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Regenerative chatter by teeth allocated in the cutting direction with position-dependent modal displacement ratios

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Cutters with high flexibility, e.g. side-and-face cutters and band saw cutters, are widely used in industries for finishing of narrow and deep grooves or parting operations. However, because of this flexibility, chatter vibration tends to occur, and it is unique in that the mode shape along the teeth affects the present vibration, the regenerative vibration, and the compliance at the present cutting tooth in both magnitude and phase, which is new to the literature. Hence, a novel regenerative chatter stability model is developed where the position-dependent modal displacement ratio in the cutting direction is considered. The experimental results show agreement with the proposed model, and unique characteristics are extracted.

Chatter; Stability; Position-dependent modal displacement ratio

1. Introduction

Regenerative chatter is a critical obstacle which often causes the limitation of the machining efficiency in industries. Since this phenomenon has a possibility to occur in most of the practical cutting methods, it has widely attracted researchers and industrial engineers for the last several decades. As a result, many studies have been dedicated to the understanding and suppression of regenerative chatter, and it is needless to say that they have contributed to the overwhelming improvement of production sites.

Beginning with the early works by Tlusty, Tobias, and Merritt in the 1950's and 60's [1-3], it was discovered that the regeneration of the vibration left on the previously cut surface causes the growth of regenerative chatter. From thereon, a variety of models have been introduced to the literature to predict the critical stability which is important both theoretically and practically. They have been modelled in the frequency domain [4], time domain [5], and semi-discrete time domain [6,7]. It can be said that the regenerative chatter is close to its complete revealment.

However, most of the introduced models have a presumption that the cutting with the flexible structure is always at one fixed position in the vibration mode, and thus the mode shape is not considered. In reality, there are such cases where cutting occurs on different positions in the vibration mode. In such cases like cutting thin walls with long slender end mills at a large axial depth of cut, the cutting occurs on different positions in the modes of both structures along the axial direction, meaning that the modes affect the stability and therefore need to be considered [6]. However, the derivation of the stability considering these mode shapes can be assumed not complicated because the stability can be predicted using the conventional models with a simple extension of the governing equation in the axial direction. As for the cases where the cutting is conducted by teeth allocated in the cutting direction with position-dependent modal displacement ratios, e.g. side-and-face cutters and band saw cutters, there has been no study in the literature, and its effect on the cutting process is unknown; the stability cannot be derived by conventional models. Note that the modal displacement ratios represent the mode shape.

In this research, the position-dependent modal displacement ratios of cutters in the cutting direction is investigated with the objective to reveal its effect on the regenerative chatter stability. The case for side-and-face cutters is considered in this paper, and their mode shapes, including the unique ones with the position-dependent modal displacement ratios in the cutting direction, are introduced at the beginning. Then, a regenerative chatter stability model is developed including the effect of the position-dependent modal displacement ratio. Analytical and experimental observations will follow, and the validity of the proposed model and the unique characteristics of the chatter are discussed. At last, conclusions will summarize the results obtained from this study.

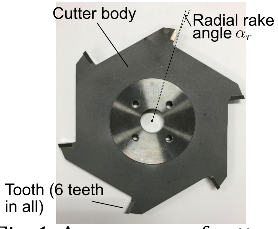

Fig. 1. Appearance of cutter.

Table 1. Specifications of cutter.

Number of teeth N_t	6	-
Diameter of cutter body	200	mm
Thickness of cutter body	10	mm
Nose radius R of insert	5	mm
Axial rake angle α_a	0	deg
Radial rake angle α_r	10	deg
Material of insert	Cemented carbide	-

2. Vibration modes of side-and-face cutters and their effect on chatter

Throughout the paper, the side-and-face cutter whose appearance and specifications are shown in Fig. 1 and Table 1, respectively, is used. It consists of six uniformly-distributed carbide teeth, and the teeth are brazed to the steel cutter body.

In order to simply introduce the mode shape to the stability model, the modal displacement ratio m_j^k (j expresses the index of the tooth and k expresses the index of the mode) is used in this paper, and it can be expressed as follows: $m_j = G_j^{re_{max}}/G_1^{re_{max}}$, where $G_1^{re_{max}}$ expresses the real part of the direct compliance at the negative peak frequency of the real part at Tooth 1, and $G_j^{re_{max}}$ is the real part of the cross compliance at Tooth j at that frequency when Tooth 1 is hit. A negative sign of the modal displacement ratio means a phase reversal, i.e. positive and negative antinodes have reverse phases. Note that this modal displacement ratio is roughly constant near the negative peak frequency, and the ratio at the negative peak frequency is used for each mode since chatter is most likely to occur near that frequency.

A hammering test is conducted to identify the mode shape. An impact hammer (086E80, PCB Inc.) and an accelerometer (352C23, PCB Inc.) are used, and the accelerometer is fixed to Tooth 1 throughout the hammering. Then, each tooth is hit to obtain its response at Tooth 1. Finally, the modal displacement ratio is determined to express the mode shape. Note that the hammering position is changed here based on the assumption that the structural compliance is symmetrical, i.e. response of Tooth 1 by impact of Tooth 2 is equal to response of Tooth 2 by impact of Tooth 1. Note also that the cutter is flexible in the axial direction, and the vibration in that direction is measured. A photograph of the hammering test is shown in Fig. 2, and the modal displacement ratios of major modes and the direct compliance of Tooth 1 are shown in Fig. 3. The modal displacement ratio for five vibration modes in the ascending order of the resonant frequency are shown.

As can be observed, side-and-face cutters have a mode (Mode 1) where the modal displacement ratio is constant in the cutting direction, i.e. the modal displacement ratio is position independent and it is the same at each tooth, and unique modes (Mode 2, 3, 4, 5) where the modal displacement ratio changes in the cutting direction, i.e. the modal displacement ratio is position dependent and differs at each tooth.

The former mode (Mode 1) is the same as those considered in the conventional regenerative chatter models [4-7], but the latter modes (Mode 2, 3, 4, 5) are unique and have not been investigated up to now. As for the former mode, the modal displacement ratio is constant in the cutting direction, and regenerative chatter is in the critical state with a constant phase shift ϵ between vibrations of adjacent teeth and the vibrations also have the same amplitude. However, since the modal displacement ratio changes in the cutting direction for the latter modes, vibrations of adjacent teeth should have an additional π rad shift if the sign of the modal displacement ratio changes. For example, the sign of the modal displacement ratio changes between Tooth 2 and Tooth 3 for Mode 3 shown in Fig. 3. Note that there should be no additional phase shift if the sign is the same. Moreover, the vibrations at the teeth have different amplitudes since the modal displacement ratio is position dependent. A schematic illustration of the vibrations in their critical state for different modes is shown in Fig. 4. In addition to the effect of the mode shape on the vibrations and resulting dynamic material removing force, the position-dependent modal displacement ratio reduces/magnifies the compliance at the present tooth depending on its position in the mode. For example, the modal displacement ratio and the corresponding compliance is smaller if it is close to a node of the mode. As a result, regenerative chatter should grow at the worst phase considering the effects of the unique mode shape on the vibrations and the compliance at the present cutting tooth.

3. Regenerative chatter stability model considering the position-dependent modal displacement ratios in the cutting direction

An analytical model to predict the regenerative chatter stability considering the position-dependent modal displacement ratios in the cutting direction is developed. A face milling process is conducted in this paper, and a conventional chatter stability model of this process can be found in for example [8]. A schematic illustration of the face milling process is shown in Fig. 5. In this section, the conventional model is reviewed, and new aspects considering the position-dependent modal displacement ratios are introduced simultaneously, which are needed to predict the stability of chatter which occurs at the unique modes. It has to be mentioned that the developed model can also predict the stability with position-independent modal displacement ratios. Note that the developed model assumes a single frequency of chatter, i.e. time-varying components of the cutting process are averaged.

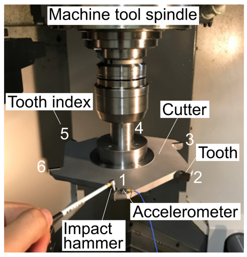

Fig. 2. Photograph of hammering test.

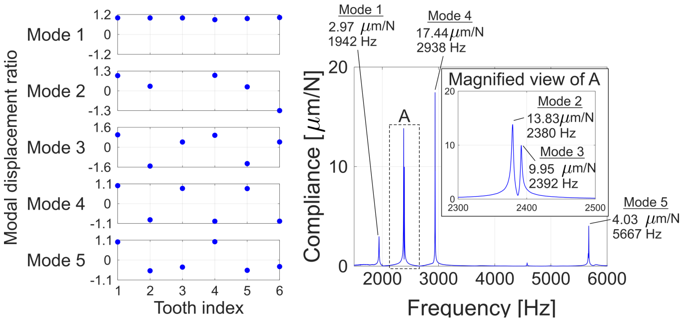

Fig. 3. Modal displacement ratios of modes and direct compliance.

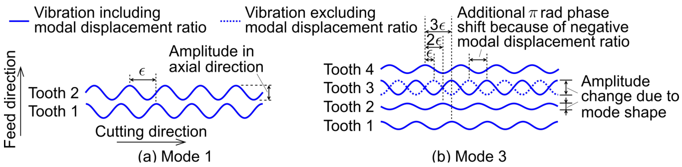

Fig. 4. Schematic illustration of vibrations in their critical state in (a) Mode 1 and (b) Mode 3.

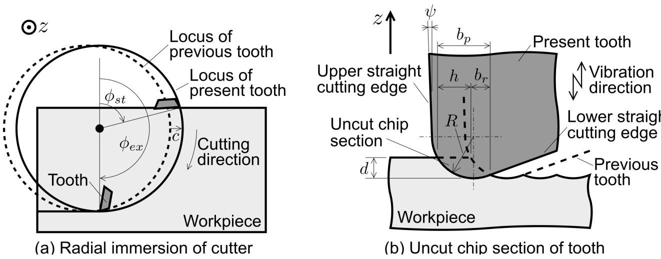

Fig. 5. Schematic illustration of face milling process [8].

Beginning with the vibration z_1 of Tooth 1 in the axial direction, the present vibration z_j of Tooth j can be expressed as $z_j = m_j z_1$ using the modal displacement ratio since the mode shape affects the vibration. This present vibration occurs on the present cutting width b_p and affects the present cutting. Similarly, the regenerative vibration of the previous tooth can be expressed as $m_{j-1}^k z_1 e^{-i\omega_c T}$, where ω_c is the chatter frequency and T is the tooth-passing period, and it also affects the present cutting which occurs on the regenerative cutting width b_r. Note that these cutting widths depend on the rotational angle ϕ of the tooth, and their averages $\overline{b_p}$ and $\overline{b_r}$ for a single tooth can be expressed as follows:

$$\overline{b_p} = \left(\bar{h}/2 + \sqrt{R^2 - (R-d)^2}\right) \times (\phi_{ex} - \phi_{st})/2\pi, \quad (1)$$

$$\overline{b_r} = \left(-\bar{h}/2 + \sqrt{R^2 - (R-d)^2}\right) \times (\phi_{ex} - \phi_{st})/2\pi. \quad (2)$$

Here, $\bar{h}$ is the average uncut chip thickness in the radial direction, d is the axial depth of cut, and ϕ_{st} and ϕ_{ex} are the starting and exiting angles of the cutter, respectively. Note that $\phi_{st} = 0$ and $\phi_{ex} = \phi_{cut}$ for up milling, and $\phi_{st} = \pi - \phi_{cut}$ and $\phi_{ex} = \pi$ for down milling where ϕ_{cut} is the radial immersion angle. $\bar{h}$ can be calculated by averaging the instantaneous uncut chip thickness $h \approx c \sin\phi$ throughout the cutting, where c is the feed per tooth.

$$\bar{h} = \frac{\int_{\phi_{st}}^{\phi_{ex}} h \, d\phi}{\phi_{ex} - \phi_{st}} = \frac{c(\cos\phi_{st} - \cos\phi_{ex})}{\phi_{ex} - \phi_{st}}. \quad (3)$$

Note that the depth of cut is assumed to be smaller than or equal to the nose radius, i.e. $d \leq R$, and the approach angle ψ is 0 deg. The radial rake angle α_r is 10 deg, but its effect on the stability is neglected since the value is fairly small. Formulations without these assumptions can be found in [8].

Now, the cutting area A_j dynamically changes as follows:

$$A_j = \left(m_{j-1}^k \overline{b_r} e^{-i\omega_c T} - m_j^k \overline{b_p}\right) z_1. \tag{4}$$

Then, the material removing force $F_{rm\,j}$ fluctuates proportionally to the dynamic cutting area expressed as follows where K_a is the specific material removing force in the axial direction:

$$F_{rm_j} = K_a A_j. \tag{5}$$

Finally, considering that the dynamic force is applied to the position of the present tooth, the compliance is reduced/magnified by the modal displacement ratio of that position and results in the vibration. Since the vibration z_1 of Tooth 1 results from the dynamic forces by all of the teeth, z_1 is expressed as follows:

$$z_1 = \sum_{j=1}^{N_t} m_j^k G_1^k F_{mr_j}. \tag{6}$$

Here, N_t is the number of teeth, G_1^k is the compliance of Tooth 1, and k is the index of the mode in concern. As a result, Eqs. (4), (5), and (6) can be summarized as the following equation:

$$z_1 = G_1^k K_a \left(\sum_{j=1}^{N_t} m_j^k m_{j-1}^k \overline{b_r} e^{-i\omega_c T} - \sum_{j=1}^{N_t} \left(m_j^k\right)^2 \overline{b_p}\right) z_1. \tag{7}$$

In this paper, $\sum_{j=1}^{N_t} m_j^k m_{j-1}^k$ is called the coefficient of regenerative vibration, and $\sum_{j=1}^{N_t} \left(m_j^k\right)^2$ is called the coefficient of present vibration. Note that $m_0^k = m_{N_t}^k$. These coefficients include the effect of the mode shape on the present vibration, the regenerative vibration, and the compliance at the present cutting tooth. The block diagram of the process is shown in Fig. 6.

Note that the modal displacement ratio depends on the mode shape meaning that the stability limit needs to be determined for each mode, and chatter should occur above the minimum stability limit among those. In this paper, the immersion angle ϕ_{cut}, which is one of the parameters that decides the machining efficiency, is varied to find the stability limit. Therefore, the spindle speed $n = 60/(N_t T)$, chatter frequency ω_c, and immersion angle ϕ_{cut} that satisfy Eq. (7) are searched to find the critical stability.

4. Analytical investigation

Examples of analytical results are introduced in this section. The analytical conditions are shown in Table 2, and modal-fitted compliances are used. Note that the specific material removing force in the axial direction is a function of the depth of cut since there is a nose part on the insert, and the value shown in the table is an experimentally obtained one at the depth of cut of 0.3 mm.

A stability lobe diagram with the spindle speed as the horizontal axis is shown in Fig. 7. The vertical axes from the top figure show the immersion angle, chatter frequency, and phase shift between the present and regenerative vibrations. The solid lines show the critical immersion angles, or stability limits, in the top figure and the colors express those for different modes. The resonant frequency for each mode is expressed by broken lines in the middle figure.

From this diagram, it can be observed that regenerative chatter should occur at different modes depending on the spindle speed, which is the same as interpreted in the conventional chatter theory when several modes have possibility to cause chatter. However, two unique characteristics can be interpreted from the stability lobe diagram as follows.

a) Larger compliance between two modes does not always mean a smaller asymptotic stability limit

This characteristic is explained using Mode 1 and 5. The magnitudes of the direct compliances of Tooth 1 at the negative

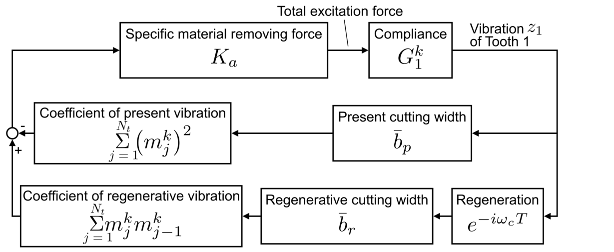

Fig. 6. Block diagram of regenerative chatter considering position-dependent modal displacement ratios in cutting direction.

Table 2. Analytical conditions.

Material property		
Specific material removing force in axial dir. K_a	450.7	MPa
Cutting conditions		
Depth of cut d	0.3	mm
Feed per tooth c	0.1	mm
Milling style	Down milling	-

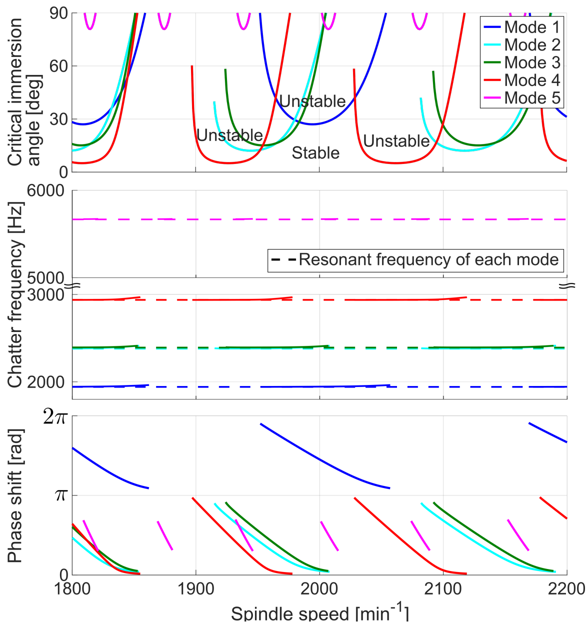

Fig. 7. Predicted stability lobe diagram considering the modal displacement ratio in the cutting direction.

peak frequencies are 2.0 and 3.0 µm/N, respectively, i.e. Mode 5 is 1.5 times larger, but the asymptotic stability limit of Mode 1 is about 3.0 times smaller than that of Mode 5. This can be explained as follows. As the mode shape affects the present and regenerative vibrations and the compliance at the present cutting tooth, their sums expressed as the coefficients $\sum_{j=1}^{N_t} m_j^k m_{j-1}^k$ and $\sum_{j=1}^{N_t} \left(m_j^k\right)^2$ affect the stability. For Mode 1, the coefficients of the regenerative and present vibrations are 5.7 and 5.7, and for Mode 5, they are −1.5 and 3.0, respectively. Since the magnitude of the coefficients for Mode 5 are smaller than those for Mode 1, which means that some of the vibrations cancel each other, the stability increases, and thus results in the reversal of stability. Do note that these coefficients for Mode 4, i.e. −5.1 and 5.1, respectively, are close to those with position-independent modal displacement ratios like Mode 1 in terms of magnitude. This is because the teeth are positioned at alternating positive/negative antinodes, and in this kind of mode, the regenerative vibrations are not cancelled out.

b) The phase shift can be less than π rad although the chatter frequency is higher than the resonant frequency

This characteristic can be observed in all modes where the coefficient $\sum_{j=1}^{N_t} m_j^k m_{j-1}^k$ of regenerative vibration is negative. In general metal cutting where the structure is mainly flexible in the depth-of-cut direction, the phase shift is within the range of π and 2π rad and the chatter frequency is higher than the resonant frequency when regenerative chatter occurs. Hence, the found characteristic is new to the literature. This can be explained as follows. Since $\sum_{j=1}^{N_t} m_j^k m_{j-1}^k$ is negative, the phase shift has to be less than π rad for the regenerative chatter to grow since the negative sign means an additional phase shift of π rad.

5. Experimental investigation

Experiments are conducted on a 3-axis machining center (MILLAC 415V, Okuma Corp.) to verify the proposed model. The experimental setup and conditions are shown in Fig. 8 and Table 3, respectively. The spindle speed and immersion angle are varied to find the critical stability, and the vibration of the cutter is measured by an eddy current sensor set on a special jig fixed to the spindle housing.

An example of the measured vibration is shown in Fig. 9. Since the eddy current sensor is fixed and the vibrating cutter is rotating, it is known that the actual vibration peak splits into two peaks in which the frequencies are the actual chatter frequency $\pm n_a \times (n/60)$ [9], where n_a is the number of waves in the vibration mode. In this example, the peak frequencies are close to Mode 3 and $n_a = 2$ for it; thus, the chatter frequency is 2404 Hz. All the other results are analysed in the same manner.

The summary of the results is shown in Fig. 10. "∘" represents that there is no chatter (maximum vibration amplitude among the frequency components is less than or equal to 0.2 μm), "△" represents that there is slight chatter (greater than 0.2 μm and less than or equal to 0.4 μm), and "×" represents that there is chatter (greater than 0.4 μm). The colors of the marks represent that the maximum vibration peak is near the negative peak frequency of the mode corresponding to the color. From these results, it can be observed that the experimental and predicted results agree.

A photograph of the cut workpiece at an unstable condition (same condition as that shown in Fig. 9) is shown in Fig. 11. Since the measured chatter frequency is 2404 Hz and the spindle speed is 2000 min-1, the calculated phase shift is 0.13 rad; it is smaller than π rad. Therefore, the chatter marks are tilted slightly counter-clockwise from the feed direction, meaning that the phase shift between the present and regenerative vibrations is less than π rad considering the feed and cutting directions; the characteristic found in the analytical investigation is true.

6. Conclusion

A regenerative chatter stability model considering the position-dependent modal displacement ratios in the cutting direction has been proposed in this paper. The cutters targeted in this research have unique mode shapes in which the modal displacement ratio changes in the cutting direction. This unique mode shape has been considered in the developed model, and it has been discovered that it causes change in the present vibration, the regenerative vibration, and the compliance of the present cutting tooth. Analyses have been carried out using the developed model, and unique characteristics have been extracted about the stability and the phase shift. Experiments have also been carried out to verify the proposed model, and the results have agreed with the predicted ones, and it is expected that the proposed model can predict the stability of chatter which occurs at the unique modes. The found unique characteristics have also been observed from the experimental results, which also verify the proposed model.

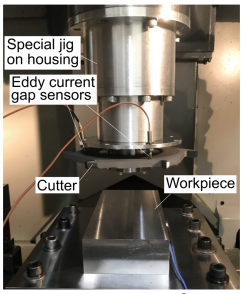

Fig. 8. Experimental setup.

Table 3. Experimental conditions.

Material property		
Material type	EN AW-7075 (DIN)	-
Cutting conditions		
Depth of cut d	0.3	mm
Feed per tooth c	0.1	mm
Spindle speed n	1875, 2000, 2020, 2070, 2150	min^{-1}
Radial immersion angle ϕ_{cut}	15, 20, 25, 30, 35, 40, 45, 50, 55, 60	deg
Milling style	Down milling	-

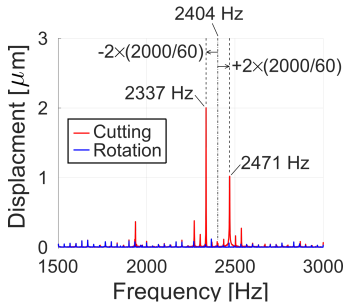

Fig. 9. Example of Fourier transform results of measured vibration at n = 2000 min-1 and ϕ_{cut} = 50 deg.

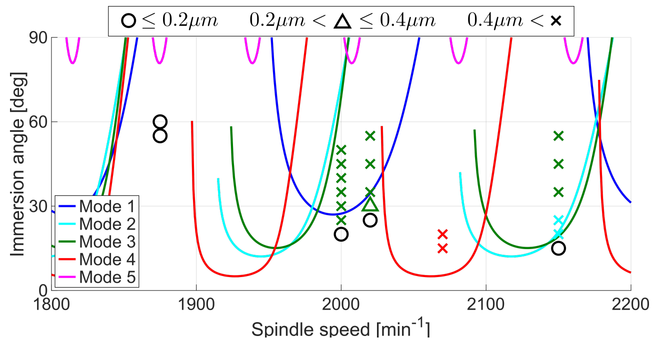

Fig. 10. Summary of experimental and predicted results.

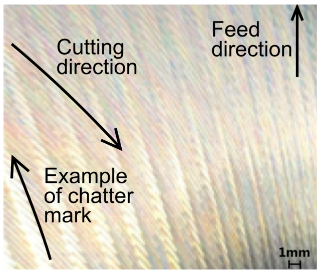

Fig. 11. Photograph of cut workpiece at unstable condition with regenerative chatter at Mode 3.

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