

別紙 4

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主 論 文 の 要 旨

論文題目 On Asymptotics of Nonlocal p-Laplacian Operators and Related Eigenvalues
(非局所的な p ラプラシアン作用素とその固有値の漸近挙動について)

氏 名 李 峰

論 文 内 容 の 要 旨

The main topic of this thesis is Nonlocal p-Laplacian equations. The thesis is divided into two parts:
(I) Asymptotic Behaviour of Fractional p-Laplacian Functionals under Dirichlet conditions;
(II) On Relative-Nonlocal p-Rayleigh Quotients.

Nonlocal p-Laplacian equations arise naturally in the study of stochastic process with jumps. Moreover nonlocal p-Laplacian equations appear naturally in other contexts such as Geometry, Fluid Mechanics, and Image Processing. The classical nonlocal p-Laplacian operator is defined as

$$(-\Delta_p)^s u(x) = \lim_{\varepsilon \searrow 0} 2 \int_{\mathbb{R}^N \setminus \mathbb{B}(x, \varepsilon)} |u(x) - u(y)|^{p-2} (u(x) - u(y)) / |x - y|^{N+sp} dy \quad (1)$$

for $0 < s < 1$ and $1 < p < +\infty$; and the variational forms restricted in Ω usually defined as

$$\int_{\Omega \times \Omega} |u(x) - u(y)|^p / |x - y|^{N+sp} dx dy \quad (2).$$

In the preliminary part, we mainly introduce some basic tools and spaces to be used later, including Γ -convergence and relative-nonlocal Sobolev space

$$\tilde{W}_{(0,tR)}^{(s,p)}(\Omega),$$

In which, $t > 1$, and R is diameter of Ω .

In chapter I of this thesis, we mainly use Γ -convergence to analysis the behaviour of fractional p-Laplacian functionals with non-homogeneous Dirichlet boundary as $p \rightarrow +\infty$, and the asymptotic

behaviour triggered by varying s with homogeneous Dirichlet boundary. In our result, by Γ -convergence we establish the convergence of the minimizers of (2) restricted on Ω with non-homogeneous Dirichlet boundary conditions to one of the minimizers of the functional

$$\| (u(x) - u(y)) / |x - y|^s \|_{L^\infty(\Omega \times \Omega)}$$

under the same Dirichlet boundary condition. Also in the De Giorgi-sense, we investigate the asymptotic behaviour of the minimizers of the operator (2) with varying s in homogeneous Dirichlet boundary condition. We will see that it is totally different for the behaviour from below compared with the case from above. And as a byproduct, we also establish an equivalent form of the sobolev space $W_0^{(s,p)}(\Omega)$ for arbitrary open bounded set Ω .

In chapter II of this dissertation, we investigate the asymptotic behaviour of nonlocal p -Rayleigh quotient as s varies, which is defined by

$$\inf_{\|u\|_{L^p(\Omega)}=1} = \int_{\mathbb{R}^N \times \mathbb{R}^N} |u(x) - u(y)|^p / |x - y|^{N+sp} dx dy .$$

We will see that the behaviour from below and above exhibit a different phenomenon.

And we also studied the convergence of corresponding nonlocal eigenfunctions. In order to investigate the asymptotic behaviour of the nonlocal p -Rayleigh quotients, we work in the relative-nonlocal space. In this part, we also establish some equivalent forms of the nonlocal Sobolev space $\tilde{W}_{(0,BR)}^{(s,p)}(\Omega)$. This result can be merged with the similar results obtained in chapter I in De Giorgi-sense, which are essentially the same.