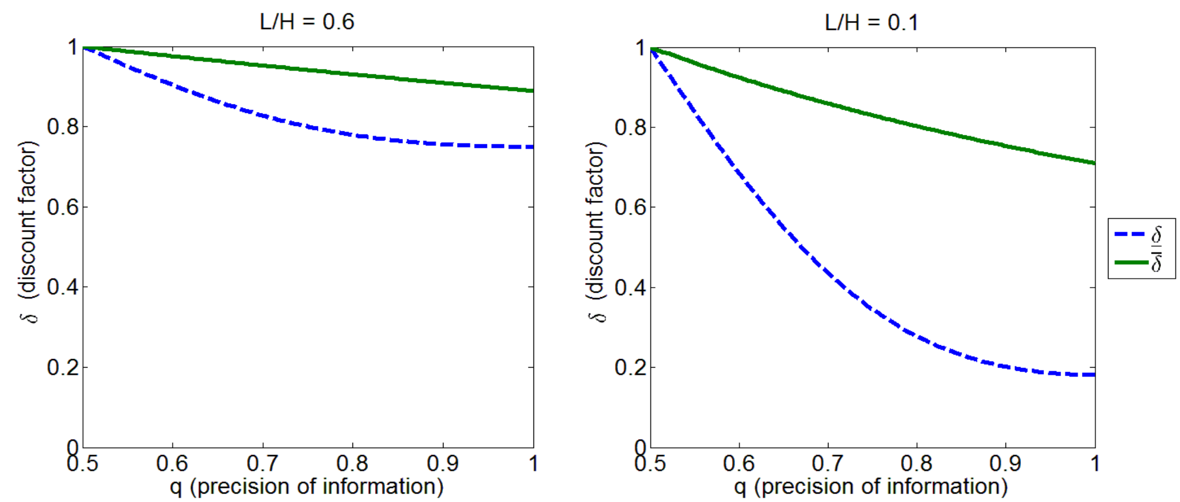

Figure 1: Efficient Agreement Schedules. The horizontal axis corresponds to the precision of information, q, and the vertical axis discount factor, δ. The area above $\overline{\delta}$ corresponds to High Delay Schedule, the area between $\overline{\delta}$ and $\underline{\delta}$ to Medium Delay Schedule, and the area below $\underline{\delta}$ to No Delay Schedule.

2.2 Efficiency via Commitment

Before we consider the attainability of efficient equilibria, we first consider a benchmark case in which players can write a binding contract prior to signal realization. We discuss this case to illustrate how players' incentive can be aligned if commitment is possible, though our main interest lies in the case where such commitment is neither feasible nor possible for various reasons.

To illustrate this point, suppose that before the players receive signals, they sign the following contract: (a) each player reports $t_i \in \{h, \ell\}$ after observing the private signal $s_i \in \{h, \ell\}$; (b) agreement takes place according to an efficient agreement schedule based on the reported signal (t_1, t_2); and (c) upon agreement, the players share the pie, each receiving a half. Given that the sharing rule is set to be half, players' preferences are aligned so that they want the agreement to take effect when the expected size of the pie is larger than the continuation value. Hence, the players find it optimal to sincerely report their private signals (that is, $t_i = s_i$) and an agreement takes effect if and only if it is efficient.[12]

The main focus of the paper, however, is the case in which such a commitment is not possible. Though players may gain from such a commitment, various frictions hinder players from committing to such a prior mechanism in an actual bargaining situation. First, the contract agreement is vulnerable to renegotiation after the players receive new information. Second, if we interpret our model as bargaining between two randomly matched players (see footnote 5), the commitment on how to share the pie is naturally very costly because the players do not know whom they will bargaining with in advance. In the example of the VC syndication in the Introduction, this commitment means that both VCs commit to a particular equity share of the future investments without knowing the details of the future investment opportunities. In reality, committing to a share before knowing important information about a potential investment opportunity is difficult because both VCs may have an opportunity to modify the terms of the agreement (including the share) when the details of the investment opportunity are revealed. Thus, we explore the model without the possibility of commitment below.

3 Efficient Equilibria

In this section, we study whether efficiency can be achieved as an equilibrium outcome. We construct an equilibrium for each efficient agreement schedule and find the conditions under which such a fully efficient equilibrium exists. We then discuss comparative statics of the equilibria we identify, focusing on the time to agreement and the proposer's offer. We also explore the possibility of an almost (or approximately) efficient equilibrium when a fully efficient equilibrium does not exist.

[12] The argument is similar to that of Proposition 4 in Eraslan et al. (2014) in which efficient equilibrium is obtained when players in a common value bankruptcy bargaining environment decide how to split the surplus first, then deicide what to do with the firm.

3.1 Full Efficiency

Let us first state our main result.

Proposition 2 *An efficient equilibrium exists if and only if parameter values* (δ, q, L, H) *satisfy either*

1. $\delta \in [\overline{\delta}(q, L/H), 1)$,

2. $\delta \in [\underline{\delta}(q, L/H), \overline{\delta}(q, L/H))$ *and* $\delta W(\delta, q, L, H) \geq A(q, L, H)$ *(Condition A), where*

$$A(q, L, H) = \left(\frac{q}{2q-1} \frac{1}{L} + \frac{q-1}{2q-1} \frac{1}{H} + \frac{1}{E[x]} \right)^{-1},$$

or,

3. $\delta \in [0, \underline{\delta}(q, L/H))$ *and* $\delta W(\delta, q, L, H) \leq B(q, L, H)$ *(Condition B), where*

$$B(q, L, H) = \frac{H+L}{2} \left(2 + \frac{2q-1}{2q(1-q)} \frac{H-L}{H+L} \frac{qH + (1-q)L}{qL + (1-q)H} \right)^{-1}.$$

This proposition describes the attainability of an efficient equilibrium. If High Delay is efficient, an efficient equilibrium always exists. If Medium Delay or No Delay is efficient, an additional condition (Condition A or B) is required for the existence of an efficient equilibrium. Figure 2 combines all the conditions for $L/H = 0.6$: an efficient equilibrium exists except in the shaded region of q and δ. It is interesting to see that for a high precision ($q \geq .78$), an efficient equilibrium exists if and only if the discount factor is outside the intermediate range.

Before verifying this proposition starting from the next subsection, we provide some intuitive arguments behind the results. For δ for which High or Medium Delay Schedules is efficient, information aggregation is necessary. To accomplish information aggregation, we need that (i) the proposer makes separating offers, and (ii) the responder is signal-responsive when she receives one of the offers (meaning that her acceptance decision when she has a good signal determines the outcome). It is easy to see that if High Delay Schedule (Medium Delay Schedule) is efficient, the responder is signal-responsive when the proposer's signal is good (bad). Accordingly, the on-path offers must satisfy the incentive condition for the proposer's separating offers and also make the responder signal-responsive.

In addition to satisfying the incentive to make separating offers and making the responder signal-responsive, we also need the proposer to have no incentive to make off-path offers, such as greedy ones. To deter the proposer from making such an offer, the responder should turn down an off-path offer if she receives it; i.e., she becomes non-signal-responsive upon

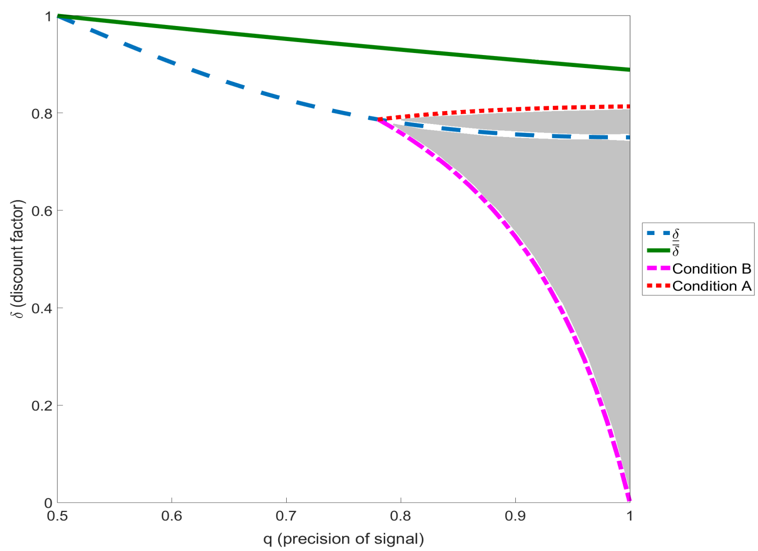

Figure 2: The figure depicts the parameter regions where an efficient equilibrium exists for the case of L/H=0.6. An efficient equilibrium corresponding to High Delay Schedule exists in the area above the solid curve $\bar{\delta}(q, H, L)$. An efficient equilibrium corresponding to Medium Delay Schedule exists in the area between the solid and dashed curves (between $\underline{\delta}(q, L, H)$ and $\bar{\delta}(q, L, H)$) that is not below the dotted curve (below Condition A). An efficient equilibrium corresponding to No Delay Schedule exists in the area below the dashed curve (below $\underline{\delta}(q, L, H)$) and above the dash-dotted curve (above Condition B).

receiving such an offer. We can show that this condition can be satisfied for an efficient equilibrium corresponding to High Delay Schedule, while it can only be satisfied in the region above the dotted line (Condition A in the figure) for an efficient equilibrium corresponding to Medium Delay Schedule.

For the parameter region where No Delay Schedule is efficient, the argument for the attainability of an efficient equilibrium is different, because no information aggregation is needed for efficiency. In fact, the responder should not be signal-responsive for efficient agreement. However, the proposer might have an incentive to make a greedier-than-on-path offer, which can only be avoided if Condition B is satisfied.

3.1.1 High Patience: $\delta \in [\overline{\delta}(q, L/H), 1)$

We start our analysis with an equilibrium in which an efficient High Delay Schedule arises. As we consider the case in which High Delay Schedule is efficient, we have $E[x] \leq 2\delta W(\delta, q, L, H)$ and

$$W(\delta, q, L, H) = \frac{\Pr(h, h)E[x|h, h]}{2(1 - (1 - \Pr(h, h))\delta)}$$

as discussed in Section 2.1 (for notational simplicity, we abuse notation W for $W(\delta, q, L, H)$).

We take a constructive approach and show that the following strategy profile and belief compose an efficient equilibrium:

The proposer's strategy is a separating one,

$$\alpha(h) = 1/2, \ \alpha(\ell) = 1,$$

while the responder's strategy is

$$\begin{array}{ll} \text{for } \alpha > 1/2, & \sigma(\alpha, s_r) = \text{reject for any } s_r = h, \ell, \\ \text{for } \alpha \in (\alpha', 1/2], & \sigma(\alpha, s_r) = \begin{cases} \text{accept} & \text{if } s_r = h, \\ \text{reject} & \text{if } s_r = \ell, \end{cases} \\ \text{for } \alpha \leq \alpha', & \sigma(\alpha, s_r) = \text{accept for any } s_r = h, \ell, \end{array}$$

where α' is defined by $(1 - \alpha')E[x] = \delta W$. The associated belief is a "pessimistic" one as[13]

$$\beta(\alpha, s_r) = \begin{cases} 1 & \text{if } \alpha \leq \alpha(h), \\ 0 & \text{otherwise.} \end{cases}$$

[13] In Appendix C, we discuss plausibility of pessimistic belief system for the efficient equilibria. In fact, whenever an equilibrium exists that delivers High or Medium Delay, there also exists an equilibrium delivering the agreement schedule that satisfies the D1 criterion of Banks and Sobel (1987).

That is, the responder believes that the proposer has h signal if and only if proposer's offer is less than or equal to $\alpha(h)$. Thus, this belief is indeed pessimistic for a greedier offer than $\alpha(h)$.

Proposition 2 (Part 1. Restatement) *Suppose $\delta \in [\overline{\delta}(q, L/H), 1)$ (High Delay Schedule is efficient). Then the above strategy profile with the pessimistic belief composes an efficient equilibrium.*

Proof. Responder's Behavior: We first establish the optimality of the responder's strategy given a proposer's offer and the pessimistic belief. It is easy to see that $\sigma(1, s_r)$ =reject. Next, consider $\alpha = 1/2$. By accepting this offer, the responder receives $(1/2)E[x|h, s_r]$, and thus acceptance is optimal if and only if $(1/2)E[x|h, s_r] \geq \delta W \Leftrightarrow E[x|h, s_r] \geq 2\delta W$. Now recall that we are considering the case that the High Delay Schedule is efficient; hence, $E[x|h, h] \geq 2\delta W \geq E[x]$ holds. This directly implies that $\sigma(1/2, h)$ =accept and $\sigma(1/2, \ell)$ =reject is optimal.

To check the optimality of the responder's response for $\alpha \neq \alpha(h), \alpha(\ell)$, note that for $\alpha \leq 1/2$ the responder believes the proposer has h signal for sure. Threshold α' is defined such that the responder with ℓ is indifferent between accepting and rejecting. Then it is straightforward to show that the above reactions for $\alpha \leq 1/2$ are optimal. For $\alpha > 1/2$, even the responder with h signal rejects because acceptance gives her only $(1-\alpha)E[x|\ell, h] < (1/2)E[x] \leq \delta W$.

Proposer's Behavior: Indeed, it suffices to compare the proposer's payoffs when he offers $\alpha = \alpha', 1/2$, and 1, and show that the on-path offer is the best. We do not need to check other offers, because (i) all the cases of $\alpha \in (1/2, 1]$ are covered by checking $\alpha = 1$ due to the fact that offering $\alpha > 1/2$ always induces rejection, (ii) all the cases of $\alpha \in (\alpha', 1/2)$ are dominated by $\alpha = 1/2$ from the proposer's perspective as the responder's reaction does not change, and (iii) all the cases of $\alpha \in [0, \alpha']$ are dominated by α' from the proposer's point of view as the responder's reaction remains unchanged.

Regardless of proposer signals, offer α' is dominated by $\alpha = 1/2$. If $s_r = h$, both offers are accepted, but clearly the proposer receives higher share for offer $\alpha = 1/2$ than α'. If $s_r = \ell$, the responder accepts offer α' while rejects $\alpha = 1/2$. Note that

$$\alpha' E[x|s_p, s_r = \ell] < \frac{1}{2}E[x|s_p, s_r = \ell] \leq \delta W,$$

where the last inequality is from $\delta \in [\overline{\delta}(q, L/H), 1)$. Thus

$$\begin{aligned}
\alpha' E[x|s_p] &= \Pr(h|s_p)\alpha' E[x|s_p, s_r = h] + \Pr(\ell|s_p)\alpha' E[x|s_p, s_r = \ell] \\
&< \Pr(h|s_p)\alpha E[x|s_p, s_r = h] + \delta W,
\end{aligned}$$

which shows that offer α' is dominated by $\alpha = 1/2$.

Finally, we show that the proposer with a h signal (ℓ, respectively) is better off by offering $\alpha = 1/2$ ($\alpha = 1$, respectively) rather than $\alpha = 1$ ($\alpha = 1/2$, respectively). The difference between these offers only arises in the responder's reaction when she has h signal. Recall $E[x|h,h] \geq 2\delta W \geq E[x]$ when the High Delay Schedule is efficient. Hence

$$\frac{1}{2}E[x|h,h] \geq \delta W \geq \frac{1}{2}E[x|\ell,h].$$

The first inequality shows that the proposer with a h signal is better off by offering $\alpha = 1/2$ than $\alpha = 1$, and the second one verifies that the proposer with a ℓ signal is better off by offering $\alpha = 1$ rather than $\alpha = 1/2$. ■

Recall that if the players can commit a share upon agreement in the benchmark case discussed in Section 2.2, they voluntarily aggregate information to reach an efficient agreement. The proof of Proposition 2 above shows that when High Delay Schedule is efficient, the same outcome can be obtained in the equilibrium where the proposer's equilibrium offer is $\alpha(h) = 1/2$.

The proposer does not have any first-mover advantage in this equilibrium.[14] The intuition is somewhat similar to the benchmark case with the commitment. The key is that the proposer with h signal cannot claim more than $1/2$.[15] This is because doing so makes the responder believe that the proposer has ℓ signal (and reject the offer). This belief makes the responder reject the offer since she does not want to agree when she believes the size of the pie is not large. In other words, the proposer's offer claiming less than half makes the responder believe that the proposer has h signal, thus the responder thinks she is signal-responsive and give her the incentive to make the best use of her signal. Moreover, this belief is consistent with the incentive of the proposer with ℓ signal: the proposer with ℓ signal is not willing to agree if he cannot claim much (as the expected size of the pie is not large given ℓ signal).

3.1.2 Medium Patience: $\delta \in [\underline{\delta}(q, L/H), \overline{\delta}(q, L/H))$

In Medium Delay Schedule, players reach an agreement unless both signals are ℓ (at least one of the signals is h). Recall that this schedule is efficient if $E[x|\ell,\ell] \leq 2\delta W(\delta, q, L, H) < E[x]$, where

$$W(\delta, q, L, H) = \frac{E[x] - \Pr(\ell,\ell)E[x|\ell,\ell]}{2(1 - \Pr(\ell,\ell)\delta)}.$$

[14] In Rubinstein bargaining, the first-mover advantage arises since the proposer can make an offer to which the responder is indifferent between accepting and rejecting.

[15] Offer $\alpha(h)$ need not be exactly $1/2$. To be more precise, $\alpha(h) \in [\mu, 1-\mu]$, where $\mu = \max\left\{1 - \frac{\delta W(\delta,q,L,H)}{E[x]}, \frac{\delta W(\delta,q,L,H)}{E[x|h,h]}\right\} < \frac{1}{2}$.

We search for an equilibrium that implements Medium Delay Schedule (again, we use W for $W(\delta, q, L, H)$ for simplicity).

The fully efficient (stationary) equilibrium that implements Medium Delay Schedule, if it exists, needs to have the following features:

- The proposer's offer strategy is separating, i.e., $\alpha(h) \neq \alpha(\ell)$
- The responder's acceptance/rejection decision for the shares given the proposer's strategy (i.e., $\alpha(h), \alpha(\ell)$) is

$$\sigma(\alpha(h), s_r) = \text{accept for } s_r = h, \ell,$$

$$\sigma(\alpha(\ell), s_r) = \begin{cases} \text{accept} & \text{for } s_r = h, \\ \text{reject} & \text{for } s_r = \ell. \end{cases}$$

In the following, we focus on the attainability of efficient equilibria when $\alpha(h) < \alpha(\ell)$. The reason we focus on the case of $\alpha(h) < \alpha(\ell)$ is that if there exists an efficient equilibrium with $\alpha(h) > \alpha(\ell)$, we can always find an efficient equilibrium with $\alpha(h) < \alpha(\ell)$. In other words, the set of parameter values that supports an efficient equilibrium with $\alpha(h) < \alpha(\ell)$ includes the set that supports an efficient equilibrium with $\alpha(\ell) < \alpha(h)$ (as shown in Lemma 4). Intuitively, for an efficient equilibrium, signaling with $\alpha(h) < \alpha(\ell)$ is more natural and effective because the proposer with h signal is more willing to claim a lower share than the proposer with ℓ signal (so that it is easier for all the responder to accept $\alpha(h)$), and the responder with ℓ signal also has less incentive to accept when the offer is $\alpha(\ell)$.

As in the case of High Delay Schedule, we consider the pessimistic belief system.

$$\beta(\alpha, s_r) = \begin{cases} 1 & \alpha \leq \alpha(h), \\ 0 & \alpha > \alpha(h). \end{cases}$$

This belief system is also natural and intuitively consistent for efficient equilibria with $\alpha(h) < \alpha(\ell)$. Indeed, we can show that adopting alternative belief systems cannot improve attainability of the efficient equilibria.[16]

We now investigate the condition under which we can find $\alpha(h)$ and $\alpha(\ell)$ that compose an efficient equilibrium with the pessimistic belief. First, we show that we can pin down $\alpha(\ell)$ in such an equilibrium. Define α^* by $(1-\alpha^*)E[x] = \delta W$,[17] which makes the responder indifferent between accepting and rejecting when she expects the size of the pie to be $E[x]$.

[16] See footnote 20.

[17] This α^* is in $[0, 1]$. First, $\alpha^* = 1 - \delta W/E[x] \geq 1/2$ because $E[x] \geq 2\delta W$ by the efficiency requirement for the case of Medium Delay Schedule. We also have $\alpha^* < 1$, because $W > 0$ and $E[x] > 0$.

Lemma 1 *Suppose $\delta \in [\underline{\delta}(q, L/H), \overline{\delta}(q, L/H))$. If there is an efficient equilibrium such that $\alpha(h) < \alpha(\ell)$ with the pessimistic belief, then $\alpha(\ell) = \alpha^*$.*

Proof. In an efficient equilibrium, the responder should accept $\alpha(\ell)$ if and only if her signal is h. This behavior is optimal for the responder if $(1 - \alpha(\ell))E[x] \geq \delta W > (1 - \alpha(\ell))E[x|\ell, \ell]$. Suppose that $(1 - \alpha(\ell))E[x] \neq \delta W$, which implies $(1 - \alpha(\ell))E[x] > \delta W$. Then consider an alternative offer $\tilde{\alpha} > \alpha(\ell)$ such that $(1 - \tilde{\alpha})E[x] > \delta W > (1 - \tilde{\alpha})E[x|\ell, \ell]$. Under the pessimistic belief, the responder who receives $\tilde{\alpha}$ believes that the proposer has ℓ signal with probability 1. This implies that the responder's optimal reaction is the same as when $\alpha(\ell)$ is offered. Then the proposer with ℓ signal would have an incentive to deviate to offering $\tilde{\alpha}$, a contradiction. We thus have $(1 - \alpha(\ell))E[x] = \delta W$, implying $\alpha(\ell) = \alpha^*$. ■

Importantly, note that for an efficient equilibrium that implements Medium Delay Schedule, the proposer retrieves his first-mover advantage. Under Medium Delay Schedule, information aggregation requires that the proposer with ℓ signal must make the responder signal-responsive. Unlike the previous discussion for the case of High Delay Schedule where the equilibrium offer is $\alpha(h) = 1/2$, the responder cannot reject a demanding offer for two reasons. First, the responder cannot reject a demanding offer unless $\alpha > \alpha^*$. In other words, she cannot "punish" a demanding proposer by rejecting an offer if $\alpha \leq \alpha^*$. Second, because the future prospect after rejection is not as good as in the previous case (medium δ), the responder has incentive to accept a demanding offer. However, due to this advantageous bargaining position of the proposer, the attainability of an efficient equilibrium may be limited in this case as we discuss below.

Given the result of the Lemma 1, we now consider an equilibrium with the following features:

- The proposer's offer strategy is $\alpha(h) < \alpha(\ell) = \alpha^*$.
- The responder's strategy is

$$\begin{aligned}
&\text{for } \alpha > \alpha(\ell) = \alpha^*, \ \sigma(\alpha, s_r) = \text{reject for } s_r = h, \ell, \\
&\text{for } \alpha \in (\alpha(h), \alpha(\ell)], \quad \sigma(\alpha, s_r) = \begin{cases} \text{accept} & \text{if } s_r = h, \\ \text{reject} & \text{if } s_r = \ell, \end{cases} \\
&\text{for } \alpha \leq \alpha(h), \qquad\quad \sigma(\alpha, s_r) = \text{accept for } s_r = h, \ell.
\end{aligned}$$

- The responder has the pessimistic belief system:

$$\beta(\alpha, s_r) = \begin{cases} 1 & \alpha \leq \alpha(h), \\ 0 & \alpha > \alpha(h). \end{cases}$$

Optimality of Responder's Strategy It is straightforward to verify that the responder's strategy is optimal as far as $\delta W > (1-\alpha(h))E[x|\ell,\ell]$. This inequality is verified in Lemma 3.[18]

Optimality of Proposer's Strategy To derive the conditions under which the proposer's strategy composes an equilibrium, we investigate two necessary conditions separately. First, the proposer with a particular signal does not have an incentive to mimic the proposer with a different signal. We call this requirement as the *nonmimicry incentive condition*. Second, the proposer has no incentive to make an offer that never arises on the equilibrium path. We call this requirement as the *off-path incentive condition*. The first requirement is more essential, as the following argument shows.

Regarding the proposer's nonmimicry incentives, the associated conditions are the following:

$$\alpha(h)E[x|h] \geq \Pr(h|h)\alpha(\ell)E[x|h,h] + \Pr(\ell|h)\delta W, \qquad (\mathrm{IC}_{h\ell})$$
$$\alpha(h)E[x|\ell] \leq \Pr(h|\ell)\alpha(\ell)E[x] \qquad + \Pr(\ell|\ell)\delta W, \qquad (\mathrm{IC}_{\ell h})$$

where $\alpha(h) < \alpha(\ell)$ and $\alpha(\ell) = \alpha^*$. ($\mathrm{IC}_{h\ell}$) is the condition for a proposer with h signal not to mimic a proposer with ℓ signal, and ($\mathrm{IC}_{h\ell}$) is for a proposer with ℓ signal not to mimic a proposer with h signal. We need to find $\alpha(h)$ satisfying both conditions. Such $\alpha(h)$ satisfying both conditions may not exist, because the minimum value of $\alpha(h)$ satisfying ($\mathrm{IC}_{h\ell}$) can be larger than the maximum value of $\alpha(h)$ satisfying ($\mathrm{IC}_{\ell h}$). The following technical lemma gives the condition under which there is an $\alpha(h)$ simultaneously satisfying ($\mathrm{IC}_{h\ell}$) and ($\mathrm{IC}_{\ell h}$).

Lemma 2 *There is an $\alpha(h)$ for which ($IC_{h\ell}$) and ($IC_{\ell h}$) simultaneously hold iff Condition A holds, i.e., $\delta W \geq A(q,L,H)$, where*

$$A(q,L,H) = \left(\frac{q}{2q-1}\frac{1}{L} + \frac{q-1}{2q-1}\frac{1}{H} + \frac{1}{E[x]}\right)^{-1}.$$

The expression $A(q,L,H)$ has the following properties: (i) $A(q,L,H)$ is increasing in q, (ii)

[18] For $\alpha > \alpha(h)$, the best response given s_r and the belief is

$$\begin{array}{ll} \text{to reject for } s_r = h,\ell, & \text{if } (1-\alpha)E[x|\ell,h] < \delta W, \\ \left\{\begin{array}{ll}\text{to accept} & \text{for } s_r = h, \\ \text{to reject} & \text{for } s_r = \ell,\end{array}\right. & \text{if } (1-\alpha)E[x|\ell,h] \geq \delta W > (1-\alpha)E[x|\ell,\ell], \end{array}$$

Note that for $\alpha > \alpha^*$, $(1-\alpha)E[x|\ell,h] < \delta W$ by the definition of α^*. Therefore, $\sigma(\alpha,s_r)$ is optimal for $\alpha > \alpha(\ell)$. To show that $\sigma(\alpha,s_r)$ is optimal for $\alpha \in (\alpha(h),\alpha(\ell)]$, we need $\delta W > (1-\alpha(h))E[x|\ell,\ell]$. For $\alpha \leq \alpha(h)$, the best response is to accept for $s_r = h,\ell$, if $\delta W \leq (1-\alpha(h))E[x|h,\ell]$, which holds since $\alpha(h) < \alpha^*$. This shows that $\sigma(\alpha,s_r)$ is optimal for $\alpha \leq \alpha(h)$.

$E[x] > 2A(q, L, H)$ for all q, and (iii) there is a $\tilde{q} \in (.5, 1)$ such that $E[x|\ell, \ell] \geq 2A(q, L, H)$ iff $q \leq \tilde{q}$.

Proof. See Appendix A.1. ■

Condition A is depicted in Figure 3 as the area above the dotted curve. It has an increasing relationship between q and δ.

For a given accuracy level, we can see that the nonmimicry conditions are more likely to hold for a higher discount factor. A higher discount factor induces a higher future value (δW) and lower $\alpha(\ell)$ ($= 1 - \delta W/E[x]$). Indeed, the closer δ gets to $\overline{\delta}(q, \ell, H)$ (so that δW approaches $E[x]/2$), the closer $\alpha(\ell)$ approaches to $1/2$. It is straightforward to see that the nonmimicry conditions hold if $\alpha(\ell)$ and $\alpha(h)$ are equal or close to $1/2$. The higher future prospect, δW, restricts the proposer's first-mover advantage in the bargaining, and it thus facilitates efficient agreement.

On the other hand, for a given discount factor, the nonmimicry conditions are less likely to hold for a higher accuracy level, q. Although a higher accuracy improves the future prospect, δW, the improvement is not drastic enough to eliminate the proposer's first-mover advantage as $q \to 1$. Meanwhile, a higher accuracy creates another temptation for the proposer with h signal to mimic; given that $\alpha(h) < \alpha(\ell)$, the proposer with h signal finds that deviation is more attractive because, with higher correlation between the two signals, it is more likely that the responder receives signal h, and therefore the probability of accepting $\alpha(\ell)(> \alpha(h))$ is higher. Indeed, $\alpha(h)$ must converge to $\alpha(\ell)$ to satisfy ($IC_{h\ell}$), as the accuracy approaches to 1. However, with $\alpha(h) \approx \alpha(\ell)$ and $\alpha(\ell) > 1/2$, ($IC_{\ell h}$) becomes

$$\alpha(\ell)E[x|\ell, \ell] \leq \delta W,$$

which can be violated for $\delta \approx \underline{\delta}(q, L/H)$ ($E[x|\ell, \ell] \approx 2\delta W$).

Intuitively, for a high accuracy level, the proposer can infer the responder's signal fairly correctly. Unless α's are so close to $1/2$ that the proposer's manipulation of information has negligible value (see the commitment case), this informational advantage creates incentives to mimic. Indeed, the proposer's bargaining advantage is higher if the discount factor is lower. Hence, the efficient equilibrium cannot exist in such cases.

The next lemma shows that we can find $\alpha(h)$ so that $\delta W > (1 - \alpha(h))E[x|\ell, \ell]$, which completes the proof that the responder's strategy is optimal.

Lemma 3 *Suppose $\delta \in [\underline{\delta}(q, L/H), \overline{\delta}(q, L/H))$ and $\delta W \geq A(q)$. Then there is $\alpha(h) < \alpha^*$ such that $(\alpha(h), \alpha(\ell) = \alpha^*)$ satisfies ($IC_{h\ell}$) and ($IC_{\ell h}$), and $\delta W > (1 - \alpha(h))E[x|\ell, \ell]$.*[19]

[19] The case where $2\delta W(\delta, q, L, H) = E[x]$ (where High Delay and Medium Delay are equally efficient) is excluded since the on-schedule ICs indeed imply $\alpha(h) = \alpha(\ell) = 1/2$, which contradicts signaling ($\alpha(h) \neq \alpha(\ell)$).

Proof. See Appendix A.2. ■

Proposer's Off-Path Incentive Conditions Given the nonmimicry conditions and the responder's strategy, it is easy to show that the proposer has no incentive to make an offer that never arises on the equilibrium path. Indeed, offer $\alpha \in (\alpha(h), \alpha(\ell))$ is dominated by $\alpha(\ell)$, given that the responder's response is the same for each $\alpha \in (\alpha(h), \alpha(\ell)]$. Similarly, offer $\alpha < \alpha(h)$ is dominated by $\alpha(h)$. These offers are never optimal as far as nonmimicry conditions hold. Therefore, we need to show the remaining case that the proposer has no incentive to make an offer $\alpha > \alpha(\ell)$. Under the pessimistic belief, this offer is always rejected, and the proposer's resulting payoff would be δW. This deviation is unprofitable if

$$\begin{aligned} &\alpha(h)E[x|h] \geq \delta W \\ &\Pr(h|\ell)\alpha(\ell)E[x] + \Pr(\ell|\ell)\delta W \geq \delta W \Leftrightarrow \alpha(\ell)E[x] \geq \delta W. \end{aligned}$$

The first line is redundant given the second line and ($\mathrm{IC}_{h\ell}$) as follows;

$$\begin{aligned} \alpha(h)E[x|h] &\geq \Pr(h|h)\alpha(\ell)E[x|h,h] + \Pr(\ell|h)\delta W, \\ &\geq \Pr(h|h)\alpha(\ell)E[x] \qquad + \Pr(\ell|h)\delta W \geq \delta W. \end{aligned}$$

The second line can be rewritten as

$$\alpha(\ell)E[x] \geq \delta W \Leftrightarrow (1 - \frac{\delta W}{E[x]})E[x] \geq \delta W \Leftrightarrow E[x] \geq 2\delta W$$

which holds if Medium Delay Schedule is efficient.

With the above lemmas, we can identify the conditions under which an efficient equilibrium for Medium Delay Schedule exists. Finally, before presenting the main results, we also show that we only need to focus on the case of $\alpha(h) < \alpha(\ell)$ and have no need to consider the case of $\alpha(h) > \alpha(\ell)$ as below.

Lemma 4 *Suppose $\delta \in [\underline{\delta}(q, L/H), \overline{\delta}(q, L/H))$. Suppose also that there is an efficient separating equilibrium in which the on-path share offered by the proposer with h signal is higher than that by the proposer with ℓ signal. Then, there is a pair $(\alpha(h), \alpha(\ell))$ with $\alpha(h) < \alpha(\ell) = a^*$ that satisfies the nonmimicry conditions, ($IC_{h\ell}$) and ($IC_{\ell h}$).*

Proof. See Appendix A.3. ■

The established nonmimicry conditions together with the pessimistic belief imply the existence of an associated efficient equilibrium with $\alpha(h) < \alpha(\ell)$.[20] Now we present the main result as follows.

[20] In this footnote, we show that adopting alternative belief systems other than the pessimistic belief

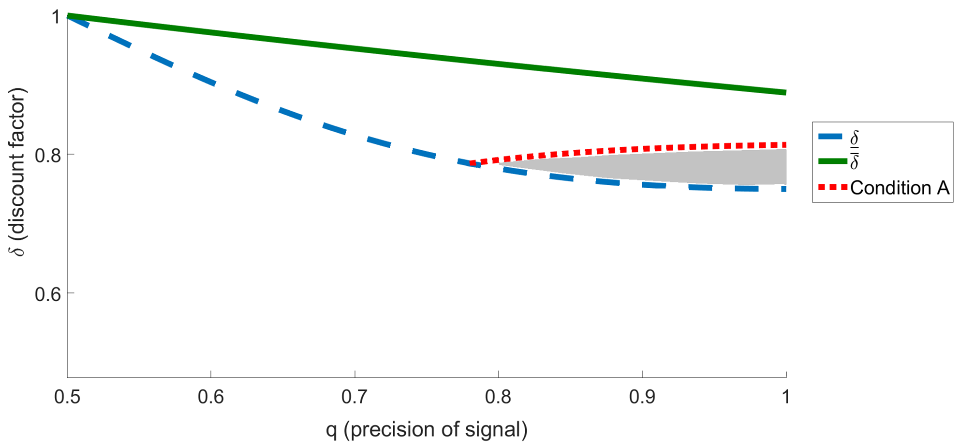

Figure 3: An efficient equilibrium for Medium Delay Schedule exists in the shaded region below Condition A (dotted curve) and above δ (the dashed line).

Proposition 2 (Part 2. Restatement) *Suppose $\delta \in [\underline{\delta}(q, L/H), \overline{\delta}(q, L/H))$ (Medium Delay Schedule is efficient). The strategy profile and the belief system defined above compose an efficient equilibrium if and only if Condition A holds, i.e., $\delta W \geq A(q, L, H)$.*

We plot the lower bound of δ for which $\delta W \geq A(q, L, H)$, i.e., ($\text{IC}_{h\ell}$) and ($\text{IC}_{\ell h}$) hold (the area above the dotted curve) in Figure 3 for the case of $L/H = .6$: The area below the dotted curve and above the dashed curve is where no equilibrium that attains Medium Delay Schedule exists. When Medium Delay Schedule is efficient (i) for a lower information accuracy level ($q \approx .78$ or smaller), efficiency is attainable in an equilibrium, while (ii) for a higher information accuracy level, it depends on the discount factor; δ must be large enough to achieve efficiency.

We have identified two essential conditions for the attainability of an efficient equilibrium for Medium Delay Schedule: first, $\alpha(\ell) = \alpha^*$ due to the off-path incentive condition. In the efficient equilibrium for Medium Delay Schedule, efficient information aggregation implies that the responder must be signal-responsive for agreement when the proposer has ℓ signal.

defined above cannot improve the attainability of efficient equilibria.

Suppose that for some belief that is different from the pessimistic one, there is an efficient equilibrium. Since an equilibrium is separating, we must have either (i) $\alpha(h) < \alpha(\ell)$ or (ii) $\alpha(h) > \alpha(\ell)$.

First, consider the case of (i). The argument in "Proposer's Off-Path Incentive Conditions" shows that all the off-path incentive conditions hold, even if we replace the belief with the pessimistic one. Hence, the attainability of an efficient equilibrium does not extend.

Second, consider the case of (ii). Lemma 4 shows that we can find $\tilde{\alpha}(h) < \alpha^*$ such that $\tilde{\alpha}(h)$ and $\alpha(\ell) = \alpha^*$ satisfy both ($\text{IC}_{h\ell}$) and ($\text{IC}_{\ell h}$). With the pessimistic belief we use for $\tilde{\alpha}(h)$ and $\alpha(\ell) = \alpha^*$, all of the off-path incentive conditions hold, and hence we can find another efficient equilibrium with the pessimistic belief. Hence the attainability of an efficient equilibrium does not extend.

Thus, the proposer's offer $\alpha(\ell)$ must be such that the responder's incentive to accept/reject is in accordance with this responder's signal-responsivity, i.e.,$\alpha(\ell)$ must satisfy $(1-\alpha(\ell))E[x] \geq \delta W \geq (1-\alpha(\ell))E[x|\ell,\ell]$. Although there are many such offers, we need $\alpha(\ell) = \alpha^*$ in equilibrium, in order that the proposer with ℓ signal has no incentive to make a greedier (off-path) offer than $\alpha(\ell)$ (Lemma 1).

Now we can see that it is more difficult to establish an efficient equilibrium if Medium Delay Schedule is efficient. The proposer's offer when he receives a bad signal must be such that the responder is signal-responsive with the offer, but is not signal-responsive with any greedier offer. Since the associated belief that the proposer has h signal when the proposer offers $\alpha(\ell)$ is zero, pessimistic belief implies that the belief for any greedier offer is also zero. Therefore, whether the responder is signal-responsive or not need to be tailored without a change in belief. In contrast, when High Delay Schedule is efficient, the responder is signal-responsive when the proposer's signal is good. In this case, the responder can be made non-signal-responsive for a greedier offer by lowering the belief. This is why the attainability of an efficient equilibrium is greater when High Delay Schedule is efficient. Indeed, the proposition shows that an efficient equilibrium always exists in this case, whereas it may not when Medium Delay Schedule is efficient.

The other essential condition is the nonmimicry condition. Unlike the case of High Delay Schedule, this condition is not trivial, because the candidate on-path shares $(\alpha(h), \alpha(\ell))$ may not be set close to $1/2$.[21] Put differently, the temptation by the proposer to mimic may arise if the proposer has a substantial bargaining advantage. As Lemma 2 (i) shows, this temptation is reinforced by the proposer's informational advantage due to a higher accuracy level. This is why the efficient equilibrium may fail to exist for a higher accuracy and a lower discount factor.

3.1.3 Low Patience: $\delta \in [0, \underline{\delta}(q, L/H))$

Recall that No Delay Schedule is efficient if $E[x|\ell,\ell] \geq 2\delta W$, where $W = E[x]/2$. In No Delay Schedule, players reach an agreement regardless of the realization of signals. An efficient equilibrium for this schedule is a *pooling* equilibrium in which the responder always accepts the offer by the proposer, and therefore information aggregation is unnecessary. Hence,

[21] Suppose that the conditions for Medium Delay to be efficient hold with strict inequalities; i.e., $E[x] > 2\delta W > E[x|\ell,\ell]$. Then

$$\frac{1}{2}E[x|h] = \frac{1}{2}\Pr(h|h)E[x|h,h] + \frac{1}{2}\Pr(\ell|h)E[x] > \frac{1}{2}\Pr(h|h)E[x|h,h] + \Pr(\ell|h)\delta W,$$
$$\frac{1}{2}E[x|\ell] = \frac{1}{2}\Pr(h|\ell)E[x] + \frac{1}{2}\Pr(\ell|\ell)E[x|\ell,\ell] < \frac{1}{2}\Pr(h|\ell)E[x] + \Pr(\ell|\ell)\delta W,$$

Now suppose $\alpha(h)$ and $\alpha(\ell)$, $\alpha(h) \neq \alpha(\ell)$, are very close to $1/2$. We can see from the above inequalities that the nonmimicry conditions are satisfied. Note that by the definition of α^*, share $\alpha(\ell)(= \alpha^*)$ is close to $1/2$ only when $E[x] \approx 2\delta W$.

no party needs to be held signal-responsive in an efficient agreement, and the nonmimicry incentive conditions are redundant.

Given that the nonmimicry conditions are redundant, the main problem for equilibrium construction is the off-path incentive condition. In particular, the proposer can make a higher off-path offer for which the responder becomes signal-responsive, i.e., she accepts the offer if and only if she has h signal. Note that this type of offer might be attractive if the proposer has h signal because the correlation of signals makes it more likely that the responder also has h signal. To deter such deviations, the highest signal-responsive offer should not be as attractive compared with the on-path offer. We maintain the pessimistic belief for high offers in order to make the highest signal-responsive offer smaller.

Now let us discuss how the discount factor δ and the accuracy parameter q affect the attractiveness of a signal-responsive offer. Indeed, deviation becomes less profitable as the discount factor or the accuracy becomes lower. When the discount factor is lower, the reservation value δW is smaller so that the on-path pooling offer satisfying $(1-\alpha^p)E[x] = \delta W$ is smaller. Intuitively, the proposer has no need to offer a high share for acceptance, and therefore the on-path offer is more attractive. Meanwhile, the attractiveness of the signal-responsive offer increases as the accuracy of the signal becomes higher, since the likelihood that the responder accepts the signal-responsive offer is high. In fact, there is a region of parameter values for which an equilibrium for No Delay Schedule fails to exist (the detailed construction of an efficient equilibrium and the condition for its existence are relegated to Appendix B).

3.2 Discussion

The model is a stylized one and has some limitations. We discuss some of the main assumptions and their implications; symmetry of information precision and bargaining power, binary signal structure, the form of offer, and communication.

3.2.1 Information Precision, Bargaining Power, and Signal Structure

First, we discuss the assumption of symmetry in information precision and bargaining power. In the model, both players receive signals with the same level of precision, q. Additionally, the recognition probability in each period, which essentially represents the bargaining power of players, is equal in the model, i.e., $r_i = r_j = 1/2$. Even if the precision differs across players ($q_i \neq q_j$) or the bargaining power differs ($r_i \neq r_j$), our analysis is robust to such changes if the difference in q_i or r_i between the players is small. However, our results are not necessarily robust to a large change in q_i or r_i. For example, if the proposer has much more precise information than the responder, he would have much less incentive to make the responder signal-responsive if the discount factor is not too high. Because relaxing these assumptions fully can result in a substantial change in our analysis, we have a companion paper Hanazono and Watanabe (2014), in which we allow asymmetry in information precision and in bargaining power in a similar setup but with a two-period timing structure.

Another assumption of the model is binary support for the signal, $\{h, \ell\}$. One natural way to modify the model would be to allow larger support for the signal, such as allowing for three signals $\{h, m, \ell\}$. The analysis becomes more complex as we need to consider a larger number of combinations of signal realizations. For example, with three signals, we need to consider six different agreement schedules. A part of our analysis, however, continues to hold for the case of a high discount factor corresponding to High Delay Schedule.[22]

3.2.2 Offer Forms and Communication

As this paper focuses on share offers and does not allow communication, we briefly discuss how our analysis is affected by allowing less restrictive offer forms and communication. Suppose that the players can sign state-contingent contracts on the realization of pie size and that they can exchange messages via cheap talk. It is now possible to achieve an efficient equilibrium even in the parameter region where no efficient equilibrium arises with a share offer in the previous analysis. In particular, consider that the proposer makes, regardless of his signal, a state-contingent contract (α^H, α^L) such that $(1-\alpha^H)H = (1-\alpha^L)L = \delta W(\delta, q, L, H)$, and if accepted, the proposer receives $\alpha^H H$ for $x = H$ and $\alpha^L L$ for $x = L$. The responder is then indifferent between accepting and rejecting the offer, no matter what signal she receives, and hence she does not care about her signal or the acceptance/rejection decision. Notice that this offer has the same effect as asking the responder for an up-front cash payment in the amount of $\delta W(\delta, q, L, H)$. Suppose now that, with the above state-contingent contract, the proposer sends a message regarding his signal and requests the responder to accept the offer if and only if the agreement is efficient (given the proposer's message and the responder's own signal). For the responder, it is optimal to grant the request. The proposer has an incentive to make such an offer and to reveal his signal, since he is the residual claimant. Therefore, the above contract with cheap talk can induce an efficient equilibrium. However, the above argument fails in the case that $L < \delta W(\delta, q, L, H)$, and further analysis is needed for when offers are contingent on the state.

To further understand the roles of contingent payment and communication, let us compare our setup with that of Eraslan et al. (2014). In their analysis of bankruptcy procedure,

[22] Suppose that it is efficient to agree if and only if both players receive h signal as in High Delay Schedule. By bundling m and ℓ signals together, we can construct an equilibrium analogous to the High Delay efficient equilibrium with two signals studied in the text. The continuation value, the threshold discount factor, and the incentive conditions are defined accordingly, by replacing $E[x]$ with $E[x|h, m]$.

information aggregation by the parties is necessary for efficient operational restructuring. Since there are multiple alternatives for operational restructuring aside from the status quo, the information conveyed only through offers and responses is insufficient to select an efficient alternative, and this is why cheap talk is needed for efficient information aggregation. In contrast, there is in our setup only one alternative aside from the "status quo" (meaning rejecting and postponing the agreement in the future). This implies that the information conveyed through offers and responses is sufficient as far as the proposer's offer is a separating one with respect to his signal. Cheap talk is only needed when the proposer's offer is pooling as in the above argument. Moreover, Eraslan et al. (2014) study bargaining with lenders and shareholders, for whom the payoffs from the status quo contract differ. They find that in such a situation, contingent payments are crucial in order to create incentives to reveal truthful information. In our setup, the parties' payoffs from the "status quo" are symmetric (both players are equally likely to become the proposer in the next period). Thus, the parties do not care about the contract form in the case of disagreement, but they do care about the expected returns. This is why other forms of contingent payment may not play a big role (at least in the region where efficiency is available through share offers).

Although our restriction to share offers is limited, there are several reasons why such a restriction could be relevant. First, it can be more costly to write a general form of contingent payment, such as debt, than an equity contract; for example, a debt or option contract must specify maturity, which may not be easily agreed upon by all parties, while an equity contract needs no maturity. This is particularly relevant for merger cases and venture capital investments because there is great uncertainty about when the intended synergies or returns will materialize. This shows how restricting offer forms to a share offer may reduce the transaction costs of writing contracts. Second, although an up-front or ex post cash payment can induce an efficient agreement, as the above argument shows, such a contract may not be available due to limited liability.

3.3 Comparative Statics

We discuss comparative statics of the efficient equilibria for efficient agreement schedules in order to consider the empirical implications of the model.[23] The two equilibrium outcomes we consider are the time to agreement and the offer proposers make. As considered in Merlo and Tang (forthcoming), the time to agreement and the terms of agreement are the typical observable endogenous variables in many bargaining data, together with some exogenous variables. We discuss how these two equilibrium outcomes change as other parameters of

[23] Note that our discussion is conditional on the equilibrium we focus on, i.e., the efficient equilibrium. This is because the players are less likely to play an inefficient equilibrium if an efficient equilibrium is attainable given the fact that the players can engage in preplay cheap talk of a very general form.

the model change, particularly the change in the signal accuracy (q) and the relative size of the pie between the two states (L/H). For No Delay Schedule, we relegate the discussion to Appendix B.

3.3.1 High Delay Schedule

First, we consider comparative statics on the expected time to agreement (denoted by $E[T]$). We consider the probability of agreement in one period because $E[T] = 1/\Pr(Agreement)$. For High Delay Schedule, the probability of agreement is the probability that the signals are (h, h), i.e., $\Pr(Agreement) = \frac{1}{2}(q^2 + (1-q)^2)$. Thus, $E[T]$ decreases as q increases. This reflects the fact that an increase in signal accuracy results in a higher likelihood of both players receiving h signal, which makes the players agree sooner. Note that the relative size of the pie (L/H) does not affect the expected time to agreement.

Second, we look at how the proposer's offer α is related to the parameters. In High Delay Schedule, there is a range that equilibrium value α can take, which is symmetric around $1/2$. Both parameters q and L/H affect the length of this region, making the region larger as q increases and as L/H decreases. As accuracy improves and the relative size of H increases, it becomes easier to satisfy the off-path IC condition.

3.3.2 Medium Delay Schedule

In Medium Delay Schedule the probability of agreement decreases as the signal accuracy (q) increases. This is because the probability of agreement is now equal to the probability of the event that signal (ℓ, ℓ) does not occur, i.e., $\Pr(Agreement) = 1 - \frac{1}{2}(q^2 + (1-q)^2)$. Thus, $E[T]$ increases as q increases, and L/H has no effect. This is because event (ℓ, ℓ) is more likely to occur as signal accuracy improves, which reduces the likelihood of agreement with (ℓ, h) and (h, ℓ). This results in a longer time to agreement as the signal accuracy improves.

Offer $\alpha^* = \alpha(\ell)$ by the proposer is uniquely determined and written as

$$(1 - \alpha^*)E[x] = \delta W \Leftrightarrow \alpha^* = 1 - \delta \frac{(1 - q^2)(L/H) + q(2 - q)}{(1 + L/H)(2 - (q^2 + (1 - q)^2)\delta)}$$

which is decreasing in q and increasing in L/H. To see why, note first that the increases in the accuracy and the size of L given H imply a higher continuation value and therefore the bargaining power of the responder improves. However, because the increase in the size of L given H also increases $E[x]$, which is indeed more significant, the resulting bargaining share of the responder decreases.

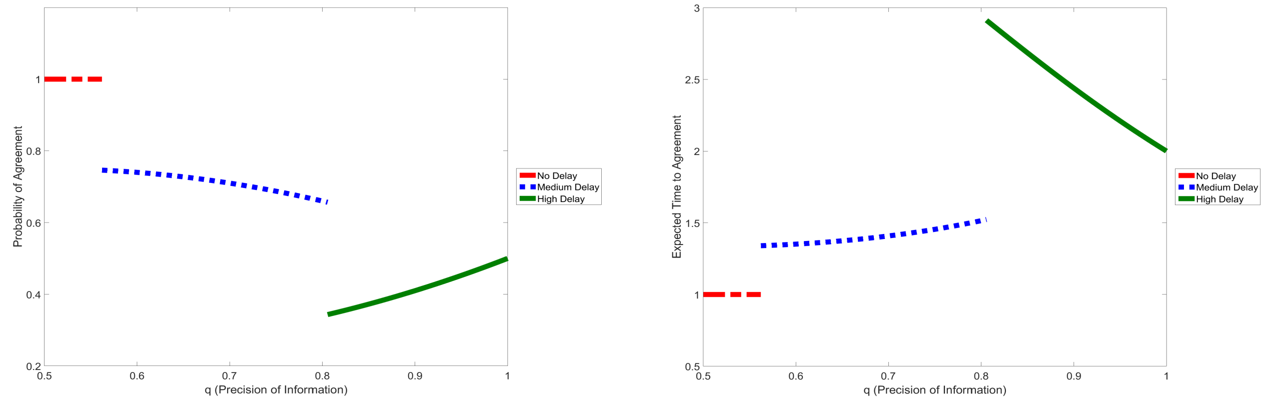

Figure 4: Comparative Statics on Agreement Probability and Time to Agreement. We use the value $L/H = 0.1$ and $\delta = 0.8$.

3.3.3 Summary of Comparative Statics

Summarizing the comparative statics across High and Medium Delay Schedules, Figure 4 draws graphs of probability of agreement and expected time to agreement as function of q. As q increases, the expected time to agreement first increases in the region of Medium Delay Schedule. After the expected time to agreement reaches its longest period at the boundary value of q between Medium and High Delay Schedules, it monotonically decreases as q increases further in the region of High Delay Schedule. The equilibrium offer is decreasing in q and increasing in L/H in Medium Delay Schedule, while the model does not have a clear prediction for High Delay Schedule other than the fact that the range of the equilibrium offer is centered around $1/2$.

The comparative statics on time to agreement are sufficient to identify which efficient equilibrium is played given how the local changes in both q and L/H affect the time to agreement ceteris paribus. Additionally, the model can be tested by the local change in q.

3.4 Approximate Efficiency

We have seen that an efficient equilibrium may fail to exist for Medium and No Delay Schedules. In this section, we consider whether the efficient outcomes can be attained in the limit of a sequence of equilibria as information becomes arbitrarily close to perfect, i.e., $q \to 1$.

For Medium Delay Schedule, consider a pooling equilibrium in which the responder rejects the equilibrium offer if it receives a low signal. The ex ante value from this agreement schedule is

$$V_p = \frac{1}{2}(\frac{1}{2}E[x|h] + \frac{1}{2}2\delta V_p) = \frac{E[x|h]}{2(2-\delta)},$$

since agreement occurs if and only if the responder's signal is h. Let α denote the pooling offer, and assume the pessimistic belief for off-path offers. The responder's on-path incentive condition is

$$(1-\alpha)E[x|h] \geq \delta V_p \geq (1-\alpha)E[x|\ell],$$

while its off-path best response is just as before. Analogous to the construction for an equilibrium for No Delay Schedule, it would be natural to think about the best offer for the proposer, because otherwise it would be harder to sustain the equilibrium. We thus take α such that

$$(1-\alpha)E[x|h] = \delta V_p = \frac{\delta E[x|h]}{2(2-\delta)} \Leftrightarrow \alpha = 1 - \frac{\delta}{2(2-\delta)}.$$

Given α as above, an offer $\alpha' > \alpha$ is always rejected. The most profitable deviation for the proposer is then α'' such that

$$(1-\alpha'')E[x|\ell,\ell] = \delta V_p \Leftrightarrow \alpha'' = 1 - \frac{\delta}{2(2-\delta)}\frac{E[x|h]}{E[x|\ell,\ell]},$$

which the responder always accepts. Thus, the incentive conditions are

$$\Pr(h|h)\alpha E[x|h,h] + \Pr(\ell|h)\delta V_p \geq \alpha'' E[x|h], \tag{IC$_h$}$$

$$\Pr(h|\ell)\alpha E[x] + \Pr(\ell|\ell)\delta V_p \geq \alpha'' E[x|\ell]. \tag{IC$_\ell$}$$

Consider the situation in which Medium Delay Schedule is efficient but the efficient equilibrium is not available. We then ask the following question: *given δ and L/H, as the accuracy of information is approaching $q = 1$, does the approximately efficient equilibrium exist in this region?* The answer is indeed affirmative. To demonstrate this, we just have to verify that the two ICs hold as $q \to 1$. Since (IC$_h$) trivially holds with strict inequality at $q = 1$, by continuity it must hold for a q sufficiently close to 1. (IC$_\ell$) holds at $q = 1$ if

$$\begin{aligned}\delta V_p \geq \alpha'' E[x|\ell] &\Leftrightarrow \delta\frac{E[x|h]}{2(2-\delta)} \geq \left(1 - \frac{\delta}{2(2-\delta)}\frac{E[x|h]}{E[x|\ell,\ell]}\right)E[x|\ell] \\ &\Leftrightarrow \delta \geq \frac{4E[x|\ell]}{E[x|h]\left(1+\frac{E[x|\ell]}{E[x|\ell,\ell]}\right)+2E[x|\ell]} = \frac{4L}{2H+2L} = \left.\frac{E[x|\ell,\ell]}{E[x]}\right|_{q=1}.\end{aligned}$$

Hence, this condition at the limit is the same as the lower-bound restriction for δ for which Medium Delay Schedule is efficient, namely $\delta \geq E[x|\ell,\ell]/E[x]$ at $q = 1$. Since $E[x|\ell,\ell]/E[x]$ is decreasing in q and (IC$_\ell$) is continuous in q, we can conclude that (IC$_\ell$) holds for a

q sufficiently close to 1 (indeed, a tedious calculation shows that (IC_ℓ) is redundant if $\delta \geq E[x|\ell,\ell]/E[x]$ for all q).

Our discussion for No Delay Schedule is in Appendix B as well.

4 Concluding Remarks

In this paper we study how and when information can be aggregated for an efficient agreement in a bilateral bargaining model with a common-value environment. We find that efficient information aggregation is attainable in an equilibrium if the discount factor is relatively high. The key idea is to consider how the proposer makes an offer in such a way that the responder will be signal-responsive if the responder's signal is crucial for efficiency. We also find that overly accurate (but not perfect) information sometimes disturbs the attainment of fully efficient equilibria because the proposer would be able to squeeze rent by distorting the efficient agreement.

One of the important issues that we did not deal with in this paper is the case where the size of the pie is not i.i.d. across periods. Once the i.i.d. assumption is relaxed, information becomes persistent over time and the strategy to be considered becomes exceedingly complex. We leave this for future research.

References

[1] Ausubel, Lawrence M., Cramton, Peter, and Deneckere, Raymond J.(2002) "Bargaining with Incomplete Information" In Aumann, R.J., Hart, S. (Eds.), Handbook of Game Theory, vol.3. Elsevier Science Publishers B.V., Greenwich, Connecticut, pp. 1897–1945.

[2] Ausubel, Lawrence M., and Deneckere, Raymond J. (1989) "A Direct Mechanism Characterization of Sequential Bargaining with One-sided Incomplete Information," *Journal of Economic Theory*, 48, pp. 18–46.

[3] Banks, Jeffrey S., and Sobel, Joel (1987) "Equilibrium Selection in Signaling Games," *Econometrica*, 55 pp.647-662.

[4] Bond, Philip, and Eraslan, Hülya (2010) "Strategic Voting over Strategic Proposals," *Review of Economic Studies*, 77(2), pp. 459-490.

[5] Brooks, Richard, Landeo, Claudia, and Spier, Kathryn (2010) "Trigger Happy or Gun Shy? Dissolving Common-Value Partnerships with Texas Shootouts," *RAND Journal of Economics,* 41(4), pp. 649-673.

[6] Chatterjee, Kalyan, and Samuelson, Larry (1987) "Bargaining with Two-Sided Incomplete Information: An Infinite Horizon Model with Alternating Offers," *Review of Economic Studies*, 54, pp. 175-192.

[7] Cramton, Peter (1984) "Bargaining with Incomplete Information: An Infinite-Horizon Model with Two-Sided Uncertainty," *Review of Economic Studies*, 51, pp. 579-593.

[8] Cramton, Peter (1992) "Strategic Delay in Bargaining with Two-Sided Uncertainty," *Review of Economic Studies*, 59, pp. 205-225.

[9] Damiano, Ettore, Li, Hao, and Suen, Wing (2010) "Delay in Strategic Information Aggregation," working paper.

[10] DeMarzo, Peter M., Ilan Kremer, and Andrzej Skrzypacz (2005) "Bidding with Securities: Auctions and Security Design," *American Economic Review*, 95(4), pp. 936-959.

[11] Deneckere, Raymond J., and Meng-Yu Liang (2006) "Bargaining with Interdependent Values." *Econometrica*, 74(5), pp. 1309-1364.

[12] Eraslan, Hülya, and Antonio Merlo (2002) "Majority Rule in a Stochastic Model of Bargaining." *Journal of Economic Theory,*103, 31-48.

[13] Eraslan, Hülya, Mylovanov, Tymofiy, and Yilmaz Bilge (2014) "Deliberation and Security Design in Bankruptcy," mimeo, Rice University

[14] Evans, Robert. (1989) "Sequential Bargaining with Correlated Values." *Review of Economic Studies*, 56(4): 499-510.

[15] Feddersen, Timothy J. and Pesendorfer Wolfgang (1996) "The Swing Voter's Curse," *American Economic Review*. 86(3), pp. 404-24

[16] Fuchs, William, and Skrzypacz, Andrzej (2010) "Bargaining with Interdependent Values, Experience Effects and the Coase Conjecture," mimeo.

[17] Fudenberg, Drew, and Tirole, Jean (1983) "Sequential Bargaining with Incomplete Information," *Review of Economic Studies*, 50, pp. 221-247.

[18] Fudenberg, Drew, and Tirole, Jean (1991) *Game Theory*, MIT Press

[19] Hanazono, Makoto, and Watanabe, Yasutora (2014) "Merger Negotiation and Merger Performance" working paper.

[20] Kennan, John and Wilson, Robert (1993) "Bargaining with private information" *Journal of Economic Literature*, 31(1), pp. 45–104

[21] Lerner, Joshua (1994) "The Syndication of Venture Capital Investments," *Financial Management*, Vol 23 (3), pp. 16-27.

[22] Merlo, Antonio and Wilson, Charles (1995) "Stochastic Model of Sequential Bargaining with Complete Information," *Econometrica*,63(2), pp. 371-399

[23] Merlo, Antonio and Wilson, Charles (1998) "Efficient Delays in a Stochastic Model of Bargaining," *Economic Theory*, 11(1), pp. 39-55.

[24] Merlo, Antonio and Hulya, Eraslan (2002) "Majority Rule in a Stochastic Model of Bargaining,"*Journal of Economic Theory,* 103 (1), pp. 31–48

[25] Merlo, Antonio and Tang, Xun (forthcoming) "Identification of Stochastic Sequential Bargaining Models" *Econometrica*

[26] Myerson, Roger B. and Mark A. Satterthwaite (1983) "Efficient Mechanisms for Bilateral Trading," *Journal of Economic Theory,* 29 (2), pp. 265–281

[27] Reinganum, Jennifer F. and Louis L. Wilde (1986) "Settlement, Litigation, and the Allocation of Litigation Costs," *RAND Journal of Economics,* 17(4), pp. 557-566.

[28] Schweinzer, Paul (2010a) "Sequential Bargaining with Common Values" *Journal of Mathematical Economics* 46 (2010) pp. 109–121.

[29] Schweinzer, Paul (2010b) "Sequential Bargaining with Common Values: The Case of Bilateral Incomplete Information," mimeo.

[30] Schweizer, Urs (1989) "Litigation and Settlement under Two-Sided Incomplete Information" *Review of Economic Studies*, 56(2), pp. 163-177.

[31] Spier, Kathryn (1992) "The Dynamics of Pretrial Negotiation" *Review of Economic Studies*, 59 (1), pp. 93-108.

[32] Vincent, Daniel R. 1989. "Bargaining with Common Values." *Journal of Economic Theory*,48(1), pp. 47-62.

A Proofs

A.1 Proof of Lemma 2

Define, from the associated ICs,

$$\alpha_1 = \frac{\Pr(h|h)\alpha(\ell)E[x|h,h] + \Pr(\ell|h)\delta W}{E[x|h]}, \quad \alpha_2 = \frac{\Pr(h|\ell)\alpha(\ell)E[x] + \Pr(\ell|\ell)\delta W}{E[x|\ell]}.$$

That is, α_1 is the lowest $\alpha(h)$ satisfying ($\mathrm{IC}_{h\ell}$) and α_2 is the highest $\alpha(h)$ satisfying ($\mathrm{IC}_{\ell h}$). To assure that there is $\alpha(h) \in [\alpha_1, \alpha_2]$, we need $\alpha_1 \leq \alpha_2$. The condition $\alpha_1 \leq \alpha_2$ can be rewritten as

$$\frac{\Pr(h|h)\alpha(\ell)E[x|h,h] + \Pr(\ell|h)\delta W}{E[x|h]} \leq \frac{\Pr(h|\ell)\alpha(\ell)E[x] + \Pr(\ell|\ell)\delta W}{E[x|\ell]}$$
$$\Leftrightarrow [\Pr(h|h)(1 - \frac{\delta W}{E[x]})E[x|h,h] + \Pr(\ell|h)\delta W]E[x|\ell] \leq [\Pr(h|\ell)(1 - \frac{\delta W}{E[x]})E[x] + \Pr(\ell|\ell)\delta W]E[x|h]$$
$$\Leftrightarrow \Pr(h|h)E[x|h,h]E[x|\ell] - \Pr(h|\ell)E[x]E[x|h]$$
$$\leq \delta W[\{\Pr(h|h)\tfrac{E[x|h,h]}{E[x]} - \Pr(\ell|h)\}E[x|\ell] + \{\Pr(\ell|\ell) - \Pr(h|\ell)\}E[x|h]]$$
$$\Leftrightarrow \delta W \geq \frac{\Pr(h|h)E[x|h,h]E[x|\ell] - \Pr(h|\ell)E[x]E[x|h]}{\{\Pr(h|h)\frac{E[x|h,h]}{E[x]} - \Pr(\ell|h)\}E[x|\ell] + \{\Pr(\ell|\ell) - \Pr(h|\ell)\}E[x|h]}$$

Define the RHS of the last inequality as $A(q, L, H)$. Since δW is increasing in δ, $\delta W \geq A(q, L, H)$ defines a lower bound of δ for which $\alpha_1 \leq \alpha_2$ holds.[24]

Simplifying the numerator of $A(q, L, H)$ yields

$$\Pr(h|h)E[x|h,h]E[x|\ell] - \Pr(h|\ell)E[x]E[x|h]$$
$$= \left(q^2H + (1-q)^2\ell\right)(q\ell + (1-q)H) - q(1-q)(H+L)(qH + (1-q)L)$$
$$= \left(q^2(H+L) - (2q-1)L\right)(q(H+L) - (2q-1)H) - q(1-q)(H+L)(q(H+L) - (2q-1)L)$$
$$= (H+L)^2(q^3 - q^2(1-q)) + HL(2q-1)^2 - (H+L)(2q-1)(qL + q^2H - q(1-q)L)$$
$$= (H+L)^2q^2(2q-1) + HL(2q-1)^2 - (H+L)^2q^2(2q-1)$$
$$= HL(2q-1)^2.$$

[24]It is straightforward to show that $\delta W \geq A(q, L, H)$ is equivalent to $\delta \geq \frac{2A(q,L,H)}{E[x] - \Pr(\ell,\ell)(E[x|\ell,\ell] - 2A(q,L,H))}$.

whereas simplifying the denominator of $A(q, L, H)$ yields

$$
\begin{aligned}
&\{\Pr(h|h)\frac{E[x|h,h]}{E[x]} - \Pr(\ell|h)\}E[x|\ell] + \{\Pr(\ell|\ell) - \Pr(h|\ell)\}E[x|h] \\
&= \left[\frac{2\left(q^2H + (1-q)^2L\right)}{H+L} - 2q(1-q)\right](qL + (1-q)H) + \left[(q^2 + (1-q)^2 - 2q(1-q)\right](qH + (1-q)L) \\
&= \left[\frac{2\left(q^2(H+L) - (2q-1)L\right)}{H+L} - 2q(1-q)\right](q(H+L) - (2q-1)H) + (2q-1)^2(q(H+L) - (2q-1)L) \\
&= \left[(2q-1)^2 + (2q-1)\frac{H-L}{H+L}\right](q(H+L) - (2q-1)H) + (2q-1)^2(q(H+L) - (2q-1)L) \\
&= (2q-1)^2(H+L) + (2q-1)\frac{H-L}{H+L}(q(H+L) - (2q-1)H) \\
&= (2q-1)(qH + (q-1)L) + 2(2q-1)^2HL/(H+L)
\end{aligned}
$$

Hence

$$
\begin{aligned}
A(q, L, H) &= \frac{HL(2q-1)^2}{(2q-1)(qH + (q-1)L) + 2(2q-1)^2HL/(H+L)} \\
&= \left(\frac{q}{2q-1}\frac{1}{L} + \frac{q-1}{2q-1}\frac{1}{H} + \frac{1}{E[x]}\right)^{-1}
\end{aligned}
$$

It is easy to see that $A(q, L, H)$ is increasing in q.

Recall that $E[x]$ is constant and $E[x|\ell, \ell]$ is decreasing in q, and that $E[x|\ell, \ell] = E[x]$ for $q = 1/2$. We can also see that

$$
A(1, L, H) = \left(\frac{1}{L} + \frac{1}{E[x]}\right)^{-1}, \quad A(1/2, L, H) = 0,
$$

and

$$
\begin{aligned}
\left(\frac{1}{L} + \frac{1}{E[x]}\right)^{-1} &< \left(\frac{2}{E[x]}\right)^{-1} = \frac{E[x]}{2}, \\
\left(\frac{1}{L} + \frac{1}{E[x]}\right)^{-1} &> \left(\frac{2}{L}\right)^{-1} = \left.\frac{E[x|\ell,\ell]}{2}\right|_{q=1}.
\end{aligned}
$$

This shows that $E[x] > 2A(q, L, H)$ for all q, and there is a threshold $\tilde{q} \in (0, 1)$ such that $E[x|\ell, \ell] > 2A(q, L, H)$ for $q < \tilde{q}$ and $E[x|\ell, \ell] < 2A(q, L, H)$ for $q > \tilde{q}$.

A.2 Proof of Lemma 3

Consider $\alpha(h)$ satisfying ($\mathrm{IC}_{h\ell}$) and ($\mathrm{IC}_{\ell h}$). We first show that if $E[x] > 2\delta W$, we can take $\alpha(h)$ to be smaller than $\alpha(\ell)$. It suffices to show that α_1 in Lemma 2 is smaller than $\alpha(\ell)$.

Indeed

$$\begin{aligned}
\alpha_1 - \alpha(\ell) &= \frac{\Pr(h|h)\alpha(\ell)E[x|h,h] + \Pr(\ell|h)\delta W}{E[x|h]} - (1 - \frac{\delta W}{E[x]}) \\
&= (\frac{\Pr(h|h)E[x|h,h]}{E[x|h]} - 1)(1 - \frac{\delta W}{E[x]}) + \frac{\Pr(\ell|h)\delta W}{E[x|h]} \\
&= (\frac{-\Pr(\ell|h)E[x]}{E[x|h]})(1 - \frac{\delta W}{E[x]}) + \frac{\Pr(\ell|h)\delta W}{E[x|h]} = \frac{\Pr(\ell|h)(-E[x] + 2\delta W)}{E[x|h]} < 0.
\end{aligned}$$

Next we show that we can take $\alpha(h)$ such that $(1 - \alpha(h))E[x|\ell,\ell] < \delta W \Leftrightarrow \alpha_2 > 1 - \delta W/E[x|\ell,\ell]$. We verify this inequality by establishing $\alpha_2 \geq 1 - \delta W/E[x|\ell]$ $(> 1 - \delta W/E[x|\ell,\ell])$. Indeed

$$\begin{aligned}
\alpha_2 - (1 - \frac{\delta W}{E[x|\ell]}) &= \frac{\Pr(h|\ell)\alpha(\ell)E[x] + \Pr(\ell|\ell)\delta W}{E[x|\ell]} - (1 - \frac{\delta W}{E[x|\ell]}) \\
&= \frac{\Pr(h|\ell)(E[x] - \delta W) + \Pr(\ell|\ell)\delta W}{E[x|\ell]} - (1 - \frac{\delta W}{E[x|\ell]}) \\
&= \frac{\Pr(h|\ell)E[x] - E[x|\ell] + 2\Pr(\ell|\ell)\delta W}{E[x|\ell]} = \frac{\Pr(\ell|\ell)(-E[x|\ell,\ell] + 2\delta W)}{E[x|\ell]} \geq 0.
\end{aligned}$$

A.3 Proof of Lemma 4

Step 1: *If an efficient equilibrium with $\alpha(h) > \alpha(\ell)$ exists, then $\alpha(h) = \alpha^*$ i.e., $(1 - \alpha(h))E[x] = \delta W$.*

Proof: Since the responder with ℓ signal must accept $\alpha(h)$ in efficient equilibrium, $(1 - \alpha(h))E[x] \geq \delta W$. Suppose $(1 - \alpha(h))E[x] > \delta W$ and consider $\alpha \in (\alpha(h), \alpha^*]$. Note that $(1-\alpha)E[x] > \delta W$ so that (i) if the responder believes the proposer has h signal, she accepts even if her signal is ℓ, and (ii) if the responder believes the proposer has ℓ signal, she accepts if and only if her signal is h. In other words, the responder's reaction to α is the same as when $\alpha(h)$ is offered (case (i)) or when $\alpha(\ell)$ is offered (case (ii)). This implies that either the proposer with h signal or the one with ℓ signal has an incentive to deviate to offer α, contradicting to equilibrium. Hence, $(1 - \alpha(h))E[x] = \delta W$.

Step 2: *If an efficient equilibrium with $\alpha(h) > \alpha(\ell)$ exists, then there is some $\alpha^{h\prime} < \alpha^*$ such that*

$$\alpha^{h\prime}E[x|h] \geq \Pr(h|h)\alpha^*E[x|h,h] + \Pr(\ell|h)\delta W, \qquad (\mathrm{IC}_{hl})$$

$$\alpha^{h\prime}E[x|\ell] \leq \Pr(h|\ell)\alpha^*E[x] \qquad + \Pr(\ell|\ell)\delta W. \qquad (\mathrm{IC}_{lh})$$

In other words, the nonmimicry conditions for an efficient equilibrium hold.

Proof: If an efficient equilibrium with $\alpha(h) = \alpha^* > \alpha(\ell)$ exists, we have the following nonmimicry conditions

$$\alpha^* E[x|h] \geq \Pr(h|h)\alpha(\ell)E[x|h,h] + \Pr(\ell|h)\delta W, \tag{$\mathrm{IC}_{hl}^{>}$}$$

$$\alpha^* E[x|\ell] \leq \Pr(h|\ell)\alpha(\ell)E[x] \qquad + \Pr(\ell|\ell)\delta W. \tag{$\mathrm{IC}_{lh}^{>}$}$$

Since $\alpha^* > \alpha(\ell)$, the latter inequality implies

$$\alpha^* E[x|\ell] < \Pr(h|\ell)\alpha^* E[x] \qquad + \Pr(\ell|\ell)\delta W.$$

Note also that $\alpha^* = 1 - \delta W/E[x] > 1/2$, since $E[x] > 2\delta W$. This implies

$$\begin{aligned}
\alpha^* E[x|h] &= \Pr(h|h)\alpha^* E[x|h,h] + \Pr(\ell|h)\alpha^* E[x] \\
&> \Pr(h|h)\alpha^* E[x|h,h] + \Pr(\ell|h)2\alpha^*\delta W \\
&> \Pr(h|h)\alpha^* E[x|h,h] + \Pr(\ell|h)\delta W
\end{aligned}$$

We then have the following inequalities:

$$\alpha^* E[x|h] > \Pr(h|h)\alpha^* E[x|h,h] + \Pr(\ell|h)\delta W,$$

$$\alpha^* E[x|\ell] < \Pr(h|\ell)\alpha^* E[x] \qquad + \Pr(\ell|\ell)\delta W.$$

Now we substitute the α^* in the left-hand sides of the inequalities with $\alpha^{h\prime}$. Since the inequalities are strict, slightly lowering $\alpha^{h\prime}$ does not change the inequalities. The resulting inequalities are indeed the nonmimicry conditions to be verified.

B Equilibrium for No Delay Schedule

Suppose that the discount factor is such that No Delay Schedule is efficient. To attain this outcome in equilibrium, the proposer's offer must be accepted regardless of his own and the responder's signals. The proposer's offer must be pooling, since otherwise he has an incentive to always make the more favorable claim of the possible offers. Let α^p be the proposer's pooling offer. The responder with ℓ signal accepts α^p if

$$(1 - \alpha^p)E[x|\ell] \geq \delta W(\delta, q, L, H) = \frac{\delta E[x]}{2}.$$

Among such shares, the best one for the proposer is the highest offer. As we will see in the following, the main problem in constructing an efficient equilibrium is to deter the proposer from making an off-path offer. Note that the higher the on-path pooling offer, the weaker the proposer's incentive to make an off-path offer. We thus focus on an equilibrium with the highest offer in order to weaken the condition to attain an efficient equilibrium:

$$(1-\alpha^p)E[x|\ell] = \delta W \Leftrightarrow \alpha^p = 1 - \frac{\delta E[x]}{2E[x|\ell]}.$$

We consider the following pessimistic off-path belief:

$$\beta(\alpha, s_r = \ell) = \begin{cases} 2q(1-q) & \text{if } \alpha \leq \alpha^p \\ 0 & \text{otherwise} \end{cases}, \quad \beta(\alpha, s_r = h) = \begin{cases} q^2 + (1-q)^2 & \text{if } \alpha \leq \alpha^p \\ 0 & \text{otherwise} \end{cases}.$$

Unlike the previous cases, the responder's on-path belief depends on her own signal due to the correlation of the signals. It is important to set zero belief for higher α to deter the proposer's deviation; as before, the responder needs to change her acceptance behavior for higher α. Note, however, that $(1-\alpha^p)E[x] > (1-\alpha^p)E[x|\ell] = \delta W$, so that the responder with h signal is willing to accept an α slightly higher than α^p even though the belief is pessimistic. Accordingly, the responder's optimal reaction to offer α is to accept always for $\alpha \leq \alpha^p$, to accept if the responder's signal is h for $\alpha \in (\alpha^p, \alpha^{**}]$ where $(1-\alpha^{**})\,E[x] = \delta W$, and to reject always for $\alpha > \alpha^{**}$ (note that $\alpha^{**} = 1 - \delta/2$ since $W = E[x]/2$).

Therefore, if an efficient equilibrium exists, it should take the following form:

- The proposer's offer strategy is $\alpha(h) = \alpha(\ell) = \alpha^p$.
- The responder's strategy is

$$\begin{aligned} &\text{for } \alpha > \alpha^*, \ \sigma(\alpha, s_r) = \text{reject for } s_r = h, \ell, \\ &\text{for } \alpha \in (\alpha^p, \alpha^{**}], \quad \sigma(\alpha, s_r) = \begin{cases} \text{accept} & \text{for } s_r = h, \\ \text{reject} & \text{for } s_r = \ell, \end{cases} \\ &\text{for } \alpha \leq \alpha^p, \qquad \sigma(\alpha, s_r) = \text{accept for } s_r = h, \ell. \end{aligned}$$

- The belief is the above pessimistic belief system.

Proposer's Off-Path Incentive Condition Since the responder's reaction is constant over each of the intervals, $[0, \alpha^p], (\alpha^p, \alpha^{**}]$, and $(\alpha^{**}, 1]$, the offers other than α^p, α^{**}, and 1 are (weakly) dominated. In addition, due to the fact that

$$\alpha^p E[x|\ell] = E[x|\ell] - \delta W > E[x|\ell, \ell] - \delta W \geq 2\delta W - \delta W = \delta W,$$

offer $\alpha = 1$ is also dominated by α^p.

To deter deviation to α^{**}, the following conditions must hold:

$$\alpha^p E[x|h] \geq \Pr(h|h)\alpha^{**}E[x|h,h] + \Pr(\ell|h)\delta W, \tag{IC$_h$}$$

$$\alpha^p E[x|\ell] \geq \Pr(h|\ell)\alpha^{**}E[x|h,\ell] + \Pr(\ell|\ell)\delta W. \tag{IC$_l$}$$

The second constraint, (IC$_\ell$), is indeed equivalent to the condition under which No Delay Schedule is efficient:

$$\left(1 - \frac{\delta W}{E[x|\ell]}\right) E[x|\ell] \geq \Pr(h|\ell)E[x]\left(1 - \frac{\delta W}{E[x]}\right) + \Pr(\ell|\ell)\delta W$$
$$\Leftrightarrow \delta W\left(-\Pr(h|\ell) + 1 + \Pr(\ell|\ell)\right) \leq -\Pr(h|\ell)E[x] + E[x|\ell] \Leftrightarrow 2\delta W \leq E[x|\ell,\ell].$$

On the other hand, the first condition, (IC$_h$), does not always hold even when No Delay Schedule is efficient. The following technical lemma gives the requirement for (IC$_h$):

Lemma 5 *(IC$_h$) holds iff Condition B is satisfied, i.e., $\delta W \leq B(q,L,H)$, where*

$$B(q,L,H) = \frac{H+L}{2}\left(2 + \frac{2q-1}{2q(1-q)}\frac{H-L}{H+L}\frac{qH+(1-q)L}{qL+(1-q)H}\right)^{-1},$$

with the following properties:

1. $2B(1,L,H) = 0$, *and* $2B(1/2,L,H) = E[x] = E[x|\ell,\ell]|_{q=1/2}$
2. *If* $L > 0$, *then there is a* $\tilde{q}$ *such that* $2B(q,L,H) > E[x|\ell,\ell]$ *for* $q \in (1/2,\tilde{q})$, *and* $2B(q,L,H) < E[x|\ell,\ell]$ *for* $q > \tilde{q}$.
3. *If* $L = 0$, $2B(q,L,H) \geq E[x|\ell,\ell]$ *for all* q *(equality holds for* $q = 1/2$, 1*).*

Proof. Rearranging (IC$_h$), we have

$$(1 - \delta W/E[x|\ell])\, E[x|h] \geq (1 - \delta W/E[x])\Pr(h|h)E[x|h,h] + \Pr(\ell|h)\delta W$$
$$\Leftrightarrow \delta W\left(\frac{E[x|h] - \Pr(\ell|h)E[x]}{E[x]} - \frac{E[x|h]}{E[x|\ell]} - \Pr(\ell|h)\right) \geq -\Pr(\ell|h)E[x]$$
$$\Leftrightarrow \delta W \leq \frac{\Pr(\ell|h)E[x]}{2\Pr(\ell|h) - E[x|h](1/E[x] - 1/E[x|\ell])}$$

Define

$$
\begin{aligned}
B(q,L,H) &= \frac{\Pr(\ell|h)E[x]}{2\Pr(\ell|h) - E[x|h]\,(1/E[x] - 1/E[x|\ell])} \\
&= \frac{q(1-q)(H+L)}{4q(1-q) - (qH+(1-q)L))(2/(H+L) - 1/(qL+(1-q)H))} \\
&= \frac{H+L}{2}\left(2 + \frac{2q-1}{2q(1-q)}\frac{H-L}{H+L}\frac{qH+(1-q)L}{qL+(1-q)H}\right)^{-1}
\end{aligned}
$$

It is easy to see

$$
\begin{aligned}
B(1,L,H) &= 0 \\
B(1/2,L,H) &= \frac{E[x]}{2} = \left.\frac{E[x|\ell,\ell]}{2}\right|_{q=1/2}.
\end{aligned}
$$

By definition,

$$
\begin{aligned}
&2B(q,L,H) - E[x|\ell,\ell] \\
&= \frac{2\Pr(\ell|h)E[x]}{2\Pr(\ell|h) - E[x|h]\,(1/E[x] - 1/E[x|\ell])} - E[x|\ell,\ell] \\
&= \frac{2E[x]^2\Pr(\ell|h)E[x|\ell] - E[x|\ell,\ell][2E[x]\Pr(\ell|h)E[x|\ell] - E[x|h]\,(E[x|\ell] - E[x])]}{2E[x]\Pr(\ell|h)E[x|\ell] - E[x|h]\,(E[x|\ell] - E[x])}
\end{aligned}
$$

Note that the denominator is positive for all q. The numerator is

$$
\begin{aligned}
&2E[x]^2\Pr(\ell|h)E[x|\ell] - E[x|\ell,\ell][2E[x]\Pr(\ell|h)E[x|\ell] - E[x|h]\,(E[x|\ell] - E[x])] \\
&= 2E[x]\Pr(\ell|h)E[x|\ell][E[x] - E[x|\ell,\ell]] - E[x|\ell,\ell]E[x|h]\,(E[x] - E[x|\ell])] \\
&= (H+L)2q(1-q)(qL+(1-q)H)[\frac{H+L}{2} - \frac{q^2L+(1-q)^2H}{q^2+(1-q)^2}] \\
&- \frac{q^2L+(1-q)^2H}{q^2+(1-q)^2}(qH+(1-q)L)[\frac{H+L}{2} - (qL+(1-q)H)] \\
&= \frac{(2q-1)(H-L)}{2(q^2+(1-q)^2)}[2(H+L)q(1-q)(qL+(1-q)H) - (q^2L+(1-q)^2H)(qH+(1-q)L)]
\end{aligned}
$$

The fraction in the last line is positive for $q > 1/2$. If $q = 1/2$, the expression in the square bracket is

$$
2(H+L)\frac{1}{4}\frac{H+L}{2} - \frac{H+L}{4}\frac{H+L}{2} = \frac{(H+L)^2}{8} > 0
$$

The expression also can be rewritten as

$$\begin{aligned}&2(H+L)q(1-q)(qL+(1-q)H)-(q^2L+(1-q)^2H)(qH+(1-q)L)\\&=((1-q)H^2+qL^2+5HL)q(1-q)-HL\end{aligned}$$

Since $((1-q)H^2+qL^2)$ and $q(1-q)$ are decreasing for $q \in (1/2,1)$, the expression is monotone decreasing in q and goes to $-HL$. Therefore, as long as $L>0$, there is $\tilde{q} \in (1/2,1)$ such that $2B(q,L,H) < E[x|\ell,\ell]$ iff $q > \tilde{q}$. ■

The condition $\delta W \leq B(q,L,H)$ and the properties of $B(q,L,H)$ provide information about when No Delay Schedule is efficient but (IC_h) fails (see also Figure 5 below). If $L=0$, it never fails since $2B(q,L,H) \geq E[x|\ell,\ell] \geq 2\delta W$ always holds under No Delay Schedule being efficient. However, if $L>0$, there is a $\tilde{q}$ such that $q>\tilde{q} \Rightarrow 2B(q,L,H) < E[x|\ell,\ell]$; hence, $2B(q,L,H) < 2\delta W \leq E[x|\ell,\ell]$ can arise.

The properties of $B(q,L,H)$ imply that (IC_h) would be harder to hold as δ becomes higher, and the threshold of δ decreases as q increases. Intuitively, if the signal accuracy is very high, the proposer with h signal would expect that it is very likely that the responder has the same signal and accepts the most profitable deviation offer $\alpha^{**} > \alpha$. To deter such deviation, the difference between $\alpha^{**} - \alpha^p = \delta(E[x]/E[x|\ell]-1)/2$ must be small enough. This is why δ needs to be small (this also makes forgoing-till-the-next-period more costly, which contributes to (IC_h) being satisfied). Moreover, the lemma confirms that if q is above a certain level, the threshold of δ is below the level for which No Delay Schedule is efficient. This immediately implies that a fully efficient equilibrium for No Delay Schedule fails to exist if the signal accuracy is relatively high and if the discount factor is not too low.

Proposition 2 (Part 3, Restatement) *Suppose $\delta \in [0, \underline{\delta}(q, L/H)]$ (No Delay Schedule is efficient). The strategy profile and the belief system defined above compose an efficient equilibrium if and only if Condition B holds, i.e., $\delta W \leq B(q,L,H)$.*

We plot the diagram for $L/H = .6$, in which $\delta W \leq B(q,L,H)$ holds in the area below the dotted (red) curve (Condition B in Figure 5). An efficient No Delay equilibrium does not exist in the region below the dashed (blue) curve and above the dotted (red) curve. Therefore, efficiency is not always attainable. As the discount factor decreases and/or the signal accuracy increases, the benefit to the proposer with a high signal of using signal-responsive offers becomes larger. Hence, the deviation temptation from a pooling offer is too high to sustain the efficient pooling equilibrium.

Consider the perfect information case, $q=1$, in which efficiency is obtained by separating offers conditional on the signals. Notice that this efficient equilibrium cannot be achieved in the limit of efficient equilibria as $q \to 1$. Instead, a sequence of separating equilibria can

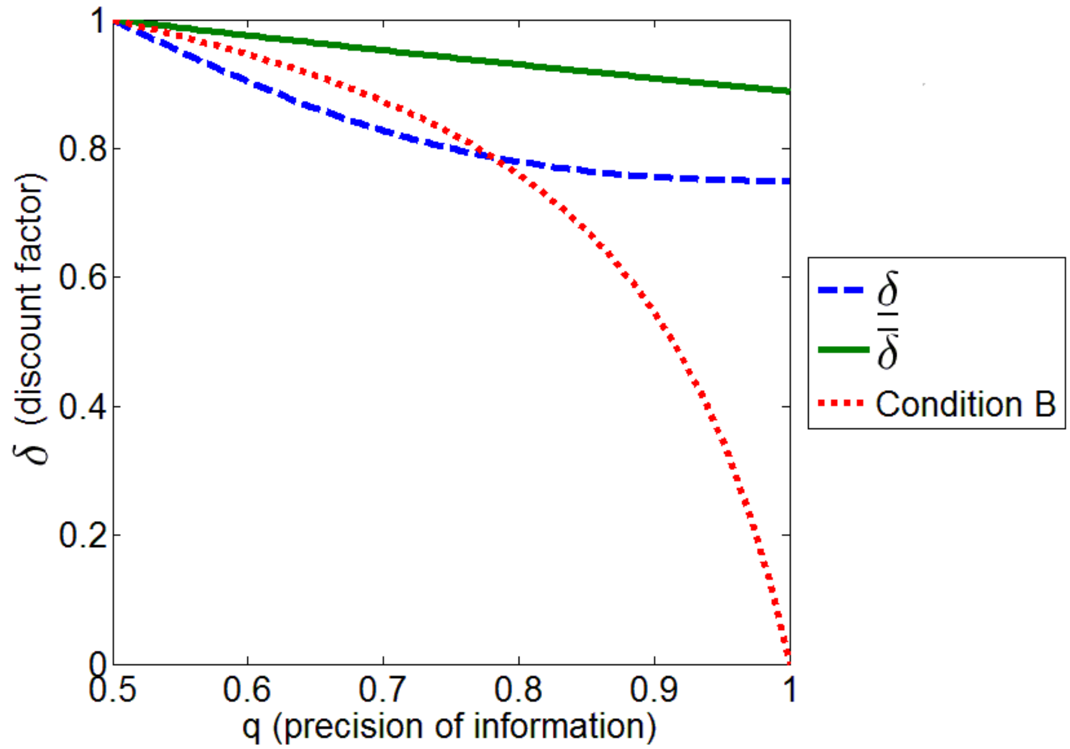

Figure 5: The Region for Efficient Equilibria for No Delay Schedule

be made arbitrarily close to the efficient equilibrium by the approximate efficiency result in the following.

Comparative Statics for No Delay Schedule For an efficient equilibrium under No Delay Schedule, we only consider how the proposer's offer α^p is affected by signal accuracy (q) and the relative size of the pie (L/H). We do not consider the effect on time to agreement because the players always agree immediately in an efficient equilibrium under No Delay Schedule (remember that the discount factor was too low so the players prefer to agree immediately under No Delay Schedule). The equilibrium offer α^p is determined by $(1-\alpha^p)E[x|\ell] = \delta W$, which can be rewritten as

$$\alpha^p = 1 - \frac{\delta E[x]}{2E[x|\ell]} = 1 - \delta \frac{L/H + 1}{2(qL/H + 1 - q)}.$$

Hence, the equilibrium offer α^p is decreasing in q and increasing in L/H. Continuation value W increases in the size of L given H, and is constant in q, whereas the expected pie size, $E[x|\ell]$, is increasing in the size of L given H, and decreasing in q. This shows why α^p

is decreasing in q. The intuition for α^p increasing in L/H is analogous to that for Medium Delay Schedule.

Approximate Efficiency of No Delay Schedule For the case of approximate efficiency in No Delay Schedule, consider a separating equilibrium in which the proposer offers $\alpha(h) > \alpha(\ell)$, and the responder accepts $\alpha(h)$ if she receives h signal and always accepts $\alpha(\ell)$. The ex ante value from this agreement schedule is

$$V_s = \frac{1}{2}(E[x] - \Pr(h,\ell)(E[x] - 2\delta V_s)) = \frac{1-\Pr(h,\ell)}{1-\delta\Pr(h,\ell)}\frac{E[x]}{2}.$$

The responder's incentive conditions are

$$\begin{aligned}(1-\alpha(h))E[x|h,h] \geq \delta V_s \geq (1-\alpha(h))E[x],\\ (1-\alpha(\ell))E[x|\ell,\ell] \geq \delta V_s.\end{aligned}$$

Again we assume the pessimistic belief for off-path offers.

The proposer's off-path incentive condition implies $(1-\alpha(\ell))E[x|\ell,\ell] \leq \delta V_s$; otherwise the proposer can deviate to offer α',which is slightly higher than $\alpha(\ell)$ but still acceptable. Hence,

$$\alpha(\ell) = 1 - \frac{\delta V_s}{E[x|\ell,\ell]}.$$

We consider the case in which the responder receives no rent, i.e.,

$$(1-\alpha(h))E[x|h,h] = \delta V_s \Leftrightarrow \alpha(h) = 1 - \frac{\delta V_s}{E[x|h,h]}.$$

Consider the nonmimicry conditions:

$$\begin{aligned}\Pr(h|h)\alpha(h)E[x|h,h] + \Pr(\ell|h)\delta V_s \geq \alpha(\ell)E[x|h]\\ \Pr(h|\ell)\alpha(h)E[x] + \Pr(\ell|\ell)\delta V_s \leq \alpha(\ell)E[x|\ell]\end{aligned}$$

The first IC is redundant for a sufficiently large q, since $q \to 1$ implies

$$\begin{aligned}\alpha(h)E[x|h,h] &= (1 - \frac{\delta V_s}{E[x|h,h]})E[x|h,h] = H - \delta\frac{H+L}{4}\\ &> H - \delta\frac{H+L}{4L}H = (1 - \frac{\delta V_s}{E[x|\ell,\ell]})E[x|h] = \alpha(\ell)E[x|h].\end{aligned}$$

The second one can be rewritten as

$$\delta\frac{H+L}{4} \le L - \delta\frac{H+L}{4} = \alpha(\ell)E[x|\ell] \Leftrightarrow \delta \le \frac{L}{(H+L)/2} = \frac{E[x|\ell,\ell]}{E[x]}.$$

An argument analogous to approximate efficiency for Medium Delay Schedule shows that as q goes to 1, an approximately efficient equilibrium exists.

C Discussion of Off-Path Beliefs

We examine whether the off-path beliefs of the above equilibria are reasonable in line with the D1 criterion of Banks and Sobel (1987). Two remarks are noted. First, the game is not a standard signaling game in which each player can move only once. However, the information in our model is not persistent, so we can still apply the refinement argument within a period (at least for a stationary equilibrium). Second, we are dealing with two-sided incomplete information with correlation. We need to consider a belief system conditional on the responder's type. To incorporate correlation, we adopt the following "two-stage" beliefs.

Let $\beta(\alpha)$ denote the belief that the proposer has h signal given offer α *ignoring the responder's signal* (think of it as an outsider's belief). Suppose α is offered with some probability. By Bayes' rule

$$\beta(\alpha) = \frac{\Pr(\alpha|s_p = h)\Pr(s_p = h)}{\Pr(\alpha|s_p = h)\Pr(s_p = h) + \Pr(\alpha|s_p = \ell)\Pr(s_p = \ell)} = \frac{\Pr(\alpha|h)}{\Pr(\alpha|h) + \Pr(\alpha|\ell)}.$$

For example, with a pooling offer α, i.e., $\Pr(\alpha|h) = \Pr(\alpha|\ell) = 1$, we have $\beta(\alpha) = .5$. With a separating offer $\alpha(h), \alpha(\ell)$, i.e., $\Pr(\alpha(h)|h) = \Pr(\alpha(\ell)|\ell) = 1$, we have $\beta(\alpha(h)) = 1$ and $\beta(\alpha(\ell)) = 0$. Note that

$$\begin{aligned}
\Pr(s_p = h, \alpha|s_r = h) &= \Pr(\alpha|s_p = h, s_r = h)\Pr(s_p = h|s_r = h) \text{ (Bayes' rule)} \\
&= \Pr(\alpha|s_p = h)\Pr(s_p = h|s_r = h) \text{ (the choice of } \alpha \text{ only depends on } s_p)
\end{aligned}$$

Responder's belief $\beta(\alpha, s_r)$ can be induced by $\beta(\alpha)$:

$$
\begin{aligned}
\beta(\alpha, h) &= \frac{\Pr(h|\alpha, s_r = h)}{\Pr(h|\alpha, s_r = h) + \Pr(\ell|\alpha, s_r = h)} \\
&= \frac{\Pr(h, \alpha|s_r = h)}{\Pr(h, \alpha|s_r = h) + \Pr(\ell, \alpha|s_r = h)} \text{ (Bayes' rule + eliminating } \Pr(\alpha|s_r = h)) \\
&= \frac{\Pr(h|s_r = h)\Pr(\alpha|s_p = h)}{\Pr(h|s_r = h)\Pr(\alpha|s_p = h) + \Pr(\ell|s_r = h)\Pr(\alpha|s_p = \ell)} \text{ (see the above note)} \\
&= \frac{\Pr(h|h)\beta(\alpha)}{\Pr(h|h)\beta(\alpha) + \Pr(\ell|h)(1-\beta(\alpha))} \\
\beta(\alpha, \ell) &= \frac{\Pr(h|\ell)\beta(\alpha)}{\Pr(h|\ell)\beta(\alpha) + \Pr(\ell|\ell)(1-\beta(\alpha))}
\end{aligned}
$$

This is consistent with the on-path belief of pooling, $\beta(\alpha, h) = q^2 + (1-q)^2$. We apply the above formulas for off-path beliefs given an arbitrary $\beta(\alpha)$. For notational simplicity, we denote β for $\beta(\alpha)$, $\beta^h = \beta(\alpha, h)$, and $\beta^\ell = \beta(\alpha, \ell)$. Straightforward algebra shows that $\beta^h \geq \beta^\ell$ for all $\beta \in [0,1]$, where equality holds at $\beta = 0, 1$. There are three possible best responses by the responder given α, β and W: (i) if

$$(1-\alpha)\left[\beta^\ell E[x] + (1-\beta^\ell)E[x|\ell,\ell]\right] \geq \delta W,$$

the responder always accepts, (ii) if

$$(1-\alpha)\left[\beta^\ell E[x] + (1-\beta^\ell)E[x|\ell,\ell]\right] < \delta W \leq (1-\alpha)\left[\beta^h E[x|h,h] + (1-\beta^h)E[x]\right]$$

only the responder with h signal accepts, and (iii) if

$$(1-\alpha)\left[\beta^h E[x|h,h] + (1-\beta^h)E[x]\right] < \delta W$$

the responder always rejects.

Now we examine the plausibility of pessimistic belief. A version of D1 criterion for our model can be stated as follows (see Fudenberg and Tirole, 1991, p.452): *fix an equilibrium and off-path offer α. Suppose that for each $\beta \in [0,1]$ such that the proposer with h* signal *(ℓ) is weakly better off by offering α than the equilibrium offer, $\alpha(h)$ ($\alpha(\ell)$), the proposer with ℓ* signal *(h) is always strictly better off by offering α than by offering the equilibrium offer, $\alpha(\ell)$ ($\alpha(h)$). Suppose, in addition, that there is β' for which only the proposer with ℓ* signal *(h) is strictly better off, then the belief after observing α should be $\beta = 0$ ($\beta = 1$). An equilibrium passes the D1 test if all the off-path beliefs satisfy this criterion.*

Efficient Equilibrium for High Patience (High Delay Schedule). First, note that in addition to the equilibrium discussed in Section 3.1.1., there are other equilibria:

the strategy profile and the belief system are the same as in the equilibrium in Section 3.1.1 except for $\alpha(h)$. We can show that if $\alpha(h)$ satisfies

$$(1-\alpha(h))E[x|h,h] \geq \delta W \geq (1-\alpha(h))E[x],$$
$$\alpha(h)E[x|h,h] \geq \delta W \geq \alpha(h)E[x],$$

the associated strategy profile and the belief system compose an equilibrium (note that these conditions hold for $\alpha(h) = 1/2$).[25]

We show that *for the highest α satisfying the above conditions, the associated equilibrium with such α passes the D1 test.* The highest α satisfies either $(1-\alpha)E[x|h,h] = \delta W$ or $\alpha E[x] = \delta W$. Consider the case where $(1-\alpha)E[x|h,h] = \delta W$ and $\alpha E[x] < \delta W$. The proposer obtains all the rent from the efficient agreement, and therefore the proposer with h signal cannot be better off by any offer given that the responder receives at least the reservation payoff. This implies that any belief system passes the D1 test.

Consider the case where $\alpha E[x] = \delta W$ and $(1-\alpha)E[x|h,h] > \delta W$. For offer α' such that $(1-\alpha')E[x|h,h] < \delta W$, the responder rejects the offer for any belief; hence the D1 test does not impose any restriction on the belief for such offer.

Consider offer $\alpha' > \alpha$ for which $(1-\alpha')E[x|h,h] \geq \delta W$. Note that $\alpha' > \alpha$ implies $\delta W \geq (1-\alpha')E[x]$. If β is high enough, the responder with h signal would accept α', and hence the proposer with h signal will be strictly better off. However, with such a belief, the proposer with ℓ signal would always be better off than the equilibrium payoff, since $\alpha' E[x] > \alpha E[x] = \delta W$. Hence, $\beta = 0$ is consistent with the D1 test.

Consider offer $\alpha' < \alpha$. For such α', the payoff of the proposer with ℓ signal would be $\alpha' E[x|\ell] < \alpha' E[x] < \alpha E[x] = \delta W$ if the responder always accepts, and $\alpha' \Pr(h|\ell)E[x] + \Pr(\ell|\ell)\delta W < \delta W$. This shows that the proposer with ℓ signal is always worse off. Thus $\beta = 1$ for offer $\alpha' < \alpha$ is consistent with the D1 test.

Efficient Equilibrium for Medium Patience (Medium Delay Schedule). We show that *the equilibrium passes the D1 test iff* $(IC_{\ell h})$ *is binding.*

Consider offer $\alpha'(< \alpha(h))$ for which the equilibrium belief is $\beta = 1$ and the responder always accepts. With any belief, the proposer is never better off by offering α'; if the belief is so high that the responder always accepts, this is the same action as in the equilibrium and the off-path ICs imply that offering α' is never profitable. If the belief is such that only the responder with h signal accepts, this offer is dominated by offering $\alpha(\ell)$, and therefore the proposer is never better off. This shows $\beta = 1$ for $\alpha' < \alpha$ is consistent with the D1 test.

Consider offer $\alpha' > \alpha(\ell)$. If $(1-\alpha')E[x|h,h] < \delta W$,the responder never accepts the offer,

[25] These are the incentive conditions on the equilibrium paths. The argument to verify the off-path incentive conditions is analogous to the one in Section 3.1.1.

regardless of the beliefs; hence, any beliefs are consistent with the D1 test. Now suppose $(1-\alpha')E[x|h,h] \geq \delta W$. If β is sufficiently high, the responder with h signal accepts it; hence, the proposer with ℓ signal strictly prefers such α' to $\alpha(\ell)$, whereas the proposer with h signal may not. Therefore $\beta = 0$ for α' is consistent with the D1 test.

Consider offer α' such that $\alpha(h) < \alpha' < \alpha(\ell)$. Since $(1-\alpha(\ell))E[x] = \delta W$, we have $(1-\alpha')E[x] > \delta W$, implying that the responder always accepts α' if β is sufficiently high. Consider an equilibrium for which ($\text{IC}_{\ell h}$) binds (it is easy to see that such an equilibrium exists if there is an $\alpha(h)$ satisfying both ($\text{IC}_{h\ell}$) and ($\text{IC}_{\ell h}$)), namely

$$\alpha(h)E[x|\ell] = \Pr(h|\ell)\alpha(\ell)E[x] + \Pr(\ell|\ell)\delta W.$$

Given this condition, the proposer regardless of the signal is strictly better off by offering α' if offer α' is always accepted, while he is worse off if offer α' is not always accepted (by the off-path incentive conditions). Hence, $\beta = 0$ for α' is consistent with the D1 test. On the other hand, for an equilibrium with ($\text{IC}_{\ell h}$) being slack, there is an α'' close to $\alpha(h)$ such that the proposer with h signal is better off by offering α'' if offer α'' is always accepted, while the proposer with ℓ signal is not, implying that $\beta = 1$ for offer α'' under the D1 criterion. Hence the equilibrium in which ($\text{IC}_{\ell h}$) binds only survives the D1 test.

Efficient Equilibrium for Low Patience (No Delay Schedule). Indeed, *some (but not all) efficient equilibrium fail to pass the D1 test.* If off-path offer $\alpha' > \alpha$ is always accepted, the proposer wishes to deviate regardless of its signal. Consider the case where $\alpha' > \alpha$ is accepted only by the responder with h signal. We check whether the proposer with h signal is better off by offering α' than with the equilibrium offer.

Consider offer α' for which $(1-\alpha')E[x|h,h] = \delta W$. Note that a higher offer than α' is always rejected. If $\beta = 1$, the proposer with h signal is better off by offering α' than with the equilibrium offer α iff (recalling that $\alpha = 1 - \delta W/E[x|\ell]$)

$$\begin{aligned}
&\alpha E[x|h] \leq \alpha' \Pr(h|h)E[x|h,h] + \Pr(\ell|h)\delta W \\
&\Leftrightarrow (1-\delta W/E[x|\ell])\, E[x|h] \leq \Pr(h|h)E[x|h,h] + (2\Pr(\ell|h) - 1)\delta W \\
&\Leftrightarrow \Pr(\ell|h)E[x] \leq (E[x|h]/E[x|\ell] + 2\Pr(\ell|h) - 1)\, \delta W \\
&\Leftrightarrow \delta W \geq \frac{\Pr(\ell|h)E[x]}{2\Pr(\ell|h) + E[x|h](1/E[x|\ell] - 1/E[x|h])}.
\end{aligned}$$

If this does not hold, $\beta = 0$ for α' is consistent with the D1 test.

Recall (IC_h) for this equilibrium:

$$\delta W \leq \frac{\Pr(\ell|h)E[x]}{2\Pr(\ell|h) + E[x|h](1/E[x|\ell] - 1/E[x])}$$

Since $-1/E[x] < -1/E[x|h]$, the above two inequalities are both satisfied in some parameter region; hence, there is some region in which the associated efficient equilibrium fails to pass the D1 test. Note also that if q is close to 1, the proposer with ℓ signal is nearly certain that the responder receives ℓ signal, so offer α' will be rejected even if $\beta = 1$. Hence, the proposer with ℓ signal is worse off by offering α' rather than by α (we could proceed to the tedious details for this argument).