Noise-aided demodulation with one-bit comparator for multilevel pulse-amplitude-modulated signals

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***Abstract*—This paper proposes a demodulation method using a one-bit comparator for signals processed by multilevel pulse amplitude modulation (PAM). The proposed method is simple and provides an alternative to using an analog-to-digital converter to describe multilevel input signals. Because of the noise present in the transmitted multilevel PAM signal, the two-level output of the one-bit comparator shows different statistical behavior for each level of the signal. Thus, it is possible to detect the signal level, or perform symbol decision, based on the maximum likelihood (ML) criterion. The present theoretical analysis reveals that reliable demodulation is possible even with a one-bit comparator if the probability mass function of the two-level outputs of each received symbol plus intentionally added noise is known.**

***Index Terms*—Stochastic resonance, nonlinear, noise, multilevel PAM, demodulation, symbol decision, one-bit comparator**

I. Introduction

In current communication systems, multilevel amplitude signals are widely used to carry information. Pulse amplitude modulation (PAM) is one method of generating such signals, and an analog-to-digital (A/D) converter is commonly used to digitally receive and demodulate these signals. Two requirements are often considered: a high resolution to describe the amplitude and a high sampling rate to reveal high-frequency signals. One noteworthy converter, the $\Delta\Sigma$ A/D converter [1]–[3], outputs a high-resolution signal with a two-level amplitude. Employing a simple one-bit comparator would aid in the realization of a high sampling rate. However, a complicated feedback structure is needed to achieve this. This work introduces an alternative method: the sampling and demodulation of the multilevel signal using only a simple one-bit comparator *with the help of noise*.

The potentially constructive role of noise has been widely discussed in the context of nonlinear physics. One impressive example is the phenomenon of Stochastic resonance (SR) [4], [5]. A typical benefit of SR is that subthreshold signals are detected by adding noise. This curious but interesting effect has been reported in various fields including neural systems [5]–[7], nonlinear electronic devices [8]–[10], signal detection and error correction algorithms [11], [12], information theory [13]–[15], and communication systems [16]–[18]. Regarding A/D converters, Tapang et al. have shown that the dynamic range is enhanced by adding noise [8]. In image sensors, a similar effect for suprathreshold signals has been reported [9]. To demodulate multilevel amplitude signals using a one-bit comparator, a different effect should be explored: the linearization of nonlinear devices by adding noise. The device is required to respond linearly to the multilevel input signal. This point has been analytically predicted in a theoretical model of dynamical systems [19] and is herein extended to practical systems using a nondynamic (static) A/D converter for communication systems.

This paper presents a noise-aided demodulation method using a one-bit comparator. In this study, through the exploitation of the noise contained in input multilevel PAM signals, the effect of SR is obtained; because of the noise, the output of the comparator behaves stochastically, and the sum of the output thus describes the amplitude of the transmitted multilevel symbols. In [20], such concept using the sum has been briefly evaluated; in this paper, to realize the noise-induced linearizion in the communication systems, a demodulation method based on the decision statistic of the sum is analytically derived. Maximum likelihood (ML) detection is focused, and the proposed method is evaluated in terms of the transmission error. With the sufficient large number of the output samples, numerical examples demonstrates that the proposed method gives the small error rate in a certain noise intensity.

This paper is organized as follows. First, the focused system is presented in Sec. II. In the next section, the proposed concept of the noise-enhanced demodulation is provided. In Sec. IV, the effectiveness of the proposed method is demonstrated through the numerical examples of the symbol error rate (SER). The conclusions are given in Sec. V.

II. System model

In the proposed receiver, the multilevel PAM signal is detected using a one-bit comparator. The structure of the proposed demodulator is presented in Fig. 1. The output of the comparator, y_{ij}, takes only two levels, indicating that the output does not fully describe the multilevel transmitted symbol. However, the sum of the output, y_i, contains the information of the transmitted symbol. The sum is used as the decision statistic, and the transmitted symbol is demodulated using the ML detection method.

The simple case of four-level PAM (4-PAM) is focused in Fig. 1. The transmitted symbol d_i can take one of four values, namely, $d_i \in \{-3A, -A, +A, +3A\}$. The subscript i represents the symbol number. The transmitted signal is $s(t) = \sum_i d_i g(t - iT_s)$, where $g(t)$ is a rectangular pulse with a duration of T_s and a unit amplitude.

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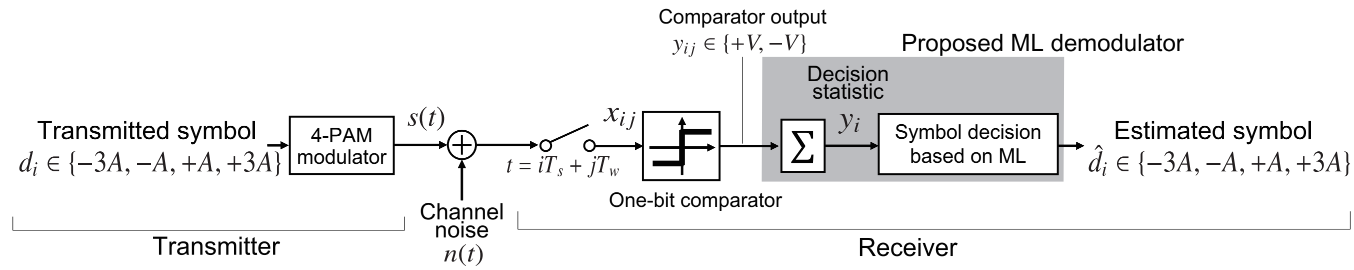


Fig. 1: System diagram for the proposed ML demodulator. Though the comparator output describes only two levels, the proposed ML detection method with the statistic y_i enables to demodulate the multilevel transmitted symbol.

In the receiver, the signal contaminated by channel noise $n(t)$ is sampled and detected by a one-bit comparator. Because the sampling rate was set to $1/T_w = N/T_s$, the resulting samples of the signal are $x_{ij} = [s(t) + n(t)]\delta[t - (iT_s + jT_w)] = d_i + n_{ij}$, where $\delta(\cdot)$ is the Kronecker delta function, N is the number of samples per symbol, and $1 \leq j \leq N - 1$. Owing to the noise term n_{ij}, the output of the comparator y_{ij} behaves stochastically. The in–out characteristic of a comparator is often considered as

$$y_{ij} = \begin{cases} +V & x_{ij} > \zeta \\ -V & x_{ij} \leq \zeta \end{cases} \tag{1}$$

for a given threshold ζ. With a large value of n_{ij}, whether a sample exceeds the threshold is determined by the stochastic term n_{ij} rather than d_i. The output $y_{ij} = +V$ for a given transmitted symbol d_i is observed with the following probability:

$$P[y_{ij} = +V|d_i] = \int_{\zeta - d_i}^{+\infty} p(n_{ij})\,\mathrm{d}n_{ij} \equiv P_{+|d_i}. \tag{2}$$

Note that the noise $n(t)$ is assumed to be white noise and has a probability density function of $p(n)$. For the other case of $y_{ij} = -V$, $P_{-|d_i} \equiv P[y_{ij} = -V|d_i] = 1 - P_{+|d_i}$. This stochastic output is used in the demodulation of the four-level transmitted symbol, which will be described in the next section.

III. Proposed noise-aided ML detection for multilevel PAM signals

This section presents the proposed demodulator. The multilevel PAM symbol is estimated from the sum of the N samples of the comparator output,

$$y_i = \sum_{j=1}^{N} y_{ij}. \tag{3}$$

Careful observation reveals that the sum y_i contains the information of the transmitted multilevel symbols. The sum is used as the decision statistic in the proposed demodulator, and the transmitted symbol is estimated based on ML detection. In Sec. III-A, the concept, and the proposed ML detection method are described, and the estimation of the multilevel symbols, is then presented in Sec. III-B.

A. Visual understanding of the proposed method

To demonstrate the concept of the proposed method, examples of the key variables, the input samples x_{ij} and the output samples y_{ij}, are shown in Fig. 2. The considered noise levels are defined with respect to the average energy per symbol, given by $E_s \equiv E[d_i^2]T_s = E[d_i^2]NT_w$, and the channel noise $n(t)$ is white Gaussian noise with a variance of $N_0/2$. The values of the parameters are $V = 1.0$, $T_w = 1$, $A = 1.0$, $N = 64$, and $(d_1, d_2, d_3, d_4) = (3.0, 1.0, -1.0, -3.0)$. The original transmitted symbols were distinguishable in the low-noise case, $E_s/N_0 = 40$ dB. However, as shown in Fig. 2(a), the amplitude information was lost once the signal passed through the comparator in this case.

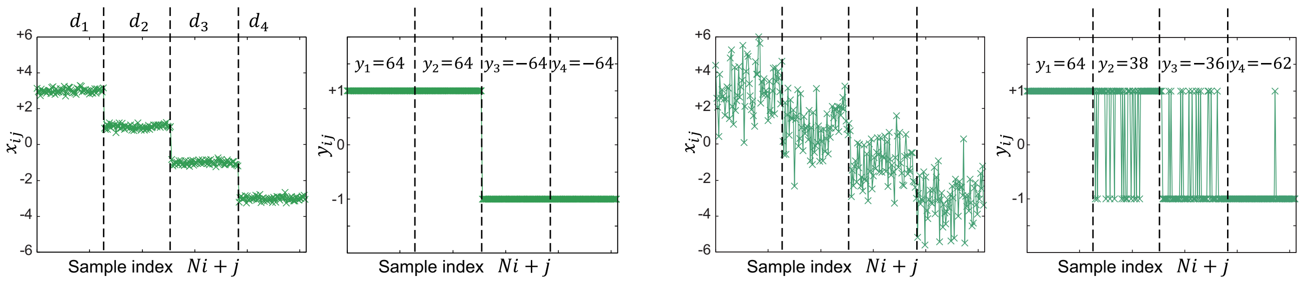
In the case of $E_s/N_0 = 20$ dB, the input signal and the resulting output sample showed clear stochastic behavior. The interesting point is that even with only a two-level output, the four amplitudes of the transmitted symbol could be extracted by considering the voltage switching of the output. For example, an input signal with $d_1 = 3.0$ takes a large amplitude, meaning that the switching of the output signal rarely occurs and many output samples remain at the high voltage $+V$. To count the number of such samples, the summation term given in (3) is introduced as the decision statistic in this paper. The value of the decision statistic for each transmitted symbol is given in Fig. 2(b). Changing the symbol from 3.0 to −3.0 reduces the value. Analytically, the probability with which the output sample takes the high voltage is given by $P_{+|d_i}$ (2). A large symbol amplitude widens the range of the integral, causing the probability, i.e., the number of output samples with $+V$, to increase. Therefore, the transmitted symbol can be obtained from the number of such samples.

When the noise has a large amplitude relative to the average energy per symbol, neither the input signal nor the resulting output sample describes the transmitted symbol. Then, the voltage switching under a large noise amplitude is determined by the noise rather than the transmitted symbol. This problem is well known in the context of SR; the response in SR systems is optimized at a certain noise intensity. As will be shown in Fig. 3, a method of adjusting the noise intensity should be considered to obtain sufficiently low symbol decision errors.

B. ML detection for noisy output samples

In the receiver, the most likely symbol is estimated based on the decision statistic y_i. This is achieved by selecting the symbol $\hat{d}_i$ that maximizes the likelihood function, i.e.,

$$\hat{d}_i = \arg\max_{d_i} \left[P[y_i|d_i]\right]. \tag{4}$$



(a) Input signal samples x_{ij} and output samples y_{ij} in $E_s/N_0 = 40$ dB

(b) Input signal samples x_{ij} and output samples y_{ij} in $E_s/N_0 = 20$ dB

Fig. 2: Examples of the key variables in the proposed ML detection method. With the sufficiently strong noise such as $E_s/N_0 = 20$ dB, the information of the multilevel amplitude is found in the statistic y_i. The proposed ML detection method exploits this behavior to demodulate the multilevel symbols.

From (1) and (2), the comparator output y_{ij} has a binomial distribution: $P[y_{ij} = +V] = P_{+|d_i}$ and $P[y_{ij} = -V] = P_{-|d_i}$. The sum y_i of the output samples is then binomially distributed for a given d_i. When k samples are $+V$ and the remaining $N-k$ samples are $-V$, $y_i = (2k - N)V$ with the probability,

$$P[y_i = (2k-N)V|d_i] = \binom{N}{k} P_{+|d_i}^{k} P_{-|d_i}^{N-k}, \quad (5)$$

for $k = 0, 1, 2, \cdots, N$. Note that $\binom{N}{k}$ is the binomial coefficient. Substituting (5) into (4), the most likely symbol $\hat{d}_i$ is estimated as

$$\hat{d}_i = \arg\max_{d_i} \left[\binom{N}{k} P_{+|d_i}^{k} P_{-|d_i}^{N-k} \right]. \quad (6)$$

The expression given in (6) is difficult to compute because when N is large, the calculations of both the binomial coefficient and the exponential terms require a large amount of computational time. Thus, a simplified form is introduced; first, because the binomial coefficient does not depend on d_i, it is removed from (6). The log-likelihood of (5) without the coefficient, $\log(P_{+|d_i}^{k} P_{-|d_i}^{N-k}) = \frac{1}{2}(y_i + N)\log P_{+|d_i} - \frac{1}{2}(y_i - N)\log(1 - P_{+|d_i})$ yields the simple form,

$$\hat{d}_i = \arg\max_{d_i} \left[(y_i + N)\log P_{+|d_i} - (y_i - N)\log(1 - P_{+|d_i})\right]. \quad (7)$$

For a given decision statistic y_i, the log-likelihoods of all possible symbols are calculated using the equation given in (7), and the most likely symbol $\hat{d}_i$, which gives the maximum log-likelihood, is selected.

As mentioned above, the variable $P_{\pm|d_i}$ is the key variable for the correct demodulation. Equation (2) shows that this parameter strongly depends on the noise level. To adjust the level, a method using the training sequence should be effective in the proposed framework. For the training sequence known in the receiver, the noise level can be adjusted in order to minimize the demodulation error.

IV. Numerical examples

In this section, the effectiveness of the proposed method is demonstrated through numerical examples. The proposed method is evaluated in terms of the Symbol Error Rate (SER), which is defined as the ratio of the number of erroneously demodulated symbols to the total number of transmitted symbols. The number of transmitted symbols in the numerical simulation was 10^6, and each symbol was randomly selected from four candidates. The threshold of the comparator was set to be $\zeta = 0$, and the number of samples per symbol was set to $N \in \{16, 64, 256\}$. The key probability $P_{+|d_i}$ was simply calculated from the probability density function of the Gaussian distribution. Note that $P_{+|d_i}$ depends on the noise distribution and varies in time. Therefore, in practical implementation, this parameter is required to be estimated in some way, e.g. estimation by a training sequence.

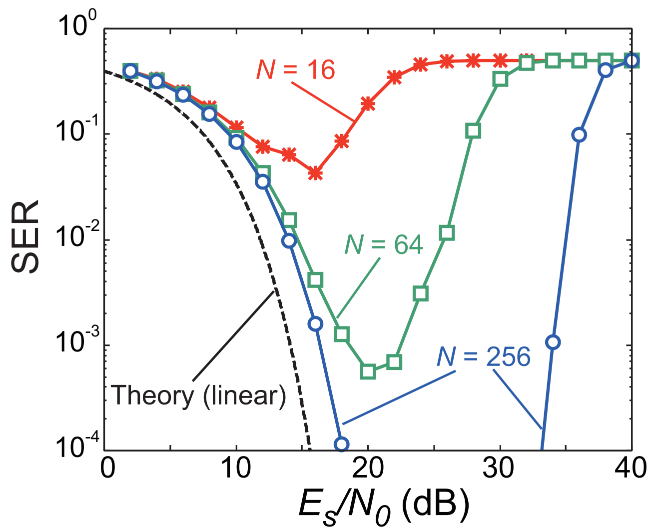


Fig. 3: SER performance of the proposed noise-aided demodulation of a 4-PAM signal plotted against E_s/N_0. The SER performance was improved in a certain range of E_s/N_0.

The results of the simulation is provided in Fig. 3. For comparison, the theoretical performance of the demodulation for 4-PAM signals is also plotted in Fig. 3 as "Theory (linear)"; with sufficiently strong noise, the performance of the proposed method is close to the theoretical one. In a certain range of E_s/N_0 values, the performance monotonically improved with decreasing E_s/N_0. These points indicate that the addition of noise enables the demodulation of the 4-PAM symbols using the proposed method with a simple one-bit comparator.

However, if the noise is weak, as in the case with E_s/N_0 = 40 dB, the demodulation performance is degraded. This point is justified in Fig. 2; in this case, the comparator output is no longer stochastic, and no information about the transmitted symbols can be found in the sum of the output. This situation has been widely discussed in the phenomenon of SR: the response of noise-enhanced systems is optimized at a certain noise level and does not continue to increase with further increases in the noise level. In the considered system, the source of the noise is the channel, which means that the noise level is not controllable. To adjust the level, an additional noise source may be installed in the case of weak noise. Employing a "without tuning" [6] method is also an option.

The dependence of the SER on the number N of samples is also shown in Fig. 3. Increasing the number yields a better SER performance. This is explained by the decision rule (7). The likelihood of obtaining an output equal to $y_{ij} = +V$ given a symbol value of d_i is represented by the term $\log P_{+|d_i}$, and that of obtaining $y_{ij} = -V$ is represented by $1 - \log P_{+|d_i}$. The difference between these likelihoods is an important factor in achieving the correct demodulation because the difference is the margin for reaching the wrong decision because of the noise. The difference is roughly proportional to $2N$. Therefore, the SER performance improves linearly with N.

V. Conclusion

This paper proposed a demodulation method for multilevel PAM signals using a one-bit comparator. First, the system model of the proposed method was presented. The proposed method can be realized with a simple device setup composed of a one-bit comparator and a noise source. Because it uses noise-aided demodulation, the proposed method does not need a complicated feedback system, unlike the ΔΣ A/D converter. Furthermore, a symbol decision algorithm was developed based on ML decision making. Numerical examples were presented to demonstrate the utility of the method in a certain SNR range. The SNR range can be extended by increasing the number of samples persymbol. The proposed method is expected to contribute to the design and development of communication systems.

In this paper, 4-PAM was focused. The proposed method should be effective for multilevel PAM modulation more than 4-PAM. Even in this case, one-bit comparator can be employed. For the correct estimation, the reliable statistic of $P_{\pm|d_i}$ should be provided. Since the variable $P_{\pm|d_i}$ depends on the noise level, the level should be tuned to minimize the error in the demodulation. This point will be investigated in a future work.

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References

[1] J. Robert, G. C. Temes, V. Valencic, R. Dessoulavy, and P. Deval, "A 16-bit low-voltage cmos A/D converter," *IEEE J. Solid-state Circuits*, vol. 22, no. 2, pp. 157–163, 1987.

[2] R. H. Walden, "Analog-to-digital converter survey and analysis," *IEEE J. Sel. Areas Commun*, vol. 17, no. 4, pp. 539–550, 1999.

[3] J. Markus, J. Silva, and G. C. Temes, "Theory and applications of incremental ΔΣ converters," *IEEE Trans. Circuits Syst.I Regular papers*, vol. 51, no. 4, pp. 678–690, 2004.

[4] L. Gammaitoni, P. Hänggi, P. Jung, and F. Marchesoni, "Stochastic resonance," *Rev. Mod. Phys.*, vol. 70, no. 1, pp. 223–287, 1998.

[5] K. Wiesenfeld and F. Moss, "Stochastic resonance and the benefits of noise: from ice ages to crayfish and SQUIDs," *Nature*, vol. 373, no. 6509, pp. 33–36, 1995.

[6] J. J. Collins, C. C. Chow, and T. T. Imhoff, "Stochastic resonance without tuning," *Nature*, vol. 376, no. 6537, pp. 236–238, 1995.

[7] L. Kish, G. P. Harmer, and D. Abbott, "Information transfer rate of neurons: Stochastic resonance of shannon's information channel capacity," *Fluct. Noise Lett.*, vol. 01, no. 01, pp. L13–L19, 2001.

[8] G. Tapang and C. Saloma, "Dynamic-range enhancement of an optimized 1-bit A/D converter," *IEEE Trans. Circuits Syst., II Analog Digit. Signal Process*, vol. 49, no. 1, pp. 42–47, 2002.

[9] Y. Tadokoro, S. Kasai, A. Ichiki, and H. Tanaka, "Design framework of image sensor system based on dynamic range extension by adding noise for saturated conditions," *IEEE Trans. Syst. Man Cybern., Syst*, vol. 46, pp. 1121–1128, 2015.

[10] S. Kasai and T. Asai, "Stochastic resonance in schottky wrap gate-controlled gaas nanowire field-effect transistors and their networks," *Appl. Phys. Express*, vol. 1, no. 8, p. 083001, 2008.

[11] S. Kay, "Can detectability be improved by adding noise?" *IEEE Signal Process. lett.*, vol. 7, no. 1, pp. 8–10, 2000.

[12] P. Ivaniš and B. Vasić, "Error errore eicitur: A stochastic resonance paradigm for reliable storage of information on unreliable media," *IEEE Trans. Commun.*, vol. 64, no. 9, pp. 3596–3608, 2016.

[13] F. Duan, F. Chapeau-Blondeau, and D. Abbott, "Capacity of very noisy communication channels based on fisher information," *Sci. Rep.*, vol. 6, p. 27946, 2016.

[14] H. Chen, L. R. Varshney, and P. K. Varshney, "Noise-enhanced information systems," *Proceedings of the IEEE*, vol. 102, no. 10, pp. 1607–1621, 2014.

[15] S. Sugiura, A. Ichiki, and Y. Tadokoro, "Stochastic-resonance based iterative detection for serially-concatenated turbo codes," *IEEE Signal Process. Lett.*, vol. 19, no. 10, pp. 655–658, 2012.

[16] H. Tanaka, K. Chiga, T. Yamazato, Y. Tadokoro, and S. Arai, "Noise-enhanced subthreshold signal reception by a stochastic resonance receiver using a non-dynamical device," *NOLTA, IEICE*, vol. 6, no. 2, pp. 303–312, 2015.

[17] Y. Nakashima, H. Tanaka, T. Yamazato, Y. Tadokoro, and S. Arai, "Effect of a root-raised-cosine filter on a BPSK stochastic resonance receiver," *NOLTA, IEICE*, vol. 8, no. 3, pp. 204–214, 2017.

[18] J. Wang, X. Ren, S. Zhang, D. Zhang, H. Li, and S. Li, "Adaptive bistable stochastic resonance aided spectrum sensing," *IEEE Trans. Wirel. Commun.*, vol. 13, no. 7, pp. 4014–4024, 2014.

[19] M. I. Dykman, D. G. Luchinsky, R. Mannella, P. V. E. McClintock, H. E. Short, N. D. Stein, and N. G. Stocks, "Noise-induced linearisation," *Phys. Lett. A*, vol. 193, no. 1, pp. 61–66, 1994.

[20] Y. Nakashima, T. Yamazato, Y. Tadokoro, and S. Arai, "A stochastic resonance receiver for 4-PAM signals," in *NOLTA2017*, 2017, pp. 291–294.