

Parity-violating gravity and GW170817

Atsushi Nishizawa^{1,*} and Tsutomu Kobayashi^{2,†}

¹*Kobayashi-Maskawa Institute for the Origin of Particles and the Universe,
Nagoya University, Nagoya 464-8602, Japan*

²*Department of Physics, Rikkyo University, Toshima, Tokyo 171-8501, Japan*



(Received 11 September 2018; published 17 December 2018)

We consider gravitational waves (GWs) in generic parity-violating gravity, including recently proposed ghost-free theories with parity violation as well as Chern-Simons (CS) modified gravity, and study the implications of observational constraints from GW170817/GRB 170817A. Whereas GWs propagate at the speed of light, c , in CS gravity, we point out that this is specific to CS gravity and the GW propagation speed deviates from c , in general, in parity-violating gravity. Therefore, contrary to the previous literature in which only CS gravity is studied as a concrete example, we show that GW170817/GRB 170817A can, in fact, be used to limit gravitational parity violation. Our argument implies that the constraint on the propagation speed of GWs can pin down the parity-violating sector, if any, to CS gravity.

DOI: [10.1103/PhysRevD.98.124018](https://doi.org/10.1103/PhysRevD.98.124018)

I. INTRODUCTION

The first detection of gravitational waves (GWs) from two merging black holes, GW150914 [1], has opened a new and intriguing arena for gravitational physics. Direct observation of GWs provides us a window into the regime of strong gravity and the propagating sector of gravity, enabling novel tests of general relativity. More recently, the nearly simultaneous detection of GWs and the gamma-ray burst from the merger of neutron stars, GW170817/GRB 170817A [2,3], gave us an unprecedented opportunity to measure the speed of GWs, c_T at a level of one part in 10^{15} .

The physics of propagation of GWs is simple and clear compared to that of generation. Before the occurrence of GW170817/GRB170817A [2], it had been expected that comparing arrival times between GW from the merger of neutron stars and high-energy photons from a short gamma-ray burst would allow us to measure GW speed precisely [4] and consequently tightly constrain the modification of gravity relevant to the cosmic accelerating expansion [5]. Indeed, the speed bound from GW170817/GRB 170817A constrains modified gravity theories at the precision of 10^{-15} and a large class of theories as alternatives to dark energy have already been almost ruled out [6–14] (see, however, [15]).

In this paper, we further pursue the implications of GW170817/GRB 170817A for modified gravity. Earlier works [6–14] focus only on parity-preserving theories. Based on one specific parity-violating realization called Chern-Simons (CS) gravity [16] (see also [17]), it is argued

that the constraint on c_T places no bounds on gravitational parity violation [18]. In this paper, we revisit this point.

As stated above, gravitational parity violation has been studied mostly through CS gravity as a concrete example, whereas recent developments in modified gravity have revealed that, in fact, one can construct theories of parity-violating gravity other than CS gravity [19]. This leads us to consider a unifying framework to study the propagation of GWs in generic parity-violating gravity. We clarify how special CS gravity is among parity-violating theories, and show that the bound on the speed of GWs yields a stringent constraint on parity violation in theories other than CS gravity.

II. PARITY-VIOLATING GRAVITY

We consider parity-violating gravity whose action is of the form

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} [R + \mathcal{L}_{\text{PV}} + \mathcal{L}_\phi], \quad (1)$$

where R is the Ricci scalar, \mathcal{L}_{PV} is a parity-violating Lagrangian, and \mathcal{L}_ϕ is the Lagrangian for a scalar field ϕ which may be coupled nonminimally to gravity.

The most frequently studied example of parity-violating gravity is CS gravity for which \mathcal{L}_{PV} is given by

$$\mathcal{L}_{\text{PV}} = \mathcal{L}_{\text{CS}} := f(\phi)P, \quad P := \varepsilon^{\mu\nu\rho\sigma} R_{\rho\sigma\alpha\beta} R^{\alpha\beta}{}_{\mu\nu}, \quad (2)$$

where $\varepsilon^{\mu\nu\rho\sigma}$ is the Levi-Civita tensor defined as $\varepsilon^{\mu\nu\rho\sigma} := \epsilon^{\mu\nu\rho\sigma} / \sqrt{-g}$ with $\epsilon^{\mu\nu\rho\sigma}$ being the antisymmetric symbol. The Pontryagin term P is a topological invariant in four

*anishi@kmi.nagoya-u.ac.jp

†tsutomu@rikkyo.ac.jp

dimensions, and for this reason we need the dynamical scalar field ϕ coupled to P via $f(\phi)$ in order for this term to contribute to the field equations. The kinetic term for ϕ is supposed to be included in \mathcal{L}_ϕ and one of the simplest possibilities is $\mathcal{L}_\phi = -(\partial\phi)^2/2 - V(\phi)$.

Since CS gravity has higher-derivative field equations as explicitly confirmed by varying the action with respect to the metric [16,20], one expects that dangerous Ostrogradsky ghosts appear in this theory. This is indeed true, as can be seen directly, e.g., from a wrong sign kinetic term in the quadratic action for perturbations around a spherically symmetric background [21] (see also [22]). This conclusion is supported by the Hamiltonian analysis performed in [19]. The ghost degrees of freedom might not be problematic if the theory (1) is treated as a low-energy truncation of a fundamental theory, but they do cause instabilities if regarded as a complete theory. Note that at least in the unitary gauge in which ϕ is homogeneous on $t = \text{const}$ hypersurfaces, CS gravity is ghost-free [19].

Recently, ghost-free parity-violating theories of gravity have been explored in [19]. At least, in the unitary gauge, it is found that one can indeed construct Ostrogradsky-stable theories other than CS gravity. One of the theories proposed in [19] is given by the following Lagrangian:

$$\mathcal{L}_{\text{PV1}} = \sum_{A=1}^4 a_A(\phi, \phi_\mu \phi^\mu) L_A, \quad (3)$$

with

$$L_1 := \varepsilon^{\mu\nu\alpha\beta} R_{\alpha\beta\rho\sigma} R_{\mu\nu}{}^\rho{}_\lambda \phi^\sigma \phi^\lambda, \quad (4)$$

$$L_2 := \varepsilon^{\mu\nu\alpha\beta} R_{\alpha\beta\rho\sigma} R_{\mu\lambda}{}^{\rho\sigma} \phi_\nu \phi^\lambda, \quad (5)$$

$$L_3 := \varepsilon^{\mu\nu\alpha\beta} R_{\alpha\beta\rho\sigma} R^\sigma{}_\nu \phi^\rho \phi_\mu, \quad (6)$$

$$L_4 := \phi_\lambda \phi^\lambda P, \quad (7)$$

where $\phi_\mu := \nabla_\mu \phi$. In order to remove the Ostrogradsky modes, it is required that $4a_1 + 2a_2 + a_3 + 8a_4 = 0$. Similarly, another ghost-free, parity-violating theory found in [19] contains second derivatives of the scalar field, $\phi_\nu^\mu := \nabla^\mu \nabla_\nu \phi$, and is described by the Lagrangian of the form

$$\mathcal{L}_{\text{PV2}} = b_1(\phi, \phi_\lambda \phi^\lambda) \varepsilon^{\mu\nu\alpha\beta} R_{\alpha\beta\rho\sigma} \phi^\rho \phi_\mu \phi_\nu^\sigma + \dots, \quad (8)$$

though its explicit expression is not important in this paper.

The purpose of the present paper is to study the propagation of GWs in such parity-violating theories of gravity. Let us consider GWs propagating on a homogeneous and isotropic background. The spatial metric is written as $g_{ij} = a^2(t)[\delta_{ij} + h_{ij}(t, \vec{x})]$ with the scale

factor $a(t)$. To derive the evolution equation for h_{ij} , we substitute the perturbed metric to Eq. (1) and expand it to second order in h_{ij} . Let us focus on \mathcal{L}_{PV1} for the moment. After some manipulation, we find¹

$$\mathcal{L}_{\text{PV1}}^{(2)} \supset \varepsilon^{ijk} \dot{h}_{il} \partial_j \dot{h}_{kl}, \quad \varepsilon^{ijk} \partial^2 h_{il} \partial_j \dot{h}_{kl}, \quad (9)$$

where ε^{ijk} is the antisymmetric symbol and the coefficients of these terms depends on time in general. In the language of the Arnowitt-Deser-Misner (ADM) formalism, the two terms come, respectively, from $\varepsilon^{ijk} K_{il} D_j K_k^l$ and $\varepsilon^{ijk} R_{il}^{(3)} D_j K_k^l$, where K_{ij} and $R_{ij}^{(3)}$ are the extrinsic and intrinsic curvature tensors of the spatial hypersurfaces and D_i is the three-dimensional covariant derivative. The second term in Eq. (9) can be recast in $\varepsilon^{ijk} \partial^2 h_{il} \partial_j h_{kl}$ by performing integration by parts. Therefore, the final form of the quadratic action for h_{ij} is of the form

$$S^{(2)} = \frac{1}{16\pi G} \int dt d^3x a^3 [\mathcal{L}_{\text{GR}}^{(2)} + \mathcal{L}_{\text{PV}}^{(2)}], \quad (10)$$

where

$$\mathcal{L}_{\text{GR}}^{(2)} = \frac{1}{4} [\dot{h}_{ij}^2 - a^{-2} (\partial_k h_{ij})^2] \quad (11)$$

is the standard Lagrangian obtained from the Einstein-Hilbert term R and

$$\mathcal{L}_{\text{PV}}^{(2)} = \frac{1}{4} \left[\frac{\alpha(t)}{a\Lambda} \varepsilon^{ijk} \dot{h}_{il} \partial_j \dot{h}_{kl} + \frac{\beta(t)}{a^3 \Lambda} \varepsilon^{ijk} \partial^2 h_{il} \partial_j h_{kl} \right] \quad (12)$$

is the Lagrangian signaling parity violation. Here α and β are dimensionless functions of time and Λ is some energy scale. Note that α and β are independent in general. At least either of α and β is taken to be an $\mathcal{O}(1)$ quantity by rescaling Λ . We obtain only the first term in Eq. (9) if we start from \mathcal{L}_{PV2} . Therefore, the Lagrangian (12) contains \mathcal{L}_{PV2} as the special case with $\beta(t) = 0$. By expanding \mathcal{L}_{CS} one sees that CS gravity corresponds to the special case of the above general action satisfying

$$\alpha(t) = \beta(t). \quad (13)$$

(See, e.g., [23].) Thus, the quadratic action (10) with (11) and (12) offers us a unifying framework to study the propagation of GWs in parity violating theories of gravity described above. The concrete forms of $\alpha(t)$ and $\beta(t)$ depend on the background cosmological evolution of $a(t)$ and $\phi(t)$ as well as the theory under consideration.

¹Hereafter, we do not discriminate the upper and lower spatial indices because they are interchanged by δ_{ij} and δ^{ij} .

Note that $\alpha(t) = \beta(t)$ could in principle occur even in the \mathcal{L}_{PV1} theory. However, an extreme fine-tuning of the time-dependent functions is required in the \mathcal{L}_{PV1} theory, while Eq. (13) is automatically satisfied in CS gravity.

The Lagrangian for CS gravity is sometimes expressed using some length scale ℓ_{CS} and the dimensionless scalar field ϑ as $\mathcal{L}_{\text{CS}} \sim (\ell_{\text{CS}}^2 \vartheta/4)P$, and ℓ_{CS} is often denoted as $\xi^{1/4}$. This notation can be converted to ours as $\alpha/\Lambda = \beta/\Lambda \sim \ell_{\text{CS}}^2 \vartheta/4$ (ignoring the cosmic expansion).

Before proceeding, let us give two comments on possible further generalization of the above framework. The first comment is that one can generalize the standard piece $\mathcal{L}_{\text{GR}}^{(2)}$ to

$$\mathcal{L}_{\text{H}}^{(2)} = \frac{1}{4} [A(t) \dot{h}_{ij}^2 - a^{-2} B(t) (\partial_k h_{ij})^2] \quad (14)$$

by considering, e.g., the Horndeski/generalized Galileon Lagrangian as \mathcal{L}_ϕ [24–26]. However, for the moment, we assume the standard Lagrangian (11) for the parity-preserving part.

The second comment is that Eq. (12) is not only the unifying description of the known parity-violating terms \mathcal{L}_{CS} , \mathcal{L}_{PV1} , and \mathcal{L}_{PV2} , but also the low-energy effective description of generic parity-violating GWs. Indeed, the same quadratic action was derived from the viewpoint of the effective field theory in [27].² In light of this viewpoint, one may, for example, further add terms like $\Lambda^{-5} \epsilon^{ijk} \dot{h}_{il} \partial^2 \partial_j \dot{h}_{kl}$, $\Lambda^{-5} \epsilon^{ijk} \partial^2 h_{il} \partial^2 \partial_j h_{kl}$, \dots , which are suppressed by powers of $1/\Lambda$. The latter was studied in the context of Hořava gravity [28].

III. PROPAGATION OF PARITY-VIOLATING GWS

Varying the action (10) with respect to h_{ij} , we obtain the equation of motion for GWs,

$$h''_{ij} + 2\mathcal{H}h'_{ij} - \partial^2 h_{ij} + \frac{1}{a\Lambda} \epsilon_{ilk} \partial_l [\alpha h''_{jk} + (\mathcal{H}\alpha + \alpha') h'_{jk} - \beta \partial^2 h_{jk}] = 0, \quad (15)$$

where the prime denotes differentiation with respect to the conformal time defined by $d\eta = a^{-1} dt$, and $\mathcal{H} := a'/a$.

We decompose h_{ij} into the circular polarization basis defined by the following linear combination of the standard $+$ and \times polarization basis:

$$e_{ij}^{\text{R}} := \frac{1}{\sqrt{2}} (e_{ij}^+ + ie_{ij}^\times), \quad e_{ij}^{\text{L}} := \frac{1}{\sqrt{2}} (e_{ij}^+ - ie_{ij}^\times). \quad (16)$$

This choice of the polarization basis is convenient for parity-violating GWs because the equations of motion for

²In this paper, the second term in (12) comes from $\epsilon^{ijk} R_{il}^{(3)} D_j K_k^l$ and integration by parts, while in [27] the same term is derived directly from the three-dimensional CS term.

the left and right circular polarizations are decoupled even though the parity-violating terms mix the $+$ and \times polarizations. Performing a Fourier decomposition, we write

$$h_{ij}(\eta, \vec{x}) = \frac{1}{(2\pi)^{3/2}} \sum_{A=\text{R,L}} \int d^3k h_k^A(\eta) e_{ij}^A e^{i\vec{k}\cdot\vec{x}}. \quad (17)$$

Using the identity

$$\epsilon_{ilk} n_l e_{jk}^{\text{R,L}} = i\lambda_{\text{R,L}} e_{ij}^{\text{R,L}}, \quad (18)$$

where $\lambda_{\text{R}} = +1$, $\lambda_{\text{L}} = -1$, and n_l is a unit vector pointing to the direction of propagation, we obtain

$$(1 - \lambda_A \tilde{k}\alpha) (h_k^A)'' + [2 - \lambda_A \tilde{k}(\alpha + \alpha' \mathcal{H}^{-1})] \mathcal{H} (h_k^A)' + (1 - \lambda_A \tilde{k}\beta) k^2 h_k^A = 0, \quad (19)$$

for $A = \text{R}$ and L . Here we defined the dimensionless wave number $\tilde{k} := k/(a\Lambda)$, which controls the magnitude of the corrections to general relativity. This parameter depends on the frequency of GWs, indicating that the parity-violating effect is more efficient at higher frequencies such as LIGO's observation band than at lower frequencies corresponding, e.g., to CMB scales. In order to compare the propagation equation derived above with the general framework of GW propagation [29], we rewrite Eq. (19) in a physically more transparent form as

$$(h_k^A)'' + (2 + \nu_A) \mathcal{H} (h_k^A)' + (c_{\text{T}}^A)^2 k^2 h_k^A = 0, \quad (20)$$

with the additional amplitude damping and the GW propagation speed squared,

$$\nu_A = \frac{\lambda_A \tilde{k}(\alpha - \alpha' \mathcal{H}^{-1})}{1 - \lambda_A \tilde{k}\alpha}, \quad (c_{\text{T}}^A)^2 = \frac{1 - \lambda_A \tilde{k}\beta}{1 - \lambda_A \tilde{k}\alpha}. \quad (21)$$

If one takes $\alpha = \beta$, CS gravity is recovered, in which case we exactly have $c_{\text{T}}^A = 1$ and only the amplitude is modified through ν_A . This agrees with the previous argument [18,30]. *In general parity-violating gravity, however, the propagation speed is also modified.* Note that since the sign of ν_A is determined by λ_A , ν_{R} and ν_{L} always have opposite signs. That is, if the amplitude of one polarization mode is enhanced, the other is suppressed. This is also true for $(c_{\text{T}}^A)^2 - 1$: *If one polarization mode is superluminal, then the other is subluminal.*

IV. OBSERVATIONAL CONSTRAINTS

Assuming that the parity-violating effect is a small correction to general relativity, namely, $\tilde{k} \ll 1$, the observables given in Eq. (21) are

$$\nu_A = \lambda_A \tilde{k}(\alpha - \alpha' \mathcal{H}^{-1}) + \mathcal{O}(\tilde{k}^2), \quad (22)$$

$$(c_{\text{T}}^A)^2 = 1 + \lambda_A \tilde{k}(\alpha - \beta) + \mathcal{O}(\tilde{k}^2). \quad (23)$$

The GW speed has already been measured from the coincident detections of GW170817/GRB 170817A [2,3] and it is constrained so tightly in the range $-7 \times 10^{-16} < 1 - c_T < 3 \times 10^{-15}$. From this and Eq. (23), we have the constraint on parity violating gravity, $\tilde{k}|\alpha - \beta| \lesssim 10^{-15}$. Since the LIGO constraint is for the GW speed at a frequency of $k/a \sim 100$ Hz, this can also be written as

$$\Lambda^{-1}|\alpha - \beta| \lesssim 10^{-11} \text{ km.} \quad (24)$$

This implies that either of the following statements holds in the low-redshift Universe: (i) the parity-violating sector is given by CS gravity; (ii) the parity-violating sector is given by a linear combination of \mathcal{L}_{PV1} and \mathcal{L}_{PV2} with $\alpha - \beta = \mathcal{O}(1)$, and the stringent constraint is obtained as $\Lambda^{-1} \lesssim 10^{-11}$ km; (iii) the parity violating sector is given by a linear combination of \mathcal{L}_{PV1} and \mathcal{L}_{PV2} , and the two time-dependent functions are extremely fine-tuned. Note that in the third case at least \mathcal{L}_{PV1} is necessary because \mathcal{L}_{PV2} generates only the α term. Note also that, of course, one has the freedom to add \mathcal{L}_{CS} in the second and third cases.

So far, we have assumed that the parity-preserving part is described by general relativity. If one generalizes this part to the Horndeski-type Lagrangian (14), ν_A and c_T^A are also affected by this modification. However, the two effects cannot be canceled out because the parity-violating modification depends on λ_A and the wave number, while the Horndeski-type modification does not. The two ways of modifying gravity are thus distinct, and therefore the above statements are robust.

The above constraint should be taken with a caution when one compares it with the existing constraints for the typical length scale Λ^{-1} in literature. The analysis is made basically for CS gravity only, and the constraints are based on the assumptions that the scalar sector is given simply by $\mathcal{L}_\phi = -(\partial\phi)^2/2 - V(\phi)$ and that ϕ has a spacelike gradient, or ϕ is considered to be a nondynamical field having the fixed configuration $f(\phi) \propto \phi = \text{const} \times t$. In the latter case, the bound on the typical length scale is given for example by $\Lambda^{-1} \lesssim 10^{-1}$ km [31], which comes from the double-binary-pulsar observation. This bound can be slightly improved by measuring a GW propagation effect with the future GW detectors [32]. However, these would depend on the form of \mathcal{L}_ϕ as well as the parity-violating sector of gravity. It should be emphasized that our constraint has been derived without assuming any specific form of the scalar-field Lagrangian.

The observational bound (24) constrains the combination $\Lambda^{-1}|\alpha - \beta|$ and we cannot distinguish the two possibilities (ii) and (iii) above. That is, if α and β are extremely fine-tuned, the constraint tells nothing about the energy scale Λ of parity violation. Let us remark that another constraint can possibly come from the measurement of the gravitational constant at a binary pulsar. In modified gravity, the effective gravitational constant for the tensor modes,

G_{GW} , can be different from that of Newtonian gravity, G_N . However, the binary-pulsar constraint from PSR B1913+16 leads to $0.995 \lesssim G_{\text{GW}}/G_N \lesssim 1.00$ [33]. This strongly limits, for instance, the viable scalar-tensor theories satisfying $c_{\text{GW}} = 1$ [34]. In our cases, from the Lagrangians (11) and (12), with $\tilde{k}\alpha$ and $\tilde{k}\beta$ now being the same at the level of 10^{-15} , the effective gravitational constant for the tensor modes is defined by including the additional contribution from the parity-violating terms as

$$G_{\text{GW}}^A = G(1 - \lambda_A \tilde{k}\alpha)^{-1}. \quad (25)$$

Even in the presence of the parity-violating terms, spherically symmetric solutions remain the same as in general relativity [16] if ϕ is minimally coupled to matter and gravity (except for the gravitational parity-violating part). In this case, we may set $G_N = G$. Then, the binary-pulsar constraint is translated to $|\tilde{k}\alpha| \lesssim 5 \times 10^{-3}$, or

$$\Lambda^{-1}|\alpha| \lesssim 10^6 \text{ km,} \quad (26)$$

using the GW frequency of $k/a \sim 4 \times 10^{-4}$ Hz [35]. This indicates that the deviation from general relativity due to the parity violation effect must be smaller than 0.5% in the Lagrangian (10).

V. CONCLUSIONS

We have studied the propagation of gravitational waves (GWs) in Chern-Simons (CS) modified gravity and recently proposed ghost-free theories of parity-violating gravity. Along with this latter extension of gravity, we have found that the propagation speed of GWs is modified in general, together with the amplitude damping, which is modified in CS gravity as well. From the measurement of the GW speed with GW170817/GRB 170817A, we conclude that the possible parity-violating extension of gravity at low redshifts has already been tightly restricted to fine-tuned models or CS gravity. We have not considered any specific Lagrangian for the scalar degree of freedom, \mathcal{L}_ϕ . Our result thus relies only on the propagation of GWs and the assumption that the scalar field ϕ has a timelike gradient, and hence is robust irrespective of this scalar sector (provided that such a scalar-field configuration is allowed).

ACKNOWLEDGMENTS

A.N. was supported by JSPS KAKENHI Grants No. JP17H06358 and No. JP18H04581. T.K. was supported by JSPS KAKENHI Grants No. JP15H05888, No. JP17H06359, No. JP16K17707, No. JP18H04355, and the MEXT-Supported Program for the Strategic Research Foundation at Private Universities, 2014-2018 (S1411024).

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