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## Thesis Abstract

Thesis topic    Topology and shape optimization theories of linear elastic body and their applications to real-world problems

Name            Wares Chancharoen

## Abstract

Non-parametric optimization uses no geometrical parameters and defines functions representing density or boundary of continua as design variables. In this thesis, we focus on non-parametric formulations of optimal product design problems from the point of view that the degree of freedom for designing shape is theoretically infinite. The mathematical problems examined in this thesis are classified into two types, namely, topology optimization problems and shape optimization problems. The former type concerns with the optimization of structures in order to improve stiffness or other objectives which may be achieved, for instance, by inserting holes to the object being optimized. Shape optimization, on the other hand, uses the boundary of the shape or at least a part of it as the main control. This thesis is dedicated to solve two types of problems in linear elastic body from the view point of engineering application. The first is a topology optimization problem in which the main objective is to increase the strength measure of a linear elastic body. This type of problem has been known as a difficult problem to solve because of the irregularity of cost function representing the strength. The other problem examined in the thesis is about a shape optimization problem in which the vibration mode similar to fish swimming is chosen as the target mode of a linear elastic body. This problem is an attempt to use the linear elastic deformation as an actuator of a machine.

The thesis consists of five chapters whose summaries are given as follows.

Chapter 1 reviews the background of non-parametric optimization problems. The general and well-known problems and case studies when designing an optimized structure are presented in this chapter. Based on previous literature, it appears that the instability of numerical and non-smooth boundary moving are

the major problems in topology and shape optimization problems, respectively. Since the major areas of optimization requires deep understanding of various mathematical background, many practical engineers are faced with the difficulty of obtaining desirable results for the optimized structures, especially with respect to the smoothness of the object's boundaries.

In Chapter 2, abstract optimum design problems are examined and the definitions of technical words, as well as the algorithms to find the optimal solutions for the present problems, are presented. The solution to these optimization problems are then found via the so-called  $H^1$  gradient and  $H^1$  Newton methods after presenting the weak formulations of the associated system of partial differential equations. For clarity and complete understanding of the two methods, detailed algorithms describing these methods are also exhibited in this chapter.

Chapter 3 tackles the background of topology optimization theories of linear elastic problem with particular focus on the methods to solve a mean compliance minimization problem and a strength maximization problem. The main results obtained in this chapter are as follows: firstly, it is shown that the use of Kreisselmeier-Steinhauser (KS) function is efficient in avoiding stress concentration in certain structures in topology optimization problem; secondly, it is found that the computation cost of the proposed  $H^1$  Newton method is more efficient than the conventional  $H^1$  gradient method; and lastly, the mathematical programming language FreeFEM++, in which the computational algorithm was developed, provides a very efficient tool to solve topology optimization problems.

Meanwhile, in Chapter 4, the shape optimization theory about boundary variations are presented. Moreover, a mean compliance minimization problem to a tailoring problem of the frequency response mode of a linear elastic body aiming to apply this formulation to design a fish robot is studied and the shape derivative of its associated cost function is computed. The discussions issued in this chapter can be summarized as follows: firstly, the tailoring problem of the linear elastic problem is investigated according to the frequency response problem. Then, the shape optimization problem is successfully formulated using the function  $\phi$  representing the domain variation of the initial domain and is successfully solved using the  $H^1$  gradient method. In finding the numerical solution of the frequency response problem, it was found that many complex numbers arises during calculations which was carried out successfully using an in-house programming library for Finite Element Method.

Finally, in Chapter 5, conclusions regarding the most important results found

in this thesis are stated. In particular, we conclude that real-world problems such as maximizing the strength of a linear elastic body or developing initial designs for fish robots can be carried out numerically through the ideas of topology and shape optimization. Through these approaches, we found rich applications of mathematical theories in solving engineering problems that models real-world problems. In addition, good understanding of the physical meaning of a mathematical problem and its working equations could greatly help a designer to implement an efficient iterative scheme to numerically solve an optimization problem through a commercial software. In other words, one can effectively design an algorithm to solve an optimization problem by first understanding the appropriate methods to apply in solving such problems.

On the other hand, there are still various problems that should be explored further. In the topology optimization problem presented in Chapter 3, additional experimental validation of those findings are suggested to determine the possibilities of applying the numerical results into practical engineering applications. Meanwhile, we recall that the problem considered in Chapter 4 was far from the real-case. We point out that our main interest is to investigate the possibilities of applying shape optimization methods to solve a tailoring problem of the frequency response mode of a linear elastic body. Because of the limitations of the present study, the proposed numerical method produced a result that is quite different from experimental data, especially with respect to the shape profile of the fish's tail. Hence, further improvements should of course be made in order to obtain a numerical result that agrees with the experimental data for the vibration and swimming modes. These further improvements may be summarized as follows. The fluid-structure interaction between the fluid and the fish's body should be considered in the state determination problem. Therefore, the state determination problem should be reformulated and so, the shape derivative of the cost function should also be recomputed. The changes in the mathematical formulations will obviously result in some modifications of the original  $H^1$  gradient method. As a consequence, of course, the finite element method would become more complicated than the present study and more suitable commercial software such as COMSOL Multiphysics must be used.

Furthermore, it may be reasonable to suppose that the results of this thesis may be used to develop an initial design for a robotic fish whose thrust is produced by the vibration mode of the linear elastic body. For example, the continuum materials of the fish body may be made using polymer or rubber, and an initial shape

design may be optimized using the numerical results found in this study. Afterwards, the actuators can be constructed using piezoelectric or shape memory polymer and be positioned in particular locations in the object's body to excite force which are stimulated via its natural vibration frequency. With that particular design developed based on the numerical findings obtained in this research, it is highly believed that the optimized shape of the fish robot could swim more efficiently than the original design. However, further tests on the experimental design should be carried out in order to validate the numerical and optimization results found in this study.