MÖBIUS AND SUPER-MÖBIUS GAUGE FIXING FOR THE CLOSED STRING AMPLITUDES ON THE DISK

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We systematically study the gauge fixing of the Möbius and the super-Möbius transformations for the N-point closed-string amplitudes on the disk. Using the Faddeev–Popov method, we obtain the explicit formulas for the Koba–Nielsen factors for these amplitudes.

Although multi-loop string amplitudes have been extensively studied recently [1], tree string amplitudes are still important in connection with the N-string vertices [2], and the cancellations of dilaton tadpoles between disk and RP_2 [3–5]. For each tree string amplitude we have to specify the Koba–Nielsen factor in order to factorize the invariant "gauge" volume of the Möbius transformations. The Koba–Nielsen factor for the bosonic closed-string amplitudes on the sphere is given by [6]

$$\Delta_S(\tilde{z}_a, \tilde{z}_b, \tilde{z}_c) = |(\tilde{z}_a - \tilde{z}_b)(\tilde{z}_b - \tilde{z}_c)(\tilde{z}_c - \tilde{z}_a)|^2 , \quad (1)$$

while that for the bosonic mixed amplitudes (with closed-string and open-string external states) on the disk is given by [7]

$$\Delta(\tilde{z}_a, \tilde{x}_b) = (1 - |\tilde{z}_a|^2)|\tilde{z}_a - \tilde{x}_b|^2 , \quad (2)$$

where $\tilde{z}_a$ is one of the closed-string gauge fixing parameters and $\tilde{x}_b$ is one of the open-string gauge fixing parameters. There is, however, no systematic study of the Koba–Nielsen factors for purely closed-string amplitudes on the disk, in spite of the importance of such amplitudes [3–5]. In this letter we thus discuss the Koba–Nielsen factors for N-point closed-string disk amplitudes for both bosonic and supersymmetric theories.

We first illustrate our method by applying it to the simple case of the N-point bosonic closed-string amplitudes on the sphere. The amplitudes are written in a generic form as

$$A_N = \int \sum_{i=1}^{N} d^2 z_i f(z) , \quad (3)$$

where $f(z) \equiv f(z_1, z_2, ..., z_N; p_1, p_2, ..., p_N)$, and the integration is performed over the whole complex plane. For instance, for N closed-tachyon external states ($\alpha' p_i^2 = 4$), we have

$$f(z) = \prod_{i<j}^{N} |z_i - z_j|^{\alpha' p_i \cdot p_j} . \quad (4)$$

The Möbius transformation on the sphere is given by

$$z \to g(z) = \frac{Az + B}{Cz + D} , \quad (5)$$

where A, B, C, and D are complex numbers satisfying $AD - BC = 1$. Since A_N is invariant under simultaneous Möbius transformations of all the variables z_i, the integral diverges due to the infinite gauge volume. We can factor out this divergence by the Faddeev–Popov method. The Faddeev–Popov determinant is given by

$$\Delta_S^{-1} = \int d\mu(g) \prod_{j=1}^{3} \delta^{(2)}(g(z_j)-\tilde{z}_j) , \qquad (6)$$

where

$$\int d\mu(g) = \int d^2A\, d^2B\, d^2C\, d^2D\, \delta^{(2)}(AD-BC-1)$$

$$\equiv V(\text{gauge}) \qquad (7)$$

is the (infinite) invariant gauge volume of the Möbius transformation and $\tilde{z}_1$, $\tilde{z}_2$, and $\tilde{z}_3$ are arbitrary constants. By directly performing the integration in eq. (6), we obtain the Koba–Nielsen factor $\Delta_S(\tilde{z}_1, \tilde{z}_2, \tilde{z}_3)$ of eq. (1). Following the usual steps of the Faddeev–Popov method, we then have

$$A_N = V(\text{gauge}) \int \prod_{i=1}^{N} d^2z_i f(z) \Delta_S(z_1, z_2, z_3)$$

$$\times \prod_{j=1}^{3} \delta^{(2)}(z_j-\tilde{z}_j) , \qquad (8)$$

where the infinity is factored out in V(gauge). Instead of direct integration over the gauge parameters in eq. (6), we can alternatively obtain Δ_S of eq. (1) as the jacobian of the change of integration variables. Since there are three complex degrees of freedom in the Möbius transformation of eq. (5), we can trade the integration over three z_i's for the integration over the gauge parameters A, B, C [with $D=(BC+1)/A$] by writing

$$z_i = g^{-1}(\tilde{z}_i) \quad (i=1, 2, 3) , \qquad (9)$$

where $\tilde{z}_i$ are arbitrary constants. Eq. (3) becomes

$$A_N = \int d^2A\, d^2B\, d^2C\, d^2D\, \delta^{(2)}(AD-BC-1) J_1$$

$$\times \int \prod_{i=4}^{N} d^2z_i f(g^{-1}(\tilde{z}_1), g^{-1}(\tilde{z}_2), g^{-1}(\tilde{z}_3),$$

$$z_4, ..., z_N; p_1, ..., p_N)$$

$$= \int d\mu(g) \int \prod_{i=1}^{N} d^2z_i f(z) J_1 \prod_{j=1}^{3} \delta^{(2)}(z_j-g^{-1}(\tilde{z}_j)) , \qquad (10)$$

where

$$J_1 = |\partial(z_1, z_2, z_3)/\partial(A, B, C)|_{D=(BC+1)/A}|^2 . \qquad (11)$$

We also have

$$\prod_{j=1}^{3} \delta^{(2)}(z_j-g^{-1}(\tilde{z}_j)) = J_2 \prod_{j=1}^{3} \delta^{(2)}(g(z_j)-\tilde{z}_j) , \qquad (12)$$

where

$$J_2 = \frac{1}{\prod_{j=1}^{3} |g^{-1\prime}(\tilde{z}_j)|^2} . \qquad (13)$$

By substituting eq. (12) into eq. (10) and comparing the result with eq. (6), we find that

$$J_1 J_2 = \Delta_S(\tilde{z}_1, \tilde{z}_2, \tilde{z}_3)$$

$$= |(\tilde{z}_1-\tilde{z}_2)(\tilde{z}_2-\tilde{z}_3)(\tilde{z}_3-\tilde{z}_1)|^2 . \qquad (14)$$

Note that since $\tilde{z}_1$, $\tilde{z}_2$, and $\tilde{z}_3$ are completely arbitrary, A_N in eq. (8) still possesses the full Möbius invariance even after the gauge fixing.

We now consider the Koba–Nielsen factor the N-point bosonic closed-string amplitudes on the disk. The amplitudes are again formally given by eq. (3), but the integration domain is now the unit disk. The Möbius transformation in this case is given by

$$z \to g(z) = \frac{Az+B}{\bar{B}z+\bar{A}} , \qquad (15)$$

where A and B are complex numbers satisfying $|A|^2-|B|^2=1$. Since there are three real gauge degrees of freedom in the transformation, we can fix three real degrees of freedom of z_i, say, z_1 and "half" of z_2. The integration over the other "half" of z_2 can be expressed as a line integral on the unit disk. We now follow the steps from eq. (9) to eq. (14). Namely, we change the integration variables from z_1 and z_2 to the gauge parameters A, B (with $|A|^2-|B|^2=1$), and $\tilde{z}_2$ by using the following relation:

$$z_i = g^{-1}(\tilde{z}_i) \quad (i=1, 2) , \qquad (16)$$

where $\tilde{z}_1$ is an arbitrary constant and $\tilde{z}_2$ is the remaining degree of freedom after the "half" of z_2 is fixed. We take the special case in which $\tilde{z}_1$ is real and the imaginary part of z_2 is fixed to be zero, i.e., $\tilde{z}_2$ is real. We then have

$$\int_{|z_i|\leq 1} d^2z_1\, d^2z_2$$

$$= \int d^2A\, d^2B\, \delta^{(2)}(|A|^2-|B|^2-1) \int_{\tilde{z}_1}^{1} d\tilde{z}_2 J_1 . \qquad (17)$$

(For the $\tilde{z}_2$ integration we can alternatively integrate from -1 to $\tilde{z}_1$.) The above choice of the integration range for $\tilde{z}_2$ guarantees the one-to-one correspondence of the integration variables between both sides of eq. (17). Simple calculations give the following Koba–Nielsen factor:

$$\Delta_D(\tilde{z}_1, \tilde{z}_2) = J_1 J_2 = (1-|\tilde{z}_1|^2)\,|(\tilde{z}_1-\tilde{z}_2)(1-\tilde{z}_1\bar{\tilde{z}}_2)|\ . \quad (18)$$

(See also ref. [8].) Note that the dependence on the gauge parameters A and B is cancelled out between J_1 and J_2 as it should be. In order to check the gauge invariance of Δ_D under the Möbius transformation of z_1 and z_2, we must express $\tilde{z}_2$ in Δ_D in terms of z_1, z_2, and an arbitrary real constant $\tilde{z}_1$ by using eq. (16). We have

$$\Delta_D = \frac{(1-\tilde{z}_1^2)^3 |z_1-z_2|/|1-z_1\bar{z}_2|}{(1\pm\tilde{z}_1|z_1-z_2|/|1-z_1\bar{z}_2|)^2}\,, \quad (19)$$

where the $\pm$sign corresponds to the two possible $\tilde{z}_2$ integration ranges: from $\tilde{z}_1$ to 1 and from -1 to $\tilde{z}_1$. Since $|z_1-z_2|/|1-z_1\bar{z}_2|$ is Möbius invariant, so is Δ_D. The N-point bosonic closed-string amplitudes on the disk are therefore given by, after dividing out V(gauge),

$$A_N(\tilde{z}_1) = \int_{\tilde{z}_1}^{1} \mathrm{d}\tilde{z}_2 \int_{|z_i|\leq 1} \prod_{i=1}^{N} \mathrm{d}^2 z_i f(z) \Delta_D(z_1, z_2) \times \delta^{(2)}(z_1-\tilde{z}_1)\delta^{(2)}(z_2-\tilde{z}_2)\,, \quad (20)$$

where $\Delta_D(z_1, z_2)$ is given by eq. (18). For the N-point closed-tachyon amplitudes, for instance, we have

$$A_N = \int_{\tilde{z}_1}^{1} \mathrm{d}\tilde{z}_2 \int_{|z_i|\leq 1} \left(\prod_{i=1}^{N} \mathrm{d}^2 z_i |1-z_i\bar{z}_i|^2\right) \times \prod_{i<j}^{N} |(z_i-z_j)(1-z_i\bar{z}_j)|^{\alpha' p_i \cdot p_j} \times (1-|\tilde{z}_1|^2)\,|(z_1-z_2)(1-z_1\bar{z}_2)| \times \delta^{(2)}(z_1-\tilde{z}_1)\delta^{(2)}(z_2-\tilde{z}_2)\ . \quad (21)$$

Note that even after our special gauge fixing (both $\tilde{z}_1$ and $\tilde{z}_2$ are real), $A_N(\tilde{z}_1)$ in eq. (20) still has a residual gauge invariance under the transformation of eq. (15) with real A and B. We shall now check the gauge fixing independence of $A_N(\tilde{z}_1)$ in eq. (20), using this residual gauge invariance. For two arbitrary real numbers $\tilde{z}_1$ and $\tilde{z}'_1$ we can always find a Möbius transformation G_0 such that

$$\tilde{z}'_1 = G_0(\tilde{z}_1) = \frac{A_0\tilde{z}_1 + B_0}{B_0\tilde{z}_1 + A_0}\,, \quad (22)$$

where A_0 and B_0 are real. We can then show that

$$A_N(\tilde{z}'_1) = A_N(G_0(\tilde{z}_1)) = A_N(\tilde{z}_1)\,, \quad (23)$$

by using the Möbius invariance of the original amplitude, eq. (3), and the following identity:

$$\int_{G_0(\tilde{z}_1)}^{1} \mathrm{d}\tilde{z}_2 \Delta_D(z_1, z_2)\delta^{(2)}(z_1-G_0(\tilde{z}_1))\delta^{(2)}(z_2-\tilde{z}_2)$$
$$= \int_{\tilde{z}_1}^{1} \mathrm{d}\tilde{z}_2 \Delta_D(G_0^{-1}(z_1), G_0^{-1}(z_2)) \times \delta^{(2)}(G_0^{-1}(z_1)-\tilde{z}_1)\delta^{(2)}(G_0^{-1}(z_2)-\tilde{z}_2)\ . \quad (24)$$

Note that the Δ_D given in ref. [8] leads to the same $A_N(\tilde{z}_1)$ as ours only when $\tilde{z}_1 = 0$.

We now extend our discussion to superstring amplitudes. Let us first consider the case for the N-point closed-string amplitudes on the sphere. The amplitudes are formally written as

$$A_N = \int \prod_{i=1}^{N} \mathrm{d}^2 z_i \mathrm{d}^2\theta_i f(z, \theta)\,, \quad (25)$$

where $f(z, \theta) \equiv f(z_1, ..., z_N; \theta_1, ..., \theta_N; p_1, ..., p_N)$, z_i are bosonic variables ranging over the whole complex plane, and θ_i are complex Grassmann variables. This A_N is invariant under the super-Möbius transformation,

$$z \to g(z, \theta) = \frac{Az+B}{Cz+D} + \frac{\theta(\gamma z+\delta)}{(Cz+D)^2}\,,$$
$$\theta \to h(z, \theta) = \frac{\gamma z+\delta}{Cz+D} + \frac{\theta(1+\frac{1}{2}\delta\gamma)}{Cz+D}\,, \quad (26)$$

where A, B, C, and D are the complex bosonic parameters, and δ and γ are complex Grassmann parameters. Note that we have in the super-Möbius transformation three complex bosonic degrees of freedom, A, B, C [with $D = (BC+1)/A$] and two complex fermionic degrees of freedom, δ and γ. We can thus fix three z_i, say, z_1, z_2, and z_3, and two θ_i, say θ_1 and θ_2, to arbitrary constants. The Koba–Nielsen factor is extracted as before. Namely, we change the integration variables from z_1, z_2, z_3, θ_1, θ_2, and θ_3 to A, B, C [with $D=(BC+1)/A$] δ, γ, and $\tilde{\theta}_3$ by using the following relation:

$$z_i = g(\tilde{z}_i, \tilde{\theta}_i)\ , \quad \theta_i = h(\tilde{z}_i, \tilde{\theta}_i) \quad (i=1,2,3)\ , \tag{27}$$

where $\tilde{z}_1$, $\tilde{z}_2$, $\tilde{z}_3$, $\tilde{\theta}_1$, and $\tilde{\theta}_2$ are arbitrary constants, but $\tilde{\theta}_3$ is an integration variable. The super jacobians J_1 and J_2 are defined in analogy with the bosonic case. For simplicity, we fix $\tilde{\theta}_1$ and $\tilde{\theta}_2$ to be zero. After a tedious but straightforward calculation, we obtain

$$J_1 J_2 = |(\tilde{z}_2 - \tilde{z}_3 - \tilde{\theta}_2\tilde{\theta}_3)(\tilde{z}_3 - \tilde{z}_1 - \tilde{\theta}_3\tilde{\theta}_1)|^2 \times |1-\delta\gamma|^2 |_{\tilde{\theta}_1=\tilde{\theta}_2=0}\ . \tag{28}$$

Absorbing the factor $|1-\delta\gamma|^2$ in eq. (28) into the invariant gauge volume,

$$V(\text{gauge}) = \int d^2A\, d^2B\, d^2C\, d^2D\, \delta^{(2)}(AD-BC-1) \times \int d^2\delta\, d^2\gamma |1-\delta\gamma|^2\ , \tag{29}$$

we have the following Koba–Nielsen factor:

$$\mathscr{D}_S(\tilde{z}_1, \tilde{z}_2, \tilde{z}_3, \tilde{\theta}_1, \tilde{\theta}_2, \theta_3) = |(\tilde{z}_2 - \tilde{z}_3 - \tilde{\theta}_2\theta_3)(\tilde{z}_3 - \tilde{z}_1 - \theta_3\tilde{\theta}_1)|^2 |_{\tilde{\theta}_1=\tilde{\theta}_2=0}\ , \tag{30}$$

where we have renamed $\tilde{\theta}_3$ as θ_3. (See also ref. [9].) The amplitude is then written as, after dividing out V(gauge),

$$A_N(\tilde{z}_1, \tilde{z}_2, \tilde{z}_3, \tilde{\theta}_1, \tilde{\theta}_2) = \int \prod_{i=1}^{N} d^2z_i d^2\theta_i f(z,\theta) \mathscr{D}_S \times \delta^{(2)}(z_1-\tilde{z}_1)\delta^{(2)}(z_2-\tilde{z}_2)\delta^{(2)}(z_3-\tilde{z}_3) \times \delta^{(2)}(\theta_1-\tilde{\theta}_1)\delta^{(2)}(\theta_2-\tilde{\theta}_2)|_{\tilde{\theta}_1=\tilde{\theta}_2=0}\ . \tag{31}$$

Note that since the numbers of fixed z_i and θ_i are different here (three and two, respectively), A_N in eq. (31) is invariant only under the following residual Möbius transformation:

$$z \to G(z,\theta) = \frac{Az+B}{Cz+D}\ , \quad \theta \to H(z,\theta) = \frac{\theta}{Cz+D}\ . \tag{32}$$

We now consider the case for the N-point closed superstring amplitudes on the disk. The amplitudes are again formally given by eq. (25), but the z_i integration is performed over the unit disk here. The super-Möbius transformation in this case is given by

$$z \to g(z,\theta) = \frac{Az+B}{\bar{B}z+\bar{A}} + \frac{\theta(\gamma z \pm \bar{\gamma})}{(\bar{B}z+\bar{A})^2}\ ,$$

$$\theta \to h(z,\theta) = \frac{\gamma z \pm \bar{\gamma}}{\bar{B}z+\bar{A}} + \frac{\theta(1 \pm \frac{1}{2}\bar{\gamma}\gamma)}{\bar{B}z+\bar{A}}\ , \tag{33}$$

where the $\pm$sign conforms to that in the involution of θ [5]:

$$(z,\theta) \to (1/\bar{z}, \pm\bar{\theta}/\bar{z})\ . \tag{34}$$

Following the same steps as before, we change the integration variables from z_1, z_2, θ_1, and θ_2 to A, B (with $|A|^2-|B|^2=1$),$\tilde{z}_2$, γ, and $\tilde{\theta}_2$ by using the following relation:

$$z_i = g(\tilde{z}_i, \tilde{\theta}_i)\ , \quad \theta_i = h(\tilde{z}_i, \tilde{\theta}_i) \quad (i=1,2)\ , \tag{35}$$

where $\tilde{z}_1$ and $\tilde{\theta}_1$ are arbitrary constants, while $\tilde{z}_2$ and $\tilde{\theta}_2$ are integration variables. We assume $\tilde{z}_1$ and $\tilde{z}_2$ to be real for simplicity. We then get, by setting $\tilde{\theta}_1=0$,

$$J_1 J_2 = |\tilde{z}_1 - \tilde{z}_2 - \tilde{\theta}_1\theta_2|\ |1-\tilde{z}_1\bar{\tilde{z}}_2 \pm \tilde{\theta}_1\bar{\theta}_2| \times (1 \pm \gamma\bar{\gamma})|_{\tilde{\theta}_1=0}\ , \tag{36}$$

where we have renamed $\tilde{\theta}_2$ as θ_2. Absorbing the factor $1\pm\gamma\bar{\gamma}$ in eq. (36) into the invariant gauge volume, V(gauge), we have the following Koba–Nielsen factor:

$$\mathscr{D}_D(\tilde{z}_1, \tilde{z}_2, \tilde{\theta}_1, \theta_2) = \mp |\tilde{z}_1 - \tilde{z}_2 - \tilde{\theta}_1\theta_2|\ |1-\tilde{z}_1\bar{\tilde{z}}_2 \pm \tilde{\theta}_1\bar{\theta}_2|\,|_{\tilde{\theta}_1=0}\ . \tag{37}$$

The $\pm$sign conforms to that in eq. (34). Thus, the amplitude is written as, after dividing out V(gauge),

$$A_N(\tilde{z}_1, \tilde{\theta}_1) = \int_{\tilde{z}_1}^{1} d\tilde{z}_2 \int_{|z_i|\leq 1} \prod_{i=1}^{N} d^2z_i d^2\theta_i f(z,\theta) \mathscr{D}_D \times \delta^{(2)}(z_1-\tilde{z}_1)\delta^{(2)}(z_2-\tilde{z}_2)\delta^{(2)}(\theta_1-\tilde{\theta}_1)|_{\tilde{\theta}_1=0}\ ,$$

where $\tilde{z}_1$ and $\tilde{z}_2$ are real. Note that even after our particular gauge fixing ($\tilde{z}_1$ and $\tilde{z}_2$ are real and $\tilde{\theta}_1=0$), the amplitude is still invariant under the residual Möbius transformation of eq. (32) with $C=B$ and $D=A$. (A and B are real.) The gauge fixing independence of $A_N(\tilde{z}_1, \tilde{\theta}_1)$ in eq. (38) can be shown in the same way as in the bosonic case. For two sets of arbitrary numbers $(\tilde{z}_1, \tilde{\theta}_1)$ and $(\tilde{z}'_1, \tilde{\theta}'_1)$, where $\tilde{z}_1$ and $\tilde{z}'_1$ are real, we can always find transformations G_0 and H_0 such that

$$\tilde{z}'_1 = G_0(\tilde{z}_1, \tilde{\theta}_1) = \frac{A_0\tilde{z}_1 + B_0}{B_0\tilde{z}_1 + A_0},$$

$$\tilde{\theta}'_1 = H_0(\tilde{z}_1, \tilde{\theta}_1) = \left.\frac{\tilde{\theta}_1}{B_0\tilde{z}_1 + A_0}\right|_{\tilde{\theta}_1=0} = 0 . \tag{39}$$

We then get

$$A_N(\tilde{z}'_1, \tilde{\theta}'_1)|_{\tilde{\theta}'_1=0} = A_N(G_0(\tilde{z}_1, \tilde{\theta}_1), H_0(\tilde{z}_1, \tilde{\theta}_1))|_{H_0=0}$$

$$= A_n(\tilde{z}_1, \tilde{\theta}_1)|_{\tilde{\theta}_1=0} , \tag{40}$$

by using the super-Möbius invariance of the original amplitude, eq. (25), and the following identity:

$$\int_{G_0(\tilde{z}_1,\tilde{\theta}_1)}^{1} d\tilde{z}_2 \int d^2\tilde{\theta}_2\, \mathscr{D}_D(z_1, z_2, \theta_1, \theta_2)$$
$$\times \delta^{(2)}(z_1 - G_0(\tilde{z}_1, \tilde{\theta}_1))\delta^{(2)}(z_2 - \tilde{z}_2)$$
$$\times \delta^{(2)}(\theta_1 - H_0(\tilde{z}_1, \tilde{\theta}_1))\delta^{(2)}(\theta_2 - \tilde{\theta}_2)|_{\tilde{\theta}_1=0}$$
$$= \int_{\tilde{z}_1}^{1} d\tilde{z}_2 \int d^2\tilde{\theta}_2\, \mathscr{D}_D(G_0^{-1}(z_1, \theta_1), G_0^{-1}(z_2, \theta_2),$$
$$H_0^{-1}(z_1, \theta_1), H_0^{-1}(z_2, \theta_2))$$
$$\times \delta^{(2)}(G_0^{-1}(z_1, \theta_1) - \tilde{z}_1)\delta^{(2)}(G_0^{-1}(z_2, \theta_2) - \tilde{z}_2)$$
$$\times \delta^{(2)}(H_0^{-1}(z_1, \theta_1) - \tilde{\theta}_1)$$
$$\times \delta^{(2)}(H_0^{-1}(z_2, \theta_2) - \tilde{\theta}_2)|_{\tilde{\theta}_1=0} . \tag{41}$$

In this letter we have obtained the Koba–Nielsen factors for closed-string amplitudes on the disk. This essentially completes the list of all the Koba–Nielsen factors for both bosonic and supersymmetric string theories [#1].

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[#1] One should note that there is a difficulty in defining supersymmetry on RP_2. (See ref. [5].)

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