COMPLEMENTARITY IN PREON MODELS WITH E_6 METACOLOR

J.-M. GÉRARD [1], Y. OKAMOTO [2] and R.E. MARSHAK [2,3]

Max-Planck-Institut für Physik und Astrophysik – Werner Heisenberg Institut für Physik –, D-8000 Munich 40, Fed. Rep. Germany

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Tumbling complementarity is studied in chiral preon models based on the metacolor group E_6. By considering the breaking of E_6 into each maximal little group in the Higgs phase, several models are found that are in accord with complementarity. Three families of massless fermions (on the metacolor scale) can be obtained, but so far we have been unable to identify an $E_6 \times SU(N)$ preon model supported by complementarity that correctly predicts the quantum numbers of ordinary quarks and leptons.

Vector-like preon models of quarks and leptons without supersymmetry are ruled out by the mass inequality condition [1]. Among non-supersymmetric chiral models, (gauged) E_6 metacolor (with preons in the 27 representation) is the unique choice if one restricts oneself to a simple group with a *single* complex irreducible representation that is anomaly-free [2]. The choice of the global color–flavor symmetry group G_{CF} is less clearcut: G_{CF} can be as large as SU(21) (from E_6 asymptotic freedom) while its gauged subgroup H_{CF} can be as small as $SU(3) \times SU(2) \times U(1)$ (from low-energy phenomenology); $E_6 \times H_{CF}$ must yield at least three families of ordinary quarks and leptons. Thus was derived the gauged $E_6 \times SO(10)$ model [2] which, in its initial version, predicted too many exotic fermions. This model was then analyzed [3,4] in the framework of 't Hooft anomaly matching [5] for different choices of G_{CF} and its unbroken global subgroup G^F_{CF}; a variety of solutions without exotics could be generated depending on the 't Hooft indices l_i.

However, several important questions were left unanswered:

(i) For a given G_{CF}, how does one choose the correct unbroken global color–flavor group G^F_{CF} among many possibilities?

(ii) How does one select the likely solution among many to the 't Hooft anomaly matching equations? In particular, can one impose any constraints [‡1] on l_i?

We believe that the combination of tumbling [7] and complementarity [8,9] should provide a clue to answering these questions for E_6 metacolor. This view is supported by the extensive studies on the application of complementarity to preon models with SU(M) metacolor [9,10] (N will always be the number of color–flavors), which establish a connection between the breaking of gauged metacolor in the Higgs phase and the pattern of global color–flavor symmetry breaking in the confining phase. The work with SU(M) metacolor has also clarified the permissible range of l_i [‡2] and

[1] Chargé de Recherches au FNRS.

[2] Permanent address: Department of Physics, Virginia Polytechnic Institute and State University, Blacksburg, VA 24061, USA.

[3] Currently Alexander von Humboldt Senior Investigator at MPI.

[‡1] Bars [6] has given arguments for $|l_i| \leqslant 1$; further arguments come from complementarity (see below).

[‡2] Except for the SU(7) model in ref. [9], which seems to lead to $l = 3$, all other SU(M) models satisfying complementarity give $|l_i| \leqslant 1$. When $|l_i| > 1$, we have degeneracies with massless fermions in the Higgs phase. However, n-fold degeneracies in the Higgs phase would imply the existence of a "new" SU(n) global symmetry, which is difficult to interpret. Our view is that some "family" symmetry, such as U(1) or SU(2), contained in the color–flavor symmetry, prevents degeneracies of surviving massless fermions in the Higgs phase, requiring that $|l_i| \leqslant 1$.

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has shown that gravitational [11] as well as chiral anomaly matching are implied by complementarity. Unfortunately, these SU(M) models have not led to a realistic composite theory of quarks and leptons. In the hope that a realistic model with E_6 metacolor can be found, we extend complementarity to E_6 preons and here we report some initial results.

Complementarity implies a one-to-one correspondence between the surviving massless fermions in the "tumbling" Higgs phase and the massless composite fermions remaining from 't Hooft anomaly matching in the confining phase. Tumbling is supposed to be triggered by the two-fermion (scalar) condensate in the most attractive channel (MAC) which is in the fundamental representation of the metacolor group G_{MC} and acts as the Higgs boson in the metacolor symmetry breaking, $G_{MC} \to G'_{MC}$. In the process of tumbling, the global chiral symmetry G_{CF} is also broken down to a subgroup G'_{CF} by the MAC condensate, and those fermions under the symmetry $G'_{MC} \times G'_{CF}$ which participated in the MAC condensate or which can form mass terms become massive. Tumbling is repeated until we reach $G^F_{MC} \times G^F_{CF}$ in which the surviving massless fermions are all G^F_{MC} singlets (or no massless fermion survives). Finally, complementarity tells us that in the confining phase we should apply 't Hooft anomaly matching to this unbroken subgroup G^F_{CF}.

We recall that our E_6 preons are in the 27 representation and that the largest global color–flavor group is SU(N) ($N < 22$). Thus our preons transform as (27; $\square$), where $\square$ is the fundamental representation of SU(N). In applying complementarity to $E_6 \times SU(N)$, the 't Hooft indices l_i must be non-negative because all the fermions are left-handed in the Higgs phase. l_i can, in principle, be greater than 1, but it is then difficult to give a physical interpretation to the implied degeneracies [‡2]. Hence, we expect that, in general, models with E_6 metacolor, satisfying complementarity, will give rise to $l_i = 0$ or 1. The crucial question will be whether the tumbling of SU(N) stops at a final unbroken color–flavor group that is consistent with low-energy phenomenology.

Following the MAC principle, the fermion–fermion condensate transforms like the symmetric $\overline{27}$, i.e.,

$$27 \times 27 \to (\overline{27})_S . \qquad (1)$$

Since $\overline{27}$ is a fundamental representation of E_6, we can proceed. Due to this $\overline{27}$ condensate, $E_6 \times SU(N)$ tumbles down to a state of lower symmetry $G'_{MC} \times G'_{CF}$. The subgroup G'_{MC} must contain a singlet in the branching of the condensate $\overline{27}$ into G'_{MC}. For this reason, we limit ourselves to the maximal little groups of E_6 and we get the following five possibilities:

$$E_6 \to F_4; \quad 27 \to 1 + 26, \qquad (2a)$$

$$E_6 \to SO(10) \times U(1);$$
$$27 \to 1(4) + 10(-2) + 16(1), \qquad (2b)$$

$$E_6 \to SU(3) \times SU(3) \times SU(3);$$
$$27 \to (1, \bar{3}, \bar{3}) + (3, 1, 3) + (\bar{3}, 3, 1), \qquad (2c)$$

$$E_6 \to G_2 \times SU(3); \quad 27 \to (1, \bar{6}) + (7, 3), \qquad (2d)$$

$$E_6 \to SU(2) \times SU(6); \quad 27 \to (1, 15) + (2, \bar{6}). \qquad (2e)$$

In eqs. (2a)–(2e), we have also listed the branching rules for the 27 of E_6. Note that U(1) in (2b), two of the three SU(3)'s in (2e), SU(3) is (2d) and SU(6) in (2e) are broken, lacking in singlets in the branching. Hence, we can take G'_{MC} to be F_4, SO(10), SU(3), G_2, or SU(2), respectively. The color–flavor group G'_{CF} is determined by the same principle. The MAC condensate corresponds to the representation $\square\square$ or $\begin{smallmatrix}\square\\ \square\end{smallmatrix}$ under G_{CF} = SU(N). The meta-Pauli principle, however, requires the use of the symmetric representation $\square\square$. This follows from the fact that the metacolor part is symmetric (see (1)), while the spin part for the two-fermion condensate (scalar) is antisymmetric. Consequently, the possible breakings of G_{CF} into G'_{CF} are [12]

$$SU(N) \to SU(N-j) \times O(j) \quad (1 \leqslant j \leqslant N). \qquad (3)$$

We shall now examine each case of metacolor symmetry breaking in the Higgs phase (see (2)). First, we consider the MAC condensate (1) (see (2a)) breaking E_6 into F_4. The color–flavor symmetry breaking is given by (3) and we take, for simplicity, $j = 1$ in (3) [‡3]; $G_{CF} = SU(N)_F$ breaks into $G'_{CF} = SU(N-1)_F$. Making use of the branching rule of $\square$ of $SU(N)_F$ into

[‡3] Essentially the same discussion follows for other values of j, owing to the fact that fundamental representations of O(j) are real.

$SU(N-1)_F$:

$$\square \to \square + 1 \tag{4}$$

our fermions transform as $(1;\square)$, $(1;1)$, $(26;\square)$, and $(26;1)$ under $F_4 \times SU(N-1)_F$. However, since F_4 has only real representations, $(1;1)$ and $(26;1)$ become massive and the surviving massless fermions are

$$(1;\square),\quad (26;\square). \tag{5}$$

The MAC for the next tumbling binds two 26 of F_4 into a singlet condensate, i.e.

$$26 \times 26 \to (1)_S . \tag{6}$$

This condensate leaves the metacolor symmetry F_4 unbroken, but it breaks the color–flavor symmetry $SU(N-1)_F$ into $SU(N-2)_F$. Here, the Pauli principle was again imposed on the two-fermion condensate. The surviving massless fermions transform as (5) under $F_4 \times SU(N-2)_F$. It is not hard to see that the sequence of tumbling does not stop until the color–flavor group is completely broken down and no massless fermions remain. Thus, the breaking of E_6 into F_4 turns out to be uninteresting.

We now consider the case in which E_6 breaks into SO(10) by means of the condensate (1) (see (2b)). As in the previous example, the color–flavor symmetry $SU(N)_F$ breaks into $SU(N-1)_F$. There are two U(1) symmetries broken here: one in (2b) and the other from $SU(N)_F$ breaking, $SU(N)_F \to SU(N-1)_F \times U(1)_F$. (Here, we have written the $U(1)_F$ symmetry explicitly.)
We can keep a linear combination of these two U(1)'s unbroken. The surviving massless fermions are then given by

$$(1;\square, 2(2N-3)),\ (1;1,6(N-1)),$$

$$(10;\square,-2N),$$

$$(16;\square,N-3),\ (16;1,3(N-1)), \tag{7}$$

under $SO(10) \times SU(N-1)_F \times U'(1)$. Here, the fermions transforming as $(10;1,0)$ have become massive as the MAC condensate. The MAC for the next tumbling binds two 10's of SO(10) into a singlet condensate, i.e.,

$$10 \times 10 \to (1)_S . \tag{8}$$

This condensate leaves the metacolor symmetry SU(10) unbroken, but it breaks the color–flavor symmetry $SU(N-1)_F \times U(1)'$ into $SU(N-2)_F \times U(1)''$ We cannot stop the sequence of tumbling until the color–flavor group is completely broken. Thus, the breaking path of E_6 into SO(10) is of no interest.

The case in which E_6 breaks into SU(3) by the MAC condensate (1) (see (2c)) is a little more involved, because we have three SU(3) symmetries in the breaking of E_6. As in the previous cases, we cannot stop the tumbling of the color–flavor group at a non-trivial subgroup.

The next case, in which E_6 breaks into G_2 (see (2d)), is more interesting; we did find a nontrivial example that is consistent with complementarity. It is the model with three preons in the $\overline{27}$ representation of E_6. We shall describe this model in both phases.

(i) Higgs phase. The preons transform as $(\overline{27};\square)$ under $E_6 \times SU(3)_F$. The MAC binds two $\overline{27}$'s of E_6 into a condensate in 27, i.e. the complex conjugate of (1). The color–flavor part of this condensate is $\yng(2)$ (= 6) of $SU(3)_F$, by the Pauli principle, and this breaks $SU(3)_F$. However, we can define the diagonal subgroup of SU(3) in the metacolor symmetry breaking (see (2d)), and of $SU(3)_F$ as the new unbroken color–flavor group SU(3)'. Namely, we can take the generators of the new SU(3)' as the sum of those in SU(3) of (2d) and those in $SU(3)_F$. This is possible because the MAC condensate transforms as $(1;\bar{6})$ under $G_2 \times SU(3)$ in (2d) for the metacolor part and as 6 under $SU(3)_F$ for the color–flavor part, which together form a singlet under the new group $G_2 \times SU(3)'$. The fermions $(\overline{27};\square)$ under $E_6 \times SU(3)_F$ then branch into

$$(7;1),\ (7;\yng(2,1)),\ (1;\yng(2,1)),\ (1;\yng(3)), \tag{9}$$

under $G_2 \times SU(3)'$ (see the branching rule in (2d)). Since G_2 has only real representations and $\yng(2,1)$ of SU(3)' is also real, the fermions which correspond to the first three terms in (9) become massive, leaving only

$$(1;\yng(3)) = (1;10) \tag{10}$$

as massless fermions. Tumbling stops here because we have only singlets under G_2.

(ii) Confining phase. In this case, none of the metacolor or color–flavor symmetries is spontaneously broken. The spectrum of the theory consists of composites

Table 1
Particle content of the $E_6 \times SU(3)_F$ model in the confining phase.

Content		E_6	$SU(3)_F$	Indices
preons	P	27	$\square$	
composites	PPP	1	$\square\square\square$	l^+
			$\begin{smallmatrix}\square\\\square\\\square\end{smallmatrix}$	l^-
			$\begin{smallmatrix}\square\square\\\square\end{smallmatrix}$	l^0

bound by the strong E_6 forces (i.e., they are E_6 singlets). The particle content is summarized in table 1. In general, in the $E_6 \times SU(N)_F$ model, the 't Hooft anomaly matching condition for the three $SU(N)_F$ currents is given by

$$\tfrac{1}{2}(N+3)(N+6)\,l^+ + \tfrac{1}{2}(N-3)(N-6)\,l^- + (N^2-9)l^0 = 27, \tag{11}$$

where the 't Hooft indices l^+, l^-, and l^0 are defined in table 1. For $N = 3$, we have

$$27\,l^+ = 27\,, \tag{12}$$

which yields the unique solution

$$l^+ = 1. \tag{13}$$

This solution indeed corresponds to the massless fermions (1; 10) of (10) that were obtained in the Higgs phase, and hence is consistent with complementarity. Note that the index of (13) satisfies our constraint on indices (0 or 1) from complementarity. However, the solution (13) does not satisfy the meta-Pauli principle (with orbital angular momentum zero). This is due to the fact that for our three-preon composites in table 1, the metacolor part is symmetric, while the spin part is mixed. Since the meta-Pauli principle requires the color–flavor part to be mixed, we see that the Pauli principle for the composite fermions in the confining phase is not automatically satisfied, even though we have used MAC condensates that satisfy the meta-Pauli principle in the Higgs phase.

The last of the models that we consider is not only consistent with complementarity but also satisfies the meta-Pauli principle. Furthermore, family replication appears in a natural way via the "family" group SU(2). In this case, we make use of the maximal little group (2e), wherein E_6 breaks into $SU(2) \times SU(6)$ in the Higgs phase. For G_{CF}, we choose $SU(N) = SU(6n)$ ($n < 4$ from metacolor asymptotic freedom).

(i) Higgs phase. The preons transform as $(27; \square)$ under $E_6 \times SU(6n)_F$. The metacolor part of the MAC condensate in this model is again $(\overline{27})_S$ of E_6 (see (1)). By the meta-Pauli principle, the color–flavor part of the condensate is $\square\square$ of $SU(6n)_F$, which breaks $SU(6n)_F$. By considering the branching of $SU(6n)_F$ into $SU(6)_F \times SU(n)_F$, we can identify the diagonal subgroup of SU(6) in (2e) and $SU(6)_F$ as the new unbroken global symmetry $SU(6)'$. In fact, this is permissible only when $n > 1$, because for $n = 1$, the $\square\square$ of $SU(6)_F$ and the $\overline{15}$ (= $\begin{smallmatrix}\square\\\square\end{smallmatrix}$) from the metacolor part of the condensate (see (2e)), cannot combine into a singlet.

For $n > 1$, we have the following branching rules of $SU(6n)_F$ into $SU(6)_F \times SU(n)_F$:

$$\square \to (\square, \square), \tag{14}$$

$$\square\square \to (\square\square\,, \square\square) + \left(\begin{smallmatrix}\square\\\square\end{smallmatrix}, \begin{smallmatrix}\square\\\square\end{smallmatrix}\right). \tag{15}$$

The second term of eq. (15) forms an $SU(6)'$ singlet with the $\overline{15}$ in the $(1, \overline{15})$ part of metacolor decomposition. Up to this point, the MAC condensate can be written as

$$\left(1; 1, \begin{smallmatrix}\square\\\square\end{smallmatrix}\right), \tag{16}$$

under $SU(2)_{MC} \times SU'(6) \times SU(n)_F$. If $n = 2$, we can stop here because $\begin{smallmatrix}\square\\\square\end{smallmatrix} = 1$ under $SU(2)_F$. If $n = 3$, $SU(3)_F$ breaks down to $SU(2)_F$ because of the following branching rule of $SU(3)_F$ breaking into $SU(2)_F$:

$$\begin{smallmatrix}\square\\\square\end{smallmatrix} \to \begin{smallmatrix}\square\\\square\end{smallmatrix}\ (= 1) + \square\,. \tag{17}$$

In sum, the fermions $(27, \square)$ under $E_6 \times SU(6n)_F$ branch as follows (see the branching rule in (2e)): for $n = 2$, (14) and (15) give

$$(2; 1, 2),\ (2; 35, 2),\ (1; 20, 2),\ (1; 70, 2), \tag{18}$$

under $SU(2)_{MC} \times SU(6)' \times SU(2)_F$.

Since 35 and 20 are real representations of $SU(6)'$, the fermions corresponding to the first three terms of (18) acquire mass, leaving only

$$(1; 70, 2) \tag{19}$$

as massless fermions. These are really two copies of

(1; 70) under $SU(2)_{MC} \times SU(6)'$. However, the non-trivial quantum number 2 of $SU(2)_F$ lifts this degeneracy. Similarly, for $n = 3$, we have, under $SU(2)_{MC} \times SU(6)' \times SU(2)_F$:

$$(1; 70, 2), (1; 70, 1), \tag{20}$$

as surviving massless fermions. Note that the three-fold degeneracy is lifted by the $SU(2)_F$ quantum numbers. [While $n \geqslant 4$ leads to a violation of metacolor asymptotic freedom in the $E_6 \times SU(6n)$ model, it is amusing to note that for general n, tumbling does not stop until the symmetry is

$$SU(2)_{MC} \times SU(6)' \times [SU(2)_F]^m, \tag{21}$$

where $m = [n/2]$. The surviving massless fermions are then (1; 70) of $SU(2)_{MC} \times SU(6)'$ with n-fold degeneracies lifted by the "family" symmetry $[SU(2)_F]^m$.]

(ii) Confining phase. We shall illustrate how 't Hooft anomaly matching works with the $n = 3$ example. The unbroken global symmetry is then $G^F_{CF} = SU(6) \times SU(2)$, and the preons (27; □) of $E_6 \times SU(18)$ branch into $P_1 = (27; □, □)$ and $P_2 = (27, □, 1)$ of $E_6 \times G^F_{CF}$. The particle content is summarized in table 2.

Since the l_2's and l_5's represent the same composite representations, we define a new index l_{25} by

$$l_{25} = l_2 + l_5. \tag{22}$$

Similarly, we define a new index l_{46} by

$$l_{46} = l_4 + l_6. \tag{23}$$

The 't Hooft anomaly matching equations come from the three SU(6) currents only, and we have (see (11))

$$54(4\,l_1^+ + 3\,l_3^+ + 2\,l_{25}^+ + l_{46}^+)$$
$$+ 27(4\,l_1^0 + 3\,l_3^0 + 2\,l_{25}^0 + l_{46}^0)$$
$$= 3 \cdot 27. \tag{24}$$

There is an infinite number of solutions of eq. (24) if we allow $l_i < 0$. However, if we impose the physical condition $l_i \geqslant 0$ and insist on the meta-Pauli principle, there are three solutions: (a) $l_{46}^0 = 3$, (b) $l_{25}^0 = l_{46}^0 = 1$, and (c) $l_3^0 = 1$, with all other $l_i = 0$. All three solutions predict three "families" of the 70 of SU(6): (a) through the index repetition of the singlet of SU(2), (b) through the doublet + singlet of SU(2) and (c) through the triplet of SU(2). Clearly, the "family" group is SU(2) but only solution (b) is consistent with complementarity (and makes a distinction between one of the three "families" and the other two!).

In this note, we have examined tumbling complementarity in preon models based on E_6 metacolor and SU(N) color flavor. The possible tumbling paths are much more restricted compared to models with SU(M) metacolor; indeed, there are only five maximal little groups of E_6. However, it turns out for many cases that we cannot stop tumbling until we break the global color–flavor group SU(N) all the way down, rendering the theory trivial. Among the non-trivial examples that we found in accord with complementarity, the $E_6 \times SU(6n)$ model with $n = 3$ is the most instructive. It leads to 3 "generations" of the 70 representation of SU(6) as well as obeying the meta-Pauli principle. The extra SU(2) symmetry in the unbroken global symmetry $G^F_{CF} = SU(6) \times SU(2)$ serve as a "family" symmetry. The 't Hooft indices satisfying complementarity are all 0 or 1, confirming our view that $l_i = 0$ or 1 is implied by complementarity. However, this model is not realistic because the 70 of SU(6) does not contain the correct quantum numbers of ordinary quarks and leptons. Perhaps a more complicated choice of the global color–flavor symmetry will overcome this deficiency in the application of the idea of complementarity to preons with E_6 metacolor.

Table 2
Particle content of the $E_6 \times SU(6) \times SU(2)$ model derived from $G_{CF} = SU(18)$.

Content		E_6	SU(6)	SU(2)	Indices
preons	P_1	27	□	□	
	P_2	27	□	1	
composites	$P_1P_1P_1$	1	[3] [1,1,1] [2,1]	[3]	l_1^+, l_1^-, l_1^0
				[2,1]	l_2^+, l_2^-, l_2^0
	$P_1P_1P_2$	1	[3] [1,1,1] [2,1]	[2]	l_3^+, l_3^-, l_3^0
				[1,1]	l_4^+, l_4^-, l_4^0
	$P_1P_2P_2$	1	[3] [1,1,1] [2,1]	□	l_5^+, l_5^-, l_5^0
	$P_2P_2P_2$	1	[3] [1,1,1] [2,1]	1	l_6^+, l_6^-, l_6^0

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References

[1] D. Weingarten, Phys. Rev. Lett. 51 (1983) 1830;
S. Nussinov, Phys. Rev. Lett. 51 (1983) 2081;
E. Witten, Phys. Rev. Lett. 51 (1983) 2351.
[2] J.M. Gipson, Y. Tosa and R.E. Marshak, Phys. Rev. D32 (1985) 284.
[3] V. Silveira and A. Zee, Phys. Lett. 157B (1985) 191.
[4] Y. Okamoto and R.E. Marshak, Phys. Lett. 162B (1985) 333; Virginia Tech preprint VPI-HEP-85/10 (1985).
[5] G. 't Hooft, in: Recent development in gauge theories, Proc. NATO Advanced Study Institute (Cargèse, 1979) eds. G. 't Hooft et al. (Plenum, New York, 1980).
[6] I. Bars, Nucl. Phys. B208 (1982) 77.
[7] S. Raby, S. Dimopoulos and L. Susskind, Nucl. Phys. B169 (1980) 373;
M. Peskin, unpublished.
[8] K. Osterwalder and E. Seiler, Ann. Phys. 110 (1978) 440;
E. Fradkin and S.H. Shenker, Phys. Rev. D19 (1979) 3682;
T. Banks and E. Rabinovici, Nucl. Phys. B160 (1979) 349.
[9] S. Dimopoulos, S. Raby and L. Susskind, Nucl. Phys. B173 (1980) 208.
[10] See, H.P. Nilles and S. Raby, Nucl. Phys. B189 (1981) 93;
J. Goity, R.D. Peccei and D. Zeppenfeld, DESY preprint 85-051 (1985).
[11] L. Alvarez-Gaumé and E. Witten, Nucl. Phys. B234 (1983) 269.
[12] L.F. Li, Phys. Rev. D9 (1974) 1723;
D.D. Wu, Nucl. Phys. B199 (1982) 523;
P. Jetzer, J.-M. Gérard and D. Wyler, Nucl. Phys. B241 (1984) 204.