ECONOMIC RESEARCH CENTER
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E-Series

No.E19-1

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April 2019

ECONOMIC RESEARCH CENTER
GRADUATE SCHOOL OF ECONOMICS
NAGOYA UNIVERSITY

Deficit-Financed Public Investment, Economic Growth, and Welfare in an Overlapping Generations Model*

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April 23, 2019

Abstract

This paper analyzes the growth and welfare effects of deficit-financed public investment using an overlapping generations model with private and public capitals. We demonstrate that the productivity effect of public capital and the weight of the utility from private consumption in the retired period are essential factors for the growth and welfare effects of deficit-financed fiscal policy. For instance, a higher intensity of public capital and survival rate support deficit-financed public investment.

Keywords: Debt; Public capital; Economic growth; Welfare

JEL classification: H54; H60; O40

*This work was supported by JSPS KAKENHI Grant Number 16H03625, 18H00865, and the Nitto Foundation.

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1 Introduction

Debt-financed public investment influences intergenerational welfare via different long-term benefits and costs. Intuitively, an increase in longevity increases the benefit to future generations from such public investments. Musgrave (1939) presented the concept of the golden rule of public finance as one way to execute capital investment along with the *pay-as-you-use principle*.[1] Numerous studies have investigated the macroeconomic effects of deficit-financed fiscal policy, including the golden rule and its variations (Greiner and Semmler 2000; Ghosh and Mourmouras 2004; Greiner 2007, 2010; Minea and Villieu 2009; Groneck 2011; Tamai 2014, 2016; Ueshina 2018).

In particular, Minea and Villieu (2009) demonstrated that the golden rule of public finance could improve intertemporal welfare compared with balanced-budget rules even though it negatively impacts long-run economic growth. Furthermore, Groneck (2011) showed that the golden rule had positive effects on both long-run growth and welfare. He mentioned that positive growth effects were observed only if public consumption expenditures were lowered in the long run.

However, the effects of debt-financing public investment on intergenerational welfare have not been sufficiently studied in terms of their properties. This reflects the fact that previous studies assumed that the representative household is infinitely lived. However, public debt serves as a form of intergenerational transfer. If the government issues public debt to invest in public capital, then the benefit accrues not only to current individuals but also to future ones. To capture this mechanism clearly, an overlapping generations model should be used. Furthermore, the present study incorporates the probability of death, making it possible to analyze how longevity affects economic circumstances and government policy under a population aging.

Recent studies have investigated the effects of public investment with debt using an OLG model (Yakita 2008; Arai 2011; Teles and Mussolini 2014). These studies analyzed the fiscal sustainability and did not focus on the intergenerational effects of debt-financed public investment. By contrast, this paper examines the growth and welfare effects of debt-financed public investment. The paper shows that there exists a the growth-maximizing tax rate and a fraction of deficit financing. Furthermore, we also derive the welfare-maximizing tax rate and fraction of deficit financing.

The remainder of this paper is organized as follows. The next section describes the basic setup of our model and characterizes the stationary equilibrium and its transitional dynamics. Section 3 examines the growth and welfare effect of debt-financed public investment. Finally, Section 4 concludes this paper.

2 The model

We consider an overlapping generations model in which each individual lives for two periods. In the first period, young individuals supply one unit of labor inelastically, and retiring during the second period. Each individual faces the risk of death. Then, the (expected) lifetime utility function for the individual born at period t is

$$U_t = \log c_t^y + p \log c_{t+1}^o \tag{1}$$

where c_t^y is the consumption for the young in the generation t, c_{t+1}^o is the consumption for the old in the generation t, and p is the survival rate ($p > 0$). The budget equations are as follows.

$$c_t^y = \widetilde{w}_t - s_t, \tag{2}$$

$$c_{t+1}^o = \frac{\widetilde{R}_{t+1} s_t}{p}, \tag{3}$$

[1] The golden rule and its variations were legally adopted by some countries between 1985 and 2014 (e.g., Brazil, Costa Rica, Germany, Japan, Luxembourg, Malaysia, and the United Kingdom). The golden rule incorporates the possibility of borrowing to finance productive public investment that has the potential to pay for itself over the long-term with a balanced current budget (IMF, 2014, Ch. 3).

where $\widetilde{w}_t$ is the post-tax wage rate, $\widetilde{R}_{t+1}$ is the post-tax interest factor, and s_t is the saving.

Solving the optimization problem, we obtain the saving function as follows.

$$s_t = \beta \widetilde{w}_t, \tag{4}$$

where

$$\beta \equiv \frac{p}{1+p}.$$

Note that the value of β increases with p.

The production function is

$$y_t = A k_t^{\alpha} g_t^{1-\alpha} l_t^{1-\alpha}, \tag{5}$$

where y_t is the output, k_t is the private capital, g_t is the public capital, and l_t is the labor. We assume that public capital is labor-augmenting and taken as given for each firm. Using the marginal principle, we obtain the interest factor and wage rate:

$$\begin{aligned} R_t &= \alpha A k_t^{\alpha-1} g_t^{1-\alpha} l_t^{1-\alpha}, && (6)\\ w_t &= (1-\alpha) A k_t^{\alpha} g_t^{1-\alpha} l_t^{-\alpha}. && (7) \end{aligned}$$

The government issues public bonds and taxes household labor income to finance the government expenditures for the interest payments and public investment in infrastructure. The government's budget equation is as follows.

$$b_{t+1} = R_t b_t + g_{t+1} - \left[\tau_t R_t (k_t + b_t) + \tau_t w_t l_t\right]. \tag{8}$$

If there is 100% depreciation of capital (not an important assumption), then $I_t = K_{t+1}$ holds. The clearing condition of asset market is

$$s_t = b_{t+1} + k_{t+1}. \tag{9}$$

Using this and equations (2), (3), (6), (7), and (8), Walras' law holds.[2]

We introduce the following deficit-financing rule. The government finances some fractions of public investment in infrastructure ($0 \leq \eta \leq 1$) by issuing public bonds. Formally, it can be described as follows.

$$b_{t+1} = \eta g_{t+1} \tag{10}$$

Equations (8) and (10) lead to

$$g_{t+1} = \frac{\tau - (1-\tau)\alpha\eta x_t}{1-\eta} y_t,$$

where $x_t \equiv g_t / k_t$. Equations (4) and (9) yield

$$\begin{aligned} k_{t+1} &= \beta \widetilde{w}_t - b_{t+1} \\ &= (1-\alpha)(1-\tau)\beta y_t - \eta g_{t+1}. \end{aligned}$$

Using the equations of private and public capital accumulation, we obtain

$$x_{t+1} = \frac{\tau - (1-\tau)\alpha\eta x_t}{(1-\alpha)(1-\eta)(1-\tau)\beta - \eta\left[\tau - (1-\tau)\alpha\eta x_t\right]} \equiv \psi(x_t), \tag{11}$$

[2] It can be verified as

$$\begin{aligned} y_t &= c_t^y + p c_t^o + k_{t+1} + g_{t+1} \\ &= w_t l_t - b_{t+1} - k_{t+1} + (b_t + k_t) R_t + k_{t+1} + (b_{t+1} - R_t b_t) \\ &= w_t l_t + R_t k_t \equiv y_t. \end{aligned}$$

where

$$\begin{aligned}
\psi(0) &= \frac{\tau}{(1-\alpha)(1-\eta)(1-\tau)\beta-\eta\tau}>0, \\
\psi(\widehat{x}) &= 0, \widehat{x} \equiv \frac{\tau}{(1-\tau)\alpha\eta}, \frac{\tau-(1-\tau)\alpha\eta x}{1-\eta}=0 \Leftrightarrow x=\widehat{x}.
\end{aligned}$$

The slope of ψ is

$$\psi'(x) = -\frac{(1-\alpha)(1-\eta)(1-\tau)^2\alpha\beta\eta}{\{(1-\alpha)(1-\eta)(1-\tau)\beta-\eta[\tau-(1-\tau)\alpha\eta x]\}^2}<0.$$

Since ψ is continuous and monotonically decreasing with x in the closed interval $[0,\widehat{x}]$, there exists a unique stationary equilibrium that satisfies $x_{t+1}=x_t$. If α is sufficiently small or if β is sufficiently large, $|\psi'(x)|<1$.

From $x=\psi(x)$ and equation (11), we obtain

$$x=\frac{\sqrt{[(1-\alpha)(1-\tau)(1-\eta)\beta+(1-\tau)\alpha\eta-\eta\tau]^2+4(1-\tau)\alpha\tau\eta^2}-[(1-\alpha)(1-\tau)(1-\eta)\beta+(1-\tau)\alpha\eta-\eta\tau]}{2(1-\tau)\alpha\eta^2}.$$

In particular, we have

$$\begin{aligned}
x &= \frac{\tau}{(1-\alpha)(1-\tau)\beta} \text{ for } \eta=0, \\
x &= \frac{\tau}{(1-\tau)\alpha} \text{ for } \eta=1.
\end{aligned}$$

The equilibrium growth rate is

$$\gamma \equiv \frac{g_{t+1}}{g_t}=\frac{k_{t+1}}{k_t}=\frac{(1-\alpha)(1-\tau)\beta A x^{1-\alpha}}{1+\eta x}.$$

3 Growth effects of fiscal policy

This section analyzes the growth effects of deficit-financed fiscal policy. Note that we have two benchmarks to evaluate the growth effects of the fiscal policy: balanced budget ($\eta=0$) and the golden rule of public finance ($\eta=1$). Using $x=\psi(x)$ and equation (11), we have

$$\begin{aligned}
\frac{\partial x}{\partial\tau} &= \frac{\alpha\eta^2x^2+(1-\alpha)(1-\eta)\beta x+(1+\alpha)\eta x+1}{2(1-\tau)\alpha\eta^2x+(1-\alpha)(1-\tau)(1-\eta)\beta+(1-\tau)\alpha\eta-\eta\tau}, && (12) \\
\frac{\partial x}{\partial\eta} &= \frac{(1-\tau)[(1-\alpha)\beta-\alpha]+\tau-2(1-\tau)\alpha\eta x}{2(1-\tau)\alpha\eta^2x+(1-\alpha)(1-\tau)(1-\eta)\beta+(1-\tau)\alpha\eta-\eta\tau}x. && (13)
\end{aligned}$$

Using equation (12), the growth effects of a change in τ is

$$\frac{1}{\gamma}\frac{\partial\gamma}{\partial\tau}=-\frac{1}{1-\tau}+(1-\alpha)\frac{1}{x}\frac{\partial x}{\partial\tau}-\frac{\eta x}{1+\eta x}\frac{1}{x}\frac{\partial x}{\partial\tau}. \tag{14}$$

Equation (14) provides the following proposition:

Proposition 1. *For any given η, there exists a growth-maximizing tax rate, satisfying*

$$\tau<\frac{1-\alpha}{1+\alpha}<1-\alpha.$$

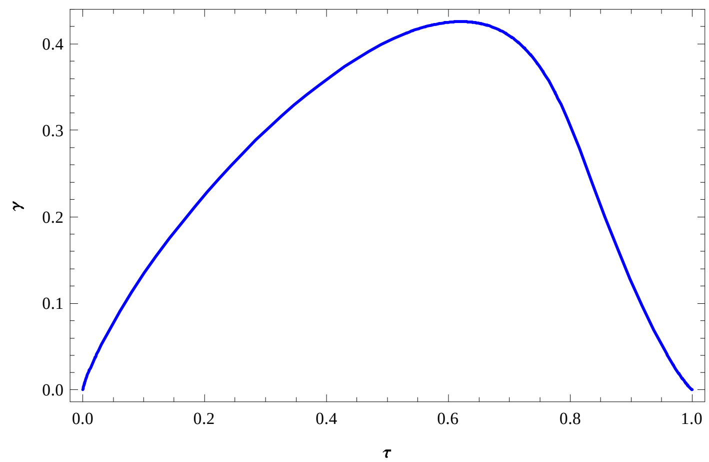

Figure 1

(Proof) Taking the limit of (14) leads to

$$\begin{aligned}\lim_{\tau\to0}\frac{1}{\gamma}\frac{\partial\gamma}{\partial\tau} &= -1+(1-\alpha)\lim_{\tau\to0}\left(\frac{1}{x}\frac{\partial x}{\partial\tau}\right)-\lim_{\tau\to0}\left[\frac{\eta x}{1+\eta x}\frac{1}{x}\frac{\partial x}{\partial\tau}\right]\\ &= -1+\frac{1-\alpha}{(1-\alpha)(1-\eta)\beta+\alpha\eta}\lim_{\tau\to0}\left(\frac{1}{x}\right)-\frac{\eta}{(1-\alpha)(1-\eta)\beta+\alpha\eta}\\ &= +\infty\end{aligned}$$

where

$$\begin{aligned}\lim_{\tau\to0}\frac{\partial x}{\partial\tau} &= \frac{\alpha\eta^2\times0+(1-\alpha)(1-\eta)\beta\times0+(1+\alpha)\eta\times0+1}{2(1-\tau)\alpha\eta^2\times0+(1-\alpha)(1-\eta)\beta+\alpha\eta}\\ &= \frac{1}{(1-\alpha)(1-\eta)\beta+\alpha\eta},\\ \lim_{\tau\to0}x &= \frac{\sqrt{[(1-\alpha)(1-\eta)\beta+\alpha\eta]^2}-[(1-\alpha)(1-\eta)\beta+\alpha\eta]}{2\alpha\eta^2}=0.\end{aligned}$$

Similarly, we have

$$\begin{aligned}\lim_{\tau\to1}\frac{1}{\gamma}\frac{\partial\gamma}{\partial\tau} &= -\lim_{\tau\to1}\frac{1}{1-\tau}+(1-\alpha)\lim_{\tau\to1}\left(\frac{1}{x}\frac{\partial x}{\partial\tau}\right)-\lim_{\tau\to1}\left[\frac{\eta x}{1+\eta x}\frac{1}{x}\frac{\partial x}{\partial\tau}\right]\\ &= -\lim_{\tau\to1}\frac{1}{1-\tau}-\alpha\lim_{\tau\to1}\left(\frac{1}{x}\frac{\partial x}{\partial\tau}\right)=-\infty.\end{aligned}$$

These results show that there exists a tax rate to maximize the equilibrium growth rate.

A change in the tax rate has a directly negative effect on private capital accumulation through a decrease in disposable income and an indirectly positive effect on private capital accumulation via an increase in public capital accumulation. Both private and public capital accumulation are sources of economic growth. Therefore, the growth-maximizing tax rate exists in the range $(0,1)$. Setting the parameters as $(\alpha,\beta,\eta,A)=(0.2,0.5,0.1,1)$, numerical analysis provides Figure 1, where the inverted-U curve is shown.

Differentiating growth rate with respect to η derives

$$\frac{1}{\gamma}\frac{\partial\gamma}{\partial\eta}=\frac{1-\alpha}{x}\frac{\partial x}{\partial\eta}-\frac{x+\eta\frac{\partial x}{\partial\eta}}{1+\eta x}. \tag{15}$$

From equation (14), we obtain the following result:

Proposition 2. *For any given τ, there exists a growth-maximizing fraction of η to finance public investment by issuing bonds if*

$$(1-\alpha)\beta > \alpha \text{ and } \tau < \frac{(1-\alpha)\left[(1-\alpha)\beta-\alpha\right]}{\alpha+(1-\alpha)\left[(1-\alpha)\beta-\alpha\right]}.$$

In contrast, a rise in η decreases the equilibrium growth rate if $(1-\alpha)\beta < \alpha$.

(Proof) The limit of equation (14) is

$$\begin{aligned}
\lim_{\eta\to 0}\frac{1}{\gamma}\frac{\partial\gamma}{\partial\eta} &= \lim_{\eta\to 0}\left[\frac{1-\alpha}{x}\frac{\partial x}{\partial\eta}-\frac{x+\eta\frac{\partial x}{\partial\eta}}{1+\eta x}\right] \\
&= \frac{(1-\alpha)(1-\tau)\left[(1-\alpha)\beta-\alpha\right]-\alpha\tau}{(1-\alpha)(1-\tau)\beta} > 0 \\
\text{if } (1-\alpha)\beta &> \alpha, \tau < \frac{(1-\alpha)\left[(1-\alpha)\beta-\alpha\right]}{\alpha+(1-\alpha)\left[(1-\alpha)\beta-\alpha\right]} = \frac{(1-\alpha)\beta-\alpha}{\frac{\alpha}{1-\alpha}+(1-\alpha)\beta-\alpha}.
\end{aligned}$$

Note that

$$\lim_{\eta\to 0}\frac{dx}{d\eta} = \frac{(1-\tau)\left[(1-\alpha)\beta-\alpha\right]+\tau}{(1-\alpha)(1-\tau)\beta}\frac{\tau}{(1-\alpha)(1-\tau)\beta}.$$

When $(1-\alpha)\beta < \alpha$, $\lim_{\eta\to 0}\partial\gamma/\partial\eta < 0$ holds.

We have

$$\begin{aligned}
\frac{(1-\alpha)\left[(1-\alpha)\beta-\alpha\right]}{\alpha+(1-\alpha)\left[(1-\alpha)\beta-\alpha\right]}-(1-\alpha) &= (1-\alpha)\left\{\frac{(1-\alpha)\beta-\alpha-\alpha-(1-\alpha)\left[(1-\alpha)\beta-\alpha\right]}{\alpha+(1-\alpha)\left[(1-\alpha)\beta-\alpha\right]}\right\} \\
&= (1-\alpha)\left\{\frac{-\alpha+\alpha\left[(1-\alpha)\beta-\alpha\right]}{\alpha+(1-\alpha)\left[(1-\alpha)\beta-\alpha\right]}\right\} \\
&= \left\{\frac{(1-\alpha)\beta-(1+\alpha)}{\alpha+(1-\alpha)\left[(1-\alpha)\beta-\alpha\right]}\right\}(1-\alpha)\alpha < 0.
\end{aligned}$$

Taking another limit, we obtain

$$\lim_{\eta\to 1}\frac{1}{\gamma}\frac{\partial\gamma}{\partial\eta} = \frac{(1-\alpha)(1-\tau)\left[1-\alpha-\tau\right]\beta-(1-\alpha)\alpha}{\left[(1-\tau)\alpha+\tau\right]^2}.$$

Note that

$$\lim_{\eta\to 1}\frac{dx}{d\eta} = \frac{(1-\tau)\left[(1-\alpha)\beta-\alpha\right]-\tau}{(1-\tau)\alpha+\tau}\frac{\tau}{(1-\tau)\alpha}.$$

$\lim_{\eta\to 1}\partial\gamma/\partial\eta < 0$ holds if

$$(1-\alpha)\beta > \alpha \text{ and } \tau < \frac{(1-\alpha)\left[(1-\alpha)\beta-\alpha\right]}{\alpha+(1-\alpha)\left[(1-\alpha)\beta-\alpha\right]}.$$

The result when $(1-\alpha)\beta < \alpha$ implies that deficit-financing slows down economic growth (e.g. Minea and Villieu 2009; Kamiguchi and Tamai 2019). However, if $(1-\alpha)\beta > \alpha$ (the productivity effect of public capital is sufficiently large), the result supports deficit-financing to enhance economic growth. This theoretical result seems to be consistent with some empirical findings asserted in IMF (2014). Based on $(\alpha, \beta, \tau, A) = (0.2, 0.5, 0.4, 1)$, the relation between η and γ is illustrated in Figure 2. The graph exhibits the inverted-U curve.

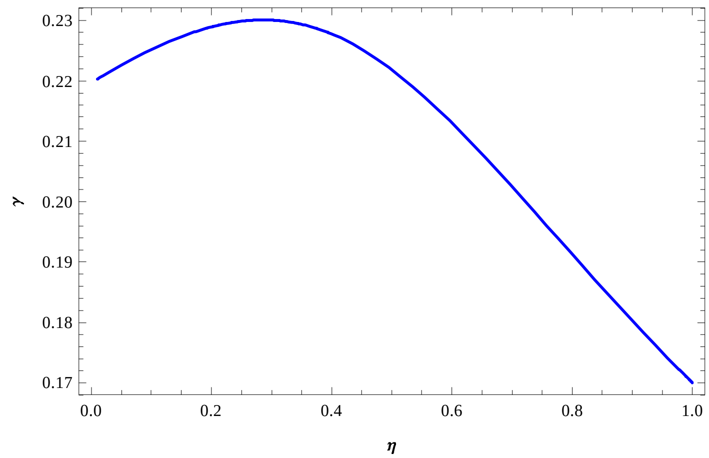

Figure 2

4 Welfare effects of fiscal policy

We assume that the government's objective function is based on existing individuals. Specifically, the welfare function is given by[3]

$$\begin{aligned} W_0 &= p \log c_0^o + \mu \left[\log c_0^y + p \log c_1^o \right] \\ &\simeq p \log (1-\tau) + \mu \left\{ (1+p) \log (1-\tau) + p \left[\log (1-\tau) + (1-\alpha) \log x \right] \right\}. \end{aligned} \tag{16}$$

The first-order derivatives of equation (14) are

$$\frac{\partial W_0}{\partial \tau} = -\frac{p + \mu (1+2p)}{1-\tau} + \frac{(1-\alpha)\mu p}{x} \frac{\partial x}{\partial \tau}, \tag{17}$$

$$\frac{\partial W_0}{\partial \eta} = \frac{(1-\alpha)\mu p}{x} \frac{\partial x}{\partial \eta}. \tag{18}$$

Using equation (17) yields the following result:

Proposition 3. *For any given η, there exists a welfare-maximizing tax rate.*

(Proof) The limits of equation (17) are

$$\lim_{\tau \to 0} \frac{\partial W_0}{\partial \tau} = -\left[p + \mu (1+2p) \right] + \lim_{\tau \to 0} \frac{(1-\alpha)\mu p}{x} \frac{\partial x}{\partial \tau} = +\infty,$$

and

$$\begin{aligned} \lim_{\tau \to 1} \frac{\partial W_0}{\partial \tau} &= -\lim_{\tau \to 1} \frac{p + \mu (1+2p)}{1-\tau} + \lim_{\tau \to 1} \frac{(1-\alpha)\mu p}{x} \frac{\partial x}{\partial \tau} \\ &= -\lim_{\tau \to 1} \frac{p + \mu \left[1 + (1+\alpha) p \right]}{1-\tau} = -\infty. \end{aligned}$$

Regardless of η, a rise in the tax rate directly reduces welfare via a decrease in disposable income and indirectly raises welfare via an increase in public investment.[4] Therefore, we can find the tax rate to maximize the welfare in the range, $(0, 1)$. Numerical simulation gives the relation between the tax rate and welfare as the graph in Figure 3, indicating the existence of the welfare-maximizing tax rate.

Finally, equation (18) leads to the following proposition:

[3]This equation takes an identical form as the *static* voting problem.

[4]For instance, in the case of a balanced-budget ($\eta = 0$) or the golden rule of public finance ($\eta = 1$), it is easily verified.

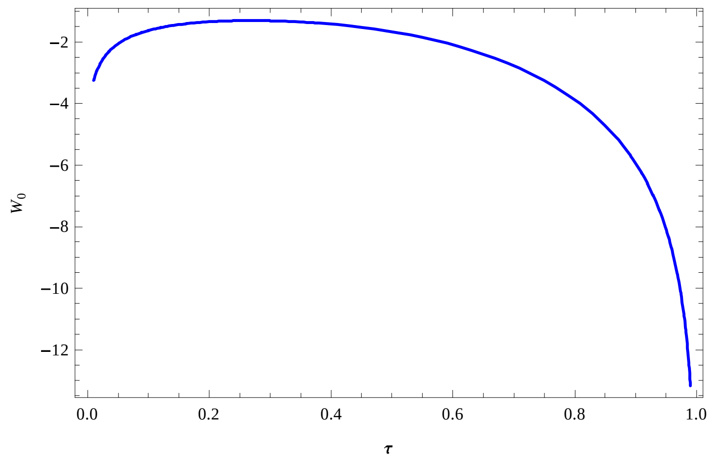

Figure 3

Proposition 4. *For any given τ, there exists a welfare-maximizing fraction, η, to finance public investment by issuing bonds if*

$$(1-\alpha)\beta > \alpha \text{ and } \tau > \frac{(1-\alpha)\beta-\alpha}{1+(1-\alpha)\beta-\alpha}.$$

On the other hand, a rise in η decreases the social welfare if $(1-\alpha)\beta < \alpha$.

(Proof) The limits of equation (18) show

$$\begin{aligned} sgn \lim_{\eta\to 0}\frac{\partial W_0}{\partial \eta} &= sgn \lim_{\eta \to 0}\frac{\partial x}{\partial \eta} > 0 \Leftrightarrow (1-\alpha)\beta > \alpha, \\ sgn \lim_{\eta\to 1}\frac{\partial W_0}{\partial \eta} &= sgn \lim_{\eta \to 1}\frac{\partial x}{\partial \eta} < 0 \Leftrightarrow \tau > \frac{(1-\alpha)\beta-\alpha}{1+(1-\alpha)\beta-\alpha} > 0. \end{aligned}$$

As shown in Proposition 2, $(1-\alpha)\beta > \alpha$ is necessary to ensure a positive growth effect of deficit-financing. The same condition must hold for a positive welfare effect because a negative growth effect has a negative welfare effect. If such a condition is not satisfied, the deficit-financing fails to improve welfare. Figure 4 illustrates the graph that exhibits the relation between η and W_0. It shows the inverted-U curve, which means the existence of a welfare-maximizing level of η. Then, we have an interest at the exact levels of τ and η to maximize welfare. Numerical analysis provides the answer; $(\tau, \eta) = (0.216, 0.852)$. The result implies that 85% of public investment expenditure should be financed by public bonds.

5 Conclusion

This paper investigated the growth and welfare effects of deficit-financed public investment using the OLG model with private and public capital. It was shown that the productivity effect of public capital and the weight of the utility from private consumption in the retired period were essential factors to determining the growth and welfare effects of deficit-financed fiscal policy. For instance, a higher intensity of public capital and survival rate support deficit-financed public investment.

Considering future direction studies, incorporating the intergenerational transfer into our model will be a worthwhile examination. Both private and public transfers play a key role in conveying the benefit from young to old and vice versa. The growth and welfare effects of fiscal policy are influenced by these transfers. Second, we simplify the government's decision-making (i.e. growth-maximizing

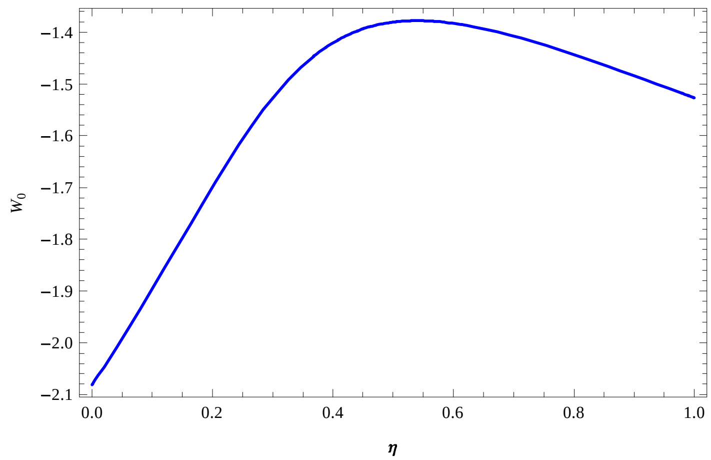

Figure 4

and welfare-maximizing) by assuming time-invariant rates of policy instruments. However, the policy will be changed over time via the economic environment and political mechanism. Thus, a dynamic approach of political economy will be fruitful to illustrate the economic situation and to obtain further insight on deficit financing. Our simple model provide a robust analytical basis to analyze these topics.

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