# A Current Harmonic Minimum PWM for Three Level Converters Aiming at the Low Frequency Fluctuation Minimum of Neutral Point Potential 

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#### Abstract

When the selected harmonic elimination PWM (SHEPWM) is adopted to three level neutral point clamped (TL-NPC) converters to reduce the switching frequency, the neutral point potential (NPP) problem becomes more severe compared with other modulations. It is because there is no degree of freedom for SHEPWM to control the NPP. SHEPWM realizes a better harmonic performance by calculating the switching angles offline based on harmonic requirements. Since it is harsh to solve the angles online, we cannot change the angles to control the NPP. Aiming at the low frequency NPP fluctuation minimum, a novel current harmonic minimum method based on the optimal 3 -order/9-order harmonic is proposed in this paper. The optimal 3 -order/9-order harmonic is derived based on the relationships among the 3-order/ 9 -order harmonic and the NPP. They can be utilized to realize the low frequency NPP fluctuation minimum without changing any switching angle online. Moreover, some comparisons, such as the solution ranges, weighted total harmonic distortion (WTHD), the initial value of iteration calculation, show that the proposed method has a good output performance. Lastly, the experimental results are given to verify the validity of the proposed method.


Index Terms-Low frequency fluctuation, neutral point potential, SHEPWM, three level, 3-order harmonic.

## I. INTRODUCTION

RECENTLY, multilevel converters and the low switching frequency (SF) modulation technologies play the vital roles in medium and high voltage high power applications. In this paper, three level neutral point clamped (TL-NPC) converter, as shown in Fig. 1, is discussed in detail. It is widely used in wind power generation, rail transport, naval propulsion, etc. Compared with two level converters, three level converters can generate more sinusoidal output voltages and reduce the required rated voltages of the switch devices. Furthermore, by

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reducing the $d u / d t$ of the output voltages, the electromagnetic interference (EMI) problem is also improved [1-3].

Pulse width modulation (PWM) is the core technology for power electronics. There are many modulation methods for different applications, such as sinusoidal PWM (SPWM), space vector PWM (SVPWM), square wave modulation, etc [4-6]. Especially since TL-NPC converter is usually utilized in medium and high voltage high power applications, it is vital to decrease the SF for reducing the loss and heat of switch devices. It directly affects the system design. Some low SF modulation methods have been put forward to improve output performance for two level converters, such as intermediate $60^{\circ}$ SPWM [7-9], selected harmonic elimination PWM (SHEPWM) [10-15], optimal PWM [16-18]. SHEPWM can eliminate the selected low-order harmonics by solving the Fourier equations of SHEPWM waveforms based on the harmonic requirements. Optimal PWM has the similar theory as SHEPWM, which tries to make weighted total harmonic distortion (WTHD) minimum. Although these modulation methods can be applied to TL-NPC converters, the neutral point potential (NPP) control becomes difficult.

For TL-NPC converters, the NPP problem is the first issue that should be solved. It can be divided into the NPP drift problem and the low frequency NPP fluctuation problem. The NPP drift problem is derived from the dead time, asymmetrical loads, the inconsistency of switch devices, etc. It may ruin the switch devices and the DC-link capacitors [19-21]. The NPP drift problem under SHEPWM has been analyzed in [22, 23]. The authors balance the NPP drift by revising the switching angles slightly. If the adjustment values of switching angles are


Fig. 1. Three-level NPC converter topology (IGBTs).
large, the modulation will be deteriorated and deviate from the real SHEPWM. If the adjustment values are small, it may not be able to deal with serious NPP drift problem. Furthermore, the method is powerless for the low frequency NPP fluctuation problem. Actually, the problem becomes more and more serious with the SF decreasing. As original from modulation technologies, NPP problem is inevitable for SPWM, SVPWM, SHEPWM and optimal PWM essentially [20].

Under the low SF, SHEPWM and optimal PWM are the mainstream choices for improving the output performances. However, the calculation of the switching angles is too difficult to realize online. The traditional method to realize SHEPWM and optimal PWM is to calculate the switching angles offline and look up the angle tables. Compared with SPWM/SVPWM, the number of switching angles $(N)$ are few. And to ensure the harmonic requirements, the angles cannot be changed for the NPP control. However, the low frequency NPP fluctuation problem makes the capacitors' reduction become impossible, which is key to reduce the weight and volume of converters. The realization of low frequency NPP fluctuation minimum is expected even under SHEPWM and optimal PWM.

A current harmonic minimum PWM (CHMPWM) aiming at the low frequency NPP fluctuation minimum is proposed in this paper. For traditional SHEPWM, the switching angles are calculated offline based on the harmonic requirements and cannot be achieved online. This paper analyzes the essential reasons that cause the low frequency NPP fluctuation under SHEPWM and shows that it evolves fundamental component and $3 n$-order harmonic components. By setting the $3 \& 9$-order harmonic components to optimal values, the low frequency NPP fluctuation minimum can be realized naturally without changing any switching angle online. However, since proposed method sacrifices 2 switching angles to ensure the optimal $3 \& 9$-order harmonic components, it means fewer switching angles can be used to eliminate other low-order harmonics. Thus, the optimal 3\&9-order harmonic components and WTHD are defined as the objective function to realize the current harmonic minimum and low frequency NPP fluctuation minimum simultaneously. It is noted that the calculation of switching angles for proposed method at this time becomes easier than traditional optimal PWM.

The contribution of this paper is to figure out the optimal $3 \& 9$-order harmonic components, which can be used to overcome the low frequency NPP fluctuation problem under SHEPWM or optimal PWM. It is noteworthy that SHEPWM and optimal PWM are not the central issue. The optimal PWM is adopted just for improving the output performance, because the fewer switching angles can be used to eliminate the low-order harmonics.
The section II introduces the basic principles of three level SHEPWM and optimal PWM, the NPP problem and the existing method. The CHMPWM aiming at the low frequency NPP fluctuation minimum is explained in section III. Then, the comparisons on the solution range of switching angles, WTHD and iteration initial value between the traditional SHEPWM and proposed method are carried out in section IV. Finally, some experimental results are shown in section $V$ to verify the


Fig. 2. The SHEPWM waveform (phase $u$ ).
validity and feasibility of proposed method.

## II. The NPP Problem and The Existing Method under Three Level SHEPWM and Optimal PWM

Although $3 n$-order harmonic components have no effect on the output performance in three phase three line systems, they could serve as an important degree of freedom to deal with the NPP problem. As a key point of this paper, the optimal $3 n$-order harmonic components are discussed and utilized to solve the low frequency NPP fluctuation problem.

## A. Three Level SHEPWM and Optimal PWM

Firstly, the basic principles of SHEPWM and optimal PWM are introduced. By calculating the switching angles of PWM waveforms based on the harmonic requirements, the selected low-order harmonics are eliminated. The SHEPWM waveform is shown as Fig. 2, taking phase $u$ as an example.

Equation (1) can be obtained by the Fourier analysis of the SHEPWM waveform, which has the quarter wave symmetry.

$$
\begin{align*}
& f(\omega t)=\sum_{n=1}^{+\infty}\left[a_{n} \sin (n \omega t)+b_{n} \cos (n \omega t)\right] \\
& a_{n}=\frac{2 u_{d c}}{n \pi} \sum_{i=1}^{N}(-1)^{i+1} \cos \left(n \alpha_{i}\right), \quad(n=1,3,5, \cdots)  \tag{1}\\
& a_{n}=0, \quad(n=2,4,6, \cdots) \\
& b_{n}=0, \quad(n=1,2,3, \cdots)
\end{align*}
$$

Assume the fundamental component as $u_{1}$ and define the modulation ratio as $m=u_{1} /\left(2 u_{d c} / \pi\right)$. Thus, the maximum $m$ is $0.866,0.9$ and 1.0 for SPWM without zero-sequence voltage injection, SVPWM and square wave modulation respectively. For SHEPWM, we set the fundamental component to $m$ and set the other low-order harmonic components to zero, as (2).

$$
\begin{align*}
& \left\{\begin{array} { l } 
{ | a _ { 1 } | = m } \\
{ | a _ { h } | = 0 }
\end{array} \Rightarrow \left\{\begin{array}{l}
\sum_{i=1}^{N}(-1)^{i+1} \cos \left(\alpha_{i}\right)=m \\
\sum_{i=1}^{N}(-1)^{i+1} \cos \left(h \alpha_{i}\right) / h=0
\end{array}\right.\right.  \tag{2}\\
& h=6 l \pm 1, \quad l=1,2,3, \cdots,(N-1) / 2
\end{align*}
$$

It is an issue for solving nonlinear equations. Since it is harsh to get the algebraic solutions of (2), some numerical methods, such as newton iteration, homotopy algorithm and genetic algorithm [12-15], are raised to solve switching angles. Thus, the algorithm convergence and initial value are important.

For optimal PWM, we usually define objective function as,

$$
\begin{equation*}
\lambda_{W T H D}=\sqrt{\sum_{h=6 l \pm 1}^{\infty}\left(\frac{I_{h}}{I_{1}}\right)^{2}}=\sqrt{\sum_{h=6 l \pm 1}^{\infty}\left(\frac{u_{h}}{h u_{1}}\right)^{2}}=\frac{1}{m} \sqrt{\sum_{h=6 l \pm 1}^{\infty}\left(\frac{u_{h(p u)}}{h}\right)^{2}} \tag{3}
\end{equation*}
$$

Thus, it becomes the constrained optimization problem for the objective function. It becomes more difficult to calculate


Fig. 3. The method [22, 23] to the NPP drift problem
the switching angles, compared with SHEPWM. As a result, it is usually used when $N$ is few.

## B. The NPP Problem under Three Level SHEPWM

The NPP problem is unavoidable for TL-NPC converters. Ignoring the harmonic components, assume the three phase fundamental voltages and currents as (4). $i_{o}$ can be achieved as (5) [19-21] for nearest three vector PWM (NTV-PWM).

$$
\left\{\begin{array}{l}
u_{u 1}=4 m / \pi \sin (\theta) \\
u_{v 1}=4 m / \pi \sin (\theta-2 \pi / 3),  \tag{5}\\
u_{w 1}=4 m / \pi \sin (\theta+2 \pi / 3)
\end{array},\left\{\begin{array}{l}
i_{u}=I_{m} \sin (\theta-\varphi) \\
i_{v}=I_{m} \sin (\theta-2 \pi / 3-\varphi) \\
i_{w}=I_{m} \sin (\theta+2 \pi / 3-\varphi)
\end{array}\right.\right.
$$

Here, $i_{o}$ is completely determined by the phase voltage and current. In fact, the NPP has a self-balancing characteristic and a low frequency fluctuation characteristic. The NPP is the integration of $i_{o}$. We calculate it from 0 to $2 \pi / 3$, as shown in (6). It is noted that $\mathrm{s}\left(i_{o}\right)$ comes back to zero during each $2 \pi / 3$. In other word, it means the fundamental voltage causes the 3 -order NPP fluctuation during $2 \pi$. However, if the voltages and currents in (4) cannot keep the sinusoidal waveforms due to a long dead time or strong asymmetrical loads, the NPP drift problem appears. At that moment, it is necessary to give some methods to amend the phase voltage for solving the NPP drift problem.

$$
\begin{align*}
s\left(i_{o}\right) & =\int_{0}^{2 \pi / 3}\left(\left(1-\left|u_{u}\right|\right) i_{u}+\left(1-\left|u_{v}\right|\right) i_{v}+\left(1-\left|u_{w}\right|\right) i_{w}\right) d \theta  \tag{6}\\
& =\int_{0}^{2 \pi / 3}\left(-\left|u_{u}\right| i_{u}-\left|u_{v}\right| i_{v}-\left|u_{w}\right| i_{w}\right) d \theta=0
\end{align*}
$$

## C. The Existing Method for The NPP Drift Problem

As mentioned above, the switching angles of SHEPWM and optimal PWM are calculated based on the harmonic requirements offline, owing to the computational complexity. Although to change the switching angles seems a bad choice, there is no other choice to solve the NPP drift problem. Reference [22,23] deals with the NPP drift problem by adjusting the switching angles subtly, as Fig. 3.

Thus, the duty cycle of the voltage (0), which is linked to the NPP, is amended by $\Delta \alpha$. The NPP is also adjusted by $\Delta \alpha$, of which the sign depends on the sign of the phase current and the NPP error $\left(\Delta v_{o}\right) . \Delta \alpha$ is vital for the method. If $\Delta \alpha$ is small, the control ability for the NPP drift is weak. If $\Delta \alpha$ is large, the
output performance will be worsen, since the PWM waveform deviates from real SHEPWM. Switching angles based on the harmonic requirements become meaningless, since switching angles are changed obviously. Due to the limit of $\Delta \alpha$, even if the method [22,23] can solve the NPP drift to some extent, the control ability is so weak that it cannot suppress low frequency NPP fluctuation mentioned above. Since low frequency NPP fluctuation exists under SHEPWM and optimal PWM, it makes the capacitors' reduction become difficult. However, it is key to reduce the weight and volume of the converters.

## III. THE PROPOSED CHMPWM AIMING AT LOW FREQUENCY NPP FLUCTUATION MINIMUM

According to the principle of SHEPWM and optimal PWM, we can know that the optimal harmonic performance is based on the switching angles. However, it is difficult to change these angles for the NPP control online, since they are calculated based on the harmonic requirements offline. Therefore, we must consider the low frequency NPP fluctuation problem before calculating these angles. In this paper, the optimal angles, which generate the smallest low frequency NPP fluctuation, are achieved. Since these angles are not changed online, a good output performance of SHEPWM or optimal PWM is ensured.

The traditional SHEPWM uses (2) to obtain the switching angles. The selected low-order harmonic components can be eliminated. Here, $3 n$-order harmonic components are not considered, since they have no effect on the line voltage for three phase three line systems. However, the $3 n$-order harmonic components are analyzed cautiously in this paper, as (7).

$$
\begin{align*}
& \left\{\begin{array} { l } 
{ | a _ { 1 } | = m } \\
{ | a _ { 3 n } | = k _ { 3 n } m } \\
{ | a _ { h } | = 0 }
\end{array} \Rightarrow \left\{\begin{array}{l}
\sum_{i=1}^{N}(-1)^{i+1} \cos \left(\alpha_{i}\right)=m \\
\sum_{i=1}^{N}(-1)^{i+1} \cos \left(3 n \alpha_{i}\right) / 3 n=k_{3 n} m \\
\sum_{i=1}^{N}(-1)^{i+1} \cos \left(h \alpha_{i}\right) / h=0
\end{array}\right.\right.  \tag{7}\\
& (h=6 l \pm 1, \quad l=1,2,3, \cdots,(N-1-n) / 2)
\end{align*}
$$

For the low frequency NPP fluctuation minimum, the ideal is to search the optimal $3 n$-order harmonic components, which can neutralize the NPP fluctuation from the fundamental component. Thereby, the relationships of $3 n$-order harmonic components and the NPP (or $i_{o}$ ) should be illustrated firstly.

## A. The Relationship Between io and 3-Order Harmonic

In this part, the 3-order harmonic component is discussed at first, which is assumed as (8).

$$
\left\{\begin{array}{l}
u_{u 3}=4 / \pi k_{3} m \sin (3 \theta)  \tag{8}\\
u_{v 3}=4 / \pi k_{3} m \sin (3 \theta-2 \pi) \\
u_{w 3}=4 / \pi k_{3} m \sin (3 \theta+2 \pi)
\end{array}\right.
$$

Substituting (4) and (8) into (5), $i_{o}$ can be written as follow.

$$
\begin{align*}
i_{o}= & -4 / \pi m\left|\sin (\theta)+k_{3} \sin (3 \theta)\right| i_{u} \\
& -4 / \pi m\left|\sin (\theta-2 \pi / 3)+k_{3} \sin (3 \theta)\right| i_{v}  \tag{9}\\
& -4 / \pi m\left|\sin (\theta+2 \pi / 3)+k_{3} \sin (3 \theta)\right| i_{w}
\end{align*}
$$



Fig. 4. The waveforms of $i_{o}, i_{o 1}$ and $i_{o 3}$ under different power factors. (a) $\mathrm{PF}=0.6$. (b) $\mathrm{PF}=0.8$. (c) $\mathrm{PF}=1.0$


Fig. 5. The waveforms of $v_{o}, v_{o 1}$ and $v_{o 3}$ under different power factors. (a) $\mathrm{PF}=0.6$. (b) $\mathrm{PF}=0.8$. (c) $\mathrm{PF}=1.0$

It can be seen from (9) that $i_{o}$ can be adjusted by $k_{3}$. If the optimal $k_{3}$ is found to keep $i_{o}$ minimal, the low frequency NPP fluctuation is also minimal. However, it is difficult to get the optimal $k_{3}$ directly from (9). Especially, if $k_{3}$ changes the sign of the absolute calculation in (9), it becomes harsher to simplify the relationship between $i_{o}$ and $k_{3}$. Thus, the limit range of $k_{3}$, which does not change the sign of the absolute calculation, is discussed at first. Equation (10) should be met to ensure the sign of absolute calculation in (9) is not changed.

$$
\begin{equation*}
-1 / 3<k_{3}<1 \tag{10}
\end{equation*}
$$

When the optimal $k_{3}$ is in the limit range of (10), we can easily simplify (9), since the sign of the absolute calculation does not change. If we divide $2 \pi$ into 6 parts ( $r 1-r 6$ ), the relationship between $i_{o}$ and $k_{3}$ can be achieved as (11).

$$
\begin{aligned}
& i_{o}=i_{o 1}+i_{o 3} \\
& \left\{\begin{array}{l}
i_{o 1}=\frac{4 m I_{m}}{\pi}\left[\frac{(-1)^{r}}{2} \cos (\varphi)+\cos \left(2 \theta-\frac{2 \pi}{3}-\varphi+\frac{\pi}{3} r\right)\right] \\
i_{o 3}=\frac{4 m I_{m}}{\pi}\left[2 k_{3} \sin (3 \theta) \sin \left(\theta-\frac{\pi}{3}-\varphi-\frac{\pi}{3} r\right)\right] \\
r=1,2,3,4,5,6
\end{array}\right.
\end{aligned}
$$

$i_{o 1}$ is the neutral point current generated from the fundamental voltage. $i_{o 3}$ is the neutral point current derived from the 3 -order harmonic voltage. The frequencies of $i_{o l}$ and $i_{o 3}$ are both three times larger than the fundamental frequency. Fig. 4 shows the waveforms of $i_{o}, i_{o 1}$ and $i_{o 3}$ under different power factors $(P F)$. It is noted from Fig. 4 that $i_{o 3}$ is opposite to $i_{o 1}$ on the whole, especially when $P F$ is large. Therefore, if we can set $i_{o 3}$ to a reasonable value, $i_{o 1}$ can be neutralized. As a result, $i_{o}$ and the NPP will become small.

The integration of $i_{o 1}$ and $i_{o 3}$ are solved as (12).

$$
\left\{\begin{array}{l}
s\left(i_{o 1}\right)=\int_{0}^{\pi / 3} i_{o 1}=\frac{4 m I_{m}}{\pi} \int_{0}^{\pi / 3}\left[\frac{-1}{2}+\cos \left(2 \theta-\frac{\pi}{3}\right)\right] d \theta=\frac{4 m I_{m}}{\pi}\left(\frac{\sqrt{3}}{2}-\frac{\pi}{6}\right)  \tag{12}\\
s\left(i_{o 3}\right)=\int_{0}^{\pi / 3} i_{o 3}=\frac{4 m I_{m}}{\pi} 2 k_{3} \int_{0}^{\pi / 3} \sin (3 \theta) \sin \left(\theta-\frac{2 \pi}{3}\right) d \theta=-\frac{4 m I_{m}}{\pi}\left(\frac{3 \sqrt{3}}{4} k_{3}\right)
\end{array}\right.
$$

If $s\left(i_{o 1}\right)$ is equal to $-s\left(i_{o 3}\right)$, the optimal $k_{3}$ can be got as,
$s\left(i_{o 1}\right)=\frac{4 m I_{m}}{\pi}\left(\frac{\sqrt{3}}{2}-\frac{\pi}{6}\right)=-s\left(i_{o 3}\right)=\frac{4 m I_{m}}{\pi}\left(\frac{3 \sqrt{3}}{4} k_{3}\right) \Rightarrow k_{3} \approx 0.2636$
It is noteworthy that the optimal $k_{3}$ of $(13)$ is the optimum not only when the sign of absolute calculation is unchanged, but also when the sign of absolute calculation is changed, which can be verified via computer-solving.

We can obtain the NPP $\left(v_{o}, v_{o 1}, v_{o 3}\right)$ by integrating $i_{o}, i_{o 1}$ and $i_{03}$ via (11), as shown in Fig. 5. Furthermore, define the suppression coefficient $\left(g_{s c}\right)$ of low frequency NPP fluctuation as $v_{o_{-} k 326} / v_{o_{-} k 30}$, where $v_{o_{-} k 326}$ and $v_{o_{-} k 30}$ are the maximum value of the NPP fluctuation when $k_{3}=0.2636$ and 0 respectively.


Fig. 6. The suppression coefficient of low frequency NPP fluctuation.


Thus, the $g_{s c}$ is calculated as Fig. 6. It can be seen from Fig. 6 that low frequency NPP fluctuation is suppressed dramatically, with the $P F$ increasing. On the other hand, since we are not able to ensure that $i_{o 3}$ is opposite to $i_{o l}$ for all the $P F$, it is not effective when the $P F$ is small. However, $i_{o 3}$ is indeed effective to suppress low frequency NPP fluctuation when the $P F$ is larger than 0.6 . it is very suitable for a majority of motor drives, such as induction motors and permanent magnet synchronous motors.

## B. The Relationship Between io and 9-Order Harmonic

If 3-order harmonic component is assigned to the optimal $k_{3}$ forcedly, it may push the energy to 9 -order harmonic component. Thus, a larger 9-order harmonic component may be generated compared with the traditional SHEPWM. It also has a large effect on the NPP. Therefore, the 9-order harmonic component is discussed in this part.

Similarly, the limit of $k g$ in (14) is necessary to ensure that the sign of absolute calculation in (5) is not changed.

$$
\begin{equation*}
-1 / 9<k_{9}<1 / 2 \tag{14}
\end{equation*}
$$

Thus, the relationship between $i_{o}$ and $k_{9}$ is expressed as (15).

$$
\begin{align*}
& i_{o}=i_{o 1}+i_{o 9} \\
& \left\{\begin{array}{l}
i_{o 1}=\frac{4 m I_{m}}{\pi}\left[\frac{(-1)^{r}}{2} \cos (\varphi)+\cos \left(2 \theta-\frac{2 \pi}{3}-\varphi+\frac{\pi}{3} r\right)\right] \\
i_{o 9}=\frac{4 m I_{m}}{\pi}\left[2 k_{9} \sin (9 \theta) \sin \left(\theta-\frac{\pi}{3}-\varphi-\frac{\pi}{3} r\right)\right] \\
r=1,2,3,4,5,6
\end{array}\right. \tag{15}
\end{align*}
$$

The waveforms of $i_{o}, i_{o 1}$ and $i_{o 9}$ under different $P F$ can be obtained as Fig. 7. Here, $i_{o 9}$ is the neutral point current derived from the 9 -order harmonic voltage. As an example in Fig. 7, k9 is also set to 0.2636 . It can be found from Fig. 7 that the frequency of $i_{o g}$ is three times larger than that of $i_{o l}$. And the difference between the phase angles of $i_{o l}$ and $i_{o 9}$ can only been adjusted to 0 or $\pi$. Thus, it is difficult to utilize $i_{o 9}$ to neutralize $i_{o l}$, no matter how to set the $k_{9}$. A large $k_{9}$ may deteriorate $i_{o}$, like the example in Fig. 7. Therefore, we can only set $k 9$ to near zero to suppress the low frequency NPP fluctuation derived from $i_{o g}$. In this paper, as the optimal values to cope with low frequency NPP fluctuation, the $k_{3}$ is set to 0.2636 and the $k_{9}$ is set to near zero.

## C. The Switching Angles Calculation Based on The Current Harmonic Minimum and Low Frequency NPP Fluctuation Minimum

For SHEPWM, two switching angles should be sacrificed to realize the low frequency NPP fluctuation minimum. There are fewer switching angles for eliminating other low-order harmonics. Therefore, instead of SHEPWM, CHMPWM is adopted here to realize the current harmonic minimum and low frequency NPP fluctuation minimum simultaneously. Since the new harmonic requirements as (16)-(17) are different from that of the traditional optimal PWM, the switching angle calculation becomes easier unexpectedly. The new harmonic requirements have a better convergence for the existing algorithms, which are used in the traditional optimal PWM.

$$
\begin{align*}
& \left\{\begin{array}{l}
\sum_{i=1}^{N}(-1)^{i+1} \cos \left(\alpha_{i}\right)=m \\
\sum_{i=1}^{N}(-1)^{i+1} \cos \left(3 \alpha_{i}\right) / 3=k_{3} m, \quad k_{3}=0.2636 \\
\sum_{i=1}^{N}(-1)^{i+1} \cos \left(9 \alpha_{i}\right) / 9=k_{9} m, \quad k_{9} \approx 0
\end{array}\right. \\
& \quad f_{l \mid m m}=\frac{1}{m} \sqrt{\sum_{h=6 l+1}^{\infty}\left(\frac{u_{h(p u)}}{h}\right)^{2}} \tag{16}
\end{align*}
$$

It is a minimization problem of (17) with the constraints of (16) essentially. Moreover, $f_{f f m}$ must be within a permissible range. If $k_{3}$ and $k_{9}$ are unreasonable values, there may be no solutions to the problem. The solution range of switching angles, the iteration initial value and the algorithm convergence are different from traditional SHEPWM and optimal PWM. The detail comparisons will be introduced in section IV.

## IV. The Comparisons of Traditional Shepwm, Traditional Optimal PWM and Proposed CHMPWM

In order to realize the current harmonic minimum and the low frequency NPP fluctuation minimum at the same time, the harmonic requirements of the proposed method have been changed compared with traditional SHEPWM and optimal PWM. In this section, the comparisons on the solution range of switching angles, the iteration initial value, the algorithm convergence and output performances are carried on.

## A. The Solution Range of Switching Angles

Although the optimal $k_{3}$ and $k_{9}$ have been achieved in section


Fig. 8. The maximum $m$ range of different modulation methods.
III, it is difficult for the traditional numerical methods to ensure the solutions. If we use the optimal $k_{3}$ and $k 9$ to calculate the switching angles, there may be no solution when $m$ is large. Fig. 8 shows the maximum $m$ range, in which the switching angles can be achieved for SVPWM, the traditional SHEPWM, the traditional optimal PWM, the proposed CHMPWM with the optimal $k_{3}$ (CHM(3-order)) and with the optimal $k_{3}$ and $k_{9}$ (CHM (3\&9-order)).

When $N$ is equal to 1 , since all the modulations go back to square wave modulation, the maximum $m$ can reach to 1 . Actually, SVPWM does not need to calculate the switching angles, so the maximum $m$ can reach to 0.9 for all the $N$. Although the maximum $m$ of SHEPWM can also reach to 0.9 when $N$ is odd, it can be about 0.5 when $N$ is even and bigger than 4. Meantime, considering the algorithm convergence and complexity of optimal PWM, the maximum $m$ is discussed only when $N$ is not bigger than 4. And in the subsequent comparisons, the traditional optimal PWM is not discussed neither. However, if we introduce optimal $k_{3}$ and $k_{9}$ to revise the harmonic requirements, it is easy to calculate the switching angles when $N$ is bigger than 4 . What's more, the maximum $m$ can reach to 0.9 for all the $N$. It is noted that when $N$ is smaller than 3, there is no solution for proposed method with optimal $k_{3}$ and $k 9$, because 2 switching angles have been used to control $3 \& 9$-order harmonic components and no switching angle can be utilized to control fundamental component. Thus, from the view of the maximum $m$, proposed method has an advantage over the traditional SHEPWM and traditional optimal PWM. But proposed method can only be used when $N$ is larger than 4.
B. The Initial Value and The Algorithm Convergence

TABLE I
The Initial Values of The Proposed Method

| [rad] | N 5 | N 6 | N 7 | N 8 | N 9 | N 10 | N 11 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\alpha_{1}$ | 0.87 | 0.54 | 0.33 | 0.50 | 0.33 | 0.26 | 0.26 |
| $\alpha_{2}$ | 0.89 | 0.57 | 0.36 | 0.54 | 0.36 | 0.27 | 0.27 |
| $\alpha_{3}$ | 1.22 | 0.85 | 0.62 | 0.85 | 0.69 | 0.52 | 0.76 |
| $\alpha_{4}$ | 1.23 | 0.89 | 0.71 | 0.89 | 0.71 | 0.54 | 0.78 |
| $\alpha_{5}$ | 1.57 | 1.37 | 1.11 | 1.11 | 0.95 | 0.80 | 0.87 |
| $\alpha_{6}$ | -- | 1.41 | 1.15 | 1.15 | 0.97 | 0.82 | 0.89 |
| $\alpha_{7}$ | -- | -- | 1.57 | 1.37 | 1.22 | 0.95 | 1.04 |
| $\alpha_{8}$ | -- | -- | -- | 1.41 | 1.23 | 0.97 | 1.06 |
| $\alpha_{9}$ | -- | -- | -- | -- | 1.57 | 1.37 | 1.22 |
| $\alpha_{10}$ | -- | -- | -- | -- | -- | 1.41 | 1.23 |
| $\alpha_{11}$ | -- | -- | -- | -- | -- | -- | 1.55 |

For the traditional SHEPWM, the initial value problem and numerical methods [12-15] have been deeply discussed, such as newton iteration method, homotopy algorithm and genetic algorithm. Although the newton iteration method is stricter about the initial value than the homotopy algorithm and genetic algorithm, it can obtain more exact solutions. Meantime, the initial value can be given as (18).

$$
\begin{align*}
& \left\{\begin{array}{l}
a_{2 k-1}=a_{2 k}=30^{\circ}+120^{\circ} k /(N+1) \\
a_{N}=90^{\circ}, k=1,2,3, \ldots,(N-1) / 2
\end{array}, \quad \text { if } N\right. \text { is odd }  \tag{18}\\
& a_{2 k-1}=a_{2 k}=120^{\circ} k / N, k=1,2,3, \ldots, N / 2, \quad \text { if } N \text { is even }
\end{align*}
$$

For the traditional optimal PWM, we must use the genetic algorithm to get an inaccurate initial value and then solve a constrained optimization problem for the objective function. Because of the algorithm complexity, the solutions are rarely discussed when $N$ is large.

Fortunately, the proposed CHMPWM with the optimal $k_{3}$ and $k 9$ has a stronger algorithm convergence. Therefore, the initial values can be easily achieved by a few attempts. The initial values of the proposed method are given as Table. I.

## C. The Output performance

As mentioned in section III, if the switching angles are calculated based on the new harmonic requirements of (16) and (17), the low frequency NPP fluctuation minimum can be realized naturally. However, it means we need to control the $3 \& 9$-order harmonic components by sacrificing some degrees of freedom. It may make the output performance worse. Given that SVPWM and SHEPWM have been deeply discussed in some reference [12], the comparisons on $\lambda_{W T H D}$ of the


Fig. 9. The comparisons on $\lambda_{\text {wTHD }}$ of the traditional SHEPWM, the CHM(3-order) and the CHM(3\&9-order). (a) $N=11$. (b) $N=9$. (c) $N=7$. (d) $N=5$.
traditional SHEPWM, CHM (3-order) and CHM (3\&9-order) are carried on in this paper, which are shown in Fig. 9.

Since the switching angles for SHEPWM can be solved in the whole linear modulation range when $N$ is odd, the comparisons of SHEPWM, CHM(3-order) and CHM(3\&9order) are also done when $N$ is odd. It is noted from Fig. 9 that CHM(3-order) has the best performance for all the $N$. However, since it cannot control the 9 -order harmonic component, the control ability for the NPP is weaker than CHM(3\&9-order). The CHM ( $3 \& 9$-order) can keep the similar performance like CHM(3-order) when $N$ is bigger than 5 , which is better than that of SHEPWM. However, compared with SHEPWM and CHM (3-order), the output performance of CHM(3\&9-order) deteriorates obviously when $N$ is smaller than 5 , since there are few degrees of freedom that can be used to improve the harmonics. Fortunately, if $m$ is bigger than 0.8 , the CHM (3\&9order) has a similar performance as CHM (3-order) and SHEPWM even when $N$ is equal to 5 or 4 . It is very vital for motor driving. This is because the output frequency is usually proportional to $m$. Thus, the small $N$ are only chosen when $m$ is large for reducing the SF .

## V. The Experimental Verification

In this section, some experiments under a small power prototype are carried on to verify the correctness of proposed method by comparing the output performance and the NPP control performance. The experimental setup is based on the DSP/TMS320C6657 and FPGA/XC6SLX45, shown as Fig. 10. DC-link voltage is set to 220 V . The upper and bottom capacitors are 1800 uF . The SF is 245 Hz and dead time is set to 4 us. The $P F$ of the R-L load ( $\mathrm{R}=10 \Omega, \mathrm{~L}=5 \mathrm{mH}$ ) is about 0.99 .

The experimental results of the SHEPWM, the optimal PWM, the proposed CHM(3-order) and CHM(3\&9-order) are given in Fig. 11, 12, 13 and 14, when $N$ is equal to 7 and $m$ is equal to 0.6. Firstly, it can be seen from Fig. 11(c) that the amplitude of 3 -order harmonic component is about 0.38 . And the difference between the phase angles of fundamental component and 3 -order harmonic component is $\pi$. Thus, if we assume the fundamental component is 1 , 3 -order harmonic component ( $k_{3}$ ) would be -0.38 . Similarly, we can see from Fig. 12(c) that the amplitude of 3-order harmonic component is about 0.3 . And the difference between the phase angles of fundamental component and 3 -order harmonic component is $\pi$. Thus, if we assume the fundamental component is 1,3 -order harmonic component ( $k_{3}$ ) would be -0.3 . It can also be found from Fig. 13(c) that the amplitude of 3 -order harmonic component is about 0.2636 . And the difference between the phase angles of fundamental component and 3-order harmonic component is 0 . 3 -order harmonic component ( $k_{3}$ ) would be 0.2636 .

Since the $k_{3}$ for the traditional SHEPWM is about -0.38 , it causes a big low frequency NPP fluctuation as Fig. 11(b), which is about 3.8 V . By the FFT analysis of the NPP as Fig. 11(f), we can find the frequency of low frequency NPP fluctuation is three times larger than output frequency, which is consistent with the theoretical analysis shown in section III. From Fig. 11(d)(e), it can be seen that the selected 5, 7, 11, 13,


Fig. 10. The experimental setup.

17, 19-order harmonic components voltages are eliminated completely. It keeps a good performance under the low SF.

For traditional optimal PWM, $k_{3}$ and $k_{9}$ are about -0.3 and 0.35 respectively. They deviate from the optimal $k_{3}$ and $k_{9}$ greatly. As a result, low frequency NPP fluctuation is about 3.5 V . Its frequency mainly focus on 3,9 -order harmonic components, as shown in Fig. 12(f). It should be noted that different quasi-optimal solutions may be obtained according to different numerical algorithms and initial values, especially when $N$ is large. Thus, we should choose a good solution from multiple quasi-optimal solutions when $N$ is large. If a careful choice is made, a better output performance in THD of $v_{u v}$ can be obtained, which are about $32.1 \%$. However, a large low frequency NPP fluctuation appears. We must make a trade-off sometimes, whether to keep a good output performance in THD or NPP problem.

For proposed CHM(3-order), 3-order harmonic component is forced to change from $k_{3}=-0.38$ to the optimal $k_{3}=0.2636$. As a result, low frequency NPP fluctuation has been suppressed obviously to about 1.5 V . By the FFT analysis of the NPP as Fig. 13(f), we can know the low frequency NPP fluctuation focuses on 9,15 -order harmonic components, which are original from the 9,15 -order voltage harmonic components in Fig. 13(c). The 3 -order harmonic component of the NPP has been improved obviously.

On the other hand, it can be seen from Fig.11(c) and Fig.13(c) that THD of phase voltages for SHEPWM and CHM (3-order) are $61.50 \%$ and $87.01 \%$ respectively. THD of phase voltage for CHM (3-order) is larger than that for SHEPWM. However, since the applications we are concerned about, are motor drives or other three phase three line systems, phase current is decided by line voltage, not phase voltage. As the energy is transferred to $3 n$-order harmonic components, which would be eliminated automatically in line voltage for three phase three line systems, THD of phase voltage for CHM (3 -order) is even smaller than that for SHEPWM, as shown in Fig. 11(d) and Fig. 13(d). At the same time, THD of phase current for CHM (3-order) is $11.59 \%$, which is also smaller than that for SHEPWM ( $13.00 \%$ ). Thus, even if we use one switching angle as the degree of freedom to suppress the low frequency NPP fluctuation, it does not cause the deterioration of the output performances. It is an effective method to control the low frequency NPP fluctuation.


Fig. 11. The experimental results of the traditional SHEPWM. (a) $v_{u}, v_{u v}, i_{u v w}$. (b) The NPP. (c) the FFT analysis of $v_{u}$. (d) the FFT analysis of $v_{u v}$. (e) the FFT analysis of $i_{u}$. (f) the FFT analysis of the NPP.


The Fig. 12. The experimental results of the traditional optimal PWM. (a) $v_{u}, v_{u v}, i_{u v w}$. (b) The NPP. (c) the FFT analysis of $v_{u}$. (d) the FFT analysis of $v_{u v}$. (e) the FFT analysis of $i_{u}$. (f) the FFT analysis of the NPP.

Moreover, if the $3 \& 9$-order harmonic components are forced to the optimal $k_{3}$ and $k 9$ like CHM ( $3 \& 9$-order), the low frequency NPP fluctuation can be further reduced to 1.3 V as Fig. 14(b). From Fig. 14(f), it is also known that the low frequency NPP fluctuation concentrates in 15 -order harmonic component and the 9 -order harmonic component of the NPP is also eliminated well as demonstrated in section III. Even
though the proposed method cannot eliminate all the 5, 7, 11, 13, 17, 19-order harmonic components voltages, it does still have a better output performance than SHEPWM, from the perspective of THD of line voltage ( $v_{u v}$ ) and current $\left(i_{u}\right)$, which are $34.25 \%$ and $12.12 \%$ respectively. The proposed method can achieve a good output performance and control performance of low frequency NPP fluctuation simultaneously.


Fig. 13. The experimental results of the proposed CHM(3-order). (a) $v_{w}, v_{u v}, i_{u w w}$. (b) The NPP. (c) the FFT analysis of $v_{u v}$. (d) the FFT analysis of $v_{u v *}$ (e) the FFT analysis of $i_{u}$. (f) the FFT analysis of the NPP.


Fig. 14. The experimental results of the proposed CHM(3\&9-order). (a) $v_{u}, v_{u v}, i_{u v w}$. (b) The NPP. (c) the FFT analysis of $v_{u}$. (d) the FFT analysis of $v_{u v}$. (e) the FFT analysis of $i_{u}$. (f) the FFT analysis of the NPP.

## VI. The Conclusions

For the low frequency NPP fluctuation problem of TL-NPC converters, a novel CHMPWM based on the optimal 3\&9-order harmonic components is proposed in this paper. Some conclusions can be achieved as follows.

1) According to the theoretical analysis of the NPP problem, it can be known that the low frequency NPP fluctuation is inevitable for those PWM methods based on NTV-PWM. And it is severer for SHEPWM, since the SF is very low.
2) It is very hard to solve the low frequency NPP fluctuation problem for SHEPWM, since there is no degree of freedom remained to control the NPP online. From theoretical analysis of low frequency NPP fluctuation, we can know it is derived from fundamental component and $3 n$-order harmonic components for SHEPWM.
3) The optimal 3-order harmonic component is about 0.2636 , which can be used to suppress the low frequency NPP fluctuation to the minimum. Utilizing the optimal PWM method, the output performance is also good.
4) The proposed method gives a new way to solve the low frequency NPP fluctuation problem as well as maintain a good output performance.

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