

2018 Doctoral Thesis

Essays on Technological Progress and
Unemployment

Graduate School of Economics

Academic Supervisor: Professor KUDOH Noritaka

Author Name: TANAKA Kosho

Acknowledgments

I am deeply grateful to my academic supervisor, Noritaka Kudoh, for his invaluable support and advice throughout the master's and doctoral programs. The last five years were full of irreplaceable and precious experiences to me. I would also like to thank my two committee members, Makoto Hanazono and Jiro Nemoto, for their insightful and constructive feedback. I am grateful to numerous individuals for comments and suggestions. I am particularly indebted to Takashi Shimizu, Hiroaki Miyamoto, Hiromi Nosaka, Atushi Ohyama and Ryo Arawatari. Their sincere comments and encouraging suggestions, actually after reading my paper, have heavily contributed to the dissertation. I also wish to thank Akihiko Yanase, Eiji Mangyo, Jun-ichi Itaya, Hikaru Hasegawa, Shinichi Kitasaka, Soichi Shinohara, Kenichi Kaminoyama, Koichi Hutagami and Akihisa Shibata for frank opinions and instructions in the last eight years. Finally, I would like to thank the members of Hanazono seminar for honest opinions and suggestions. Its members are one of the indispensable elements to have gotten this far.

Contents

1	Overview	7
1.1	The puzzle and big-picture motivation	7
1.2	Ideas and the literature	8
1.3	Organization	11
2	Technological Progress, Firm Selection, and Unemployment	15
2.1	Introduction	15
2.2	The model	16
2.2.1	Wage determination and job creation	19
2.2.2	Firm entry and exit	20
2.3	Stationary equilibrium	21
2.4	Numerical simulation	22
2.5	Conclusion	26
3	Technological Progress and Unemployment Revisited: Obsolescence, Technology Adoption, and Selection in Search Equilibrium	27
3.1	Introduction	27
3.2	The model	30
3.2.1	Set up of the model	30
3.2.2	Value functions	33
3.2.3	Wage determination	34
3.2.4	Optimal labor demand	36
3.2.5	Average productivity levels of update and obsolete firms	37
3.2.6	Firm entry and exit	39
3.3	Stationary equilibrium	43

3.4	Comparative statics	45
3.4.1	Changes in the cutoff productivity levels	45
3.4.2	The impact of technological progress on unemployment	46
3.5	Numerical simulation	48
3.5.1	Calibration	48
3.5.2	Results	50
3.5.3	Sensitivity analysis	52
3.6	Conclusion	53
4	Creative Destruction and Capitalization with On-the-job Search	55
4.1	Introduction	55
4.2	The model	57
4.2.1	Setup	57
4.2.2	Wage determination	61
4.2.3	Equilibrium	62
4.2.4	Labor market dynamics	64
4.3	Numerical simulation	65
4.3.1	Calibration	66
4.3.2	Results	67
4.3.3	Is technology embodied?	69
4.3.4	Sensitivity to b	70
4.4	Conclusion	71
A	Appendix of Chapter 2	73
A.1	Derivation of the inverse demand	73
A.2	Derivation of the wage and labor demand curves	74
A.3	Derivation of the free entry condition	77
B	Appendix of Chapter 3	79
B.1	First order and envelope conditions of firms	79
B.2	Wage determination	80
B.3	Derivation of the average productivity levels	81
B.4	Derivation of the values of firms	82
B.5	Derivation of the firm entry condition	83

B.6	Derivation of P_u/P	85
B.7	The standard capitalization effect	86

Chapter 1

Overview

1.1 The puzzle and big-picture motivation

This dissertation gives novel theoretical explanations for the long-term impact of technological progress on unemployment. Based on trend data of developed countries, the empirical studies support the large negative impact of the rate of technological progress on the unemployment rate. In particular, there are the two influential results in the empirical literature, as follows. Blanchard and Wolfers (2000) estimate that one percent-point decline in the total-factor-productivity growth rate increases the unemployment rate by 0.25-0.71 percent. Pissarides and Valanti (2007) find the negative effect to be 1.3-1.5 percent.

On the theory side, however, the following “knowledge hole” exists as a puzzle: the standard search-matching models fail to predict the actual impact of technological progress on unemployment, both qualitatively and quantitatively.

A search-matching model is the model to incorporate frictional phenomena in which an agent is difficult to meet another agent. As the pioneers, Peter A. Diamond, Dale T. Mortensen, and Christopher A. Pissarides jointly won the Nobel Prize in 2010. This idea is applied in the labor market, and now many analyses of unemployment is based on the search-matching framework. This dissertation also focuses on this framework.

The research problem to be solved, through this dissertation, is that the standard search-matching models (henceforth, the standard models) almost lose ability

to replicate the empirically-observed impact of technological progress on unemployment. In particular, this problem/puzzle falls into two points. The first point is that the standard models tend to generate the opposite qualitative prediction to data once technology obsolescence is taken into account. The second point is that, even if technology obsolescence is abstracted from a standard model, the model can replicate the quantitative magnitude of the prediction less than 1/10 of the above estimated impacts at most.

Why this is important to explain? The answer is that unemployment variations are considered to affect the country's real economic performance significantly, and technological progress is one of the promising candidates to uncover them.

For instance, a decrease in the unemployment rate directly decreases the country's employed workers and thus leading to a lower level of Gross-Domestic-Product. Especially, in Japan, such a problem is worsened by declining birthrate and aging population. Of course, the value of understanding unemployment variations is not confined to the above example, but the followings are the same. Can we predict unemployment variations? What kind of policies improve economic performance?

In addition, technological progress has been considered to be a major determinant of long-term unemployment. Specifically, the rapid diffusion of information and communication technology dramatically changes the standard life-styles and business-styles in the world; at the same time, some firms easily go bankrupt due to misunderstandings of the future possibilities to use it while other firms grow rapidly, represented as "Apple" or "Amazon.com". Together with associated turnovers of jobs and job-types, technological progress appears to seriously affect unemployment. This dissertation seeks plausible mechanisms under such a situation.

1.2 Ideas and the literature

The core conjecture of this dissertation is that the impact of technological progress on unemployment is generated not only through improved-productivity within each firm but also through improved-productivity at the aggregate level.

Compared to the literature, there are two new points here: (1) multi-worker firms and (2) heterogeneity of such firms.

The first point is multi-worker firms. Early studies consider a pair-matching of one job and one worker; in other words, each firm can hire only one worker no more and no less. However, this simplified matching is harmful with the two reasons: firm-size distribution in terms of employees is abstracted; job creation and destruction lead to the same as firm entry and exit. To clearly understand aggregate employment variations, this simplified setting should be replaced by the setting of multi-worker firms in which each firm can optimally choose the number of employees. Especially, the subsequent analyses reveal that the above second reason is a key to solve the puzzle.

The second point is heterogeneity of multi-worker firms. This dissertation considers winners and losers after technological change, by taking firm heterogeneity into account. In the literature, each firm/job is often set to be homogeneous and a market-level change with respect to firm shakeout is abstracted. In particular, introducing firm heterogeneity helps to describe different ways to use technologies and to adopt them by each firm. Such heterogeneity changes model predictions based on previous studies.

The main part of this dissertation is Chapter 2 and 3, in which the above ideas are modeled and examined. Chapter 2 analyzes a model without technology obsolescence. Chapter 3 analyzes a model with technology obsolescence; thus, in this sense, the model of Chapter 2 is generalized in Chapter 3.

On the other hand, in Chapter 4, I examine the role of on-the-job search, which generates job-to-job direct transitions, under the case with technology obsolescence. The objective of Chapter 4 is different from Chapter 2 and 3. Previous studies imply that the incorporation of on-the-job search appears to be one of the most good ideas to reconcile theory and evidence. In Chapter 4, this view is tested by revisiting conventional settings but with more general structure in terms of technology obsolescence. Before going to brief summary of each chapter, I review seminal and related papers.

In the literature, there are two canonical effects, known as: the creative destruction effect and the capitalization effect. Aghion and Howitt (1994) uncover the creative destruction effect under which rapid technological progress leads to rapid job-turnover. In their model, technology obsolescence is introduced and each job becomes obsolete. Importantly, the creative destruction effect is the positive

effect of technological progress on unemployment. On the other hand, Pissarides (2000) shows the capitalization effect under which rapid technological progress uniformly improves the productivity-level of each firm and technology obsolescence is abstracted. In this case, the value of firm/job entry increases and the number of aggregate jobs increases; that is, the capitalization effect is the negative effect of technological progress on unemployment. It is worth noting that the creative destruction effect and the capitalization effect are competing each other.

A natural question arises then: which effect is dominant? Pissarides and Valanti (2007) develop a model with *technology obsolescence*, but not with full obsolescence. They consider that only a fixed-fraction of productivity becomes obsolete in order to describe a middle situation of the mentioned canonical models. However, they find that the creative destruction effect is too strong and its effect has to be fully abstracted when to *qualitatively*-replicate the estimated changes in unemployment. Moreover, even if the creative destruction effect is arbitrarily-excluded, the capitalization effect is still insufficient to predict actual unemployment variations. Specifically, their standard model without technology obsolescence predicts that one percent-point increase in the rate of technological progress decreases the unemployment rate by 0.02 percent. The *quantitative* size of this negative effect is less than 1/10 of the estimated impacts. This puzzle remains to be solved.

Mortensen and Pissarides (1998) incorporate technology-update decision by each firm/job into a model with technology obsolescence. By allowing technology-updates, rapid technological progress not necessarily decreases firm/job lifetime. In addition, they consider firm/job heterogeneity with respect to productivity and show that there is a unique level of cutoff productivity above which technology-updates occur. However, their model predicts that technological progress increases the cutoff productivity level and increases the ratio of short lifetime firms/jobs. Thus, model performance is exacerbated by introducing technology-updates per se. This dissertation suggests that its worsened model performance stems from the equivalence between jobs and firms. By distinguishing each other, model responses through job cutbacks and firm exits are separated. Combined with this arrangement, the model of Chapter 3 conclude the opposite result to that of Mortensen and Pissarides (1998). On the empirical side, Duernecker (2014) points out that many European countries have lagged behind the United States in terms of technol-

ogy adoption. He finds that European countries with higher technology adoption, similar to that observed in the United States, exhibit lower unemployment rates. The dissertation's result is rather plausible in line with this empirical observation.

Back to the puzzle story by Pissarides and Vallanti (2007). One of the most plausible ways to solve the puzzle appears to incorporate on-the-job search. According to the Survey of Income and Program Participation, 49% of total exits from employers is associated with job-to-job transitions and its value increases to 71% over the exits which remain in the labor force (Nagypal, 2008). This empirical observation supports the view that on-the-job search is important in the determination of unemployment. Miyamoto and Takahashi (2011) develop a model with no technology obsolescence and on-the-job search. Their model predicts that one percent-point increase in the rate of technological progress decreases the unemployment rate by 0.23 percent, which is much closer to the empirical estimations. On the other hand, Michau (2013) develop a model with full technology obsolescence and on-the-job search. His model predicts that the creative destruction effect becomes almost negligible mainly through job-to-job direct transitions. Thus, these papers both imply that incorporating on-the-job search has power to solve the puzzle, but in an implicit manner. This dissertation tests how explicitly powerful on-the-job search is by considering a generalized case of these models in terms of technology obsolescence. Interestingly, the generalization uncovers that incorporating on-the-job search leads to puzzle-solving results only under limited cases.

1.3 Organization

The remainder of this dissertation is as follows. Again, the main part is Chapter 2 and 3; these chapters introduce the two novel settings, compared to the literature, (1) multi-worker firms and (2) heterogeneity of such firms.

In Chapter 2, I firstly tackle with the quantitative part of the puzzle. In particular, technology obsolescence is abstracted. Under this simplification and new settings of (1) and (2), the model of this chapter describes the firm selection effect through which an increase in the rate of technological progress induces firms with low productivity levels to exit and increases aggregate-level productivity.

With this effect, one percent-point increase in the rate of technological progress decreases the unemployment rate by 0.28 percent, which is about 40 times as strong as the effect in the same model without the selection effect.

In Chapter 3, the model of Chapter 2 is generalized by taking technology obsolescence into account in order to solve the entire puzzle and understand the role of technology obsolescence in the determination of aggregate employment. In the model of Chapter 3, each firm faces two key decisions: whether to remain in the market and whether to adopt new technology. The latter decision is the extended decision to the model of Chapter 2. In short, these decisions affect the aggregate composition of firm-types, leading to the additional impact on unemployment.

New theoretical implications are in order. First, rapid technology obsolescence not only accelerates job cutbacks but also improves aggregate-level productivity; in this sense, technology obsolescence has a bright side in the long run. Second, distinction between job cutback and firm exit leads to the literature-opposite prediction of a change in the aggregate technology adoption rate. Through a benchmark simulation, one percent-point increase in the rate of technological progress decreases the unemployment rate by 0.48 percent. In contrast to the literature view, the model implies that a large part of surviving firms are exposed to technology obsolescence and this is rather essential to replicate actual unemployment variations both qualitatively and quantitatively.

The heart of Chapter 2 and 3 is to introduce ideas that start from the field of industrial organization and to apply these to the current dissertation's field. As a result, the analyses of Chapter 2 and 3 suggest that the dominant impact of technological progress on unemployment is not through improved productivity within each firm but through improved productivity at the aggregate level. Then, this view leads to one plausible solution to the puzzle as the main contribution to the literature.

However, these results depend on a lot of assumptions. In particular, the models of Chapter 2 and 3 assume that firm-specific productivity is drawn at the time of firm entry, and its corresponding productivity level remains constant forever after the entry event. This means that “winners” after a technological change are always winners after any technological progress. In addition, the models assume competition as Dixit-Stiglitz-style monopolistic competition; there is no strategic

interaction between firms. These problems, which may change the dissertation's conclusion, remain to be studied as future works.

Chapter 4 is quite different from Chapter 2 and 3. The objective of Chapter 4 is to test the implicit view in the literature such that incorporating on-the-job search has power to solve the puzzle. Toward this goal, I develop a generalized model based on the combination of models in Pissarides and Vallanti (2007), Miyamoto and Takahashi (2011) and Michau (2013). In particular, the model considers that a fixed-fraction of productivity becomes obsolete in addition to the setting of on-the-job search. As a result, I confirm the result in Miyamoto and Takahashi (2011) such that the incorporation of on-the-job search magnifies the negative impact of technological progress on unemployment if an almost zero-fraction of productivity becomes obsolete. However, except for this limited situation, the creative destruction effect becomes dominant; thus, the puzzle is still unsolved. The point is as follows. To get the result in Miyamoto and Takahashi (2011), the setting of endogenous job separation by idiosyncratic shock, as in Mortensen and Pissarides (1994), has to be also incorporated. However, the endogenous job separation is abstracted from the model in Michau (2013), and introducing endogenous job separation strengthens the creative destruction effect.

The difficulty of Chapter 4 is in the numerical computation part. A serious problem is that the feasible parameter-space to simulate the model is restricted. Its restriction is to the extent that simulation results become perhaps unreliable. Thus, although the conclusion in Chapter 4 is quite intuitive, model settings and calibration strategy have to be reconsidered one by one in order to get more robust conclusion.

Chapter 2

Technological Progress, Firm Selection, and Unemployment

2.1 Introduction

Recent empirical research using longitudinal data from several developed countries suggests that the negative relationship between technological progress and unemployment. Blanchard and Wolfers (2000) estimate that a decrease in the growth rate of total factor productivity of one percent-point translates into an increase in the unemployment rate of 0.25-0.71 percent. Pissarides and Vallanti (2007) suggest that the same decrease in the growth rate leads to more than one percent increase in the unemployment rate. Associated with these papers, technological progress is considered to be one of the important determinants of unemployment.

On the theory side, however, the standard search-matching models cannot replicate unemployment variations as implied in the estimations. By imposing the assumption that technology obsolescence is negligible in the steady state, the standard models can replicate the *qualitative* impact of technological progress on unemployment. The point, which this chapter focuses, is that the standard models are insufficient to predict the actual *quantitative* size of the impact of technological progress. Actually, a standard model as in Pissarides and Vallanti (2007) predicts that one percent-point increase in the rate of technological progress leads to a decrease in the unemployment rate by around 0.02 percent at most. This predicted

quantitative magnitude is less than 1/10 of the above estimations. This chapter challenges this puzzle.

The novelty of this chapter is to consider firm heterogeneity. As empirically reported in Bartelsman and Doms (2000), productivity differences across establishments and firms are large and persistent. On the other hand, in theory, firm heterogeneity is associated with the determination of aggregate productivity via inter-firm resource reallocations (Melitz, 2003); and, aggregate productivity naturally links to aggregate job creation (Felbermayr and Prat, 2011). The model incorporates these views into the current-focusing literature.

This chapter shows that rapid technological progress increases aggregate job creation through improving aggregate productivity where less productive firms get to be difficult to survive. Here, a market-wide change, as the current novelty, occurs rather than a change within each firm, as the conventional view known as the capitalization effect. In particular, the conventional capitalization effect explains such that technological progress uniformly increases the productivity level within each homogeneous firm and better prospects for entering the market increases aggregate job creation.

The current novelty is the firm selection effect. A higher rate of technological progress lets the labor market become tighter; in other words, the total cost of employing a worker leads to be higher. In this situation, the recruitment cost-and-benefit-balance of less productive firms worsens. Thus, in the new equilibrium, relatively-productive firms only survive as if these firms are selected to remain. Surprisingly, through a simulation, the model predicts that one percent-point increase in the rate of technological progress leads to a decrease in the unemployment rate by 0.28 percent. In addition, within the same model, the impact leads to be 0.007 percent if the current new effect is abstracted. The subsequent sensitivity analyses conclude that the main result is robust.

2.2 The model

The economy is composed of a unit measure of infinitely lived workers and operating heterogeneous firms that the total mass is n . All agents discount the future at the common rate r . Time is discrete and indexed by t .

Production requires only labor input l_t and output is given by $a_t\psi l_t$. ψ denotes a firm-specific productivity level drawn from a Pareto distribution with c.d.f. $F(\psi) = 1 - (\psi_{\min}/\psi)^\alpha$ and p.d.f. $f(\psi)$. Firms are heterogeneous in terms of the firm-specific productivity component which is realized after in the time of entry after incurring the entry cost K_t . For simplicity, I assume that the firm-specific component is constant over time. The rest, a_t , captures technology-related productivity at time t . All firms have the same level of technology-related productivity and its level grows at exogenous rate such that $g = (a_{t+1} - a_t)/a_t$.¹

In the product market, the firms are monopolistically competitive without strategic interactions each other. The aggregate demand of workers/individuals for each differentiated good is given by solving the maximization problem of workers with the Dixit-Stiglitz preference over a continuum of differentiated goods:

$$\max_{Q_{i,t}^j} \left[\int (Q_{i,t}^j)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}}, \quad (2.1)$$

subject to $y_t^j = \int (Q_{i,t}^j p_i/P) di$ where $Q_{i,t}^j$ denotes the demand level of individual j for good i . y_t^j is the real income level of individual j . A price index is $P = [(1/n) \int p_i^{1-\sigma} di]^{1/(1-\sigma)}$ where the price of good i represents p_i . After solving the problem, aggregate demand for good i at time t takes the form:

$$\int Q_{i,t}^j dj = \left(\frac{p_i}{P} \right)^{-\sigma} \frac{Y_t}{n}, \quad (2.2)$$

where $Y_t = \int y_t^j dj$. For given this aggregate demand, each monopolistic firm determines the number of job vacancies v to maximize the total profit in the future.

The value of an operating firm with type ψ at time t is

$$J_t(l, \psi) = \max_v \left[R_t(l; \psi) - w_t(l; \psi)l - \gamma_t v - I_t + \left(\frac{1-\delta}{1+r} \right) J_{t+1}(l', \psi) \right], \quad (2.3)$$

where $R_t(l; \psi) = [p(l; \psi)/P]a_t\psi l$ represents the revenue for given l and ψ . Here,

¹This setting is known as disembodied technological progress.

$p(l; \psi)/P = (a_t \psi l_t / [Y_t/n])^{-1/\sigma}$ holds from the derived inverse demand. In addition, $w_t(l; \psi)$ denotes the wage; γ_t is the cost of posting a job vacancy. Each firm is destroyed with exogenous probability δ ; and independently, each employed worker also faces job separation shock with exogenous probability λ . Thus, while employed, the workers become unemployed after production with the total separation probability $s = \lambda + \delta - \lambda\delta$. The employment level evolves such that $l' = (1 - \lambda)l + q(\theta)v$. The term $q(\theta)v$ represents new employees because posted vacancies are filled with probability $q(\theta)$. The labor market involves search frictions where the total number of the unemployed workers u and the aggregate number of vacancies V jointly determine the total new jobs/matches made; by following the literature, the number of newly created jobs is represented by the function $m(u, V)$, called as matching function. This is an increasing function with respect to both arguments and the function is assumed to be constant returns to scale in line with empirical evidence. The extent of "labor market friction" is measured by the indicator $\theta \equiv V/u$, called as labor market tightness. This indicator is endogenous and the most important variable in this chapter because unemployment u depends on only θ , endogenously. The vacancy filling probability is given as $m(u, V)/V = m(1/\theta, 1) = q(\theta)$ where $q(\cdot)$ is a decreasing function. Similarly, the job finding probability is $m(u, V)/u = \theta q(\theta)$.

The values of workers at time t are

$$E_t(l; \psi) = w_t(l; \psi) + \frac{1}{1+r} [(1-s)E_{t+1}(l'; \psi) + sU_{t+1}], \quad (2.4)$$

for an employed worker that works for a firm with type ψ , and

$$U_t = z_t + \frac{1}{1+r} [\theta q(\theta) \tilde{E}_{t+1} + (1 - \theta q(\theta))U_{t+1}], \quad (2.5)$$

for an unemployed worker. z_t denotes the flow value of unemployment including unemployment insurance and a home production value; \tilde{E}_{t+1} is the expected value in the state of being employed at the next period.

The steady state in the economy reflects a balanced growth path where all variables grow at the rate g . After normalized by the general productivity level a at a base period, the variables become constant over time as a steady-state equilibrium. To ensure this, I assume that exogenous variables are also constant

after the normalization such that $\gamma_t/a_t = \gamma$, $z_t/a_t = z$, $I_t/a_t = I$ and $K_t/a_t = K$.²

In equilibrium, the value of each firm evolves at rate g so that $J_t(l, \psi)/a_t = J(l, \psi)$. One important remark is that the mentioned functional problem on the firm side after the normalization involves the net discount factor:

$$\Lambda_g = \frac{1 - \delta}{1 + r}(1 + g).$$

A change in the rate of technological progress affects firm's labor demand through a change in the net discount factor as discussed later.

2.2.1 Wage determination and job creation

I assume that wages are determined through the bargaining problem in which a fixed fraction β of the match surplus goes to the worker and a fraction $1 - \beta$ goes to the firm. As the key point to solve such a problem, each worker is treated as the marginal worker (Stole and Zwiebel 1996); implying the surplus sharing rule:

$$E_t(l; \psi) - U_t = \beta \left[\frac{\partial J_t(l; \psi)}{\partial l} + E_t(l; \psi) - U_t \right], \quad (2.6)$$

where $\partial J_t(l; \psi)/\partial l + E_t(l; \psi) - U_t$ represents the match surplus which takes into account the future events. Combining Eqs. (2.3)-(2.6), I get the following wage curve (see Appendix):

$$w = z + \frac{\beta}{(1 - \beta)(1 - \delta)} \frac{\gamma}{q(\theta)} [\theta q(\theta) + (1 - \delta)(\Lambda_g^{-1} - 1 + \lambda)]. \quad (2.7)$$

This wage function is the value after the normalization such that $w = w_t/a_t$. As you can see, w is constant for given θ . Thus, the economy exhibits one wage equilibrium because the hiring cost, which depends on the market tightness θ , is the same across firms. It is not the specific property in the current model, and here I avoid detail discussions about this point. The wage increases with the market tightness because the value of the unemployed increases.

On the other hand, the market tightness is determined by solving:

²See, for example, Mortensen and Pissarides (1998), Pissarides and Vallanti (2007) and Hornstein, Krusell and Violante (2007).

$$\frac{\gamma}{q(\theta)} = \sum_{t=1}^{\infty} \Lambda_g^t \left[\frac{\sigma-1}{\sigma-\beta} \tilde{\psi} - w - \frac{\lambda\gamma}{q(\theta)} \right], \quad (2.8)$$

where $\tilde{\psi}$ denotes the average level of firm-specific productivity. This equation is derived from the firm's maximization problem in determining the level of labor demand and its problem is evaluated at the averaged firm. An average firm with type $\tilde{\psi}$ is the firm that sets the optimal price being equal to the price index. The above equation means that the recruitment cost $\gamma/q(\theta)$ is equal to the job creation value; the terms in brackets represent the revenue per worker minus the wage and the cost of separation shock.

2.2.2 Firm entry and exit

A large pool of ex-ante identical potential entrants aim for entering the market, and each entry requires the sunk cost K_t ; then, each entrant realizes a level of ψ drawn from the common distribution $F(\psi)$. The total expected profit, just after drawing specific ψ , is defined as $J(0, \psi)$. There is no employees at this moment and $J(0, \psi)$ can be viewed as the value of entry for given ψ . Thus, the free entry of firms implies the following free entry condition:

$$\left(\frac{\psi^*}{\psi_{\min}} \right)^\alpha K = J(0, \tilde{\psi}), \quad (2.9)$$

where ψ is evaluated at the averaged firm.³ In short, new entrants enter the market by considering the expected profit.

However, if an entrant realizes a too low level of ψ , the entrant rather exits. Such a decision-cutoff level of the firm-specific component is derived from the condition:

$$J(0, \psi^*) = 0, \quad (2.10)$$

where the corresponding reservation rule is $\psi^* = \inf \{ \psi : J(0, \psi) \geq 0 \}$. From some computations, ψ^* can be uniquely determined by:

³Here, $\int_{\psi^*}^{\infty} J(0, \psi) f(\psi) d\psi = (\psi_{\min}/\psi^*)^\alpha J(0, \tilde{\psi}) = K$ holds.

$$\left(\frac{\psi^*}{\psi_{\min}}\right)^\alpha K = \left[1 + \sum_{t=1}^{\infty} \Lambda_g^t\right] \frac{\sigma - 1}{\alpha + 1 - \sigma} I. \quad (2.11)$$

Note that ψ^* pins down $\tilde{\psi}$ such that $\tilde{\psi}(\psi^*) = \left[\int_{\psi^*}^{\infty} \psi^{\sigma-1} [f(\psi)/1 - F(\psi^*)] d\psi\right]^{1/(\sigma-1)}$. As in Melitz (2003), the economy can be interpreted to be composed of *homogenous* firms with the same productivity level $\tilde{\psi}$, but the incorporation of heterogeneous firms lets the model describe an endogenous change in its average productivity level which reflects adjustments of firm entry and exit.

2.3 Stationary equilibrium

An *equilibrium solution* is a list of the variables (w, θ, ψ^*) that satisfy Eqs. (2.7), (2.8) and (2.11). The equilibrium unemployment rate u is derived so that the flows in and out of employment equate as below:

$$u\theta q(\theta) = (1 - u)s \Leftrightarrow u = \frac{s}{\theta q(\theta) + s}, \quad (2.12)$$

where u decreases with θ . Finally, the mass of firms is given by the equality condition of aggregate labor demand and the total employed workers:

$$nl(\tilde{\psi}) = 1 - u \Leftrightarrow n = \frac{1 - u}{l(\tilde{\psi})}, \quad (2.13)$$

where the average firm's revenue $\tilde{\psi}l(\tilde{\psi})$ is pinned down by Eq. (2.10) and the average firm's optimal employment $l(\tilde{\psi})$ is determined for given $\tilde{\psi}$. In equilibrium, an increase in the average productivity level serves to decrease the average employment as a labor saving effect.

I now turn to explaining the impact of technological progress. Figure 2.1 depicts stationary equilibria under different rates of technological progress, g . The *JC* line is Eq. (2.8) after plugging the wage function. The *JC* line is upward sloping because a higher average productivity level increases the market tightness with a higher aggregate labor demand. The *FE-ZCP* line is given from Eq. (2.11).

Consider the solid lines are benchmark. If g increases, then the solid lines shift to the dashed lines respectively. In this case, the steady-state equilibrium changes

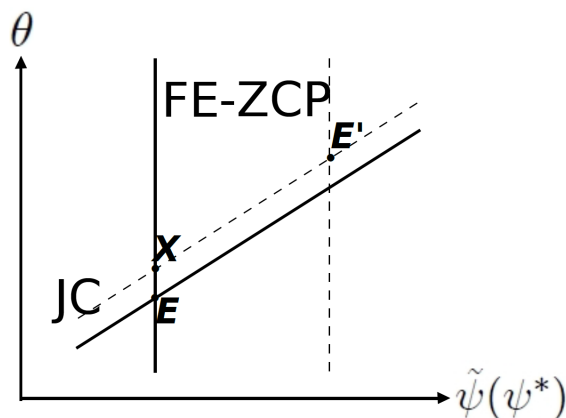


Figure 2.1: Balanced growth equilibria under different rates of technological progress

from E to E' as $\Delta g > 0$. The point is that the change from E to E' can be decomposed into the changes: from E to X and from X to E' . The former change is from the shift of JC line and the latter change is from the shift of $FE-ZCP$ line. While the former change is the conventional channel, the latter change is the new channel in this chapter. In the next subsection, I illustrate the quantitative importance of each channel.

2.4 Numerical simulation

The model is calibrated to the U.S. economy, and its time period is taken to be one-year. The matching function is assumed to be Cobb-Douglas $m(u, V) = m_0 u^\eta V^{1-\eta}$. Parameter values are summarized in Table 2.1. I choose the yearly interest rate as $r = 0.05$ and the rate of technological progress as $g = 0.02$. As in Pissarides (2009), the monthly job separation rate is set to be $s/12 = 0.036$; its derivation method is based on Shimer (2012). A firm destruction rate is set as $\delta = 0.087$, computed from averaging its available values for 1977-2014 in Business Dynamics Statistics. Matching function's elasticity and bargaining power are set as $\eta = 0.5$, in line with Petrongolo and Pissarides (2001), and $\beta = 0.5$, as the standard practice in the literature. In Ebell and Haefke (2009), the entry cost is directly given as 0.6 months of aggregate income per capita based on data for 1997, and they suggest that its

Table 2.1: Parameter values, annual

Variable	Symbol	Value
Interest rate	r	0.05
Rate of technological progress	g	0.02
Probability of being unemployed	$s/12$	0.036
Probability of firm destruction	δ	0.087
Elasticity of matching function	η	0.5
Worker's bargaining power	β	0.5
Scale of matching function	m_0	8.40
Flow value of unemployment	z	0.58
Cost of posting a vacancy	γ	0.28
Elasticity of substitution between differentiated goods	σ	3.5
Shape of firm-specific productivity distribution	α	2.65
Minimum level of firm-specific productivity	ψ_{\min}	0.05
Entry cost	K	0.24
Flow fixed cost	I	0.18

value is 5.2 months of the income for 1978 by estimation. For determining the entry cost level, I use the mean value of these i.e., $K = [(0.6 + 5.2)/2] \times 1/12 = 0.24$.

To pin down the minimum productivity level ψ_{\min} , I normalize the average productivity level $\tilde{\psi}$ to one.

I get the scale parameter of matching function as $m_0 = 8.40$, the flow value of unemployment as $z = 0.58$ and the cost of posting a vacancy as $\gamma = 0.28$. These are given by using the three observational targets: $\theta = 0.72$ (Pissarides, 2009), $z/w = 0.71$ (Hall and Milgrom, 2008) and $\theta q(\theta) = 0.594$ (Shimer, 2012; Pissarides, 2009). To check the plausibility, I compute the recruitment cost $\gamma/q(\theta)$ and have its value as being equal to 14.0% of the quarterly wage; the outcome is the same percentage as targeted in Elsby and Michaels (2013, p.19); in this sense, the above calibration appears to be plausible. Here, the calculated wage and the unemployment rate are $w = 0.82$ and $u = 0.057$.

I choose the elasticity of substitution between differentiated goods as $\sigma = 3.5$, which gives the markup under bargaining as $(\sigma - \beta)/(\sigma - 1) = 1.2$, consistent with the estimates by Martins et al. (1996) and Christopoulou and Vermeulen (2008). According to Axtell (2001), the U.S. firm sizes are approximately Zipf-distributed

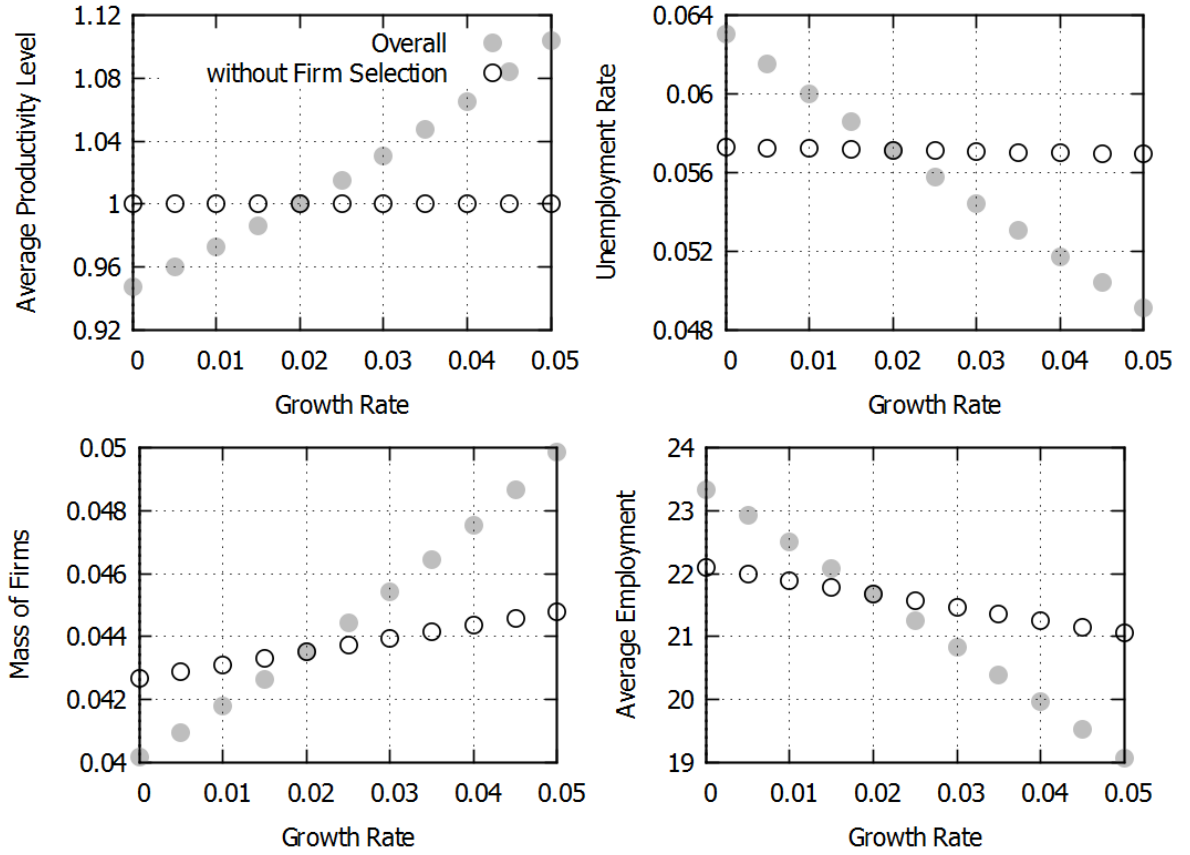


Figure 2.2: Impacts of technological progress

as a robust implication over time; based on this finding, the shape parameter of firm size distribution is set as $\alpha/(\sigma - 1) = 1.06$; that is, $\alpha = 2.65$. A flow fixed cost is set as $I = 0.18$ in order to preserve the average firm size such that $l(\tilde{\psi}) = 21.67$. 21.67 is its mean value for 1977-2014 in Business Dynamics Statistics. Finally, the level of ψ_{\min} leads to 0.10. The model solutions of the cutoff productivity level, the mass of firms and the level of aggregate vacancies are $\psi^* = 0.32$, $n = 0.044$ and $V = 0.04$.⁴

Figure 2.2 reports the key equilibrium outcomes: the average firm-specific productivity level, the unemployment rate, the mass of firms and the average size of labor demand. Each graph compares the complete effect with the effect without

⁴For the computations, I use the software wxMaxima 16.04.2.

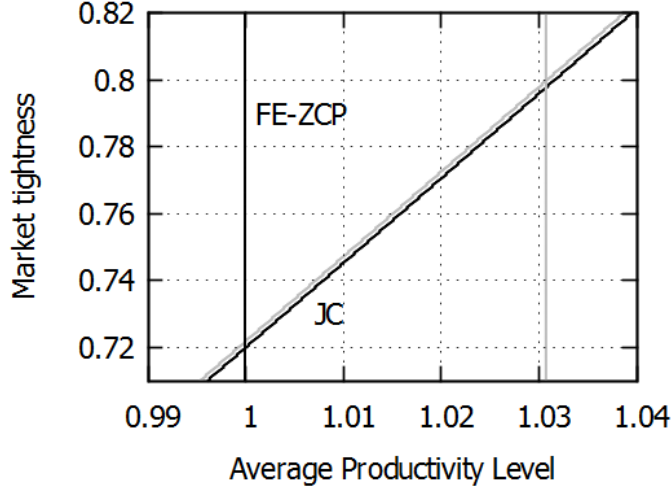


Figure 2.3: Simulated balanced growth equilibria under $g = 0.02$ (black) and $g = 0.03$ (gray)

Table 2.2: Sensitivity to z/w

z/w	z	c	$\frac{c/q(\theta)}{\text{Quarterly wage}}$	du/dg
0.71 (baseline)	0.58	0.28	0.14	-0.28
0.4	0.32	0.57	0.28	-0.14

a shift in *FE-ZCP* line. The simulation results illustrate that incorporating firm heterogeneity magnifies the impacts of technological progress. By using the same calibration strategy to fit the observational targets used here, a standard model as in Pissarides (2000) predicts $du/dg = -0.007$. This is fairly similar to the size of the impact here in the case without a shift in *FE-ZCP* line. Actually, its effect is computed as about $du/dg = -0.007$. On the other hand, in the complete effect, its value becomes much larger such that $du/dg = -0.282$. The estimations in Blanchard and Wolfers (2000) imply the effect to be between -0.25 and -0.71 .

The outcomes reported in Figure 2.3 are more straightforward. This figure is the simulated version of the previous figure. The black lines are under $g = 0.02$ and the gray lines are under $g = 0.03$. Clearly, the new effect, as the shift in *FE-ZCP* line, is much significant than the conventional effect, as the shift in *JC* line. Moreover, the next tables conclude that these results are robust.

Table 2.3: Sensitivity to σ

Markup	σ	α	z	c	du/dg
1.1	6	5.3	0.63	0.31	-0.14
1.2 (baseline)	3.5	2.65	0.58	0.28	-0.28
1.3	2.7	1.80	0.54	0.26	-0.41

In the benchmark case, I set the observational target as $z/w = 0.71$ because z should include not only unemployment insurance but also a home production benefit. Although this stance appears to be plausible but reporting the other case as $z/w = 0.4$ is informative since this setting is often used in the literature. Table 2.2 shows the result. If the target for z/w is 0.4, the impact of an increase in the rate of technological progress leads to $du/dg = -0.14$. This is still much sizable than the standard models.

It is also informative to see the results under different values of entry cost K and fixed cost I . Interestingly, under the above calibration strategy, any change in the value of K/I do not change the values of c and z , leading to the same value of du/dg as in the benchmark case.

Finally, Table 2.3 reports sensitivity to the markup value. In the benchmark case, the value of σ is set to be 3.5, which is below the reported range of the empirical estimations in the trade literature, which is in between 5 to 10 (Anderson and Wincoop, 2004). The value for σ leads to 6 if the observational target of markup is 1.1. In this case, the impact of technological progress leads to $du/dg = -0.14$. This value is also still sizable.

2.5 Conclusion

This chapter considered the impact of technological progress on unemployment by incorporating firm heterogeneity and modeling the firm selection effect. The simulation results showed that this extension magnifies the size of the impact of technological progress on unemployment as consistent with the empirical findings. However, technology obsolescence was abstracted from the current model and there was no firm's decision with respect to technology adoption. In this sense, a part of the puzzle as concluded in Pissarides and Vallanti (2007) remains to be solved.

Chapter 3

Technological Progress and Unemployment Revisited: Obsolescence, Technology Adoption, and Selection in Search Equilibrium

3.1 Introduction

Several OECD countries have experienced persistent high levels of unemployment since the 1980s. What is a major source for this? One of the answers is considered to be a decline in the pace of technological progress. Blanchard and Wolfers (2000) estimate that one percent-point decrease in the rate of total factor productivity growth translates into an increase in the unemployment rate of 0.25-0.71 percent. They show that technological progress has a statistically-significant negative impact on unemployment, based on panel data for 20 OECD nations.

However, theoretical predictions of the impact of technological progress on unemployment remain to be explained. Pissarides and Vallanti (2007) simulate that one percent-point decrease in the rate of technological progress increases the unemployment rate by only 0.02 percent in the standard search model. There are

two points here. First, the standard search model can replicate the magnitude of this effect less than 1/10 of the total estimated impact. Second, to generate its qualitative negative effect, it has to be assumed that technological progress is almost disembodied.¹

To address these issues, the current chapter considers heterogeneous multi-worker firms. I assume that technological progress is embodied and each firm can choose to either adopt new technology or not. To be concrete, the novelty of this chapter is to introduce heterogeneous multi-worker firms into the model in Mortensen and Pissarides (1998).² In their model, the economy comprises a mixture of jobs with and without technology updates. In contrast, the current model introduces *multi-worker* firms where a firm size in terms of employees depend on the level of productivity. Along with this, by taking *firm heterogeneity* into account, the model involves heterogeneous decisions for technology adoption and firm exit with the selection effect.³ In other words, the current chapter newly examines the question of how a change in *the composition of firms* of different types affects the relationship between technological change and unemployment.

In this chapter, I assume that technology is embodied not in jobs but in firms. Bloom, Sadun, and Reenen (2012) suggest that the intensive uses of information technology increase labor productivity. Moreover, they find that people management practices are important, even after controlling technology-skill complementarity in their regression analyses. In line with this study, there is growing evidence that the effective benefits from information technology are linked to the internal organization of firms. Following this view, the model considers that new technology is embodied in new firms and technology update per se is independent of a firm size.

In the model, as the speed of technological progress accelerates, old technologies

¹In theory, technological progress is distinguished into two types. If progress is disembodied, its progress raises productivity uniformly in new and incumbent jobs/firms. If progress is embodied, its progress increases productivity in new jobs/firms but does not automatically improve productivity in incumbent jobs/firms. In comparison to the former, the latter delivers technology obsolescence, which serves to devalue old jobs/firms if they do not renew its technology.

²In a standard search model, one employer/firm hires one worker, leading to a new job. In a model with multi-worker firms, one firm is allowed to hire more than one worker.

³The selection effect is introduced by Melitz (2003) in the international trade literature and Felbermayr and Prat (2011) in the search matching literature, in which firms with heterogeneous productivity levels self-select into exit firms and surviving firms.

become obsolete faster while the benefits from technology adoption increase. As opposed to the result in Mortensen and Pissarides (1998), this increases the proportion of firms with technology adoption. Together with the selection effect, this effect also increases the average productivity of whole surviving firms, leading in turn to an increase in aggregate labor demand. In line with this, Duernecker (2014) points out that many European countries have lagged behind the United States in terms of technology adoption. In addition, he finds that European countries with higher technology adoption, similar to that observed in the United States, exhibit lower unemployment rates.

Through a simulation, an increase in the growth rate of technological progress from 1.5 to 2.5 percent decreases the unemployment rate by 0.48 percent, as the net effect. This is consistent with empirical estimates in Blanchard and Wolfers (2000), both qualitatively and quantitatively. In addition, the result holds under technology obsolescence of firms. The above increase in the growth rate increases the proportion of firms with technology updates from 3.0 percent to 5.1 percent, in which the small fraction of firms contributes substantially to job creation. On the contrary, a large part of firms with technology obsolescence reduces their employees. The mentioned increase in the growth rate increases the annual job cut-back rate of such firms from 3.7 to 6.0 percent and, *ceteris paribus*, this increases the unemployment rate by 0.61 percent. However, the net effect of technological progress on unemployment becomes negative. The subsequent sensitivity analyses show that the simulation results are robust.

Finally, this chapter also considers product market competition with respect to the substitutability between differentiated products. To achieve low unemployment, researchers and policy makers have focused on labor market institutions. However, based on recent empirical findings, it appears to be insufficient to explain the evolution of unemployment over time and thus attention has increasingly shifted to product market institutions (Blanchard and Giavazzi, 2003; Ebell and Haefke, 2009; Felbermayr and Prat, 2011). In the current model, product market competition strengthens firm obsolescence but, at the same time, it increases the aggregate technology adoption rate.

3.2 The model

3.2.1 Set up of the model

The economy is composed of a unit measure of identical individuals and operating heterogeneous firms that the mass is n . Individuals live forever, and each firm is destroyed with exogenous probability δ . All agents discount the future at the common rate r . Time is discrete and indexed by t .

The labor market is frictional. Each individual can be either employed or unemployed, and the unemployed can apply for a job vacancy. The aggregate number of jobs filled is represented by a matching technology $m(u, V)$ as a function of the measures of unemployed workers u and vacancies posted by all firms V . The function is increasing its arguments and assumes constant returns to scale. The vacancy filling probability is given by $m(u, V)/V = q(\theta)$ as the market tightness $\theta = V/u$ and a decreasing function $q(\cdot)$. Then, the job finding probability is $m(u, V)/u = \theta q(\theta)$. An unemployed worker receives the income z_t which includes unemployment insurance and the flow value of home production. While employed, a worker earns the wage w_t which is determined through the bargaining problem specified later.

Demand of individuals. Each individual is risk neutral and has the Dixit-Stiglitz preference over a continuum of differentiated goods:

$$\max_{Q_{i,t}^j} \left[n^{-\frac{1}{\sigma}} \int (Q_{i,t}^j)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}} \quad (3.1)$$

subject to $y_t^j = \int (Q_{i,t}^j p_i / P) di$ where $Q_{i,t}^j$ denotes demand of individual j for good i and y_t^j is the real income of individual j , both at time t . A price index is $P = [(1/n) \int p_i^{1-\sigma} di]^{1/(1-\sigma)}$ where the price of good i represents p_i . After solving the problem, aggregate demand for good i at time t takes the form:

$$\int Q_{i,t}^j dj = \left(\frac{p_i}{P} \right)^{-\sigma} \frac{Y_t}{n} \text{ where } Y_t = \int y_t^j dj. \quad (3.2)$$

The elasticity of substitution among differentiated goods is given by $\sigma > 1$ as a measure of monopolistic competition. Eq. (3.2) holds each period but the aggregate real income Y_t grows over time.

Flow profits of firms. At the beginning of each period, the economy experiences technological progress. a_t denotes the general productivity level that is associated with the frontier technology at time t . This evolves at the exogenous rate $g = (a_{t+1} - a_t)/a_t$, which is referred to as the growth rate of technological progress in this chapter. New technology and the increased general productivity level are available to each firm by bearing the flow cost of introducing new technology I_t . A firm can choose either to update its technology or not, and I define the two types of firms, update firms and obsolete firms, as follows. The *update firms* are the firms that decide to update its technology and improve the general productivity every period. On the other hand, the *obsolete firms* avoid updating new technology but continue to use technology at the time of entry.⁴ These are to clearly distinguish two bipolar types of surviving firms.

Each firm has a linear production function that uses only labor input l , and its labor productivity level is given by the product $a_t\psi$. ψ represents the firm-specific component drawn from a Pareto distribution with c.d.f. $F(\psi) = 1 - (\psi_{\min}/\psi)^\alpha$ and p.d.f. $f(\psi)$ after incurring the entry cost K_t in the time of entry. I assume that ψ is constant over time by following Melitz (2003) to focus on the main topic in the current chapter.⁵ The heterogeneity of firm-specific productivity is considered as the heterogeneity of technology use such that a firm with a higher ψ exploits technology more effectively.

By using (3.2), the inverse demand function at time t takes the form:

$$\frac{p(l_t; \psi, t - \tau)}{P} = \left(\frac{a_\tau \psi l_t}{Y_t/n} \right)^{-\frac{1}{\sigma}} \quad (3.3)$$

where a real price is a function of labor input, firm-specific productivity, and the extent to which the currently-used technology created at date τ becomes old as

⁴This setting is abstracted from the endogeneity of update frequency, which is introduced in Mortensen and Pissarides (1998), but it can be confirmed that exogenously-fixed update cycles generate the same basic conclusion in this chapter.

⁵For more details, please refer to Hopenhayn (1992).

discussed later.

An operating firm pays wages, vacancy costs, and, if necessary, the update cost. I define the flow profit of an obsolete firm at time t created at date τ so that:

$$\frac{p(l_t; \psi, t - \tau)}{P} a_\tau \psi l_t - w_t^o(l_t; \psi, t - \tau) l_t - \gamma_t v_t \quad (3.4)$$

where $a_\tau \psi l_t$ denotes output, γ_t denotes the cost of posting a vacancy, and v_t represents the number of job vacancies posted. I use the superscript (o) for obsolete firms to distinguish from update firms. Similarly, the flow profit of an update firm at time t is:

$$\frac{p(l_t; \psi, 0)}{P} a_t \psi l_t - w_t(l_t; \psi, 0) l_t - \gamma_t v_t - I_t. \quad (3.5)$$

In comparison to obsolete firms, update firms pay the update cost and stay in the state of the art.

Rendering the growth model stationary. I focus on a steady state. This corresponds to a balanced growth path in which the economy evolves at the growth rate of general productivity, g . By following the literature, I assume $\gamma_t = a_t \gamma$, $z_t = a_t z$, $I_t = a_t I$ and $K_t = a_t K$ so that these exogenous variables grow at the same rate g in order to ensure a stationary equilibrium.⁶ For example, consider a case that the flow value of unemployment, z_t , increases at a rate more than the economy's growth rate, g . The unemployed workers never search jobs. In the opposite case that z_t increases at a rate less than the growth rate g , z_t does not matter in the economy and is treated as being zero. For the other exogenous variables, similar situations emerge as trivial ones.

This assumption guarantees that wages and the aggregate real income grow at the rate g . Thus, a_τ / Y_t in (3.3) leads to $(1 + g)^{-(t-\tau)} / Y$ as the function of technology obsolescence represented by the time distance $t - \tau$. In the model, similar to the literature, obsolete firms determine each destruction age associated with obsolescence. This age is denoted by T to maximize the total profit after the entry.

⁶See, for example, Mortensen and Pissarides (1998) and Pissarides and Vallanti (2007).

3.2.2 Value functions

By using (3.4), the value function of an operating obsolete firm at time t created at date τ is given as:

$$J_t^o(l; \psi, t - \tau) = \max \left[\max_v \left[\frac{p(l; \psi, t - \tau)}{P} a_\tau \psi l - w_t^o(l; \psi, t - \tau) l - \gamma_t v + \left(\frac{1 - \delta}{1 + r} \right) J_{t+1}^o(l', \psi, t + 1 - \tau) \right], 0 \right], \quad (3.6)$$

where l' denotes the employment level in the next period and δ is the probability of exogenous firm destruction. Similarly, by using (3.5), the value of an operating update firm at time t takes the form:

$$J_t(l; \psi) = \max \left[\max_v \left[\frac{p(l; \psi, 0)}{P} a_t \psi l - w_t(l; \psi) l - \gamma_t v - I_t + \left(\frac{1 - \delta}{1 + r} \right) J_{t+1}(l', \psi) \right], 0 \right], \quad (3.7)$$

where the extent of technology obsolescence is zero for the update firms by definition. The employment level, for these values, evolves such that:

$$l' = (1 - \lambda)l + q(\theta)v,$$

where λ denotes the probability of exogenous job separations which are independent of the firm destruction. Thus, each employed worker becomes unemployed with probability $\lambda + \delta - \lambda\delta$. I define the job separation probability such that $s = \lambda + \delta - \lambda\delta$.

On the other hand, the value of an employed worker who works for an obsolete firm at time t created at date τ is:

$$E_t^o(l; \psi, t - \tau) = \max \left[w_t^o(l; \psi, t - \tau) + \frac{1}{1 + r} \left[(1 - s) E_{t+1}^o(l'; \psi, t + 1 - \tau) + s U_{t+1} \right], U_t \right]. \quad (3.8)$$

The value of an employed worker who works for an update firm at time t is given as:

$$E_t(l; \psi) = \max \left[w_t(l; \psi) + \frac{1}{1+r} [(1-s)E_{t+1}(l'; \psi) + sU_{t+1}], U_t \right]. \quad (3.9)$$

Finally, the value of an unemployed worker is:

$$U_t = z_t + \frac{1}{1+r} \left[\theta_q(\theta) \tilde{E}_{t+1} + (1 - \theta_q(\theta)) U_{t+1} \right], \quad (3.10)$$

where \tilde{E}_{t+1} denotes the expected value of being employed at time $t + 1$.

3.2.3 Wage determination

In the frictional labor market, wages are determined through a bargaining in which the surplus of each job are shared between the worker and firm. As in the literature, this chapter assumes the surplus sharing rule such that a fixed proportion β of the surplus goes to the worker and a proportion $(1 - \beta)$ goes to the firm. Although the former corresponds to the gain of being employed for an unemployed worker, the latter is the marginal benefit from employment. As the key point to solve such a problem in a model with multi-worker firms, each worker is treated as the marginal worker (Stole and Zwiebel 1996), implying:

$$\beta \frac{\partial J_t^o(l; \psi, t - \tau)}{\partial l} = (1 - \beta) [E_t^o(l; \psi, t - \tau) - U_t], \quad (3.11)$$

or

$$\beta \frac{\partial J_t(l; \psi)}{\partial l} = (1 - \beta) [E_t(l; \psi) - U_t], \quad (3.12)$$

for each firm type.

Proposition 1 *The wage rate of an obsolete firm at time t created at date τ satisfies:*

$$w^o(l; \psi, t - \tau) = \omega(\theta) + \beta \left[\frac{\sigma - 1}{\sigma - \beta} \frac{p(l; \psi, t - \tau)}{P} \left(\frac{1}{1+g} \right)^{t-\tau} \psi - \omega(\theta) \right], \quad (3.13)$$

where the strictly increasing function of the market tightness is defined as:

$$\omega(\theta) = z + \frac{\beta}{1 - \beta} \frac{\theta\gamma}{1 - \delta}, \quad (3.14)$$

for worker's outside options.

Proof. In the appendix. ■

This expression is after the normalization by the general productivity level, α_t , such that $w^o(l; \psi, t - \tau) = w_t^o(l; \psi, t - \tau)/\alpha_t$. The normalized wage is independent of time t itself but includes the term $(1 + g)^{-(t-\tau)}$ which potentially attenuates the benefit from employing a worker as time goes by. This implies that technology obsolescence deepens while the flow value of worker's outside options is constant over time under constant market tightness.

On the other hand, the corresponding normalized wage for an update firm takes the form:

$$\begin{aligned} w(l; \psi) &= \omega(\theta) + \beta \left[\frac{\sigma - 1}{\sigma - \beta} \frac{p(l; \psi, 0)}{P} \psi - \omega(\theta) \right] \\ &= w^o(l; \psi, 0). \end{aligned} \quad (3.15)$$

The equality $w^o(l; \psi, 0) = w(l; \psi)$ holds since the cost of employing a worker is the same across firms. Each multi-worker firm optimally considers the hiring decision so that employment is adjusted to achieve the same labor profitability, equal to the cost which consists of not private variables but the aggregate variable θ .⁷ Along with the determination of optimal labor demand in the next subsection, it can be shown that wages are the same across firms also independently of technological obsolescence, i.e., $w = w^o(l; \psi, t - \tau) = w(l; \psi)$ holds for any $t - \tau$. The wage w only depends on labor market tightness θ .⁸

⁷For additional information, see Ebell and Haefke (2009). They explain a similar wage determination in their model with homogeneous multi-worker firms.

⁸This is from the same reason as early mentioned and is repeated in a number of models with multi-worker firms.

3.2.4 Optimal labor demand

Each firm maximizes its lifetime profit such that employment is adjusted in order to satisfy the following Euler equation. By using the first order and envelope conditions for (3.6), I get:

$$\begin{aligned} \frac{\gamma_{t-1}}{q(\theta)} \left(\frac{1-\delta}{1+r} \right)^{-1} = \\ \frac{\sigma-1}{\sigma} \frac{p(l; \psi, t-\tau)}{P} a_{\tau} \psi - \frac{\partial w_t^o(l; \psi, t-\tau)}{\partial l} l - w_t^o(l; \psi, t-\tau) + \frac{(1-\lambda)\gamma_t}{q(\theta)}. \end{aligned} \quad (3.16)$$

By plugging the wage function, this can be rewritten as:

$$\begin{aligned} \frac{\gamma}{q(\theta)} \left[\left(\frac{(1+g)(1-\delta)}{1+r} \right)^{-1} - 1 + \lambda \right] = \\ (1-\beta) \left[\frac{\sigma-1}{\sigma-\beta} \frac{p(l; \psi, t-\tau)}{P} \left(\frac{1}{1+g} \right)^{t-\tau} \psi - \omega(\theta) \right]. \end{aligned} \quad (3.17)$$

The left hand side of this expression represents the cost of employing a worker, and the right hand side of this expression is a fraction $1-\beta$ of the flow surplus from a marginal job match. As $t-\tau=0$, this equation also holds for update firms, similar to wage determination. In what follows, $l(\psi, t-\tau)$ and $p(\psi, t-\tau)$ denote corresponding optimal levels to firm-specific productivity, ψ , and the extent of obsolescence, $t-\tau$. From (3.17) and (3.3), the relationships between prices and labor demands across firms can be written as:⁹

$$\frac{p(\psi', x')}{p(\psi, x)} = \left(\frac{\psi'}{\psi} \frac{1}{(1+g)^{x'-x}} \right)^{-1} \quad \text{and} \quad \frac{l(\psi', x')}{l(\psi, x)} = \left(\frac{\psi'}{\psi} \frac{1}{(1+g)^{x'-x}} \right)^{\sigma-1}. \quad (3.18)$$

A firm with higher productivity implies a lower price and more employees. In addition, a firm with older technology implies a higher price and less employees. Here, the extents of technological progress and product market competition affect

⁹Here, I replace $t-\tau$ by x for simplicity.

the relationship between labor demands of firms.¹⁰

For subsequent use, I define the two important parameters:

$$\Lambda_g = \frac{(1+g)(1-\delta)}{1+r} < 1, \quad (3.19)$$

and

$$\tilde{g} = (1+g)^{\sigma-1} > 1.$$

The former denotes the net discount factor of firms, and a change in its parameter is the only source which generates the *negative* impact of technological progress on unemployment in the standard search model as in Pissarides and Vallanti (2007). Next, one minus the inverse of the latter parameter represents the job cutback rate of obsolete firms, i.e., $l(\psi, x+1) = \tilde{g}^{-1}l(\psi, x)$ holds, and it delivers the *positive* impact of technological progress on unemployment. However, this chapter focuses on an endogenous change in aggregate productivity through a change in the composition of heterogeneous firms, and asks the importance of it.

3.2.5 Average productivity levels of update and obsolete firms

For each firm type, I define the average level of firm-specific productivity. At this moment, I impose presumptive existences of the cutoff productivity levels: *obsolescence-cutoff* level ψ_o^* below which all are obsolete firms and *exit-cutoff* level ψ^* below which all exit the market. In subsequent pages, I confirm these presumptions.

The aggregate price index can be rewritten as

$$P = \left[\int_{\psi^*}^{\infty} \sum_{x=0}^T (p(\psi, x))^{1-\sigma} h(x|x \leq T) \mu(\psi) d\psi \right]^{\frac{1}{1-\sigma}}, \quad (3.20)$$

where $\mu(\psi)$ denotes the equilibrium density of firm-specific productivity condi-

¹⁰In the standard search model, labor profitability is exogenously fixed opposed to the current model. This difference is the key for the determination for optimal destruction age, T , in terms of technology obsolescence. I discuss this topic later.

tional on surviving firms such that $\mu(\psi) = f(\psi)/[1 - F(\psi^*)]$ as $\psi^* \leq \psi$, and $h(x|x \leq T)$ is the probability of survival for an obsolete firm with respect to the degree of obsolescence such that $h(x|x \leq T) = \delta(1 - \delta)^x/[1 - (1 - \delta)^{T+1}]$.¹¹

The model distinguishes between two price indices: P_u denotes the price index of update firms and P_o is the price index of obsolete firms. A relationship to the aggregate price is:

$$P^{1-\sigma} = \frac{1 - F(\psi_o^*)}{1 - F(\psi^*)} P_u^{1-\sigma} + \frac{F(\psi_o^*) - F(\psi^*)}{1 - F(\psi^*)} P_o^{1-\sigma}, \quad (3.21)$$

where

$$P_u^{1-\sigma} = \int_{\psi_o^*}^{\infty} (p(\psi, 0))^{1-\sigma} \frac{f(\psi)}{1 - F(\psi_o^*)} d\psi, \quad (3.22)$$

and

$$P_o^{1-\sigma} = \int_{\psi^*}^{\psi_o^*} \sum_{x=0}^T (p(\psi, x))^{1-\sigma} h(x|x \leq T) \frac{f(\psi)}{F(\psi_o^*) - F(\psi^*)} d\psi. \quad (3.23)$$

Here, I define the two average productivity levels, $\tilde{\psi}_u$ for update firms and $\tilde{\psi}_o$ for obsolete firms, to satisfy:

$$P_u = p(\tilde{\psi}_u, 0), \quad (3.24)$$

and

$$P_o = p(\tilde{\psi}_o, \tilde{x}), \quad (3.25)$$

where \tilde{x} denotes a certain extent of obsolescence.

Proposition 2 *The average productivity of update firms takes the form:*

$$\tilde{\psi}_u = \left(\frac{\alpha}{\alpha + 1 - \sigma} \right)^{\frac{1}{\sigma-1}} \psi_o^*. \quad (3.26)$$

¹¹A firm is destroyed with probability δ each period. This leads to the above Geometric distribution. As derived later, T is given independent of ψ .

On the other hand, the average productivity of obsolete firms is calculated as:

$$\tilde{\psi}_o = \left(\tilde{g}^{\tilde{x}} \times \frac{\delta}{1 - (1 - \delta)^{T+1}} \frac{1 - [\tilde{g}^{-1}(1 - \delta)]^{T+1} \left(\frac{\psi_o^*}{\psi^*} \right)^{\alpha+1-\sigma} - 1}{1 - \tilde{g}^{-1}(1 - \delta)} \frac{\left(\frac{\psi_o^*}{\psi^*} \right)^{\alpha+1-\sigma} - 1}{\left(\frac{\psi_o^*}{\psi^*} \right)^\alpha - 1} \right)^{\frac{1}{\sigma-1}} \tilde{\psi}_u. \quad (3.27)$$

Proof. In the appendix. ■

Note that, for given the unique cutoff productivity levels, $\tilde{\psi}_u$ becomes unique but a pair $(\tilde{\psi}_o, \tilde{x})$ is jointly determined to satisfy $P_o = p(\tilde{\psi}_o, \tilde{x})$.¹² The first equation of this proposition is a quite similar to that of early studies as in Felbermayr and Prat (2011). The current model reduces to their model if all firms are update firms and the growth rate of technological progress is zero. In contrast, the second equation of this proposition includes both the growth rate, g , and product market competition, σ . For each firm type, the average productivity level is given as a weighted average of firm-specific productivity. These levels are independent of the mass of firms or unemployment.¹³ The market share, in terms of numbers, between the two types of firms depends on the ratio ψ^*/ψ_o^* .

3.2.6 Firm entry and exit

A large pool of ex-ante identical potential entrants aims for entering the market to start production and gain profit. The firm entry requires the unrecoverable cost $a_t K$ as a sunk cost with uncertainty about the firm-specific productivity, which is drawn from the common distribution $F(\psi) = 1 - (\psi_{\min}/\psi)^\alpha$ in the time of entry. To understand gains from the choice of firm type, I firstly calculate the values of firms as below.

Proposition 3 *The value of an update firm is computed as:*

$$J(\psi) = \frac{1}{1 - \Lambda_g} \left[\left[\frac{1 - \beta}{\sigma - \beta} \frac{p(\psi, 0)}{P} \psi + [\Lambda_g^{-1} - 1] \frac{\gamma}{q(\theta)} \right] l(\psi, 0) - I \right], \quad (3.28)$$

where $J(\psi) = J_t(l(\psi, 0); \psi)/a_t$. On the other hand, the value of an obsolete firm

¹²In this case, P_o is assured to be a unique value.

¹³As discussed later, the cutoff productivity levels are determined independently of labor market outcomes, which are solved next.

is given as:

$$J^o(\psi, 0) = \frac{1}{1 - \Lambda_g \tilde{g}^{-1}} \left[\frac{1 - \beta}{\sigma - \beta} \frac{p(\psi, 0)}{P} \psi + [\Lambda_g^{-1} - \tilde{g}^{-1}] \frac{\gamma}{q(\theta)} \right] l(\psi, 0), \quad (3.29)$$

where $J^o(\psi, 0) = J_t^o(l(\psi, 0); \psi, 0)/a_t$.

Proof. In the appendix. ■

The value of a firm is calculated such that the flow profit is multiplied by the term with net discount factor Λ_g . The difference between firm types is as follows. The value of an update firm includes the cost of updating technology, I . In contrast, the value of an obsolete firm includes the parameter $\tilde{g}^{-1} < 1$ which is equivalent to one minus the job cutback rate. Importantly, $1/(1 - \Lambda_g \tilde{g}^{-1}) < 1/(1 - \Lambda_g)$ holds.

Corollary 4 *An obsolete firm chooses the optimal destruction age to be $T = \infty$.*

The corollary comes because an obsolete firm can optimally choose the number of employees in order to achieve the constant target of labor productivity level. In other words, the employees of the firm reduces as its technology is obsolete but firm destruction never occur except for an arrival of exogenous shock.

In the time of entry, each new entrant can choose to become either an update firm or an obsolete firm by comparing the corresponding total expected values:

$$\begin{aligned} \Pi(\psi) &= -\frac{\gamma}{q(\theta)} l(\psi, 0) - I + \Lambda_g J(\psi) \\ &= \frac{\Lambda_g}{1 - \Lambda_g} \left[\frac{1 - \beta}{\sigma - \beta} R(\psi, 0) - I \right] - I, \end{aligned} \quad (3.30)$$

for an update firm, and

$$\begin{aligned} \Pi^o(\psi) &= -\frac{\gamma}{q(\theta)} l(\psi, 0) - I + \Lambda_g J^o(\psi, 0) \\ &= \frac{\Lambda_g}{1 - \Lambda_g \tilde{g}^{-1}} \frac{1 - \beta}{\sigma - \beta} R(\psi, 0) - I, \end{aligned} \quad (3.31)$$

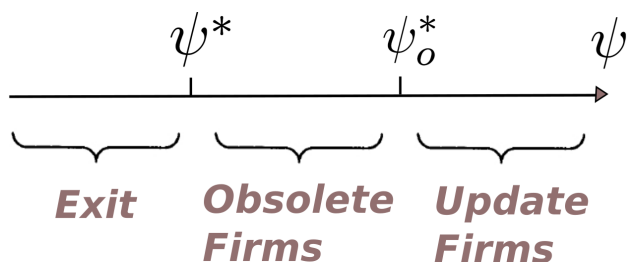


Figure 3.1: Two cutoff levels of firm-specific productivity

for an obsolete firm. In these equations, I define the revenue to be $R(\psi, x) = [p(\psi, x)/P][1/(1 + g)]^x \psi l(\psi, x)$. From (3.17), $R(\psi, x)/l(\psi, x)$ is given independently of ψ and x . At the beginning, each firm has no employee and expends on introducing new technology and posting vacancies in order to start production in the next period of entry.^{14 15}

The difference between these total expected values increases with the level of firm-specific productivity, i.e., $\partial[\Pi(\psi) - \Pi^o(\psi)]/\partial\psi > 0$. The total value for being an obsolete firm and the revenue both also increase with it, i.e., $\partial\Pi^o(\psi)/\partial\psi > 0$, and $\partial R(\psi, 0)/\partial\psi > 0$.¹⁶ Thus, there are the unique two cutoff levels associated with the two reservation rules:

$$\text{Obsolescence-cutoff} : \psi_o^* = \inf \{ \psi : \Pi(\psi) - \Pi^o(\psi) \geq 0 \}, \quad (3.32)$$

and

$$\text{Exit-cutoff} : \psi^* = \inf \{ \psi : \Pi^o(\psi) \geq 0 \}. \quad (3.33)$$

Figure 3.1 shows the relationship between *Obsolescence-cutoff* and *Exit-cutoff* from these reservation rules. In equilibrium, a fraction $1 - F(\psi_o^*)$ of the firms self-select into update firms, a fraction $F(\psi_o^*) - F(\psi^*)$ becomes obsolete firms, and the rest $F(\psi^*)$ exit from the market. For example, the mass of update firms are

¹⁴To employ one worker, a firm has to post vacancies of $1/q(\theta)$. Thus, a hiring cost per worker is $\gamma/q(\theta)$.

¹⁵The model assumes the linear cost with respect to posted vacancies, in which an optimal level of employment is obtained in the next period of entry.

¹⁶In terms of ψ , $\Pi(\psi)$ has a higher slope and a lower intercept than $\Pi^o(\psi)$.

computed as the product of the mass of all surviving firms and the proportion of update firms, i.e., $n \times [1 - F(\psi_o^*)]/[1 - F(\psi^*)] = n \times (\psi^*/\psi_o^*)^\alpha$.

Proposition 5 *Those two cutoff productivity levels are characterized by the following equations:*

$$\frac{I}{1 - \frac{1-\Lambda_g}{1-\Lambda_g\tilde{g}^{-1}}} = \frac{1-\beta}{\sigma-\beta} R(\psi_o^*, 0), \quad (3.34)$$

and

$$\left(\frac{\psi_o^*}{\psi^*}\right)^{\sigma-1} = \frac{1}{1-\tilde{g}^{-1}}. \quad (3.35)$$

Proof. The first equation is immediately given by using (3.30) and the condition $\Pi(\psi_o^*) - \Pi^o(\psi_o^*) = 0$ from (3.32). By combining the first equation and the condition $\Pi^o(\psi^*) = 0$ from (3.33), the second equation is also derived. ■

For this first equation, the numerator and the denominator of the left hand side respectively correspond to the cost and the benefit in terms of technology adoption. This suggests that a lower update cost, a lower I , and obsolescence acceleration, a higher \tilde{g} , both increase the value of being an update firm and potentially decrease the mass of obsolete firms. More surprisingly, the second equation of this proposition shows that the steady-state proportion of firm type is entirely determined by the two parameters: the growth rate of technological progress and product market competition.¹⁷

Corollary 6 *The ratio of labor demands, $l(\psi_o^*, 0)/l(\psi^*, 0)$, is given as the inverse of the job cutback rate.*

Finally, the entry of firms occurs as long as the expected value of entry exceeds the corresponding cost such that:

$$K < \int_{\psi_o^*}^{\infty} \Pi(\psi)f(\psi)d\psi + \int_{\psi^*}^{\psi_o^*} \Pi^o(\psi)f(\psi)d\psi. \quad (3.36)$$

As the mass of the firms increases, the expected value of entry decreases through a change in the level of Exit-cutoff. Thus, the mass of the firms increases until (3.36) holds with equality.

¹⁷ $(\psi_o^*/\psi^*)^{\sigma-1}$ is equal to $R(\psi_o^*, 0)/R(\psi^*, 0)$ and $l(\psi_o^*, 0)/l(\psi^*, 0)$.

Proposition 7 *The firm entry condition, which is (3.36) with equality, can be transformed as below:*

$$\left(\frac{\psi^*}{\psi_{min}}\right)^\alpha K = \left[1 + \left(\frac{\psi_o^*}{\psi^*}\right)^{-\alpha} \frac{\Lambda_g}{1 - \Lambda_g}\right] \frac{\sigma - 1}{\alpha + 1 - \sigma} I. \quad (3.37)$$

Proof. In the appendix. ■

The term $(\psi^*/\psi_{min})^\alpha$ is the inverse of the rate of survival. Thus, the right hand side of this condition is the expected value of entry conditional on survival. Importantly, the first term in brackets in the right hand side is associated with the profitability of being an obsolete firm, and the second term in brackets is associated with the profitability of being an update firm. The latter is given as the product of the proportion of update firms and the summation $\sum_{t=1}^{\infty} (\Lambda_g)^t$ which represents the future benefits from technology updates.

3.3 Stationary equilibrium

Proposition 8 *An equilibrium solution to the model is a list $(\psi^*, \psi_o^*, w, \theta)$ that satisfies:*

$$w = \omega(\theta) + \frac{\beta}{1 - \beta} \frac{\gamma}{q(\theta)} [\Lambda_g^{-1} - 1 + \lambda], \quad (3.38)$$

$$w = \frac{\sigma - 1}{\sigma - \beta} \frac{P_u}{P} \left(\frac{\alpha}{\alpha + 1 - \sigma}\right)^{\frac{1}{\sigma-1}} \psi_o^* - \frac{\gamma}{q(\theta)} [\Lambda_g^{-1} - 1 + \lambda], \quad (3.39)$$

$$\left(\frac{\psi_o^*}{\psi^*}\right)^{\sigma-1} = \frac{1}{1 - \tilde{g}^{-1}}, \quad (3.40)$$

$$\left(\frac{\psi^*}{\psi_{min}}\right)^\alpha K = \left[1 + \left(\frac{\psi_o^*}{\psi^*}\right)^{-\alpha} \frac{\Lambda_g}{1 - \Lambda_g}\right] \frac{\sigma - 1}{\alpha + 1 - \sigma} I, \quad (3.41)$$

where P_u/P is given as

$$\frac{P_u}{P} = \left(\frac{\psi_o^*}{\psi^*}\right)^{-1} \left[\frac{\delta}{1 - \tilde{g}^{-1}(1 - \delta)} \left[1 + \frac{1 - \delta}{\delta} \left(\frac{\psi_o^*}{\psi^*}\right)^{-\alpha} \right] \right]^{\frac{1}{\sigma-1}}. \quad (3.42)$$

Proof. The first and the second equations, which represent the wage curve and the labor demand curve respectively, are given by combining (3.13) and (3.17). The third and the fourth equations are (3.35) and (3.37). P_u/P is computed in the appendix. ■

In this system of equations, the two levels of cutoffs are uniquely determined from (3.40) and (3.41) independently of labor market outcomes. For given these results, the wage and the labor market tightness are solved from (3.38) and (3.39). For the wage curve, (3.38), the wage increases with market tightness since, at the same time, the value of worker's outside options increase and the optimal target level of labor productivity should be high. The latter is implied by the Euler equation for employment, (3.17). On the other hand, for the labor demand curve, (3.39), the wage decreases with market tightness because the hiring cost also increases with it. Thus, there is a unique pair (w, θ) as long as $[(\sigma - 1)/(\sigma - \beta)]R(\psi, 0)/l(\psi, 0) > z$ holds.¹⁸

This model encompasses the model structure without technological progress as in Ebell and Haefke (2009) and Felbermayr and Prat (2011). Moreover, as all firms have the same productivity $\psi = 1$ and the product market is competitive $\sigma \rightarrow \infty$, it can be shown that the current determination of market tightness reduces to an almost similar expression to the basic search model with fully-disembodied technological progress.¹⁹ This is discussed in the appendix accompanied by its quantitative impact.

The remaining variables are given as follows. By using (3.33), an optimal labor demand leads to:

$$l(\psi, x) = \left(\frac{\psi}{\psi^*}\right)^{\sigma-1} \tilde{g}^{-x} \frac{1 - \Lambda_g \tilde{g}^{-1}}{\Lambda_g} \frac{\sigma - \beta}{1 - \beta} \frac{P}{P_u} \frac{I}{\tilde{\psi}_u}. \quad (3.43)$$

The equilibrium unemployment equates the flows in and out of employment as below:²⁰

¹⁸The labor demand curve can be rewritten such that $[(\sigma - \beta)/(\sigma - 1)]([\Lambda_g^{-1} - 1 + \lambda] \gamma/q(\theta) + w) = R(\psi, x)/l(\psi, x)$, where $(\sigma - \beta)/(\sigma - 1)$ represents the markup under bargaining.

¹⁹As $\psi = 1$, $\tilde{\psi}_u = P_u/P = 1$. In this case, all surviving firms are update firms.

²⁰In line with a large number of empirical studies, the unemployment rate is positively correlated with the unemployment duration of workers. This duration, $1/\theta q(\theta)$, is negatively related with labor market tightness. Thus, the market tightness decreases the unemployment rate.

$$\frac{u}{1-u} = \frac{s}{\theta q(\theta)} \Leftrightarrow u = \frac{s}{\theta q(\theta) + s}. \quad (3.44)$$

Finally, the mass of firms, n , is given by the equality of aggregate labor demand and the number of the employed such that:

$$\begin{aligned} \frac{1-u}{n} &= \int_{\psi^*}^{\infty} \sum_{x=0}^{\infty} l(\psi, x) h(x) \mu(\psi) d\psi \\ &= \left(\frac{\psi^*}{\psi_o^*}\right)^{\alpha} l(\tilde{\psi}_u, 0) + \left[1 - \left(\frac{\psi^*}{\psi_o^*}\right)^{\alpha}\right] l(\tilde{\psi}_o, \tilde{x}) \\ &= \left(\frac{P_u}{P}\right)^{\sigma-1} l(\tilde{\psi}_u, 0). \end{aligned} \quad (3.45)$$

3.4 Comparative statics

In this section, I examine the impact of technological progress. In addition, some novel predictions which are related to product market policies are discussed. I firstly analyze changes in the cutoff productivity levels and then study the impact of technological progress on unemployment.

3.4.1 Changes in the cutoff productivity levels

The proportion of update firms, which is referred to as the economy's technology adoption rate, is obtained from (3.40). In this determination, I get the following corollary.

Corollary 9 *It requires rapid technological progress or strong monopolistic competition to increase the proportion of firms with technology updates. In other words, the effectiveness of policies which are unrelated to these is likely to be limited.*

As demonstrated by numerical simulations later, the majority of the firms are obsolete firms in the economy but a small fraction of update firms substantially contribute to aggregate employment.

Naturally, if there is no technological progress, all firms are obsolete firms. On the other hand, as long as the growth rate of technological progress is positive,

the presence of update firms is assured and the economy involves the two types of firms. Similarly, all firms are obsolete firms as the product market is monopolistic, i.e., $\sigma \rightarrow 1$.²¹ As opposed to this, all firms are update firms as the product market is competitive, i.e., $\sigma \rightarrow \infty$. Along with (3.40), the corollary holds from (3.41).

Corollary 10 *An increase in the growth rate of technological progress, g , unambiguously increases the level of Exit-cutoff, ψ^* , as long as update firms exist.*

The swift pace of technological advance increases the value of entry, through an increase in the value of an update firm and an increase in the conditional probability that an entrant becomes an update firm. This promotes firm entry which continues until the value of entry is back to the level of entry cost through an increase in the Exit-cutoff. Importantly, if all firms are obsolete, the effect of technological progress on the Exit-cutoff is zero, i.e., $\partial\psi^*/\partial g \rightarrow 0$ as $(\psi_o^*/\psi^*)^{-\alpha} \rightarrow 0$. (3.41) also gives the corollary.

Corollary 11 *A decrease in the entry cost, K , leads to an increase in the level of Exit-cutoff, ψ^* . This also increase the level of Obsolescence-cutoff, ψ_o^* while the relationship between these cutoffs, ψ_o^*/ψ^* , remains unchanged.*

3.4.2 The impact of technological progress on unemployment

By using (3.38), (3.39) and (3.42), I have:

$$\begin{aligned} \frac{R(\psi, x)}{l(\psi, x)} &\equiv \left[\left(\frac{\psi^*}{\psi_o^*} \right)^\alpha \tilde{\psi}_u^{\sigma-1} + \left[1 - \left(\frac{\psi^*}{\psi_o^*} \right)^\alpha \right] \tilde{\psi}_o^{\sigma-1} \tilde{g}^{-\tilde{x}} \right]^{\frac{1}{\sigma-1}} \\ &\equiv \left(\frac{\alpha}{\alpha + 1 - \sigma} \right)^{\frac{1}{\sigma-1}} \psi^* \left[\frac{\delta}{1 - \tilde{g}^{-1}(1 - \delta)} \left[1 + \frac{1 - \delta}{\delta} \left(\frac{\psi_o^*}{\psi^*} \right)^{-\alpha} \right] \right]^{\frac{1}{\sigma-1}} \\ &= \frac{\sigma - \beta}{\sigma - 1} \left[\omega(\theta) + \frac{\gamma}{q(\theta)} \frac{\Lambda_g^{-1} - 1 + \lambda}{1 - \beta} \right], \end{aligned} \tag{3.46}$$

²¹In this case, $\psi_o^* \rightarrow \infty$ is obtained.

where the right hand side of this equation increases with θ .²² First, changes in the cutoff productivity levels affect labor market tightness as follows.

Corollary 12 *An increase in either the Exit-cutoff, ψ^* , or the proportion of update firms, $(\psi_o^*/\psi^*)^{-\alpha}$, increases the market tightness, θ .*

Through these channels, the growth rate g increases θ and decreases the unemployment rate, u . Here, more productive firms afford to cover higher employment costs in the tighter labor market which corresponds to a higher θ .

For the opposed effect to this, the next corollary is also obtained.

Corollary 13 *An increase in the job cutback rate of obsolete firms, given as $1 - \tilde{g}^{-1}$, decreases θ .*

The term $\delta/[1 - \tilde{g}^{-1}(1 - \delta)]$ in the second line of (3.46) is associated with the weight of obsolete firms. An increase in g decreases this term and θ , through an increase in the job cutback rate and a decrease in the labor demand of obsolete firms. Note that the growth rate of technological progress and product market competition both strengthen firm obsolescence. However, at the same time, these both decrease the proportion of obsolete firms. The latter result is a new finding of this chapter.²³ These effects both improve the worker reallocation from obsolete to new and renewed jobs and not necessarily increase the unemployment rate.

Finally, An increase in the net discount factor, Λ_g , increases θ . This is well known as capitalization effect in the literature. It is uncovered in the derivation of (3.17) that this effect uniformly holds for the firms, independently of firm type. The incorporation of multi-worker firms distinguishes the problem of optimal hiring from firm entry and exit. This is considered to generate the current result. More importantly, the impact of this effect on unemployment is almost zero, as suggested in the literature and replicated in the next section.

²²The first line of this equation is given from (B.23) in the appendix.

²³Mortensen and Pissarides (1998) explain the opposite case in which technological progress increases the proportion of obsolete jobs. Their model solves the cutoff productivity level, below which jobs are obsolete, based on cycles of technology updates. The current model solves the Obsolescence-cutoff by focusing on values of firm types. I consider that this is one main reason of the difference.

Table 3.1: Calibrated parameters, annual

Variable	Symbol	Value
Interest rate	r	0.05
Rate of technological progress	g	0.02
Probability of job separation	$s/12$	0.036
Probability of firm destruction	δ	0.087
Elasticity of matching function	η	0.5
Worker's bargaining power	β	0.5
Scale of matching function	m_0	8.40
Flow value of unemployment	z	0.58
Cost of posting a vacancy	γ	0.28
Elasticity of substitution among differentiated goods	σ	3.5
Shape of firm-specific productivity distribution	α	2.65
Minimum level of firm-specific productivity	ψ_{\min}	0.05
Entry cost	K	0.24
Technology adoption cost	I	1.23

3.5 Numerical simulation

In this section, I examine the following questions quantitatively. How does technological progress affect unemployment? Does a change in the composition of firms matter for steady-state unemployment? How does the average labor demand and the mass of firms, distinguished by firm type, change?

3.5.1 Calibration

The model is calibrated to the U.S. economy, and its time period is taken to be one-year. The matching function is assumed to be Cobb-Douglas $m(u, V) = m_0 u^\eta V^{1-\eta}$.

Parameter values are summarized in Table 3.1. I choose the yearly interest rate to be $r = 0.05$ and the rate of technological progress to be $g = 0.02$. I follow Pissarides (2009) to set the monthly job separation rate as $s/12 = 0.036$. The annual firm destruction rate $\delta = 0.087$ is computed as the sample mean over the 1977-2014 period from the Business Dynamics Statistics.²⁴ I follow Petrongolo and

²⁴The firm destruction rate in each year is computed as Firmdeath_Firms divided by Firms. These series are obtained from https://www.census.gov/ces/dataproducts/bds/data_firm.html

Pissarides (2001) to set the elasticity in the matching function to be $\eta = 0.5$. As in the literature, I set the exogenous bargaining power to satisfy $\beta = \eta$.

According to Ebell and Haefke (2009), the entry cost in the US in 1997 equals 0.6 months of per-capita income, and the entry cost in 1978 amounts to 5.2 months of per-capita income. This suggests that there is a large variation in the entry cost over time. Here, I simply use the mean value of these estimates: $K = [(0.6 + 5.2)/2] \times 1/12 = 0.24$.

I normalize the revenue per worker, given as $R(\psi, x)/l(\psi, x)$, into one.²⁵ I obtain the scale parameter of the matching function, $m_0 = 8.40$, the flow value of unemployment, $z = 0.58$, and the cost of posting a vacancy, $c = 0.28$, from the three targets: $\theta = 0.72$ (Pissarides, 2009), $z/w = 0.71$ (Hall and Milgrom, 2008) and $\theta q(\theta)/12 = 0.594$ (Pissarides, 2009). The implied recruitment cost $c/q(\theta)$ from these parameters is 14.0 percent of the quarterly wage, which is consistent with Elsby and Michaels (2013).

It is well known that the estimate for the elasticity of substitution σ has a range. Rather than choosing an arbitrary value, I target the markup to pin down σ . Specifically, I choose $\sigma = 3.5$ so that the implied markup is $(\sigma - \beta)/(\sigma - 1) = 1.2$, which is consistent with the estimates by Martins *et al.* (1996) and by Christopoulou and Vermeulen (2008).

According to Axtell (2001), the size distribution for US firms is approximately Zipf. I follow Axtell (2001) and set the shape parameter for the model's size distribution function to be $\alpha/(\sigma - 1) = 1.06$, from which I obtain $\alpha = 2.65$.²⁶

The technology adoption cost, $I = 1.23$, is chosen to set the average firm size to be $(\psi^*/\psi_o^*)^\alpha l(\tilde{\psi}_u, 0) + [1 - (\psi^*/\psi_o^*)^\alpha] l(\tilde{\psi}_o, \tilde{x}) = 21.67$, which is the sample mean for the 1977-2014 period in the Business Dynamics Statistics.²⁷ Finally, I get the minimum level of firm-specific productivity to be $\psi_{\min} = 0.05$.

²⁵From (3.17), $R(\psi, x)/l(\psi, x)$ is constant across firms.

²⁶As a caveat, the exact shape of firm size distribution differs from the level of $\alpha/(\sigma - 1)$. In the model, this level corresponds to the shape for the update firms. From a technical reason, I approximately use 1.06 for it. Although the average firm size of obsolete firms is 2.6 persons from the current calibration, Luttmer (2004, Figure 1) shows that the shape of firm size distribution is similar to the value which is reported in Axtell (2001) as he uses the size categories of 5 employees and higher. I examine the sensitivity later.

²⁷The average firm size in each year is computed as Emp divided by Firms. These series are obtained from https://www.census.gov/ces/dataproducts/bds/data_firm.html

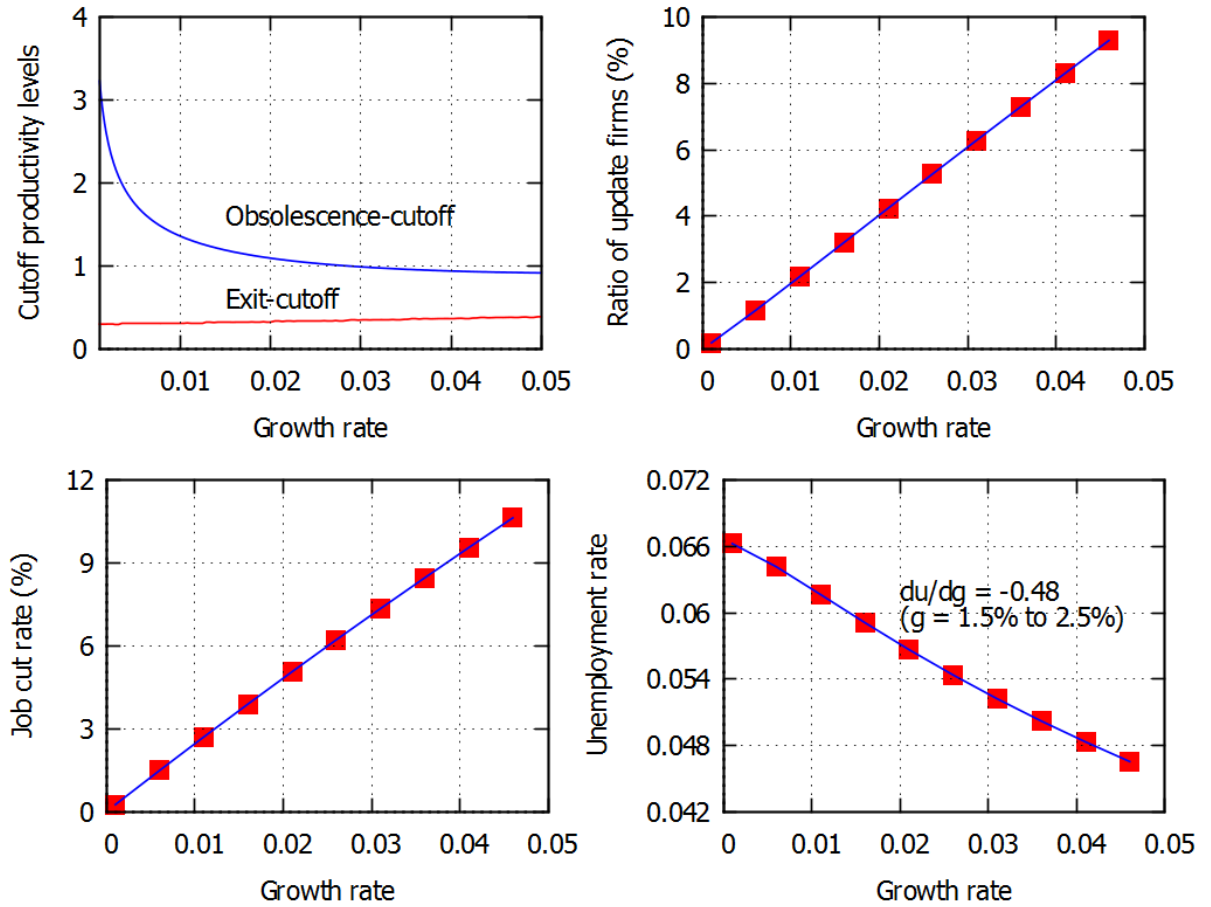


Figure 3.2: Impacts of technological progress

The model solutions are as follows: the wage $w = 0.82$, the unemployment rate $u = 0.057$, the mass of firms $n = (1 - u)/21.67 = 0.0435$, the Obsolescence-cutoff $\psi_o^* = 1.09$, and the Exit-cutoff $\psi^* = 0.32$.²⁸

3.5.2 Results

Figure 3.2 demonstrates the quantitative results for changes in the growth rate of technological progress. I compute the two cutoff productivity levels, the proportion of update firms to surviving firms, the job cutback rate, and the unemployment

²⁸I used wxMaxima 16.04.2 to obtain the quantitative results. All codes are available upon request.

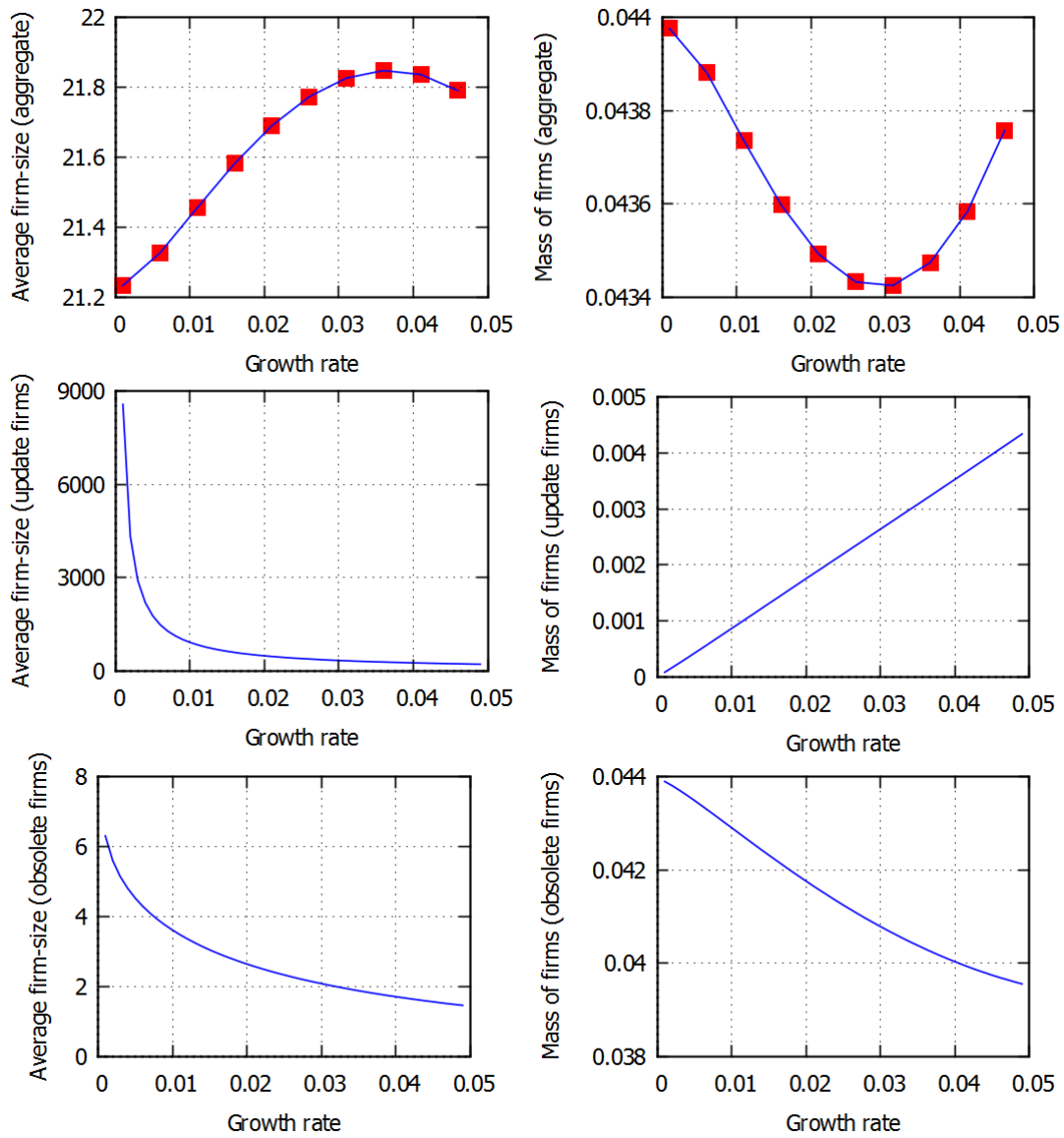


Figure 3.3: Impacts of technological progress

rate.

The three panels, except for the lower right panel, in Figure 3.2 confirm the previous analyses. The lower right panel shows that the relationship between the growth rate and the unemployment rate is entirely negative. A change in the growth rate from 1.5% to 2.5% decreases the unemployment rate by 0.48%. This result is within the range of estimates by Blanchard and Wolfers (2000).

Based on (3.46), this net impact $du/dg = -0.48$ is approximately decomposed into the following four effects: an increase in the proportion of update firms $du/dg|_{\text{ceteris paribus}} = -0.58\%$, an increase in the Exit-cutoff $du/dg|_{\text{ceteris paribus}} = -0.51\%$, an increase in the annual job cutback rate $du/dg|_{\text{ceteris paribus}} = +0.61\%$, and an increase in the net discount factor $du/dg|_{\text{ceteris paribus}} = -0.007\%$. These results suggest that a change in the composition of firms, through changes in the cutoffs, is an essential ingredient for the determination of unemployment.

Figure 3.3 presents the corresponding results, distinguished by firm type. The first column is for the average firm size and the second column is for the mass of firms. Although the aggregate outcomes mask changes with respect to firm type, Figure 3.3 clearly shows that an increase in the mass of update firms is the driving force for a lower unemployment. On the other hand, the average firm sizes of the two types both decrease with technological progress.

3.5.3 Sensitivity analysis

To examine the sensitivity of the simulation results, I recalibrate the net effect of the rate of technological progress on the unemployment rate du/dg . This is computed under the following different values and targets: the entry cost K , the worker's bargaining power β , the elasticity of substitution among differentiated goods σ , the flow value of unemployment divided by the wage z/w , and the shape of firm-specific productivity distribution $\alpha/(1 - \sigma)$. Table 3.2 reports the results. To make it easy to read, each result is given as $-du/dg$, as multiplied by -1 . Each value with the superscript (*) is the baseline which is used in previous subsections. In addition, each value in brackets represents the standard capitalization effect decomposed. For changes in K or β , the results are very robust. On the other hand, for changes in the rest, the robustness is weaker but still holds. Throughout,

Table 3.2: Impacts of technological progress on unemployment

	K			β		
	0.05	0.24*	0.43	0.2	0.5*	0.8
$- du/dg$	0.48 (0.01)	0.48 (0.01)	0.48 (0.01)	0.43 (0.02)	0.48 (0.01)	0.49 (0.00)
	σ			z/w		
	3.5*	6.5	11	0.3	0.71*	0.8
$- du/dg$	0.48 (0.01)	0.40 (0.01)	0.30 (0.01)	0.21 (0.01)	0.48 (0.01)	0.69 (0.01)
	$\alpha/(1 - \sigma)$					
	1.06*	1.1	1.2			
$- du/dg$	0.48 (0.01)	0.40 (0.01)	0.22 (0.01)			

the standard capitalization effect has its size much smaller than the overall impact.

3.6 Conclusion

New technologies are utilized by firms in different ways. This generates different productivity levels of firms and is linked to the content of firm entry and exit. I asked the question of how these affect the relationship between technological progress and unemployment. To address this, the chapter considered heterogeneous multi-worker firms and revealed that technological progress changes a composition of firms with different technological uses and it considerably affects the determination of steady-state unemployment. The results showed that a small fraction of firms with technology updates substantially contributes to job creation while the other firms with obsolescence contributes to job cutback. Here, the presence of the latter firms is important. It uncovers that rapid technological progress accelerates firm obsolescence but, at the same time, reduces such firms. This has a role to improve aggregate productivity and lead to lower unemployment.

Chapter 4

Creative Destruction and Capitalization with On-the-job Search

4.1 Introduction

Many economists seek an answer to the question of how the unemployment rate is determined. One of the main factors for explaining long-term unemployment dynamics has long been considered to be technological change. In general, technological progress brings about two competing opposite effects on unemployment, known as capitalization and creative destruction effects. This chapter examines the importance of these two effects.

This chapter revisits this issue by allowing job-to-job transitions. According to the Survey of Income and Program Participation, 49% of total exits from employers is associated with job-to-job transitions and its value increases to 71% over the exits which remain in the labor force (Nagypal, 2008). This implies that on-the-job search has had an important role in the past determination of unemployment.

The incorporation of on-the-job search changes the job flows. In the model, total job creation consists of hiring of unemployed and hiring of employed job seekers, and total job destruction consists of endogenous separation by an idiosyncratic shock, resignation of employed job seekers, and job destruction after the comple-

tion of technological obsolescence. In addition, as a key feature behind these flows, the current extension modifies the worker's outside options so that a change in a job finding rate of workers affects bargained wages but this effect is weaker than the model without on-the-job search.

The sign of the impact of technological progress on unemployment depends on the extent to which new technology is embodied in new jobs. Here, there are two types of the progress through which innovation serves to increase either the productivity of all jobs (disembodied technological progress), or the productivity of newly created jobs only (embodied technological progress). The former generates the negative impact on unemployment, and at the same time, the latter brings about the positive impact on unemployment.

Through a simulation, I examine the model's predictions under different proportions of embodied technology to disembodied technology utilized in jobs. For a lower proportion, almost-zero, of embodied technology, rapid technological progress decreases the unemployment rate, mainly through an increase in the value of job creation and a decrease in endogenous job separations in which a job-match with better profitability endures separation shocks longer. In comparison, for a sufficiently-higher proportion of embodied technology, rapid technological progress increases the unemployment rate from the opposite reason to the above.

The main findings by taking on-the-job search into account are three folds. First, in the economy with almost-full disembodied technology, the model can replicate the magnitude of the negative impact of technological progress on unemployment consistent with recent empirical estimates, similar to the conclusion in Miyamoto and Takahashi (2011). The model's simulations demonstrate that one percent-point increase in the rate of technological progress decreases the unemployment by 1.14%. Pissarides and Vallanti (2011) show this value to be 0.02% in their basic model without on-the-job search while empirically suggest that one percent-point increase in the growth rate of total factor productivity decreases the unemployment rate by 1.3% to 1.5%.

Second, a change in a job destruction flow through technological obsolescence hardly affects equilibrium unemployment because jobs are recreated at job-to-job transitions before their technologies become outdated, as concluded in Michau (2013). In this sense, job-to-job transition seems to have a similar feature to

the setting, allowing the update of embodied technology such that technology is renewed without an increase in net job destruction. However, in the current model, this not necessarily weakens the positive impact of technological progress on unemployment because endogenous job separations by shocks are taken into account. In particular, the simulation results show that one percent-point increase in the rate of technological progress increases the unemployment rate by 0.90% as the case of fully embodied technology. In Michau (2013), the corresponding value is 0.07% under his model without the endogenous separations.

Finally, the model's prediction is compatible with the evidence which support the negative relationship between technological progress and unemployment, only if to a large extent technology is disembodied. The literature indirectly implies that the incorporation of on-the-job search seems to fill the gap between theory and evidence through generating the above negative relationship easier, but there may still be a lack of understanding. The results suggest that a critical point for the proportion of embodied technology at which the effect of technological progress on unemployment is zero exists between 0 and 0.1, similar to a model without on-the-job search in Pissarides and Vallanti (2007).

4.2 The model

4.2.1 Setup

The economy evolves with technological progress and the progress can be either *disembodied* or *embodied*, distinguished into these *two* components, as in Pissarides and Vallanti (2007). I incorporate on-the-job search into the above. The structure of the model is given by combining the model basics in Miyamoto and Takahashi (2011) and Michau (2013).

The economy consists of a unit measure of identical workers and a large measure of ex-ante identical jobs/employers. Time is continuous and horizon is infinite. All agents are risk neutral and discount the future at the common rate r .

The output of each filled job in period t is represented by $y_t(a)$, which depends on job's age a . The model assumes that the output is the product of two

components as below.¹

$$y_t(a) = z_t k_t(a), \quad (4.1)$$

where $z_t = [e^g]^{\lambda t}$ and $k_t(a) = [e^g]^{(1-\lambda)(t-a)}$. Consider the case as $a = 0$, in which the whole economy's output grows exogenously at rate g . On the other hand, the output of each job evolves slower than this because the latter part of production technology is embodied in the job and its technology level is constant over time after the filled job is born. Thus, the component z_t is associated with disembodied technology and the component $k_t(a)$ is associated with embodied technology. I follow Pissarides and Vallanti (2007) so that $1 - \lambda$ denotes the exogenous extent to which new technology is embodied in new jobs. As a natural consequence, the maximum lifetime of each filled job corresponds to the length until job obsolescence is complete at which own technology is too old to continue production.

Workers can be either unemployed or employed. With on-the-job search, all workers can switch jobs for a higher wage. Embodied technology and job obsolescence generates the output differences between jobs and it leads to the wage differences. Thus, in the model, each worker can be motivated to move from old to young jobs.

The labor market is frictional. The aggregate number of matches made is determined by the standard constant-returns-to-scale matching function $M(u + m, v)$, where u denotes the measure of the unemployed, m denotes the measure of the employed job seekers and v denotes the aggregate job vacancies. From the matching function, the vacancy filling rate is given by $M(u + m, v)/v = q(\theta)$, where $\theta \equiv v/(u + sm)$. Similarly, the job finding rate of unemployed workers is given by $M(u + m, v)/(u + m) = \theta q(\theta) \equiv p(\theta)$. The corresponding job finding rate of employed job seekers is also $p(\theta)$. A job search cost of employed workers can differ from that of the unemployed. To describe this, I follow Michau (2013) such that the opportunity cost of on-the-job search is introduced and denoted by σ_t . Wages, denoted by $w_t(a)$, are determined through bargaining specified later.

To take into account a model's channel as in a estimation that labor produc-

¹The setting is mainly based on Hornstein, Krusell and Violante (2007) and also similar to Pissarides and Vallanti (2007).

tivity growth decreases the job separation rate (Miyamoto and Takahashi, 2011), I consider an endogenous job separation through which a job is likely to be destroyed by an idiosyncratic shock before the completion of job obsolescence. The important thing is that the opportunity of on-the-job search reduces (increases) the relative value of being unemployed (employed) and its fact affects the response of job separations.

It is known that incorporating both on-the-job search and endogenous job separation highly increases its model complexity. When it comes to embodied technological progress, the above problem becomes more serious. The current chapter tackles with this by trying an alternative way to incorporate endogenous separation. In particular, after an idiosyncratic shock occurs at rate δ , each employer decides to either destroy the job or pay a cost, denoted by H_t , for continuing its production. Along with job obsolescence, endogenous separation is translated into a change in the job's cutoff age at which the above decisions by employers are indifferent.

Throughout, I focus on the *balanced growth equilibrium* in which all values grow at the same rate. To ensure the existence of a balanced growth equilibrium, I follow the literature to assume that the vacancy cost, the unemployment benefit, and the cost of continuing production after an idiosyncratic shock all grow at rate g . Specifically, I assume that the unit vacancy cost is $y_t(0)c$, the unemployment benefit is $y_t(0)b$, the search cost of employed job seekers $y_t(0)\sigma$, the cost after a shock $y_t(0)H$, where c , b , σ , and H are parameters (Mortensen and Pissarides, 1998). In addition, I assume $g < r$.

The value of posting a vacancy at period t is

$$V_t = -c_t dt + \frac{1}{1+rdt} [q(\theta)dt J_{t+dt}(0) + (1-q(\theta)dt)V_{t+dt}], \quad (4.2)$$

and the value of a filled job of age a at period t is

$$J_t(a) = [y_t(a) - w_t(a)] dt + \frac{1 - 1^w(a)p(\theta)dt}{1+rdt} \times \left[\begin{array}{c} (1 - \delta dt) J_{t+dt}(a + dt) \\ + 1^f(a)\delta dt [J_{t+dt}(a + dt) - H_{t+dt}] \end{array} \right], \quad (4.3)$$

where dt denotes a time interval of length and the above equation includes two indicator functions as follows. $1^w(a)$ is an indicator which is equal to 1 if the worker on the job searches another job and to 0 otherwise. In other words, the job continues at rate of $1 - 1^w(a)p(\theta)dt$. Next, $1^f(a)$ is an indicator which is equal to 1 if the employer continues production by paying the cost H to recover after a shock and to 0 otherwise. When a shock occurs at rate δdt , the future value is $1^f(a) [J_{t+dt}(a + dt) - H_{t+dt}]$.

On the other hand, the value of an unemployed worker at period t is

$$U_t = b_t dt + \frac{1}{1 + r dt} [p(\theta) dt W_{t+dt}(0) + (1 - p(\theta) dt) U_{t+dt}], \quad (4.4)$$

and the value of an employed worker is

$$\begin{aligned} W_t(a) = & [w_t(a) - 1^w(a)\sigma_t] dt \\ & + \frac{1}{1 + r dt} [1^w(a)p(\theta) dt W_{t+dt}(0)] \\ & + \frac{1 - 1^w(a)p(\theta) dt}{1 + r dt} \left[\begin{array}{l} [1 - (1 - 1^f(a))\delta dt] W_{t+dt}(a + dt) \\ + (1 - 1^f(a))\delta dt U_{t+dt} \end{array} \right], \end{aligned} \quad (4.5)$$

where if the worker succeed at on-the-job search his/her future value changes to $W_{t+dt}(0)$ as in the second line of this equation.

These value functions of the discrete form are summarized into those of the continuous form as below, along with using the assumption for exogenous parameters' growth and the free entry of posting a vacancy. I follow Hornstein, Krusell and Violante (2007, p.1096) to transform these equations and then get

$$0 = -c + q(\theta)J(0), \quad (4.6)$$

$$\begin{aligned} (r - g)J(a) - J'(a) = & e^{-(1-\lambda)ga} - w(a) - 1^f(a)\delta H \\ & - [(1 - 1^f(a))\delta + 1^w(a)p(\theta)] J(a), \end{aligned} \quad (4.7)$$

$$(r - g)U = b + p(\theta) [W(0) - U], \quad (4.8)$$

$$(r - g)W(a) - W'(a) = w(a) - 1^w(a)\sigma + 1^w(a)p(\theta) [W(0) - W(a)] - (1 - 1^f(a))\delta [W(a) - U], \quad (4.9)$$

where, as assured in a balanced growth equilibrium in which all values grow at the same rate as $y_t(0)$, $W(a) = W_t(a)/y_t(0)$, $U = U_t/y_t(0)$, $J(a) = J_t(a)/y_t(0)$, $V = V_t/y_t(0)$, and $V = 0$. In addition, $w(a) = w_t(a)/y_t(0)$ and $e^{-(1-\lambda)ga} = y_t(a)/y_t(0)$.

4.2.2 Wage determination

As is standard in the literature, wages are determined through Nash bargaining. Thus, the below surplus sharing rule is the condition to derive a wage function.

$$W(a) - U = \beta [J(a) + W(a) - U] \equiv \beta S(a), \quad (4.10)$$

where $S(a)$ represents the total surplus and the worker's bargaining power is β so that a fraction β of the surplus goes to the worker.

Thus, the wage rate satisfies²

$$w(a) = \beta [e^{-(1-\lambda)ga} - 1^f(a)\delta H] + (1 - \beta) \left[\omega(\theta) - 1^w(a) \left[\frac{\beta c \theta}{1 - \beta} - \sigma \right] \right], \quad (4.11)$$

where $\omega(\theta) \equiv b + \beta c \theta / (1 - \beta)$ as the value of worker's outside options in the model without on-the-job search.

If there is a on-the-job search, i.e., $1^w(a) = 1$, the current value of worker's outside option is lower than $\omega(\theta)$ as long as the search cost on the job, σ , is sufficiently-low. This implies that a standard model without on-the-job search evaluates unreasonably high value of being unemployment because only the unemployed can search jobs and gain the expected value from it. By relaxing this, the impact of technological progress is not mitigated through the above problem of worker's outside options.

²Here, the same logic as in Michau (2013, Appendix A) holds and the bargaining set is convex.

4.2.3 Equilibrium

In the model, there are three channels of job destruction at which on-the-job search succeeds, a low productivity job is hit by an idiosyncratic shock, and a job becomes completely obsolete. To characterize the equilibrium of the model, I firstly consider the worker's choice of on-the-job search.

Each worker tries to change jobs as long as its action is profitable. Along with (4.9), this decision making is described as

$$1^w(a) = \begin{cases} 1 & \text{if } p(\theta) [W(0) - W(a)] \geq \sigma, \\ 0 & \text{if } p(\theta) [W(0) - W(a)] < \sigma, \end{cases} \quad (4.12)$$

where if the value of a new position, $W(0)$, sufficiently exceeds the value of the current position, $W(a)$, the worker starts on-the-job search and vice versa.

By using (4.6) and (4.10), the condition (4.12) can be written as

$$1^w(a) = \begin{cases} 1 & \text{if } \frac{1}{\beta p(\theta)} \left[\frac{\beta c \theta}{1-\beta} - \sigma \right] \geq S(a), \\ 0 & \text{if } \frac{1}{\beta p(\theta)} \left[\frac{\beta c \theta}{1-\beta} - \sigma \right] < S(a). \end{cases} \quad (4.13)$$

If the left hand side of this equation has a value between $S(\bar{a})$ and $S(0)$, a cutoff age a^w at which the above choices are indifferent exists between 0 and \bar{a} .

Second is endogenous job separation. Each employer decides to destroy the job after an idiosyncratic shock hits, if the cost H for recovering from the shock and continuing production exceeds the future value of the employer $J(a)$, and vice versa. By using (4.10), it is represented as³

$$1^f(a) = \begin{cases} 1 & \text{if } S(a) \geq \frac{H}{1-\beta}, \\ 0 & \text{if } S(a) < \frac{H}{1-\beta}, \end{cases} \quad (4.14)$$

where $1^f(a)$ is 1 as the employer continues production. Similarly, If the right hand side of this equation has a value between $S(\bar{a})$ to $S(0)$, a cutoff age a^w at which the above decisions are indifferent exists between 0 and \bar{a} .

Finally, a job can also be destroyed at which job's output level is down to the

³Here, only the employer incurs the cost and it is attributed to the setting that the current wage is independent of the separation decision.

flow value of worker's outside options such that

$$e^{-(1-\lambda)g\bar{a}} = b + \sigma, \quad (4.15)$$

where I assume $\beta c\theta(1-\beta) \geq \sigma$ in order to allow on-the-job search in this economy.⁴

Proposition 14 *A balanced growth equilibrium is defined by a list $(\theta, a^w, a^f, \bar{a})$ that satisfies*

$$\frac{c}{q(\theta)} = (1-\beta)S(0) \quad (4.16)$$

$$\frac{1}{\beta p(\theta)} \left[\frac{\beta c\theta}{1-\beta} - \sigma \right] = S(a^w), \quad (4.17)$$

$$S(a^f) = \frac{H}{1-\beta}, \quad (4.18)$$

$$\bar{a} = \frac{1}{(1-\lambda)g} \ln \left(\frac{1}{b+\sigma} \right). \quad (4.19)$$

Proof. *The first equation is derived by using (4.7) and (4.10). The rest is from (4.13), (4.14) and (4.15). ■*

By plugging (4.11) into (4.7) and combining it with (4.10), the surplus function is also solved as

$$\begin{aligned} S(a) = & \int_a^{\bar{a}} \exp \left[-(r + \delta - g)(x - a) + \delta \int_a^x 1^f(i)di - p(\theta) \int_a^x 1^w(j)dj \right] \\ & \times \left[e^{-(1-\lambda)gx} - 1^f(x)\delta H - \omega(\theta) + 1^w(x) \left[\frac{\beta c\theta}{1-\beta} - \sigma \right] \right] dx, \end{aligned} \quad (4.20)$$

where the initial condition is $S(\bar{a}) = 0$.

⁴Here, \bar{a} is independent of θ . The previous version of Michau (2013) shows that the incorporation of endogenous search efforts brings about the same result.

4.2.4 Labor market dynamics

Case 1: $a^w < a^f$

The equilibrium rate of unemployment, denoted by u , is given such that job creation and destruction are equal. This condition is

$$JC = JD, \quad (4.21)$$

where

$$JC = p(\theta)u + p(\theta)m, \quad (4.22)$$

and

$$\begin{aligned} JD = & p(\theta)m + \delta JC \exp[-p(\theta)(a^f - a^w)] \int_{a^f}^{\bar{a}} \exp[-(p(\theta) + \delta)(x - a^f)] dx \\ & + JC \exp[-\delta(\bar{a} - a^f) - p(\theta)(\bar{a} - a^w)]. \end{aligned} \quad (4.23)$$

The jobs are created through search activities by both the unemployed and employed job seekers. The jobs are destroyed through on-the-job search, endogenous separation after an idiosyncratic shock, and the completion of job obsolescence.

In the current case, the above job flows are equivalent to the flows in and out of employed job seekers. Here, the number of employed job seekers, denoted by m , is determined by the condition such that:

$$1 - u = m + a^w JC. \quad (4.24)$$

This equation represents that the total employed workers consist of job seekers, with on-the-job search, and the rest, without on-the-job search.

Case 2: $a^w \geq a^f$

Similarly, the equilibrium rate of unemployment is given by

$$JC = JD, \quad (4.25)$$

where

$$JC = p(\theta)u + p(\theta)m, \quad (4.26)$$

and

$$JD = p(\theta)m + \delta [1 - u - a^f JC] + JC \exp [-\delta(\bar{a} - a^f) - p(\theta)(\bar{a} - a^w)]. \quad (4.27)$$

One changed point is the above term of endogenous separation, $\delta[1 - u - a^f JC]$, where the total jobs with age more than a^f are equal to the total employed minus the jobs with age less than a^f , i.e., $1 - u - a^f JC$.

On the other hand, the number of employed job seekers is determined such that the equal flows in terms of the set of employed job seekers hold. In particular, this condition is

$$JC \exp [\delta(a^w - a^f)] = p(\theta)m + \delta m + JC \exp [-\delta(\bar{a} - a^f) - p(\theta)(\bar{a} - a^w)]. \quad (4.28)$$

While the employed start to search jobs, as in the left hand side of the above expressions, employed job seekers stop on-the-job search because its searches succeed, job separations occur, or jobs become completely obsolete and are destroyed, as in each term of the right hand side in order.

4.3 Numerical simulation

The objective of this chapter is to evaluate the quantitative importance of the capitalization and creative destruction effect in the model with on-the-job search. In this section, I describe how the model parameters are chosen and present the quantitative results.

Table 4.1: Parameter values

Variable	Symbol	Value
Interest rate	r	0.04
Rate of technological progress	g	0.02
Proportion of disembodied technology	λ	0.999
Rate at which an idiosyncratic shock occurs	δ	0.5
Elasticity of matching function	η	0.5
Worker's bargaining power	β	0.5
Flow value of unemployment	b	0.4
Scale of matching function	m_0	1.672
Cost of posting a vacancy	c	0.40
Search cost for employed job seekers	σ	0.27
Cost at the shock to continue production	H	0.07

4.3.1 Calibration

The model is calibrated to the US economy, and its time period is chosen to be one year. I follow the standard calibration procedure whenever possible. The baseline parameter values are summarized in Table 4.1.⁵

I choose the annual interest rate to be $r = 0.04$ and the rate of technological progress to be $g = 0.02$. In baseline calibration, I assume $\lambda = 0.999$ as an alternative to full disembodied technology and examine its sensitivity later. An arrival rate of idiosyncratic shock is set to be $\delta = 0.5$.⁶ I follow Petrongolo and Pissarides (2001) to set the elasticity in the matching function to be $\eta = 0.5$. As in the literature, I set the exogenous bargaining power to satisfy $\beta = \eta$. I follow Shimer (2005) to set the flow value of unemployment as $b = 0.4$.

In the subsequent calibration, I get parameter values by assuming that $a^w < a^f$ at $g = 0.02$, under which the level of the parameter H is restricted to be lower than $(1 - \beta)S(a^w)$. This means that damage from a job separation shock is small scale and on-the-job search starts before job separation by the shock emerges.

I obtain the scale parameter of the matching function, $m_0 = 1.627$, the cost of posting a vacancy, $c = 0.40$, the search cost for employed job seekers, $\sigma = 0.27$, and the cost after an idiosyncratic shock occurs to continue production, $H = 0.07$, from

⁵For the following numerical computations, all codes by Maxima are available upon request.

⁶Miyamoto and Takahashi (2011) get $\lambda = 0.552$ through their calibration.

Table 4.2: Simulated equilibrium values

	u	m	θ	a^w	a^f	\bar{a}
$\lambda = 0.999$						
$g = 0.02$	0.0500	0.173	1.00	2.087	2.102	19.7×10^3
$g = 0.03$	0.0386	0.182	1.01	2.096	2.258	13.1×10^3
$\lambda = 0.7$						
$g = 0.02$	0.0558	0.166	0.98	2.111	2.060	65.6
$g = 0.03$	0.0583	0.166	0.98	2.089	2.017	43.7
$\lambda = 0.3$						
$g = 0.02$	0.0621	0.159	0.96	2.152	2.024	28.1
$g = 0.03$	0.0681	0.155	0.95	2.153	1.959	18.7
$\lambda = 0$						
$g = 0.02$	0.0672	0.153	0.95	2.185	1.993	19.7
$g = 0.03$	0.0762	0.145	0.92	2.206	1.905	13.1

the four targets: market tightness is normalized to 1 (Shimer, 2005), the monthly transition rate from employment to unemployment is 0.73% (Nagypal, 2008), the monthly transition rate from employment to employment is 2.5% (Nagypal, 2008), and the unemployment rate is 5%.⁷

4.3.2 Results

Table 4.2, Table 4.3, and Table 4.4 present the main results of this chapter. These show simulated values for different levels of the rate of technological progress g and the extent to which new technology is embodied in new jobs $1 - \lambda$. In Table 4.2 and Table 4.3, I compute equilibrium values of the unemployment rate u , the ratio of employed job seekers to the workforce m , labor market tightness θ , the job's age at which on-the-job search starts α^w , the job's age after which job separation occurs by a shock α^f , the job's age at which obsolescence is complete and the job is destroyed \bar{a} , and the surplus values for each age $S(\cdot)$. For more detail information, I also calculate the corresponding job flows in Table 4.4.

These results imply the four views. First, the effect of job obsolescence, as a channel of job destruction, is negligible, similar to Michau (2013). This is because

⁷As in Michau (2013, footnote 14), annual transition rates are computed as 0.088 and 0.304, respectively.

Table 4.3: Surplus values

	$S(0)$	$S(a^w)$	$S(a^f)$
$\lambda = 0.999$			
$g = 0.02$	0.4775	0.1489	0.1483
$g = 0.03$	0.4803	0.1537	0.1483
$\lambda = 0.7$			
$g = 0.02$	0.4736	0.1423	0.1483
$g = 0.03$	0.4722	0.1399	0.1483
$\lambda = 0.3$			
$g = 0.02$	0.4687	0.1339	0.1483
$g = 0.03$	0.4647	0.1272	0.1483
$\lambda = 0$			
$g = 0.02$	0.4649	0.1274	0.1483
$g = 0.03$	0.4590	0.1172	0.1483

workers can change jobs before the completion of obsolescence in the model with on-the-job search and also the surplus value at age 0, represented as the expected value of job creation, decreases slightly.

Second, as opposed to Michau (2013), rapid technological progress effectively decreases the unemployment rate for sufficient low λ . Although Michau (2013) assumes exogenous job separation by shock, the current chapter considers endogenous job separation. Here, jobs can be destroyed more easily before on-the-job search succeeds, and it leads to the lower job creation value. Thus, incorporating on-the-job search is not always to mitigate the creative destruction effect.

Third, as in Table 4.4, the unemployment rate is lower (higher) while transitions from employment to employment occurs more (less) and transitions from unemployment to employment occurs less (more). It suggests that job-to-job transition matters in the determination of steady-state unemployment. Interestingly, the relationship between total job creation/destruction and the unemployment rate is non-linear.

Finally, for sufficient high λ , the presence of on-the-job search magnifies the negative impact of technological progress on unemployment, which substantially improves the model's performance in line with recent empirical findings. Similar to Miyamoto and Takahashi (2011), the result shows that a decrease in unemployment is not only through an increase in job creation but also through a decrease in job

Table 4.4: Job flows

	Total job creation/ destruction	Job creation		Job destruction		
		Hiring of unemployed	Job-to-job transition	Endogenous separations	Obsolescence	Job-to-job transition
$\lambda = 0.999$						
$g = 0.02$	0.3724	0.0836	0.2888	0.0836	0	0.2888
$g = 0.03$	0.3716	0.0648	0.3068	0.0648	0	0.3068
$\lambda = 0.7$						
$g = 0.02$	0.3686	0.0925	0.2761	0.0925	1.1×10^{-60}	0.2761
$g = 0.03$	0.3713	0.0963	0.2750	0.0963	4.1×10^{-40}	0.2750
$\lambda = 0.3$						
$g = 0.02$	0.3627	0.1019	0.2608	0.1019	2.4×10^{-25}	0.2608
$g = 0.03$	0.3625	0.1109	0.2516	0.1109	1.6×10^{-16}	0.2516
$\lambda = 0$						
$g = 0.02$	0.3583	0.1093	0.2490	0.1093	2.2×10^{-17}	0.2490
$g = 0.03$	0.3563	0.1225	0.2337	0.1225	3.2×10^{-11}	0.2337

separation. In the case $\lambda = 0.999$, the change in g from 0.02 to 0.03 increases α^f from 2.102 to 2.258. It decreases job separations and serves to decrease the unemployment rate.

4.3.3 Is technology embodied?

In what follows, I present sensitivity analyses. As in Pissarides and Vallanti (2007), I choose the value for λ to be 0.999 in the baseline case in order to understand the maximum impact of which technological progress reduces unemployment. However, the proportion of embodied technology $1 - \lambda$ is not necessarily zero. Table 4.5 shows simulation results after re-calibrations under different values for λ . In the Table, du denotes a change in the unemployment rate and $d\theta$ denotes a change in market tightness when the growth rate of technological progress increases from 2% to 3%.⁸

The sign of the impact du/dg changes from $\lambda = 0.999$ to $\lambda = 0.9$. Recent empirical estimates support the negative sign of the impact du/dg (Blanchard and Wolfers, 2000; Pissarides and Vallanti, 2007), and thus the model's prediction is compatible only if technological progress is considered as almost disembodied

⁸ $dg = 0.03 - 0.02 = 0.01$ and $du/dg = du/0.01$.

Table 4.5: Sensitivity analysis A

	Impact of growth on unemployment du/dg	Change in market tightness $d\theta$	Re-calibrated parameters		
			c	H	σ
$\lambda = 0.999$	-1.144	+0.012	0.40	0.07	0.27
$\lambda = 0.9$	+0.228	-0.002	0.40	0.07	0.28
$\lambda = 0.7$	+0.371	-0.007	0.40	0.07	0.28
$\lambda = 0.3$	+0.686	-0.017	0.39	0.07	0.28
$\lambda = 0$	+0.949	-0.025	0.38	0.06	0.28

Table 4.6: Sensitivity analysis B

	Impact of growth on unemployment du/dg	Change in market tightness $d\theta$	Re-calibrated parameters		
			c	H	σ
$b = 0.1$	-1.153	+0.012	0.60	0.11	0.41
$b = 0.3$	-1.148	+0.012	0.47	0.09	0.32
$b = 0.5$	-1.140	+0.012	0.33	0.06	0.23
$b = 0.7$	-1.120	+0.012	0.20	0.04	0.14
$b = 0.9$	-1.022	+0.011	0.07	0.01	0.05

one. Thus, this conclusion is robust in the class of canonical Diamond-Mortensen-Pissarides-style models whether or not on-the-job search is incorporated. However, allowing job-to-job transition magnifies the impact du/dg of the standard models, because the worker's outside options are modified and then the structure of the job flows changes.

4.3.4 Sensitivity to b

In the baseline case, the value of b is 0.4, which might be low if the flow value of the unemployment includes the value of leisure, or home production (Hall and Milgrom, 2008). Thus, it is informative to report the results after re-calibrations under different values for b . Table 4.6 presents the results. It shows that the previous simulations are robust.

4.4 Conclusion

This chapter tried to understand theoretical predictions of the impact of technological progress on unemployment under different extents to which new technology is embodied in new jobs, by using the model with on-the-job search. The incorporation of job-to-job transitions magnified the size of the above impact, but preserved the conclusion of Pissarides and Vallanti (2007). In particular, the current chapter also concluded that embodied technology and creative destruction should hardly play a role in the determination of steady-state unemployment in order to generate plausible simulation results compatible with recent empirical findings.

Appendix A

Appendix of Chapter 2

A.1 Derivation of the inverse demand

Each individual is risk neutral and has the Dixit-Stiglitz preference over a continuum of differentiated goods:

$$\max_{Q_{i,t}^j} \left[\int (Q_{i,t}^j)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}} \quad (\text{A.1})$$

$$\text{subject to } y_t^j = \int Q_{i,t}^j \frac{p_i}{P} di,$$

where $P = [(1/n) \int p_i^{1-\sigma} di]^{1/(1-\sigma)}$. Thus, I have

$$Q_{i,t}^j = \left(\frac{p_i}{P} \right)^{-\sigma} \frac{y_t^j}{\int \left(\frac{p_i}{P} \right)^{1-\sigma} di} = \left(\frac{p_i}{P} \right)^{-\sigma} \frac{y_t^j}{n}, \quad (\text{A.2})$$

for individual j . The second equality uses the definition of the price index P . Then, the aggregate demand for good i is given as Eq. (2.2).

Because a firm with type ψ has the production technology $a_t \psi l_t$, as the function of labor input l_t , the optimal price of firm i with type ψ is set to satisfy

$$\int Q_{i,t}^j dj = \left(\frac{p_i}{P} \right)^{-\sigma} \frac{Y_t}{n} = a_t \psi l_t \Leftrightarrow \frac{p_i}{P} = \left(\frac{a_t \psi l_t}{Y_t/n} \right)^{-\frac{1}{\sigma}}, \quad (\text{A.3})$$

so that each monopolistic firm takes the demand curve into account in the deter-

mination of optimal employment level. In the above equation, $p_i = p(l; \psi)$ holds.

A.2 Derivation of the wage and labor demand curves

The value of an operating firm with type ψ at time t is

$$J_t(l, \psi) = \max_v \left[R_t(l; \psi) - w_t(l; \psi)l - \gamma_t v - I_t + \left(\frac{1 - \delta}{1 + r} \right) J_{t+1}(l', \psi) \right] \quad (\text{A.4})$$

$$\text{subject to } l' = (1 - \lambda)l + q(\theta)v,$$

where $R_t(l; \psi) = [p(l; \psi)/P]a_t\psi l$ and $p(l; \psi)/P = (a_t\psi l_t/[Y_t/n])^{-1/\sigma}$. The first order condition yields

$$\frac{\gamma_t}{q(\theta)} = \frac{1 - \delta}{1 + r} \frac{\partial J_{t+1}(l'; \psi)}{\partial l'}. \quad (\text{A.5})$$

The envelope condition is

$$\frac{\partial J_t(l; \psi, 0)}{\partial l} = \frac{\sigma - 1}{\sigma} \frac{R_t(l; \psi)}{l} - \frac{\partial}{\partial l} [w_t(l; \psi)l] + \frac{1 - s}{1 + r} \frac{\partial J_{t+1}(l'; \psi)}{\partial l'}, \quad (\text{A.6})$$

where $1 - s = (1 - \lambda)(1 - \delta)$. By using these two conditions, I get the Euler equation for employment:

$$\frac{\gamma_{t-1}}{q(\theta)} \left(\frac{1 - \delta}{1 + r} \right)^{-1} = \frac{\sigma - 1}{\sigma} \frac{R_t(l; \psi)}{l} - \frac{\partial}{\partial l} [w_t(l; \psi)l] + \frac{\gamma_t}{q(\theta)} (1 - \lambda). \quad (\text{A.7})$$

For determining wages, the surplus sharing rule leads to $\beta \partial J_t(l; \psi) / \partial l = (1 - \beta) [E_t(l; \psi) - U_t] \Leftrightarrow$

$$\begin{aligned}
& \beta \left[\frac{\sigma - 1}{\sigma} \frac{R_t(l; \psi)}{l} - \frac{\partial w_t(l; \psi)}{\partial l} l - w_t(l; \psi) + \frac{1 - s}{1 + r} \frac{\partial J_{t+1}(l'; \psi)}{\partial l'} \right] \\
& = (1 - \beta) \left[w_t(l; \psi) - z_t + \frac{1}{1 + r} \left[\begin{array}{c} (1 - s)[E_{t+1}(l'; \psi) - U_{t+1}] \\ -\theta q(\theta) [\tilde{E}_{t+1} - U_{t+1}] \end{array} \right] \right].
\end{aligned} \tag{A.8}$$

Because Eq. (A.5) holds for any firm, the below equation also holds for any employed worker.

$$\begin{aligned}
& \beta \frac{\gamma_t}{q(\theta)} \left(\frac{1 - \delta}{1 + r} \right)^{-1} = (1 - \beta) [E_{t+1}(l'; \psi) - U_t] \\
& \Leftrightarrow E_{t+1}(l'; \psi) - U_t = \frac{\beta}{1 - \beta} \frac{\gamma_t}{q(\theta)} \left(\frac{1 - \delta}{1 + r} \right)^{-1},
\end{aligned}$$

which can satisfy

$$\tilde{E}_{t+1} - U_t = \frac{\beta}{1 - \beta} \frac{\gamma_t}{q(\theta)} \left(\frac{1 - \delta}{1 + r} \right)^{-1}. \tag{A.9}$$

Plugging this equation and $\beta \partial J_{t+1}(l'; \psi) / \partial l' = (1 - \beta) [E_{t+1}(l'; \psi) - U_{t+1}]$, Eq. (A.8) leads to

$$\begin{aligned}
& \beta \left[\frac{\sigma - 1}{\sigma} \frac{R_t(l; \psi)}{l} - \frac{\partial w_t(l; \psi)}{\partial l} l - w_t(l; \psi) \right] \\
& = (1 - \beta) \left[w_t(l; \psi) - z_t + \frac{\beta}{1 - \beta} \frac{\theta \gamma_t}{1 - \delta} \right] \\
& \Leftrightarrow \frac{\partial w_t(l; \psi)}{\partial l} + \frac{1}{\beta l} w_t(l; \psi) = \frac{1}{l} \left[\frac{\sigma - 1}{\sigma} \frac{R_t(l; \psi)}{l} + \frac{1 - \beta}{\beta} z_t + \frac{\theta \gamma_t}{1 - \delta} \right].
\end{aligned} \tag{A.10}$$

Then, I have the following wage function:

$$\begin{aligned}
w_t(l; \psi) &= l^{-\frac{1}{\beta}} \int_0^l \left[\frac{\sigma - 1}{\sigma} \frac{R_t(x; \psi)}{x} + \frac{1 - \beta}{\beta} z_t + \frac{\theta \gamma_t}{1 - \delta} \right] x^{\frac{1}{\beta} - 1} dx \\
&= (1 - \beta) z_t + \frac{\beta \theta \gamma_t}{1 - \delta} + \frac{\sigma - 1}{\sigma - \beta} \frac{\beta R_t(l; \psi)}{l},
\end{aligned} \tag{A.11}$$

where the first equality uses the below lemma.

Lemma 15 $w(l)$ and $\kappa(l)$ are defined as functions of l and k is defined as a constant value. Consider the following equation:

$$\frac{\partial w(l)}{\partial l} + \frac{\kappa}{l} w(l) = \kappa(l). \tag{A.12}$$

Then, the solution of this differential equation is

$$w(l) = l^{-\kappa} \int_0^l \kappa(x) x^{\kappa} dx. \tag{A.13}$$

Proof. By multiplying both sides by $\exp\left(\int_{\kappa_2}^l (\kappa/x) dx\right)$, Eq. (A.12) leads to

$$\left(\frac{\partial w(l)}{\partial l} + \frac{\kappa}{l} w(l) \right) \exp\left(\int_{\kappa_2}^l \frac{\kappa}{x} dx\right) = \kappa(l) \exp\left(\int_{\kappa_2}^l \frac{\kappa}{x} dx\right).$$

The left hand side is equal to $d\left[w(l) \exp\left(\int_{\kappa_2}^l (\kappa/x) dx\right)\right] / dl$ where $\exp\left(\int_{\kappa_2}^l (\kappa/x) dx\right) = l^{\kappa} \kappa_2^{-\kappa}$ and κ_2 denotes a constant value. By integrating both sides in the above equation, I obtain

$$\begin{aligned}
w(l) l^{\kappa} \kappa_2^{-\kappa} &= \int_0^l \kappa(x) x^{\kappa} \kappa_2^{-\kappa} dx \\
\Leftrightarrow w(l) &= l^{-\kappa} \int_0^l \kappa(x) x^{\kappa} dx.
\end{aligned}$$

■

Finally, the wage and labor demand curves are given by using Eqs. (A.7) and (A.11). For the labor demand curve, Eq. (2.8) can be written as

$$\tilde{\psi} = \frac{R(\tilde{\psi})}{l(\tilde{\psi})} = \frac{\sigma - \beta}{\sigma - 1} \left[w + (\Lambda_g^{-1} - 1 + \lambda) \frac{\gamma}{q(\theta)} \right],$$

where $(\sigma - \beta)/(\sigma - 1)$ represents the markup under the bargaining.

A.3 Derivation of the free entry condition

To get Eq. (2.11), I am going to show

$$J(0, \tilde{\psi}) = \left[1 + \sum_{t=1}^{\infty} \Lambda_g^t \right] \frac{\sigma - 1}{\alpha + 1 - \sigma} I. \quad (\text{A.14})$$

It is convenient to firstly solve the normalized value $J(\psi) = J(l(\psi), \psi) = J_t(l(\psi), \psi)/a_t$ in which a firm with type ψ keeps the optimal level of employment $l(\psi)$. This value is given by

$$J(\psi) = R(\psi) - wl(\psi) - \gamma v - I + \Lambda_g J(\psi),$$

where $R(\psi)$ also represents its optimal level and $v = \lambda l(\psi)/q(\theta)$ derived from $l(\psi) = (1 - \lambda)l(\psi) + q(\theta)v$. Plugging the wage function into the above, I have

$$\Lambda_g J(\psi) = \frac{\Lambda_g}{1 - \Lambda_g} \left[\frac{1 - \beta}{\sigma - \beta} R(\psi) - I \right] + \frac{\gamma}{q(\theta)} l(\psi). \quad (\text{A.15})$$

While enter the market, each firm posts vacancies in order to start production from the next period, the value of a such firm with type ψ in the entry period is given as

$$\begin{aligned} J(0, \psi) &= -\frac{\gamma}{q(\theta)} l(\psi) - I + \Lambda_g J(\psi) \\ &= \frac{\Lambda_g}{1 - \Lambda_g} \left[\frac{1 - \beta}{\sigma - \beta} R(\psi) - I \right] - I. \end{aligned} \quad (\text{A.16})$$

To employ one worker, a firm needs to post $1/q(\psi)$ vacancies in the frictional labor market; it costs $\gamma/q(\psi)$.

The zero cutoff profit condition leads to

$$J(0, \psi^*) = 0$$

$$\Leftrightarrow \frac{\Lambda_g}{1 - \Lambda_g} \left[\frac{1 - \beta}{\sigma - \beta} R(\psi^*) - I \right] = I.$$

From this condition, the average revenue $R(\tilde{\psi}) = \tilde{\psi}l(\tilde{\psi})$ is also pinned down through the relationship $R(\tilde{\psi})/R(\psi^*) = (\tilde{\psi}/\psi^*)^{\sigma-1}$. In addition, by using the definition of the average productivity level, i.e., $P = p(l(\tilde{\psi}); \tilde{\psi})$, the value $(\tilde{\psi}/\psi^*)^{\sigma-1}$ can be solved as being equal to $\alpha/(\alpha + 1 - \sigma)$. In the end, $J(0, \tilde{\psi})$ is derived as in Eq. (A.14).

Appendix B

Appendix of Chapter 3

B.1 First order and envelope conditions of firms

By using (3.6), the maximization problem of an obsolete firm becomes:

$$J_t^o(l; \psi, t - \tau) = \max_v \left[\begin{array}{l} \frac{p(l; \psi, t - \tau)}{P} a_\tau \psi l - w_t^o(l; \psi, t - \tau) l - \gamma_t v \\ + \left(\frac{1 - \delta}{1 + r} \right) J_{t+1}^o(l'; \psi, t + 1 - \tau), 0 \end{array} \right],$$

subject to $p(l; \psi, t - \tau)/P = [a_\tau \psi l / (Y_t/n)]^{-\frac{1}{\sigma}}$ and $l' = (1 - \lambda)l + q(\theta)v$. Thus, the first order condition is given as:

$$\frac{\gamma_t}{q(\theta)} = \frac{1 - \delta}{1 + r} \frac{\partial J_{t+1}^o(l'; \psi, t + 1 - \tau)}{\partial l}. \quad (\text{B.1})$$

The envelope condition yields:

$$\begin{aligned} & \frac{\partial J_t^o(l; \psi, t - \tau)}{\partial l} \\ &= \frac{\sigma - 1}{\sigma} \frac{p(l; \psi, t - \tau)}{P} a_\tau \psi - \frac{\partial w_t^o(l; \psi, t - \tau)}{\partial l} l - w_t^o(l; \psi, t - \tau) + \frac{(1 - \lambda)\gamma_t}{q(\theta)} \end{aligned} \quad (\text{B.2})$$

where the last term of the right hand side is given by (B.1).

On the other hand, by using (3.7), the maximization problem of an update firm becomes:

$$J_t(l; \psi) = \max_v \left[\frac{p(l; \psi, 0)}{P} a_t \psi l - w_t(l; \psi) l - \gamma_t v - I_t + \left(\frac{1 - \delta}{1 + r} \right) J_{t+1}(l'; \psi) \right], \quad (\text{B.3})$$

subject to the same equations. Similarly, I have the first order and envelope conditions such that:

$$\frac{\gamma_t}{q(\theta)} = \frac{1 - \delta}{1 + r} \frac{\partial J_{t+1}(l'; \psi)}{\partial l}, \quad (\text{B.4})$$

and

$$\frac{\partial J_t(l; \psi)}{\partial l} = \frac{\sigma - 1}{\sigma} \frac{p(l; \psi, 0)}{P} a_t \psi - \frac{\partial w_t(l; \psi)}{\partial l} l - w_t(l; \psi) + \frac{(1 - \lambda) \gamma_t}{q(\theta)}. \quad (\text{B.5})$$

B.2 Wage determination

By using (3.9), (3.10), and (B.2), (3.11) can be written as follows:

$$\begin{aligned} & \beta \left[\frac{\sigma - 1}{\sigma} \frac{p(l; \psi, t - \tau)}{P} a_\tau \psi - \frac{\partial w_t^\circ(l; \psi, t - \tau)}{\partial l} l - w_t^\circ(l; \psi, t - \tau) + \frac{(1 - \lambda) \gamma_t}{q(\theta)} \right] \\ & = (1 - \beta) \left[\begin{array}{c} w_t^\circ(l; \psi, t - \tau) - z_t \\ + \frac{1}{1+r} [(1 - s - \theta q(\theta)) (E_{t+1}^\circ(l'; \psi, t + 1 - \tau) - U_{t+1})] \end{array} \right], \quad (\text{B.6}) \end{aligned}$$

where $\tilde{E}_{t+1} = E_{t+1}^\circ(l'; \psi, t + 1 - \tau)$ since $\beta [\partial J_{t+1}^\circ(l'; \psi, t + 1 - \tau) / \partial l] = (1 - \beta) [E_{t+1}^\circ(l'; \psi, t + 1 - \tau) - U_{t+1}]$ holds in the next period. By using (B.1), the following expression also holds.

$$\frac{\beta}{1 - \beta} \frac{\gamma_t}{q(\theta)} \left(\frac{1 - \delta}{1 + r} \right)^{-1} = E_{t+1}^\circ(l'; \psi, t + 1 - \tau) - U_{t+1}. \quad (\text{B.7})$$

Note that $E_{t+1}^\circ(l'; \psi, t + 1 - \tau)$ is pinned down by not private variables but the aggregate variable θ . Thus, by combining (B.6) and (B.7), I get

$$\frac{\partial w_t^o(l; \psi, t - \tau)}{\partial l} + \frac{1}{\beta l} w_t^o(l; \psi, t - \tau) = \frac{1}{l} \left[\frac{\sigma - 1}{\sigma} \frac{p(l; \psi, t - \tau)}{P} a_\tau \psi + \frac{1 - \beta}{\beta} z_t + \frac{\theta \gamma_t}{1 - \delta} \right]. \quad (\text{B.8})$$

Together with the mentioned lemma in Chapter 2 and the inverse demand function, the wage function takes the form:

$$\begin{aligned} w_t^o(l; \psi, t - \tau) &= l^{-\frac{1}{\beta}} \int_0^l \left[\frac{\sigma - 1}{\sigma} \frac{p(x; \psi, t - \tau)}{P} a_\tau \psi + \frac{1 - \beta}{\beta} z_t + \frac{\theta \gamma_t}{1 - \delta} \right] x^{\frac{1}{\beta} - 1} dx \\ &= (1 - \beta) \left[z_t + \frac{\beta \theta \gamma_t}{(1 - \beta)(1 - \delta)} \right] + \beta \left[\frac{\sigma - 1}{\sigma - \beta} \frac{p(l; \psi, t - \tau)}{P} a_\tau \psi \right]. \end{aligned} \quad (\text{B.9})$$

By plugging $z_t = a_t z$ and $\gamma_t = a_t \gamma$ into this equation, the wage equation is obtained. Divided by a_t , this leads to (3.13).

B.3 Derivation of the average productivity levels

By using (3.24), (3.22) leads to:

$$\int_{\psi_o^*}^{\infty} (p(\psi, 0))^{1 - \sigma} \frac{f(\psi)}{1 - F(\psi_o^*)} d\psi = p(\tilde{\psi}_u, 0)^{1 - \sigma}. \quad (\text{B.10})$$

Because $p(\psi, 0)/p(\tilde{\psi}_u, 0) = (\psi/\tilde{\psi}_u)^{-1}$ and $F(\psi) = 1 - (\psi_{\min}/\psi)^\alpha$, this equation also gives:

$$\begin{aligned} \tilde{\psi}_u^{\sigma - 1} &= (\psi_o^*)^\alpha \alpha \int_{\psi_o^*}^{\infty} \psi^{\sigma - 1} \psi^{-\alpha - 1} d\psi \\ &= \frac{\alpha}{\alpha + 1 - \sigma} (\psi_o^*)^{\sigma - 1}. \end{aligned} \quad (\text{B.11})$$

On the other hand, by using (3.24) and $h(x|x \leq T) = \delta(1 - \delta)^x/[1 - (1 - \delta)^{T+1}]$, (3.23) leads to:

$$\begin{aligned}
& \frac{\delta}{1 - (1 - \delta)^{T+1}} \int_{\psi^*}^{\psi_o^*} \sum_{x=0}^T (p(\psi, x))^{1-\sigma} (1 - \delta)^x \frac{f(\psi)}{F(\psi_o^*) - F(\psi^*)} d\psi \quad (\text{B.12}) \\
&= \frac{\delta}{1 - (1 - \delta)^{T+1}} \int_{\psi^*}^{\psi_o^*} p(\psi, 0)^{1-\sigma} \sum_{x=0}^T [\tilde{g}^{-1}(1 - \delta)]^x \frac{f(\psi)}{F(\psi_o^*) - F(\psi^*)} d\psi \\
&= \frac{\delta}{1 - (1 - \delta)^{T+1}} \frac{1 - [\tilde{g}^{-1}(1 - \delta)]^{T+1}}{1 - \tilde{g}^{-1}(1 - \delta)} \int_{\psi^*}^{\psi_o^*} p(\psi, 0)^{1-\sigma} \frac{f(\psi)}{F(\psi_o^*) - F(\psi^*)} d\psi \\
&= p(\tilde{\psi}_o, \tilde{x})^{1-\sigma},
\end{aligned}$$

where the first equality holds since $p(\psi, x)^{1-\sigma}/p(\psi, 0)^{1-\sigma} = \tilde{g}^{-x}$. Similarly, this equation gives:

$$\begin{aligned}
\tilde{\psi}_o^{\sigma-1} \tilde{g}^{-\tilde{x}} &= \frac{\delta}{1 - (1 - \delta)^{T+1}} \frac{1 - [\tilde{g}^{-1}(1 - \delta)]^{T+1}}{1 - \tilde{g}^{-1}(1 - \delta)} \alpha \int_{\psi_o^*}^{\infty} \psi^{\sigma-1} \frac{\psi^{-\alpha-1}}{\left(\frac{1}{\psi^*}\right)^\alpha - \left(\frac{1}{\psi_o^*}\right)^\alpha} d\psi \\
&= \frac{\delta}{1 - (1 - \delta)^{T+1}} \frac{1 - [\tilde{g}^{-1}(1 - \delta)]^{T+1}}{1 - \tilde{g}^{-1}(1 - \delta)} \tilde{\psi}_u^{\sigma-1} \left[\frac{\left(\frac{\psi_o^*}{\psi^*}\right)^{\alpha+1-\sigma} - 1}{\left(\frac{\psi_o^*}{\psi^*}\right)^\alpha - 1} \right], \quad (\text{B.13})
\end{aligned}$$

where the second equality uses (B.11).

B.4 Derivation of the values of firms

From (3.7), the value of an update firm is given as:

$$J(\psi) = \frac{p(\psi, 0)}{P} \psi l(\psi, 0) - w(l(\psi, 0); \psi) l(\psi, 0) - \gamma \frac{\lambda l(\psi, 0)}{q(\theta)} - I + \Lambda_g J(\psi), \quad (\text{B.14})$$

where $J(\psi) = J_t(l(\psi, 0); \psi)/a_t$ and $v = \lambda l(\psi, 0)/q(\theta)$. By using (3.15) and (3.17), $w(l(\psi, 0); \psi)$ and $\omega(\theta)$ are replaced to get:

$$\begin{aligned}
[1 - \Lambda_g] J(\psi) &= \left(1 - \frac{\sigma - 1}{\sigma - \beta}\right) \frac{p(\psi, 0)}{P} \psi l(\psi, 0) + [\Lambda_g^{-1} - 1] \frac{\gamma l(\psi, 0)}{q(\theta)} - I, \quad (\text{B.15}) \\
&= \left[\frac{1 - \beta}{\sigma - \beta} \frac{p(\psi, 0)}{P} \psi + [\Lambda_g^{-1} - 1] \frac{\gamma}{q(\theta)} \right] l(\psi, 0) - I.
\end{aligned}$$

On the other hand, from (3.6), the value of an obsolete firm at time τ created at date τ is:

$$\begin{aligned}
J^o(\psi, 0) &= \max_T \left[\sum_{t=\tau}^{\tau+T} \left(\frac{1 - \delta}{1 + r} \right)^{t-\tau} \frac{a_t}{a_\tau} \begin{bmatrix} \frac{p(\psi, t-\tau)}{P} \left(\frac{1}{1+g} \right)^{t-\tau} \psi l(\psi, t-\tau) \\ -w l(\psi, t-\tau) \\ -\frac{\gamma}{q(\theta)} \begin{bmatrix} l(\psi, t+1-\tau) \\ -(1-\lambda) l(\psi, t-\tau) \end{bmatrix} \end{bmatrix} \right] \quad (\text{B.16}) \\
&= \max_T \left[\sum_{t=\tau}^{\tau+T} \Lambda_g^{t-\tau} \left[+ \left[\Lambda_g^{-1} - \frac{l(\psi, t+1-\tau)}{l(\psi, t-\tau)} \right] \frac{\gamma}{q(\theta)} \right] l(\psi, t-\tau) \right],
\end{aligned}$$

where $J^o(\psi, x) = J_t^o(l(\psi, x); \psi, x)/a_t$ and $p(\psi, t-\tau) (1/(1+g))^{t-\tau} = p(\psi, 0)$. Plugging $l(\psi, t-\tau) = l(\psi, 0) \tilde{g}^{-(t-\tau)}$, the equation leads to:

$$\begin{aligned}
J^o(\psi, 0) &= \max_T \left[\sum_{t=\tau}^{\tau+T} (\Lambda_g \tilde{g}^{-1})^{t-\tau} \left[\frac{1 - \beta}{\sigma - \beta} \frac{p(\psi, 0)}{P} \psi + [\Lambda_g^{-1} - \tilde{g}^{-1}] \frac{\gamma}{q(\theta)} \right] l(\psi, 0) \right] \quad (\text{B.17}) \\
&= \frac{1}{1 - \Lambda_g \tilde{g}^{-1}} \left[\frac{1 - \beta}{\sigma - \beta} \frac{p(\psi, 0)}{P} \psi + [\Lambda_g^{-1} - \tilde{g}^{-1}] \frac{\gamma}{q(\theta)} \right] l(\psi, 0),
\end{aligned}$$

where the optimal destruction age, T , is infinite.

B.5 Derivation of the firm entry condition

By using (3.30) and (3.31), the free entry condition leads to:

$$\begin{aligned}
K &= \int_{\psi_o^*}^{\infty} \Pi(\psi) f(\psi) d\psi + \int_{\psi^*}^{\psi_o^*} \Pi^o(\psi) f(\psi) d\psi & (B.18) \\
&= -[1 - F(\psi^*)]I + \int_{\psi_o^*}^{\infty} \frac{\Lambda_g}{1 - \Lambda_g} \left[\frac{1 - \beta}{\sigma - \beta} \left(\frac{\psi}{\psi^*} \right)^{\sigma-1} R(\psi^*, 0) - I \right] f(\psi) d\psi \\
&\quad + \int_{\psi^*}^{\psi_o^*} \frac{\Lambda_g}{1 - \Lambda_g \tilde{g}^{-1}} \frac{1 - \beta}{\sigma - \beta} \left(\frac{\psi}{\psi^*} \right)^{\sigma-1} R(\psi^*, 0) f(\psi) d\psi,
\end{aligned}$$

where $R(\psi, 0) = (\psi/\psi^*)^{\sigma-1} R(\psi^*, 0)$. It is convenient to calculate in advance as follows:

$$\int_{\psi_o^*}^{\infty} \left(\frac{\psi}{\psi^*} \right)^{\sigma-1} f(\psi) d\psi = \frac{\alpha}{\alpha + 1 - \sigma} \left(\frac{\psi_{\min}}{\psi_o^*} \right)^{\alpha} \left(\frac{\psi_o^*}{\psi^*} \right)^{\sigma-1}, \quad (B.19)$$

and

$$\int_{\psi^*}^{\psi_o^*} \left(\frac{\psi}{\psi^*} \right)^{\sigma-1} f(\psi) d\psi = \frac{\alpha}{\alpha + 1 - \sigma} \left[\left(\frac{\psi_{\min}}{\psi^*} \right)^{\alpha} - \left(\frac{\psi_{\min}}{\psi_o^*} \right)^{\alpha} \left(\frac{\psi_o^*}{\psi^*} \right)^{\sigma-1} \right]. \quad (B.20)$$

By plugging these results into (B.18), I get:

$$\begin{aligned}
K &= - \left(\frac{\psi_{\min}}{\psi^*} \right)^{\alpha} I + \frac{\Lambda_g}{1 - \Lambda_g} \left(\frac{\psi_{\min}}{\psi_o^*} \right)^{\alpha} \left[\frac{1 - \beta}{\sigma - \beta} \frac{\alpha}{\alpha + 1 - \sigma} \left(\frac{\psi_o^*}{\psi^*} \right)^{\sigma-1} R(\psi^*, 0) - I \right] \\
&\quad + \frac{\Lambda_g}{1 - \Lambda_g \tilde{g}^{-1}} \frac{1 - \beta}{\sigma - \beta} \frac{\alpha}{\alpha + 1 - \sigma} \left[\left(\frac{\psi_{\min}}{\psi^*} \right)^{\alpha} - \left(\frac{\psi_{\min}}{\psi_o^*} \right)^{\alpha} \left(\frac{\psi_o^*}{\psi^*} \right)^{\sigma-1} \right] R(\psi^*, 0) \\
&= - \left(\frac{\psi_{\min}}{\psi^*} \right)^{\alpha} I - \frac{\Lambda_g}{1 - \Lambda_g} \left(\frac{\psi_{\min}}{\psi_o^*} \right)^{\alpha} I \\
&\quad + \left(\frac{\psi_{\min}}{\psi_o^*} \right)^{\alpha} \frac{\alpha}{\alpha + 1 - \sigma} \left(\frac{\psi_o^*}{\psi^*} \right)^{\sigma-1} \frac{1 - \Lambda_g \tilde{g}^{-1}}{1 - \Lambda_g} I \\
&\quad + \frac{\alpha}{\alpha + 1 - \sigma} \left(\frac{\psi_{\min}}{\psi^*} \right)^{\alpha} \left[1 - \left(\frac{\psi^*}{\psi_o^*} \right)^{\alpha} \left(\frac{\psi_o^*}{\psi^*} \right)^{\sigma-1} \right] I,
\end{aligned}$$

where the second equality uses:

$$\Pi^o(\psi^*) = 0 \Leftrightarrow \frac{1-\beta}{\sigma-\beta} R(\psi_o^*, 0) = \frac{1-\Lambda_g \tilde{g}^{-1}}{\Lambda_g} I. \quad (\text{B.21})$$

The firm entry condition also leads to:

$$\begin{aligned} K &= \left(\frac{\psi_{\min}}{\psi^*} \right)^\alpha I \left[-1 - \frac{\Lambda_g}{1-\Lambda_g} \left(\frac{\psi^*}{\psi_o^*} \right)^\alpha + \left(\frac{\psi^*}{\psi_o^*} \right)^\alpha \frac{\alpha}{\alpha+1-\sigma} \left(\frac{\psi_o^*}{\psi^*} \right)^{\sigma-1} \frac{1-\Lambda_g \tilde{g}^{-1}}{1-\Lambda_g} + \right. \\ &\quad \left. \frac{\alpha}{\alpha+1-\sigma} - \frac{\alpha}{\alpha+1-\sigma} \left(\frac{\psi^*}{\psi_o^*} \right)^\alpha \left(\frac{\psi_o^*}{\psi^*} \right)^{\sigma-1} \right] \\ &= \left(\frac{\psi_{\min}}{\psi^*} \right)^\alpha I \left(\frac{\alpha}{\alpha+1-\sigma} - 1 \right) \left[1 + \frac{\Lambda_g}{1-\Lambda_g} \left(\frac{\psi^*}{\psi_o^*} \right)^\alpha \right], \end{aligned}$$

where the second equality uses the below equation from (3.35):

$$\left(\frac{\psi_o^*}{\psi^*} \right)^{\sigma-1} \left[\frac{1-\Lambda_g \tilde{g}^{-1}}{1-\Lambda_g} - 1 \right] = \frac{\Lambda_g}{1-\Lambda_g}. \quad (\text{B.22})$$

B.6 Derivation of P_u/P

P_u/P is derived by (3.21) so that:

$$\begin{aligned} P^{1-\sigma} &= \frac{1-F(\psi_o^*)}{1-F(\psi^*)} P_u^{1-\sigma} + \frac{F(\psi_o^*)-F(\psi^*)}{1-F(\psi^*)} P_o^{1-\sigma} \quad (\text{B.23}) \\ &\Leftrightarrow \left(\frac{P}{P_u} \right)^{1-\sigma} = \left(\frac{\psi^*}{\psi_o^*} \right)^\alpha + \left[1 - \left(\frac{\psi^*}{\psi_o^*} \right)^\alpha \right] \left(\frac{P_o}{P_u} \right)^{1-\sigma} \\ &\Leftrightarrow \left(\frac{P_u}{P} \right)^{\sigma-1} = \left(\frac{\psi^*}{\psi_o^*} \right)^\alpha + \left[1 - \left(\frac{\psi^*}{\psi_o^*} \right)^\alpha \right] \left(\frac{\tilde{\psi}_o}{\tilde{\psi}_u} \right)^{\sigma-1} \tilde{g}^{-\tilde{x}}. \end{aligned}$$

The second row uses the Pareto distribution, and the third row uses (3.24) and (3.18). By plugging (B.13), this equation can be transformed into:

$$\begin{aligned}
\left(\frac{P_u}{P}\right)^{\sigma-1} &= \left(\frac{\psi^*}{\psi_o^*}\right)^\alpha \left[1 + \kappa_g^{-1} \left[\left(\frac{\psi_o^*}{\psi^*}\right)^{\alpha+1-\sigma} - 1\right]\right] \\
&= (1 - \kappa_g^{-1}) \left(\frac{\psi^*}{\psi_o^*}\right)^\alpha + \kappa_g^{-1} \left(\frac{\psi^*}{\psi_o^*}\right)^{\sigma-1},
\end{aligned} \tag{B.24}$$

where

$$\kappa_g^{-1} = \frac{\delta}{1 - (1 - \delta)^{T+1}} \frac{1 - [\tilde{g}^{-1}(1 - \delta)]^{T+1}}{1 - \tilde{g}^{-1}(1 - \delta)}. \tag{B.25}$$

As T is infinite, κ_g^{-1} becomes $\delta/[1 - \tilde{g}^{-1}(1 - \delta)]$. Thus, $1 - \kappa_g^{-1}$ also leads to $(1 - \tilde{g}^{-1})(1 - \delta)/[1 - \tilde{g}^{-1}(1 - \delta)] = (\psi^*/\psi_o^*)^{\sigma-1}(1 - \delta)/[1 - \tilde{g}^{-1}(1 - \delta)]$ where this equality uses the subsequent outcome in (3.35). The reason of this transformation is to clarify respective effects on Pu/P with easy interpretation. The relative price is given by:

$$\begin{aligned}
\left(\frac{P_u}{P}\right)^{\sigma-1} &= \frac{1}{1 - \tilde{g}^{-1}(1 - \delta)} \left[(1 - \delta) \left(\frac{\psi^*}{\psi_o^*}\right)^{\sigma-1} \left(\frac{\psi^*}{\psi_o^*}\right)^\alpha + \delta \left(\frac{\psi^*}{\psi_o^*}\right)^{\sigma-1} \right] \\
&= \frac{\delta}{1 - \tilde{g}^{-1}(1 - \delta)} \left(\frac{\psi^*}{\psi_o^*}\right)^{\sigma-1} \left[\frac{1 - \delta}{\delta} \left(\frac{\psi^*}{\psi_o^*}\right)^{\sigma-1} + 1 \right].
\end{aligned} \tag{B.26}$$

B.7 The standard capitalization effect

In this appendix, I use a basic search model in which progress is fully disembodied. Then, I compute the effect of technological progress on unemployment. In the literature, this effect, under the basic model with disembodied technological progress, is known as the capitalization effect. The result shows that the capitalization effect is quantitatively almost zero, as obtained in Pissarides and Vallanti (2007).

I use the same notation in the current chapter, except for V_t and J_t . V_t denotes the value of creating a new vacancy at time t . J_t denotes the value of a filled job at time t . The value functions are given as:

$$\begin{aligned}
V_t &= -\gamma_t + \frac{1}{1+r} [q(\theta)J_{t+1} + (1 - \theta q(\theta))V_{t+1}], \\
J_t &= a_t - w_t + \frac{1-\lambda}{1+r} J_{t+1}, \\
E_t &= w_t + \frac{1}{1+r} [(1-\lambda)E_{t+1} + \lambda U_{t+1}], \\
U_t &= z_t + \frac{1}{1+r} [\theta q(\theta)E_{t+1} + (1 - \theta q(\theta))U_{t+1}],
\end{aligned}$$

where the free entry condition, represented as $V_t = 0$, holds.

Wages are determined to satisfy the surplus sharing rule:

$$E_t - U_t = \beta [J_t + E_t - U_t],$$

where the worker's bargaining power is β .

To ensure the existence of a balanced growth equilibrium, I assume that $z_t = a_t z$ and $\gamma_t = a_t \gamma$. From this assumption, wages and the above values grow at rate $g = (a_{t+1} - a_t)/a_t$. As a result, I get the job creation condition:

$$\frac{\gamma}{q(\theta)} \frac{\left(\frac{1+g}{1+r}\right)^{-1} - 1 + \lambda}{1 - \beta} + z + \frac{\beta \theta \gamma}{1 - \beta} = 1.$$

(3.17) in the current chapter reduces to this condition when it is assumed that all firms have the same productivity $\psi = 1$, the product market is competitive $\sigma \rightarrow \infty$ and the explicit firm destruction rate is zero $\delta = 0$. These assumption implies $R(\psi, x)/l(\psi, x) = 1$, which corresponds to the right hand side of the above expression. In this case, the net discount factor is $(1 + g)/(1 + r)$.

I calibrate the parameters as in the body of this chapter. The specification of $q(\theta)$ is $m_0 \theta^{-\eta}$. The firstly-given parameters are $r = 0.05$, $g = 0.02$, $s = 12 \times 0.036$, $\beta = \eta = 0.5$. Then, I obtain $m_0 = 8.4$, $z = 0.70$, $\gamma = 0.37$, from the three targets: $\theta = 0.72$, $\theta q(\theta) = 12 \times 0.594$, $z/w = 0.71$.

The numerical result is that one percent-point increase in the rate of technological progress, from 1.5% to 2.5%, decreases the unemployment rate by 0.007%. This is consistent with Pissarides and Vallanti (2007) and Miyamoto and Takahashi (2011). In addition, if I alternatively use the target $z/w = 0.3$ in the calibration,

the corresponding result is unchanged such that $du/dg = -0.007$.

Bibliography

- [1] Aghion, P., & Howitt, P. (1994). "Growth and unemployment". *Review of Economic Studies*, 61(3), 477-494.
- [2] Anderson, J.E., and van Wincoop, E. (2004) "Trade Costs" *Journal of Economic Literature* 42(3), 691–751.
- [3] Axtell, R. L. (2001). "Zipf distribution of US firm sizes". *Science*, 293(5536), 1818-1820.
- [4] Bartelsman, E. J., & Doms, M. (2000). "Understanding productivity: Lessons from longitudinal microdata". *Journal of Economic literature*, 38(3), 569-594.
- [5] Blanchard, O. (2006). "European unemployment: the evolution of facts and ideas". *Economic policy*, 21(45), 6-59.
- [6] Blanchard, O., & Wolfers, J. (2000). "The role of shocks and institutions in the rise of European unemployment: the aggregate evidence". *Economic Journal*, 110(462), 1-33.
- [7] Bloom, N., Sadun, R., & Van Reenen, J. (2012). "Americans do IT better: US multinationals and the productivity miracle". *American Economic Review*, 102(1), 167-201.
- [8] Caballero, R. J., & Hammour, M. L. (1994). "The cleansing effect of recessions". *American Economic Review* 84(5), 1350–1368.
- [9] Caballero, R. J., & Hammour, M. L. (1996). "On the timing and efficiency of creative destruction". *Quarterly Journal of Economics*, 111(3), 805-852.

- [10] Christopoulou, R., & Vermeulen, P. (2012). "Markups in the Euro area and the US over the period 1981–2004: a comparison of 50 sectors". *Empirical Economics*, 42(1), 53-77.
- [11] Colecchia, A., & Schreyer, P. (2002). "ICT investment and economic growth in the 1990s: is the United States a unique case?: a comparative study of nine OECD countries". *Review of Economic Dynamics*, 5(2), 408-442.
- [12] Cummins, J. G., & Violante, G. L. (2002). "Investment-specific technical change in the United States (1947–2000): Measurement and macroeconomic consequences". *Review of Economic dynamics*, 5(2), 243-284.
- [13] Duernecker, G. (2014). "Technology Adoption, Turbulence, and the Dynamics of Unemployment". *Journal of European Economic Association*, 12(3), 724-754.
- [14] Ebell, M., & Haefke, C. (2009). "Product market deregulation and the US employment miracle". *Review of Economic Dynamics*, 12(3), 479-504.
- [15] Elsby, M. W., & Michaels, R. (2013). "Marginal jobs, heterogeneous firms, and unemployment flows". *American Economic Journal: Macroeconomics*, 5(1), 1-48.
- [16] Felbermayr, G., & Prat, J. (2011). "Product market regulation, firm selection, and unemployment". *Journal of European Economic Association*, 9(2), 278-317.
- [17] Hall, R. E., & Milgrom, P. R. (2008). "The limited influence of unemployment on the wage bargain". *American Economic Review*, 98(4), 1653-1674.
- [18] Hopenhayn, H. A. (1992). "Entry, exit, and firm dynamics in long run equilibrium". *Econometrica*, 60, 1127-1150.
- [19] Hornstein, A., Krusell, P., & Violante, G. L. (2007). "Technology—Policy Interaction in Frictional Labour-Markets". *Review of Economic Studies*, 74(4), 1089-1124.

- [20] Jorgenson, D. W., Ho, M. S., & Stiroh, K. J. (2008). "A retrospective look at the US productivity growth resurgence". *Journal of Economic Perspectives*, 22(1), 3-24.
- [21] Luttmer, Erzo G. J. (2004). "The Size Distribution of Firms in an Economy with Fixed and Entry Costs". Federal Reserve Bank of Minneapolis, Working Paper No. 633.
- [22] Luttmer, E. G. (2007). "Selection, growth, and the size distribution of firms". *Quarterly Journal of Economics*, 122(3), 1103-1144.
- [23] Kudoh, N., Miyamoto, H., & Sasaki, M. (2018). "Employment and hours over the business cycle in a model with search frictions". *Review of Economic Dynamics*, forthcoming.
- [24] Martins, J. O., Scarpetta, S., & Pilat, D. (1996). "Mark-up ratios in manufacturing industries". OECD Economics Dept, Working Paper No. 162.
- [25] Melitz, M. J. (2003). "The impact of trade on intra-industry reallocations and aggregate industry productivity". *Econometrica*, 71(6), 1695-1725.
- [26] Michau, J. B. (2013). "Creative destruction with on-the-job search". *Review of Economic Dynamics*, 16(4), 691-707.
- [27] Miyamoto, H., & Takahashi, Y. (2011). "Productivity growth, on-the-job search, and unemployment". *Journal of Monetary Economics*, 58(6), 666-680.
- [28] Mortensen, D. T., & Pissarides, C. A. (1994). "Job creation and job destruction in the theory of unemployment". *Review of economic studies*, 61(3), 397-415.
- [29] Mortensen, D. T., & Pissarides, C. A. (1998). "Technological progress, job creation, and job destruction". *Review of Economic Dynamics*, 1(4), 733-753.
- [30] Nagypal, E., (2008). "Worker reallocation over the business cycle: The importance of employer-to-employer transitions". Manuscript, Northwestern University.

- [31] Petrongolo, B., & Pissarides, C. A. (2001). "Looking into the black box: A survey of the matching function". *Journal of Economic literature*, 39(2), 390-431.
- [32] Pissarides, C. A. (2000). *Equilibrium unemployment theory*, 2nd ed MIT press.
- [33] Pissarides, C. A. (2009). "The unemployment volatility puzzle: Is wage stickiness the answer?". *Econometrica*, 77(5), 1339-1369.
- [34] Pissarides, C. A., & Vallanti, G. (2007). "The Impact of TFP Growth on Steady-State Unemployment". *International Economic Review*, 48(2), 607-640.
- [35] Postel-Vinay, F. (2002). "The dynamics of technological unemployment". *International Economic Review*, 43(3), 737-760.
- [36] Shimer, R. (2005). "The cyclical behavior of equilibrium unemployment and vacancies". *American economic review*, 25-49.
- [37] Shimer, R. (2012). "Reassessing the ins and outs of unemployment". *Review of Economic Dynamics*, 15(2), 127-148.
- [38] Stole, L. A., & Zwiebel, J. (1996). "Intra-firm bargaining under non-binding contracts". *Review of Economic Studies*, 63(3), 375-410.