Effects of modified gravity on the turnaround radius in cosmology

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(Received 5 June 2018; published 2 July 2018)

We revisit the concept of turnaround radius in cosmology, in the context of modified gravity. While preliminary analyses were limited to scalar-tensor/$F(R)$ gravity, we extend the definition and the study of this quantity to a much broader class of theories including also quantum R^2 gravity. The turnaround radius is computed in terms of the parameters of the theory, and it is shown that a deviation not larger than 10% of this quantity from its value in Einstein's theory could constrain the model parameters and even rule out some current theories.

DOI: 10.1103/PhysRevD.98.024005

I. INTRODUCTION

Modifying general relativity (GR) is a necessity from the theoretical physics point of view. In fact, virtually all attempts to quantize GR modify the Einstein-Hilbert action by adding extra dynamical fields or nonlocal terms, or by introducing higher order derivatives in the field equations (see [1] for review). These corrections are not necessarily Planck-scale suppressed. For example, the simplest string theory, the bosonic string theory, reduces to an $\omega = -1$ Brans-Dicke gravity in its low-energy limit [2,3], and the $F(R)$ theories of gravity that are nowadays popular to explain away dark energy are nothing but scalar-tensor theories in disguise [4–10].

A significant body of experimental efforts aiming to test gravity at all possible astrophysical and cosmological scales has emerged in the past decade. There is little doubt, however, that the main motivation to question GR comes from cosmology. The present acceleration of the universe discovered in 1998 with type Ia supernovae requires an explanation. While a cosmological constant Λ offers a possible explanation in principle, it is peppered with enormous fine-tuning problems, which has led to the introduction of the completely *ad hoc* concept of dark energy (see [11] for a review). Many authors, dissatisfied with these approaches, have turned to the possibility of modifying gravity at large scales [12,13] (see also [4–10,14] for reviews). Modulo some fine-tuning, the idea works in principle, but many modified gravity models (and many dark energy models as well) fit the observational data. Therefore, one would like to avail oneself of all the tests of gravity that become available, at all scales and in all regimes, to obtain the correct scenario. In this context, the turnaround radius may be useful.

The concept of turnaround radius has been around for many years under various names (see, e.g., [15–22]): radius of the "zero velocity surface" or of the "effective sphere of influence of a cosmic structure," "zero gravity radius," "critical radius," "maximum size of large scale structures," "maximum size of bound cosmic structures," and "maximum turnaround radius." The literature seems to have settled on the term "turnaround radius," which we adopt. Its study has emerged only recently as a possible way to test dark energy in GR by comparing theoretical predictions with astronomical observations [23–25].

In an accelerating Friedmann-Lemaître-Robertson-Walker (FLRW) universe, there is a maximum physical (areal) radius, called *turnaround radius* r_{TA} such that any spherical shell of dust (composed of test particles following radial timelike geodesics) located outside r_{TA} and given zero radial velocity initially cannot collapse but is forced to expand forever by the cosmic acceleration. A similar dust shell located inside the turnaround radius, instead, will collapse. The turnaround radius constitutes the maximum possible radius of a bound structure in an accelerating FLRW universe. Early comparisons of the theoretical turnaround radius in the GR-based ΛCDM model with celestial objects have been carried out [23–25] but the astronomical error is quite large. Nevertheless, the method is quite promising in principle. Within GR, the concept of the turnaround radius has been made precise, more rigorous, and gauge invariant (to first order in the perturbations of an exact FLRW cosmos) by using an approach [26,27] based on the Hawking-Hayward quasilocal energy [28–30]. The numerical value of the turnaround radius estimated in this way, however, turns out to be quite close to that estimated with the previous method [26,27]. The definition of quasilocal energy in GR is not unique (see [31] for a review), but it is reassuring that the Hawking-Hayward construct and the Brown-York quasilocal mass (in an appropriate gauge) provide the same answer to first order in the cosmological perturbations [32] (higher order calculations are futile in view of the large observational errors in the determination of the turnaround radius).

Even more interesting is the fact that the turnaround radius can, in principle, be used to discriminate between GR and alternative theories of gravity. Preliminary analyses of the turnaround radius in scalar-tensor and $F(R)$ gravity and others were performed in Refs. [33–36]. Unfortunately, the status of quasilocal energy (which is already nonunique in GR [31]) is not clear in modified gravity, in spite of some attempts to generalize this definition within the restricted context of scalar-tensor theories [37–42]. Therefore, in modified gravity one is forced, at least for the moment, to give up the quasilocal energy approach and to pursue other approaches. Recently, it was reported that the upper bound set by GR on the turnaround radius is significantly exceeded in the galaxy group NGC 5353/4 [43,44]. The need to take into account the error introduced by the nonsphericity of the system has been emphasized [45], together with the fact that one should expect a distribution of the value of the turnaround radius among different astronomical systems and, therefore, an excess in this quantity would be significant from the statistical point of view rather than for individual systems [46,47]. Currently, the observational search is focusing on galaxy groups with weblike structures in their neighboring zones, and six more groups exceeding the general-relativistic prediction for the turnaround radius have been reported [47]. In view of these very promising observational developments and of the potential consequences as a probe of the correct theory of gravity it is worth studying this subject more in depth.

In GR and in an asymptotically de Sitter spacetime the turnaround radius depends on the cosmological constant, the gravitational coupling, and the mass contained inside this radius. It is important to realize that, in modified gravity, both the effective cosmological constant and the gravitational coupling are changed from their GR values [27,33]. Therefore, by comparing the turnaround radius in modified gravity theories with the size of large scale structures, we constrain the modified gravity theory.

Lacking a clear concept of quasilocal energy when we leave the GR context, as already noted, we resort to a different definition of turnaround radius than the one of [27]. The idea is that, at the turning radius, the gravitational force balances the inertial force generated by the accelerating expansion of the universe. Here, we consider the generalization of the turnaround radius for $F(R)$ gravity and $R + R^2 + R_{\mu\nu}R^{\mu\nu}$ gravity and its generalizations (like one-loop corrected quantum R^2 gravity). In Ref. [34], only the asymptotically de Sitter spacetime background was considered, where the effective cosmological constant and the effective gravitational coupling are constant. In this paper instead, we consider power-law expansion in $F(R)$ gravity, where the effective gravitational "constant" is time dependent and therefore the expression of the turnaround radius changes from that in GR coupled with a cosmological fluid. Even in the case of $R + R^2 + R_{\mu\nu}R^{\mu\nu}$ gravity, which includes the square of the Ricci tensor introducing new degrees of freedom, the Schwarzschild–de Sitter spacetime is an exact solution. We investigate the possible observational constraints on the parameters of the models coming from the turnaround radius. A problem arising in these models is that we observe the effective coupling constant as defined by Newton's law and, therefore, it is difficult to distinguish the modified gravity theory from Einstein gravity in the de Sitter spacetime background using the turnaround radius. This fact tells us that we need to find the coupling constant in the Einstein-Hilbert term with independent methods.

The plan of this paper is as follows. In the next section, we review the turnaround radius and discuss the case of $F(R)$ gravity, especially in a power-law expanding universe. Section III derives general formulas for a broad class of gravitational theories, while Sec. IV focuses on a particular model of R^2 gravity including the Ricci-squared term. Section V constrains a yet more general class of models that represent one-loop corrected R^2 gravity, and Sec. VI contains a discussion and the conclusions. We use the metric signature $-+++$ and units in which the speed of light c assumes the value unity. G_N is Newton's constant, and otherwise we follow the notation of Ref. [48].

II. GENERALIZATION OF TURNAROUND RADIUS

For the spherical and (locally) static Schwarzschild-like metric written in curvature coordinates

$$ds^2 = -A(r)dt^2 + \frac{1}{A(r)}dr^2 + r^2 d\Omega_{(2)}^2, \qquad (2.1)$$

where $d\Omega_{(2)}^2 = d\theta^2 + \sin^2\theta d\varphi^2$ is the line element on the unit 2-sphere, the turnaround radius r_{TA} (an areal radius) is defined by $r = r_{\mathrm{TA}}$ which satisfies the condition [34]

$$0 = A'(r_{\mathrm{TA}}). \qquad (2.2)$$

This is because $A(r)$ is related with the effective gravitational potential ϕ by

$$A(r) = 1 + 2\phi. \tag{2.3}$$

In particular, in the case of the Schwarzschild–de Sitter spacetime,

$$A(r) = 1 - \frac{2G_N M}{r} - \frac{r^2}{l^2}, \tag{2.4}$$

with Newton's gravitational constant G_N, the de Sitter length parameter l, and the mass M of the gravitational source, we find [24,25]

$$r_{TA}^3 = G_N M l^2. \tag{2.5}$$

Following previous literature, we discuss a spherical inhomogeneity embedded in a spatially flat FLRW background universe with line element

$$ds^2 = -dt^2 + a^2(t)(dr^2 + r^2 d\Omega_{(2)}^2) \tag{2.6}$$

and scale factor $a(t)$. The turnaround radius can be regarded as the radius where the gravitational force

$$F_g = \frac{G_N m M}{r^2}, \tag{2.7}$$

acting on a test mass m or an observer, balances the inertial force generated by the expansion of the universe and discussed for the big rip [49] and the little rip [50,51],

$$F_{in} = rm\frac{\ddot{a}}{a} = rm(\dot{H} + H^2). \tag{2.8}$$

Here H is the Hubble rate, $H \equiv \dot{a}/a$ where an overdot denotes differentiation with respect to the comoving time t of the FLRW background. In the case of the de Sitter universe, where $H = 1/l$, the equation expressing the balance $F_g = F_{in}$ reproduces the well known result of Eq. (2.5). In a more general expanding universe, we obtain the following expression of r_{TA}:

$$r_{TA}^3 = \frac{G_N M}{\dot{H} + H^2}. \tag{2.9}$$

Equation (2.9) can be used, for example, in the universe with the power-law expansion. This criterion for the turnaround radius is conceptually different from previous definitions given in the literature [24,25,27] in the context of GR, although the numerical value of this quantity can be numerically close to that computed with other definitions in some physically interesting situations.

As is clear from the expression of the inertial force (2.8), F_{in} is repulsive in an accelerating expanding universe, $\ddot{a} > 0$, but in a decelerating expanding universe, $\ddot{a} < 0$, as in the matter/radiation-dominated universe, the inertial force F_{in} becomes attractive, the turnaround radius r_{TA} does not exist, and we do not obtain any constraint. Even in an accelerating universe, the inertial force becomes smaller as time passes if the effective equation of state (EoS) parameter $w \equiv P/\rho$ (where ρ and P are the energy density and pressure of the cosmic fluid, respectively) is larger than -1, i.e., for $-1 < w < -1/3$, and the turnaround radius becomes larger. An interesting point in $F(R)$ gravity is that the effective gravitational coupling $G_{eff} \propto 1/F'(R)$ is time dependent. For example, if $F(R)$ behaves as $F(R) \sim R^\alpha$ with a constant α, $F'(R) \propto R^{\alpha-1} \propto t^{-2(\alpha-1)}$ because $R = 6(\dot{H} + 2H^2)$ behaves as $R \propto t^{-2}$. Therefore, we find

$$r_{TA}^3 \propto t^{2(\alpha-2)}. \tag{2.10}$$

Then, if $\alpha > 2$, the turnaround radius r_{TA} becomes larger as time passes but becomes smaller if $\alpha < 2$. We should note that if $F(R)$ behaves as $F(R) \sim R^\alpha$, the scale factor a behaves as [6,13]

$$a(t) \propto t^{\frac{(\alpha-1)(2\alpha-1)}{\alpha-2}}, \tag{2.11}$$

when we neglect the contribution from the matter; therefore $\alpha > 2$ corresponds to a phantom universe and $1 < \alpha < 2$ to a quintessence one. A general α corresponds to Einstein gravity coupled with a perfect fluid with the effective equation of state parameter,

$$w_{eff} = -\frac{(6\alpha^2 - 11\alpha + 7)}{3(\alpha-1)(2\alpha-1)}. \tag{2.12}$$

In the case that matter with the EoS parameter w couples with $F(R)$ gravity, the effective EoS parameter is

$$w_{eff} = -1 + \frac{1+w}{\alpha}. \tag{2.13}$$

In both cases (2.12) or (2.13), if $w_{eff} < -1/3$, the inertial force (2.8) becomes repulsive and there appears the turnaround radius (2.9). We should note that, even if the expansion of the universe is identical, the behavior of the turnaround radius is different in $F(R)$ gravity and in Einstein gravity coupled with a perfect fluid. When Einstein gravity couples with only one kind of perfect fluid with a constant EoS parameter $w \neq -1$, the Hubble rate H always behaves as $H \propto t^{-1}$. Then, Eq. (2.9) tells us that, in GR,

$$r_{TA}^3 \propto t^2, \tag{2.14}$$

which is different from the expression (2.10) of $F(R)$ gravity. If $\alpha > 6$, the turnaround radius $r_{\rm TA}$ in $F(R)$ gravity is larger than the corresponding radius (2.14) in Einstein gravity, and, therefore, comparatively larger bound structures can form more easily in the universe. On the other hand, if $\alpha < 6$, the turnaround radius $r_{\rm TA}$ in $F(R)$ gravity is smaller than the radius (2.14) in Einstein gravity and the size of bound structures in the universe becomes smaller.

The concept of the inertial force (2.8) generated by the expansion of the universe has been introduced in the investigation of the little rip [50,51]. In the little rip scenario, the Hubble rate H goes to infinity in the infinite future $t \to +\infty$, while H diverges at a finite future $t \to t_s$ in the big rip scenario. Anyway, as H becomes larger and larger, which means $\dot{H} > 0$, the turnaround radius (2.9) becomes smaller and smaller. In Eq. (2.9), we have considered the balance between the Newtonian force and the inertial force (2.8). If the radius becomes of the order of the human size, say, the electromagnetic force between molecules becomes stronger than the gravitational force, and then we need to consider the balance between the electromagnetic force and the inertial force. If the size of the turnaround radius becomes of the order of the nuclear size, we further need to consider the balance between nuclear and inertial forces.

III. A CLASS OF MODELS

We now consider the following class of models of gravity:

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}[F(R) + G(R)R_{\mu\nu}R^{\mu\nu}] + S_{\rm matter}. \tag{3.1}$$

By varying the action (3.1) with respect to the (inverse) metric $g^{\mu\nu}$, one obtains the fourth order field equations

$$\begin{aligned}
0 = {} & \frac{1}{2}g_{\mu\nu}[F(R) + G(R)R_{\rho\sigma}R^{\rho\sigma}] - 2G(R)R_\mu^\rho R_{\nu\rho} \\
& - [F'(R) + G'(R)R_{\rho\sigma}R^{\rho\sigma}]R_{\mu\nu} \\
& + (\nabla_\mu\nabla_\nu - g_{\mu\nu}\nabla^2)[F'(R) + G'(R)R_{\rho\sigma}R^{\rho\sigma}] \\
& + \nabla_\mu\nabla^\rho[G(R)R_{\rho\nu}) + \nabla_\nu\nabla^\rho(G(R)R_{\rho\mu}] \\
& - \nabla^2[G(R)R_{\mu\nu}] - g_{\mu\nu}\nabla^\rho\nabla^\sigma[G(R)R_{\rho\sigma}] + \kappa^2 T_{\mu\nu}.
\end{aligned} \tag{3.2}$$

Here we have used the following formulas:

$$\delta R_{\mu\nu} = \frac{1}{2}[\nabla^\rho(\nabla_\mu\delta g_{\nu\rho} + \nabla_\nu\delta g_{\mu\rho}) - \nabla^2\delta g_{\mu\nu} - \nabla_\mu\nabla_\nu(g^{\rho\lambda}\delta g_{\rho\lambda})], \tag{3.3}$$

$$\delta R = -\delta g_{\mu\nu}R^{\mu\nu} + \nabla^\mu\nabla^\nu\delta g_{\mu\nu} - \nabla^2(g^{\mu\nu}\delta g_{\mu\nu}). \tag{3.4}$$

When $T_{\mu\nu} = 0$, if we assume that the scalar curvature and the Ricci tensor are covariantly constant,

$$R = \frac{12}{l^2}, \qquad R_{\mu\nu} = \frac{3}{l^2}g_{\mu\nu}, \tag{3.5}$$

we obtain the algebraic equation for $1/l^2$,

$$0 = \frac{1}{2}F(R_0) - \left[F'(R_0) + \frac{36G'(R_0)}{l^4}\right]\frac{3}{l^2}, \qquad R_0 \equiv \frac{12}{l^2}. \tag{3.6}$$

If a real positive solution $1/l^2$ exists, the de Sitter and the Schwarzschild–de Sitter spacetimes (2.4) are solutions of Eq. (3.2), with

$$A(r) = 1 - \frac{2G_{\rm eff}M}{r} - \frac{r^2}{l^2}, \tag{3.7}$$

except for the fact that Newton's gravitational constant $G_{\rm N}$ is now replaced by the effective one $G_{\rm eff}$.

In order to define the effective gravitational constant $G_{\rm eff}$, we consider the perturbation of Eq. (3.2),

$$g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu}. \tag{3.8}$$

Because

$$\delta\Gamma^\kappa_{\mu\nu} = \frac{1}{2}g^{\kappa\lambda}(\nabla_\mu h_{\nu\lambda} + \nabla_\nu h_{\mu\lambda} - \nabla_\lambda h_{\mu\nu}), \tag{3.9}$$

we obtain

$$\begin{aligned}
\delta R &= -h_{\mu\nu}R^{\mu\nu} + \nabla^\mu\nabla^\nu h_{\mu\nu} - \nabla^2(g^{\mu\nu}h_{\mu\nu}), \\
\delta R_{\mu\nu} &= \frac{1}{2}[\nabla^\rho(\nabla_\mu h_{\nu\rho} + \nabla_\nu h_{\mu\rho}) - \nabla^2 h_{\mu\nu} - \nabla_\mu\nabla_\nu(g^{\rho\lambda}h_{\rho\lambda})] \\
&= \frac{1}{2}[\nabla_\mu\nabla^\rho h_{\nu\rho} + \nabla_\nu\nabla^\rho h_{\mu\rho} - \nabla^2 h_{\mu\nu} - \nabla_\mu\nabla_\nu(g^{\rho\lambda}h_{\rho\lambda}) \\
&\quad - 2R^\lambda{}_\nu{}^\rho{}_\mu h_{\lambda\rho} + R^\rho{}_\mu h_{\rho\nu} + R^\rho{}_\mu h_{\rho\nu}].
\end{aligned} \tag{3.10}$$

Then we find

$$
\begin{aligned}
0 =& \frac{1}{2}h_{\mu\nu}\{F(R)+G(R)R_{\rho\sigma}R^{\rho\sigma}\}+\frac{1}{2}g_{\mu\nu}\{F'(R)+G'(R)R_{\rho\sigma}R^{\rho\sigma}\}(-h_{\xi\eta}R^{\xi\eta}+\nabla^{\xi}\nabla^{\eta}h_{\xi\eta}-\nabla^2(g^{\xi\eta}h_{\xi\eta}))\\
&-2G'(R)R^{\rho}_{\mu}R_{\nu\rho}(-h_{\xi\eta}R^{\xi\eta}+\nabla^{\xi}\nabla^{\eta}h_{\xi\eta}-\nabla^2(g^{\xi\eta}h_{\xi\eta}))\\
&-\frac{1}{2}\{F'(R)+G'(R)R_{\rho\sigma}R^{\rho\sigma}\}\{\nabla^{\rho}(\nabla_{\mu}h_{\nu\rho}+\nabla_{\nu}h_{\mu\rho})-\nabla^2h_{\mu\nu}-\nabla_{\mu}\nabla_{\nu}(g^{\rho\lambda}h_{\rho\lambda})\}\\
&-2G(R_0)R_{\mu}{}^{\eta}\{\nabla^{\rho}(\nabla_{\eta}h_{\nu\rho}+\nabla_{\nu}h_{\eta\rho})-\nabla^2h_{\eta\nu}-\nabla_{\eta}\nabla_{\nu}(g^{\rho\lambda}h_{\rho\lambda})\}\\
&-2G(R_0)R_{\nu}{}^{\eta}\{\nabla^{\rho}(\nabla_{\eta}h_{\mu\rho}+\nabla_{\mu}h_{\eta\rho})-\nabla^2h_{\eta\mu}-\nabla_{\eta}\nabla_{\mu}(g^{\rho\lambda}h_{\rho\lambda})\}\\
&-\{F''(R)+G''(R)R_{\rho\sigma}R^{\rho\sigma}\}R_{\mu\nu}(-h_{\xi\eta}R^{\xi\eta}+\nabla^{\xi}\nabla^{\eta}h_{\xi\eta}-\nabla^2(g^{\xi\eta}h_{\xi\eta}))\\
&+(\nabla_{\mu}\nabla_{\nu}-g_{\mu\nu}\nabla^2)\{(F''(R)+G''(R)R_{\rho\sigma}R^{\rho\sigma})(-h_{\xi\eta}R^{\xi\eta}+\nabla^{\xi}\nabla^{\eta}h_{\xi\eta}-\nabla^2(g^{\xi\eta}h_{\xi\eta}))\}\\
&+\nabla_{\mu}\nabla^{\rho}\{G'(R)R_{\rho\nu}(-h_{\xi\eta}R^{\xi\eta}+\nabla^{\xi}\nabla^{\eta}h_{\xi\eta}-\nabla^2(g^{\xi\eta}h_{\xi\eta}))\}\\
&+\nabla_{\nu}\nabla^{\rho}\{G'(R)R_{\rho\mu}(-h_{\xi\eta}R^{\xi\eta}+\nabla^{\xi}\nabla^{\eta}h_{\xi\eta}-\nabla^2(g^{\xi\eta}h_{\xi\eta}))\}\\
&-\nabla^2\{G'(R)R_{\mu\nu}(-h_{\xi\eta}R^{\xi\eta}+\nabla^{\xi}\nabla^{\eta}h_{\xi\eta}-\nabla^2(g^{\xi\eta}h_{\xi\eta}))\}\\
&-g_{\mu\nu}\nabla^{\rho}\nabla^{\sigma}\{G'(R)R_{\rho\sigma}(-h_{\xi\eta}R^{\xi\eta}+\nabla^{\xi}\nabla^{\eta}h_{\xi\eta}-\nabla^2(g^{\xi\eta}h_{\xi\eta}))\}\\
&-g_{\mu\nu}G(R)R_{\rho}{}^{\xi}R^{\rho\eta}h_{\xi\eta}+2G(R)R_{\mu}{}^{\xi}R_{\nu}{}^{\eta}h_{\xi\eta}+2G'(R)R_{\rho}{}^{\xi}R^{\rho\eta}h_{\xi\eta}R_{\mu\nu}\\
&+(\nabla_{\mu}\nabla_{\nu}-g_{\mu\nu}\nabla^2)(G'(R)R_{\rho}{}^{\xi}R^{\rho\eta}h_{\xi\eta})\\
&+\frac{1}{2}g_{\mu\nu}G(R)R^{\rho\sigma}\{\nabla^{\xi}(\nabla_{\rho}h_{\sigma\xi}+\nabla_{\sigma}h_{\rho\xi})-\nabla^2h_{\rho\sigma}-\nabla_{\rho}\nabla_{\sigma}(g^{\xi\eta}h_{\xi\eta})\}\\
&-G(R)\{\nabla^{\xi}(\nabla_{\rho}h_{\mu\xi}+\nabla_{\mu}h_{\rho\xi})-\nabla^2h_{\rho\mu}-\nabla_{\rho}\nabla_{\mu}(g^{\xi\eta}h_{\xi\eta})\}R_{\nu}{}^{\rho}\\
&-G(R)\{\nabla^{\xi}(\nabla_{\rho}h_{\nu\xi}+\nabla_{\nu}h_{\rho\xi})-\nabla^2h_{\rho\nu}-\nabla_{\rho}\nabla_{\nu}(g^{\xi\eta}h_{\xi\eta})\}RR_{\mu}{}^{\rho}\\
&-G'(R)R^{\rho\sigma}\{\nabla^{\xi}(\nabla_{\rho}h_{\sigma\xi}+\nabla_{\sigma}h_{\rho\xi})-\nabla^2h_{\rho\sigma}-\nabla_{\rho}\nabla_{\sigma}(g^{\xi\eta}h_{\xi\eta})\}R_{\mu\nu}\\
&+(\nabla_{\mu}\nabla_{\nu}-g_{\mu\nu}\nabla^2)\{G'(R)R^{\rho\sigma}\{\nabla^{\xi}(\nabla_{\rho}h_{\sigma\xi}+\nabla_{\sigma}h_{\rho\xi})-\nabla^2h_{\rho\sigma}-\nabla_{\rho}\nabla_{\sigma}(g^{\xi\eta}h_{\xi\eta})\}\}\\
&+\nabla_{\mu}\nabla^{\rho}\{G(R)\{\nabla^{\xi}(\nabla_{\rho}h_{\nu\xi}+\nabla_{\nu}h_{\rho\xi})-\nabla^2h_{\rho\nu}-\nabla_{\rho}\nabla_{\nu}(g^{\xi\eta}h_{\xi\eta})\}\}\\
&+\nabla_{\nu}\nabla^{\rho}\{G(R)\{\nabla^{\xi}(\nabla_{\rho}h_{\mu\xi}+\nabla_{\mu}h_{\rho\xi})-\nabla^2h_{\rho\mu}-\nabla_{\rho}\nabla_{\nu}(g^{\xi\eta}h_{\xi\eta})\}\}\\
&-\nabla^2\{G(R)\{\nabla^{\xi}(\nabla_{\mu}h_{\nu\xi}+\nabla_{\nu}h_{\mu\xi})-\nabla^2h_{\mu\nu}-\nabla_{\mu}\nabla_{\nu}(g^{\xi\eta}h_{\xi\eta})\}\}\\
&-g_{\mu\nu}\nabla^{\rho}\nabla^{\sigma}\{G(R)\{\nabla^{\xi}(\nabla_{\rho}h_{\sigma\xi}+\nabla_{\sigma}h_{\rho\xi})-\nabla^2h_{\rho\sigma}-\nabla_{\rho}\nabla_{\sigma}(g^{\xi\eta}h_{\xi\eta})\}\}\\
&-(h_{\mu\nu}\nabla^2-g_{\mu\nu}h_{\xi\eta}\nabla^{\xi}\nabla^{\eta})\{F'(R)+G'(R)R_{\rho\sigma}R^{\rho\sigma}\}\\
&-\frac{1}{2}(\delta_{\mu}{}^{\xi}\delta_{\nu}{}^{\eta}-g_{\mu\nu}g^{\xi\eta})(\nabla_{\xi}h_{\eta\lambda}+\nabla_{\eta}h_{\xi\lambda}-\nabla_{\lambda}h_{\xi\eta})\partial^{\lambda}\{F'(R)+G'(R)R_{\rho\sigma}R^{\rho\sigma}\}\\
&-\frac{1}{2}\{2g^{\kappa\lambda}(\nabla_{\mu}h_{\nu\lambda}+\nabla_{\nu}h_{\mu\lambda}-\nabla_{\lambda}h_{\mu\nu})\nabla^{\rho}(G(R)R_{\rho\kappa})+\nabla_{\mu}(h_{\xi\eta}\nabla^{\xi}(G(R)R^{\eta}_{\nu}))+\nabla_{\nu}(h_{\xi\eta}\nabla^{\xi}(G(R)R^{\eta}{}_{\mu}))\\
&+\nabla_{\mu}(g^{\rho\sigma}(g^{\kappa\lambda}(\nabla_{\rho}h_{\sigma\lambda}+\nabla_{\sigma}h_{\rho\lambda}-\nabla_{\lambda}h_{\sigma\rho})(G(R)R_{\kappa\nu})+g^{\kappa\lambda}(\nabla_{\rho}h_{\nu\lambda}+\nabla_{\nu}h_{\rho\lambda}-\nabla_{\lambda}h_{\nu\rho})(G(R)R_{\sigma\kappa})))\\
&+\nabla_{\nu}(g^{\rho\sigma}(g^{\kappa\lambda}(\nabla_{\rho}h_{\sigma\lambda}+\nabla_{\sigma}h_{\rho\lambda}-\nabla_{\lambda}hg_{\sigma\rho})(G(R)R_{\kappa\mu})+g^{\kappa\lambda}(\nabla_{\rho}h_{\mu\lambda}+\nabla_{\mu}h_{\rho\lambda}-\nabla_{\lambda}h_{\mu\rho})(G(R)R_{\sigma\kappa})))\\
&-2h_{\xi\eta}\nabla^{\xi}\nabla^{\eta}(G(R)R_{\mu\nu})-g^{\xi\eta}(g^{\kappa\lambda}(\nabla_{\xi}h_{\eta\lambda}+\nabla_{\eta}h_{\xi\lambda}-\nabla_{\lambda}h_{\xi\eta})\nabla_{\kappa}(G(R)R_{\mu\nu})\\
&+g^{\kappa\lambda}(\nabla_{\xi}h_{\mu\lambda}+\nabla_{\mu}h_{\xi\lambda}-\nabla_{\lambda}h_{\xi\mu})\nabla_{\eta}(G(R)R_{\kappa\nu})+g^{\kappa\lambda}(\nabla_{\xi}h_{\nu\lambda}+\nabla_{\nu}h_{\xi\lambda}-\nabla_{\lambda}h_{\xi\nu})\nabla_{\eta}(G(R)R_{\mu\kappa}))\\
&-g^{\xi\eta}\nabla_{\xi}(g^{\kappa\lambda}(\nabla_{\eta}h_{\mu\lambda}+\nabla_{\mu}h_{\eta\lambda}-\nabla_{\lambda}h_{\eta\mu})(G(R)R_{\kappa\nu})+g^{\kappa\lambda}(\nabla_{\eta}h_{\nu\lambda}+\nabla_{\nu}h_{\eta\lambda}-\nabla_{\lambda}h_{\eta\nu})(G(R)R_{\mu\kappa}))\\
&+2h_{\mu\nu}\nabla^{\rho}\nabla^{\sigma}(G(R)R_{\rho\sigma})-2g_{\mu\nu}h_{\xi\eta}\nabla^{\xi}\nabla^{\sigma}(G(R)R^{\eta}{}_{\sigma})-g_{\mu\nu}g^{\tau\rho}g^{\kappa\lambda}(\nabla_{\tau}h_{\rho\lambda}+\nabla_{\rho}h_{\tau\lambda}-\nabla_{\lambda}h_{\tau\rho})\nabla^{\sigma}(G(R)R_{\kappa\sigma})\\
&-g_{\mu\nu}\nabla^{\rho}(g^{\tau\sigma}(g^{\kappa\lambda}(\nabla_{\tau}h_{\rho\lambda}+\nabla_{\rho}h_{\tau\lambda}-\nabla_{\lambda}h_{\tau\rho})(G(R)R_{\kappa\sigma})+g^{\kappa\lambda}(\nabla_{\tau}h_{\sigma\lambda}+\nabla_{\sigma}h_{\tau\lambda}-\nabla_{\lambda}h_{\tau\sigma})(G(R)R_{\rho\kappa})))\}\\
&+\kappa^2T_{\mu\nu}. \qquad (3.11)
\end{aligned}
$$

In the de Sitter background (3.5), Eq. (3.11) assumes the simplified form,

$$\begin{aligned}
0 =& \frac{1}{2}h_{\mu\nu}\left(F(R_0)+\frac{36}{l^4}G(R_0)\right)+\frac{1}{2}g_{\mu\nu}F'(R_0)\left(-\frac{3}{l^2}h+\nabla^\xi\nabla^\eta h_{\xi\eta}-\nabla^2 h\right)\\
&-\frac{1}{2}\left(F'(R_0)+\frac{36}{l^4}G'(R_0)+\frac{72}{l^2}G(R_0)\right)\{\nabla^\rho(\nabla_\mu h_{\nu\rho}+\nabla_\nu h_{\mu\rho})-\nabla^2 h_{\mu\nu}-\nabla_\mu\nabla_\nu(g^{\rho\lambda}h_{\rho\lambda})\}\\
&+\left(F''(R_0)+\frac{36}{l^4}G''(R_0)\right)\left(\nabla_\mu\nabla_\nu-g_{\mu\nu}\nabla^2-\frac{3}{l^2}g_{\mu\nu}\right)\left(-\frac{3}{l^2}h+\nabla^\xi\nabla^\eta h_{\xi\eta}-\nabla^2 h\right)\\
&+\frac{6}{l^2}G'(R_0)(\nabla_\mu\nabla_\nu-g_{\mu\nu}\nabla^2)\left(-\frac{3}{l^2}h+\nabla^\xi\nabla^\eta h_{\xi\eta}-\nabla^2 h\right)-\frac{9}{l^4}g_{\mu\nu}G(R_0)h+\frac{18}{l^4}G(R_0)h_{\mu\nu}+\frac{54}{l^6}g_{\mu\nu}G'(R_0)h\\
&+\frac{9}{l^4}G'(R_0)(\nabla_\mu\nabla_\nu-g_{\mu\nu}\nabla^2)h+\frac{3}{l^2}G(R_0)g_{\mu\nu}(2\nabla^\xi\nabla^\eta h_{\xi\eta}-\nabla^2 h)-\frac{6}{l^2}G(R_0)\{\nabla^\xi(\nabla_\nu h_{\mu\xi}+\nabla_\mu h_{\nu\xi})-\nabla^2 h_{\mu\nu}-\nabla_\mu\nabla_\nu h\}\\
&+\frac{6}{l^2}G'(R_0)\left(\nabla_\mu\nabla_\nu-g_{\mu\nu}\nabla^2-\frac{3}{l^2}g_{\mu\nu}\right)(\nabla^\xi\nabla^\eta h_{\xi\eta}-\nabla^2 h)+G(R_0)\nabla_\mu\nabla^\rho\{\{\nabla^\xi(\nabla_\rho h_{\nu\xi}+\nabla_\nu h_{\rho\xi})-\nabla^2 h_{\rho\nu}-\nabla_\rho\nabla_\nu h\}\}\\
&+G(R_0)\nabla_\nu\nabla^\rho\{\nabla^\xi(\nabla_\rho h_{\mu\xi}+\nabla_\mu h_{\rho\xi})-\nabla^2 h_{\rho\mu}-\nabla_\rho\nabla_\nu h\}-G(R_0)\nabla^2\{\nabla^\xi(\nabla_\mu h_{\nu\xi}+\nabla_\nu h_{\mu\xi})-\nabla^2 h_{\mu\nu}-\nabla_\mu\nabla_\nu h\}\\
&-G(R_0)g_{\mu\nu}\nabla^\rho\nabla^\sigma\{\nabla^\xi(\nabla_\rho h_{\sigma\xi}+\nabla_\sigma h_{\rho\xi})-\nabla^2 h_{\rho\sigma}-\nabla_\rho\nabla_\sigma h\}+\frac{6}{l^2}G(R_0)(\nabla_\mu\nabla^\rho h_{\rho\nu}+\nabla_\nu\nabla^\rho h_{\rho\mu}-\nabla^2 h_{\mu\nu}+g_{\mu\nu}\nabla^\rho\nabla^\sigma h_{\rho\sigma})\}\\
&+\kappa^2 T_{\mu\nu}. \qquad (3.12)
\end{aligned}$$

In the weak-field, slow-motion limit of GR the linearized Einstein equations reduce to $\nabla^2 h_{\mu\nu}=-2\kappa^2 T_{\mu\nu}$ [48]. Hence, the coupling of the graviton to matter is given by the coefficient of $\nabla^2 h_{\mu\nu}$ in the linearized field equations (3.12) of our gravity model (where we discard time derivatives and spatial derivatives of order higher than second). The result is

$$\frac{1}{8\pi G_{\text{eff}}}=\frac{1}{\kappa^2}\left[F'(R_0)+\frac{36}{l^2}\left(2G(R_0)+\frac{G'(R_0)}{l^2}\right)\right]. \qquad (3.13)$$

Then the turnaround radius (2.5) is expressed as

$$\begin{aligned}
r_{\text{TA}}^3 &= G_{\text{eff}}Ml^2\\
&=\frac{\kappa^2 Ml^2}{8\pi\left[F'(R_0)+\frac{36}{l^2}\left(\frac{G'(R_0)}{l^2}+2G(R_0)\right)\right]}. \qquad (3.14)
\end{aligned}$$

As a partial consistency check of our Eq. (3.13), consider the special case of $F(R)=R^2$ gravity. While this model is a good approximation of Starobinsky inflation $F(R)=R+\mu R^2$ in the early, strongly curved universe, it is well known that this theory (or, in d spacetime dimensions, $F(R)=R^{d/2}$ [52]) does not admit a Newtonian limit [53] and suffers from other problems as well [54–56]. Indeed, the exponent n of $F(R)=R^n$ gravity is severely constrained by the precession of Mercury's perihelion to be [57–60]

$$n-1=(2.7\pm 4.5)\times 10^{-19}, \qquad (3.15)$$

while the criterion $F''(R)\geq 0$ necessary to avoid the notorious Dolgov-Kawasaki instability requires $n\geq 1$ [13,61,62]. For the pathological model with $F(R)=R^2$ and $G(R)=0$, Eq. (3.13) gives

$$G_{\text{eff}}=\frac{\kappa^2}{16\pi R_0}. \qquad (3.16)$$

In order to take the Newtonian limit, one must be able to consider a Minkowskian background, an assumption that complements our Eq. (3.8) and that is implemented when $g_{\mu\nu}$ becomes the Minkowski metric $\eta_{\mu\nu}$. This Minkowski background can be seen as a de Sitter space with zero curvature and G_{eff} diverges as $R_0\to 0$, which shows that the pathological theory $F(R)=R^2$ without Newtonian limit leads to inconsistencies in our equations, as it should be.

In the following sections we consider more concrete models.

IV. $R+R^2+R_{\mu\nu}R^{\mu\nu}$ MODEL

Let us consider now the model [1,63]

$$S=\frac{1}{2\kappa^2}\int d^4x\sqrt{-g}[R-\Lambda+aR^2+bR_{\mu\nu}R^{\mu\nu}]+S_{\text{matter}}, \qquad (4.1)$$

where a and b are constants and S_{matter} denotes the matter action. This theory is known to be multiplicatively renormalizable quantum gravity (for a review, see [1]) which still has some unresolved issues with unitarity.

One could add to this Lagrangian density a term proportional to the Kretschmann scalar $R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$, but this does not make the action more general. In fact, in four spacetime dimensions, the integral of the Gauss-Bonnet combination

$$\chi \equiv \int d^4x\sqrt{-g}(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}) \tag{4.2}$$

is a constant topological invariant, which allows one to eliminate the Kretschmann term and reduce the action

$$S' = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}[R - \Lambda + aR^2 + bR_{\mu\nu}R^{\mu\nu} + cR_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}] + S_{\mathrm{matter}}, \tag{4.3}$$

where c is another constant, to an action integral of the form (4.1) with new coefficients $a' = a - c$ and $b' = b + 4c$.

It is known that the model (4.1) contains a scalar mode and also a massive spin two ghost mode, in addition to the massless spin two mode, which is the usual graviton familiar from GR. The existence of this ghost mode tells us that this model is not unitary and is therefore inconsistent. This model is, however, regarded as a low-energy effective theory and, if we include the higher order corrections and nonperturbative effects, we may obtain a consistent theory.

Because

$$F(R) = R - \Lambda + aR^2, \qquad G(R) = b, \tag{4.4}$$

Eq. (3.6) is reduced to

$$0 = \frac{3}{l^2} - \frac{\Lambda}{2}; \tag{4.5}$$

that is, if $1/l^2 \neq 0$,

$$l^2 = \frac{6}{\Lambda}. \tag{4.6}$$

Equation (3.13) gives also the effective gravitational coupling G_{eff} as

$$8\pi G_{\mathrm{eff}} = \frac{\kappa^2}{1 + \frac{24}{l^2}(a + 3b)}, \tag{4.7}$$

while the turnaround radius is

$$r_{\mathrm{TA}}^3 = \frac{3\kappa^2 M}{4\pi\Lambda[1 + \frac{24}{l^2}(a + 3b)]}. \tag{4.8}$$

In Ref. [34], it is required that the maximum turnaround radius in any alternative theory of gravity be, at most, 10% smaller than the corresponding radius (2.5) in GR,

$$r_{\mathrm{TA}} \geq 0.9(G_{\mathrm{N}}Ml^2)^{1/3}. \tag{4.9}$$

Applying this criterion here yields the constraint

$$\frac{G_{\mathrm{eff}}}{R_{\mathrm{dS}}} \geq \frac{0.182G_{\mathrm{N}}}{\Lambda}. \tag{4.10}$$

Here R_{dS} is the scalar curvature of the geometry describing the de Sitter spacetime that solves the alternative theory of gravity and Λ is the cosmological constant in GR, $\Lambda = 3/l^2$ in the definition of [34]. In the case of the action (4.1), Eq. (4.8) in conjunction with the constraint (4.9) yields

$$\frac{1}{1 + \frac{24}{l^2}(a + 3b)} \geq 0.7, \tag{4.11}$$

which gives a constraint on the parameters a and b in the model (4.1). We should note, however, that we have estimated the effective gravitational coupling G_{eff} as given by the coupling of $h_{\mu\nu}$, which may give Newton's law. If we can know directly any parameter κ^2, a, or b by any independent procedure, Eq. (4.11) produces a more realistic constraint on the model. For example, it is not so clear whether the effective gravitational coupling G_{eff} in the solution describing the Schwarzschild–de Sitter spacetime (3.7) is identical with G_{eff} in (4.7). The effective gravitational coupling G_{eff} in (3.7) depends on the definition of the mass M in the modified gravity theory.

In the case of the critical gravity theory [64],

$$a = -\frac{1}{\Lambda} = -\frac{l^2}{6}, \qquad b = -3a = \frac{3}{\Lambda} = \frac{l^2}{2}, \tag{4.12}$$

the scalar mode does not propagate and the massive spin two mode of the general $R + R^2 + R_{\mu\nu}R^{\mu\nu}$ gravity becomes massless. Then, Eq. (4.12) tells us that

$$\frac{1}{1 + \frac{24}{l^2}(a + 3b)} = \frac{1}{33}, \qquad 8\pi G_{\mathrm{eff}} = \frac{\kappa^2}{33}, \tag{4.13}$$

and therefore Eq. (4.11) is not satisfied.

V. A MORE GENERAL MODEL

Instead of the action (4.1), we consider the case that the constants Λ, a, and b depend on R,

$$S = \int d^4x \frac{\sqrt{-g}}{2\kappa(R)^2}[R - \Lambda(R) + a(R)R^2 + b(R)R_{\mu\nu}R^{\mu\nu}] + S_{\mathrm{matter}}, \tag{5.1}$$

that is,

$$F(R) = \frac{\kappa_0^2}{\kappa(R)^2}[R - \Lambda(R) + a(R)R^2],$$
$$G(R) = \frac{\kappa_0^2}{\kappa(R)^2} b(R). \tag{5.2}$$

(Similar to the previous section, adding a term $cR_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$ with constant coefficient c to the Lagrangian density does not add in generality.) We now write κ in Eq. (3.1) as κ_0. In the model consisting of one-loop corrected quantum R^2 gravity of Refs. [1,65], we have

$$\kappa(R)^2 \sim \kappa_0^2(1 + \lambda_0\beta_2\tau)^{0.77},$$
$$\kappa(R)^2\Lambda(R) \sim \kappa_0^2\Lambda_0(1 + \lambda_0\beta_2\tau)^{-0.55}, \tag{5.3}$$

$$\frac{a(R)}{\kappa(R)^2} \sim \frac{a_0}{\kappa_0^2}(1 + \lambda_0\beta_2\tau),$$
$$\frac{b(R)}{\kappa(R)^2} \sim \frac{b_0}{\kappa_0^2}(1 + \lambda_0\beta_2\tau), \tag{5.4}$$

where

$$\beta_2 = \frac{133}{10}, \qquad \tau = \tau_1 \ln\left|\frac{R}{R_1}\right|. \tag{5.5}$$

The R-dependent coefficients represent one-loop renormalization group (RG) coupling constants. The second interpretation of the same model is just a more complicated version of modified gravity that includes the Ricci-squared term.

Therefore, we obtain

$$F(R) \sim R(1 + \lambda_0\beta_2\tau)^{0.77} - \Lambda_0(1 + \lambda_0\beta_2\tau)^{-0.55} + a_0R^2(1 + \lambda_0\beta_2\tau),$$
$$G(R) \sim b_0(1 + \lambda_0\beta_2\tau), \tag{5.6}$$

and Eq. (3.6) then yields

$$0 = \frac{1}{2}[R_0(1 + \lambda_0\beta_2\tau_0)^{0.77} - \Lambda_0(1 + \lambda_0\beta_2\tau_0)^{-0.55} + a_0R_0^2(1 + \lambda_0\beta_2\tau_0)]$$
$$- \frac{3}{l^2}\Big[(1 + \lambda_0\beta_2\tau_0)^{0.77} + 2a_0R_0(1 + \lambda_0\beta_2\tau_0) + 0.77\lambda_0\beta_2(1 + \lambda_0\beta_2\tau_0)^{-0.23} + 0.55\frac{\Lambda_0\lambda_0\beta_2}{R_0}(1 + \lambda_0\beta_2\tau_0)^{-1.55} + a_0\lambda_0\beta_2R_0\Big], \tag{5.7}$$

where

$$\tau_0 = \tau_1 \ln\left|\frac{R_0}{R_1}\right|. \tag{5.8}$$

It is difficult to solve Eq. (5.7) but if we choose $R_1 = R_0 = 12/l^2$, that is, $\tau_0 = 0$, Eq. (5.7) reduces to

$$0 = \frac{3 - 0.77\lambda_0\beta_2}{l^2} - \Lambda_0\left(\frac{1}{2} + 0.14\lambda_0\beta_2\right) - \frac{36a_0\lambda_0\beta_2}{l^4}, \tag{5.9}$$

which can be solved with respect to l^2, obtaining

$$l^2 = \frac{3 - 0.77\lambda_0\beta_2 \pm \sqrt{(3 - 0.77\lambda_0\beta_2)^2 - 72a_0\lambda_0\beta_2\Lambda_0(1 + 0.28\lambda_0\beta_2)}}{2\Lambda_0(1 + 0.28\lambda_0\beta_2)}. \tag{5.10}$$

In Eq. (5.10), the upper (+) sign corresponds to the classical limit (4.6). On the other hand, Eq. (3.13) gives

$$\frac{1}{8\pi G_{\mathrm{eff}}} = \frac{1}{\kappa^2}\Big[(1 + \lambda_0\beta_2\tau_0)^{0.77} + 2a_0R_0(1 + \lambda_0\beta_2\tau_0) + 0.77\lambda_0\beta_2(1 + \lambda_0\beta_2\tau_0)^{-0.23} + 0.55\frac{\Lambda_0\lambda_0\beta_2}{R_0}(1 + \lambda_0\beta_2\tau_0)^{-1.55} + a_0\lambda_0\beta_2R_0 + \frac{72}{l^2}b_0(1 + \lambda_0\beta_2\tau_0)\Big]. \tag{5.11}$$

Then, if we choose $R_1 = R_0$, we find

$$\frac{1}{8\pi G_{\mathrm{eff}}} = \frac{1}{\kappa^2}\left\{1 + \frac{24a_0}{l^2} + 0.77\lambda_0\beta_2 + \frac{0.55\Lambda_0\lambda_0\beta_2 l^2}{12} + \frac{12a_0\lambda_0\beta_2}{l^2} + \frac{72}{l^2}b_0\right\}, \tag{5.12}$$

and the turnaround radius is given by

$$r_{\mathrm{TA}}^3 = \frac{8\pi\kappa^2 M l^2}{1 + \frac{24a_0}{l^2} + 0.77\lambda_0\beta_2 + \frac{0.55\Lambda_0\lambda_0\beta_2 l^2}{12} + \frac{12a_0\lambda_0\beta_2}{l^2} + \frac{72}{l^2}b_0}, \tag{5.13}$$

which provides the constraint, as in Eq. (4.11),

$$\frac{l^2}{1 + \frac{24a_0}{l^2} + 0.77\lambda_0\beta_2 + \frac{0.55\Lambda_0\lambda_0\beta_2 l^2}{12} + \frac{12a_0\lambda_0\beta_2}{l^2} + \frac{72}{l^2}b_0} \geq \frac{4.2}{\Lambda}. \tag{5.14}$$

If we assume that the correction from Einstein gravity with a truly constant cosmological constant $\Lambda = \Lambda_0$, Eq. (5.10) gives

$$l^2 = \frac{3}{\Lambda}(1 - 0.31\lambda_0\beta_2 - 4a_0\lambda_0\beta_2\Lambda_0). \qquad (5.15)$$

Then combining with (5.14), we obtain the constraint

$$\frac{24a_0}{l^2} + 1.08\lambda_0\beta_2 + \frac{0.55\Lambda_0\lambda_0\beta_2 l^2}{12} + 8a_0\lambda_0\beta_2\Lambda_0 + 24\Lambda_0 b_0 \le 0.4, \qquad (5.16)$$

as in (4.11).

VI. DISCUSSION AND CONCLUSIONS

Given the degeneracy between dark energy and modified gravity models attempting to explain the present acceleration of the universe, and the current level of theoretical and experimental effort aiming to detect and study, or to constrain, possible deviations of gravity from Einstein's theory [14,66–68], the turnaround radius of large structures in cosmology could be very useful. Two approaches to the turnaround radius in the context of GR ([24,25] and [26,27]) produce more or less the same numerical results. The second approach, being based on the Hawking-Hayward quasilocal energy is gauge independent to any degree of approximation compatible with current and foreseeable astronomical observations [26,27]), but it becomes ill defined in modified gravity. For this reason, we used an alternative definition of turnaround radius in our analysis in the context of modified gravity models.

Three previous works [33–35] were restricted to scalar-tensor or $F(R)$ gravity (the latter is an incarnation of the former class of theories). Here we discuss more general classes of theories containing also the square of the Ricci tensor and mixed terms. Allowing terms in $R_{\mu\nu}R^{\mu\nu}$ to be present in the action introduces extra degrees of freedom in comparison with pure $F(R)$ or scalar-tensor gravity.

An important realization is that, even when the cosmic expansion is identical in GR and in a modified gravity model, in general the time dependence of the turnaround radius in the latter is different from that of the corresponding turnaround radius in GR coupled with a perfect fluid, because the effective gravitational coupling becomes time dependent.

To fix the ideas, we have imposed that the deviation of the turnaround radius in modified gravity from its GR value is not larger than 10% (this figure may be debatable given the large error in the observational determination of the turnaround radius [45], but it serves the purpose of illustration). The constraint that we derive would already put the critical gravity scenario of Ref. [64] in jeopardy. Similarly, more complicated models will be constrained by the turnaround radius if and when reliable astronomical observations of this quantity become available.

ACKNOWLEDGMENTS

This work is supported in part by MINECO (Spain), Projects No. FIS2016-76363-P and SGR247 (AGAUR, Catalonia), (S. D. O.), by MEXT Grant-in-Aid for Scientific Research on Innovative Areas "Cosmic Acceleration" (No. 15H05890) and the JSPS Grant-in-Aid for Scientific Research (C) No. 18K03615 (S. N.), and V. F. is supported by the Natural Sciences and Engineering Research Council of Canada (Grant No. 2016-03803).

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