Anisotropy freezing of passive scalar fields in anisotropy growing homogeneous turbulence

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We study the mixing of a passive scalar in homogeneous turbulent flow with and without anisotropic external force. It is assumed that no scalar source exists and the scalar spectrum at an initial time instant t_0 is given by the form $Ck^2 + o(k^2)$ at the wave number $k \to 0$, where C is independent of k. We have performed direct numerical simulations (DNSs) of the mixing, in which the initial integral length scales of the scalar field are comparable to those of the velocity field and the Schmidt number is unity. The DNSs show that even though the large-scale anisotropy of the velocity field grows with time owing to the external force, its scalar field counterpart remains almost unchanged, i.e., frozen with respect to time, in a state where the scalar field evolves in a self-similar manner. The degree of the anisotropy of each of the velocity and scalar fields is measured by the ratios of the integral length scales in different Cartesian directions. The DNSs also suggest that the scalar spectrum keeps the form $Ck^2 + o(k^2)$ at small k for time $t (\geqslant t_0)$, and that C is time independent.

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I. INTRODUCTION

Turbulent mixing of scalar quantities is a phenomenon of multiscales and strong nonlinearity, and is encountered in many flow circumstances in engineering and geophysics [1–3]. Temperature fluctuations in a weakly heated flow, dye concentration in liquids, and so on can be modeled as passive scalars. The passive scalars are convected by flow, but do not influence the flow dynamics.

The large-scale structure of passive scalar fields $\theta(\boldsymbol{x}, t)$ without any scalar source plays significant roles in the decay of the scalar variance $\langle \theta^2 \rangle$ in three-dimensional (3D) incompressible homogeneous turbulence [4–10], where $\boldsymbol{x}$ is the position, t is time, $\langle \dots \rangle$ denotes the ensemble average of ... and we assume here $\langle \theta \rangle = 0$. In homogeneous turbulence, the ensemble average $\langle \dots \rangle$ is independent of the position $\boldsymbol{x}$. The arguments $\boldsymbol{x}$ and t are omitted at will. The structure is well characterized by the spectral scalar correlation $\hat{\Theta}(\boldsymbol{k}, t)$ at $\boldsymbol{k} \to \boldsymbol{0}$, where $\boldsymbol{k}$ is the wave vector, and $\hat{\Theta}(\boldsymbol{k}, t)$ is defined by the Fourier transform of the second order two-point scalar correlation: $(2\pi)^{-3} \int_{\mathbb{R}^3} \Theta(\boldsymbol{r}, t) \exp(-i\boldsymbol{k} \cdot \boldsymbol{r}) d\boldsymbol{r}$ and $\Theta(\boldsymbol{r}, t) = \langle \theta(\boldsymbol{x})\theta(\boldsymbol{x} + \boldsymbol{r}) \rangle$.

A recent study [10] generalized the argument by Corrsin [4] for homogeneous isotropic passive scalar turbulence, i.e., isotropic passive scalar fields in homogeneous isotropic turbulence, to homogeneous anisotropic passive scalar turbulence. At an initial time instant t_0, $\hat{\Theta}(\boldsymbol{k}, t_0)$ is assumed to be $O(k^0)$ at $\boldsymbol{k} \to \boldsymbol{0}$, where $k = |\boldsymbol{k}|$. It was shown that for any $t \geqslant t_0$, the leading order term

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of $\hat{\Theta}(\boldsymbol{k},t)$ at $\boldsymbol{k}\to\boldsymbol{0}$ keeps the zeroth order in k and is time independent, if $|\int_{\mathbb{R}^3}\langle \boldsymbol{u}(\boldsymbol{x})\theta(\boldsymbol{x})\theta(\boldsymbol{x}+\boldsymbol{r})\rangle \exp(-i\boldsymbol{k}\cdot\boldsymbol{r})d\boldsymbol{r}| = O(k^0)$ at $\boldsymbol{k}\to\boldsymbol{0}$. Here, $\boldsymbol{u}$ is the fluctuating part of the velocity field of the incompressible fluids. The time independence, called dynamical invariance, implies that the scalar spectrum $E^\theta(k)$ takes the form given by

$$E^\theta(k) = Ck^2 + o(k^2) \tag{1}$$

at $k\to 0$ for any $t \geqslant t_0$ and that C is an invariant, where $E^\theta(k) = (1/2)\int \hat{\Theta}(\boldsymbol{q})dS_k$ and $\int \dots dS_k$ denotes the integral of ... over the spherical surface with a radius of $|\boldsymbol{q}| = k$ and a center at $\boldsymbol{q} = \boldsymbol{0}$. We set here $C \neq 0$. The invariant C is equivalent to the integral $\int_{\mathbb{R}^3}\Theta(\boldsymbol{r},t)d\boldsymbol{r}$, which is called Corrsin's integral invariant [4] if the scalar field is isotropic. It was also shown in Ref. [10] that if the scalar field satisfies a certain kind of self-similarity at large scales where $|\boldsymbol{r}|$ is greater than or comparable to L_j^θ, i.e., in the wave-number range where k is smaller than or comparable to $1/L_j^\theta$, then the degree of the anisotropy of the scalar field L_i^θ/L_j^θ is time independent, i.e.,

$$L_i^\theta/L_j^\theta \simeq \text{const}, \tag{2}$$

where L_j^θ is an integral length scale of $\theta(\boldsymbol{x},t)$ in the jth Cartesian direction, and $i, j = 1, 2, 3$ $(i \neq j)$. See the Appendix, regarding the meaning of the self-similarity. The anisotropy of the scalar field is frozen with respect to time in the state satisfying the self-similarity. In addition, it was pointed out that Eq. (2) holds, irrespective of the velocity field, if the self-similarity and the invariance of C are satisfied [10]. This implies that the velocity field may be subjected to external force, such as buoyancy, the Coriolis force, and the Lorentz force. These forces are in general anisotropic, and therefore statistics of the velocity field are anisotropic in general. The anisotropy of the velocity field in the energy containing range, which is measured by L_i^u/L_j^u's, can grow or decay monotonically with time in the presence of anisotropic forcing (e.g., Refs. [11,12]), where L_j^u is an integral length scale of $\boldsymbol{u}$. Therefore, Ref. [10] suggests that even if L_i^u/L_j^u's grow or decay, Eq. (2) can hold.

In contrast, it seems natural to think that the statistics of the passive scalar field must more or less reflect the statistics of the velocity field. For the evolution of the passive scalar field in the range where $kL_j^\theta \sim 1$, one may think that the anisotropy of the scalar field evolves in accordance with the anisotropy of the velocity field, i.e., $L_i^\theta/L_j^\theta \propto L_i^u/L_j^u$, at least in the case that $L_j^\theta/L_j^u \sim 1$ at an initial time instant.

We then pose the following questions. Can we observe the freezing anisotropy of the passive scalar field? If yes, do the invariance of C and the self-similarity hold well? In order to get some idea on these questions, we study here the passive scalar mixing by turbulent flow under a simple and anisotropic external force by using direct numerical simulation (DNS). As such a force, we choose the Lorentz force under a uniformly imposed magnetic field $\boldsymbol{B}$, and consider passive scalar fluctuations without any scalar sources in two types of incompressible homogeneous turbulence: one is freely decaying hydrodynamic (HD) turbulence in the absence of $\boldsymbol{B}$, and the other is decaying magnetohydrodynamic (MHD) turbulence subjected to $\boldsymbol{B}$ at low magnetic Reynolds number such that the quasistatic (QS) approximation [11,13] is applicable.

II. BASIC EQUATIONS AND DIRECT NUMERICAL SIMULATION

The velocity field $\boldsymbol{u}$ and passive scalar field θ obey

$$\partial_t\boldsymbol{u} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} = -\frac{1}{\rho}\nabla p + \nu\nabla^2\boldsymbol{u} - \frac{\sigma B^2}{\rho}\nabla^{-2}\partial_3^2\boldsymbol{u}, \tag{3}$$

$$\partial_t\theta + (\boldsymbol{u}\cdot\nabla)\theta = \kappa\nabla^2\theta, \tag{4}$$

and the incompressible condition $\nabla\cdot\boldsymbol{u} = 0$, where $p(\boldsymbol{x},t)$ is the modified pressure including the magnetic pressure, ν is the kinematic viscosity, ρ is the fluid density, σ is the electric conductivity, κ is the scalar diffusivity, $\partial_t = \partial/\partial t$, $\nabla = (\partial_1,\partial_2,\partial_3)$, $\partial_i = \partial/\partial x_i$, ∇^{-2} represents the inverse

Laplace operator, and $\langle \boldsymbol{u} \rangle = \langle \theta \rangle = 0$. The direction of $\boldsymbol{B}$ is taken to be in the x_3 direction so that $\boldsymbol{B} = (0, 0, B)$. The kinetic energy $\langle |\boldsymbol{u}|^2 \rangle$ and the scalar variance $\langle \theta^2 \rangle$ decay with time monotonically. In homogeneous turbulence, the ensemble average $\langle \dots \rangle$ can be regarded as the space average of … under appropriate assumptions (see, e.g., the textbook by Batchelor [14]).

In deriving the external forcing term $-(\sigma B^2/\rho)\nabla^{-2}\partial_3^2 \boldsymbol{u}$, in Eq. (3), we applied the QS approximation to the induction equation and the Lorentz force term. The induction equation is given as $\partial_t \boldsymbol{b} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{b} = (\boldsymbol{b}\cdot\nabla)\boldsymbol{u} + \eta\nabla^2\boldsymbol{b}$, where $\boldsymbol{b}(\boldsymbol{x}, t) = \boldsymbol{B} + \boldsymbol{b}'(\boldsymbol{x}, t)$, $\nabla\cdot\boldsymbol{b} = 0$, $\boldsymbol{b}'(\boldsymbol{x}, t)$ is the fluctuating magnetic field, $\eta = 1/(\sigma\mu)$, η is the magnetic diffusivity, and μ is the magnetic permeability. We assume that $|\boldsymbol{b}'| \ll |\boldsymbol{B}|$, the magnetic Reynolds number is much smaller than unity, and $|\partial_t \boldsymbol{b}'|$ is negligible in the induction equation. After a little algebra, the induction equation reads as $\boldsymbol{b}' = -\sigma\mu B\nabla^{-2}\partial_3\boldsymbol{u}$. In addition, the rotational part of the Lorentz force per unit mass results in $-(\sigma B^2/\rho)\nabla^{-2}\partial_3^2\boldsymbol{u}$. For the details, one may refer to Refs. [11,13].

We performed four DNSs of the turbulence in a $(2\pi)^3$ periodic box using 1024^3 grid points: one (run 1) simulates HD passive scalar turbulence and the other three (runs 2, 3, and 4) simulate QS MHD passive scalar turbulence. The DNS method and other parameters such as ν are the same as those presented in Ref. [15]. The Schmidt number ν/κ is set to unity. The initial velocity field is identical for all the runs, and the same is true for the passive scalar field. They are statistically quasi-isotropic and random. The initial energy spectrum and passive scalar spectrum $E^\theta(k)$, respectively, take forms proportional to $k^2\exp(-k^2/k_p^2)$ and $\langle |\boldsymbol{u}|^2 \rangle = \langle \theta^2 \rangle = 1$ at the initial time $t = t_0$. Here and in the following, $\langle \dots \rangle$ denotes the space average over the periodic box, and $t_0 = 0$. We put $k_p = 60$ (see, e.g., Ref. [16] for velocity fields in freely decaying HD turbulence). The initial Reynolds number defined by $u'\mathcal{L}^u/\nu$ is 107, where $u' = 1/\sqrt{3}$, $\mathcal{L}^u = L_3^{u_3}(0)$, and the definition of $L_3^{u_3}(0)$ is given in the next paragraph. The initial $L_j^\theta(0)$ is comparable to $L_j^{u_j}(0)$. The difference among the runs lies in the difference of the interaction parameters N defined as $N = \sigma B^2\mathcal{L}^u/(\rho u')$ at $t = 0$, where $N = 0,\ 0.1,\ 0.25$, and 0.5 in runs 1, 2, 3, and 4, respectively.

Since the turbulence is statistically axisymmetric, it is convenient to introduce the integral length scales in the direction perpendicular to and parallel to $\boldsymbol{B}$;

$$L_\perp^\theta(t) = \tfrac{1}{2}\left(L_1^\theta + L_2^\theta\right), \quad L_\parallel^\theta(t) = L_3^\theta, \quad L_\perp^u(t) = \tfrac{1}{2}\left(L_1^{u_1} + L_2^{u_2}\right), \quad L_\parallel^u(t) = L_3^{u_3}. \tag{5}$$

for the scalar and velocity fields. Here, $L_j^\xi(t)$ $(\xi = \theta, u_j)$ is defined as

$$L_j^\xi(t) = \frac{\int_0^\pi \langle \xi(\boldsymbol{x}, t)\xi(\boldsymbol{x} + r\boldsymbol{e}_j, t)\rangle dr}{\langle \xi^2(\boldsymbol{x}, t)\rangle}, \tag{6}$$

where $\boldsymbol{e}_j$ is the unit vector pointing to the jth direction. Corresponding to Eq. (5), we introduce $\langle u_\perp^2 \rangle = (\langle u_1^2 \rangle + \langle u_2^2 \rangle)/2$ and $\langle u_\parallel^2 \rangle = \langle u_3^2 \rangle$. For all runs, it was confirmed that $L_1^\theta \approx L_2^\theta, L_1^{u_1} \approx L_2^{u_2}$, and $\langle u_1^2 \rangle \approx \langle u_2^2 \rangle$ in fully developed states of the turbulent scalar and velocity fields (figure omitted).

III. NUMERICAL RESULTS

In Sec. III A, we briefly describe the DNS results about the evolution of the anisotropy of the velocity field in the energy containing range. Next, in Secs. III B and III C, we examine whether Eq. (2), the invariance of C, and the self-similarity of the scalar field evolution at the large scales well hold, by using the DNSs.

A. Growing anisotropy of the velocity field

Figures 1(a) and 1(b), respectively, show the evolution of the anisotropy represented by the ratio of the integral length scales $L_\parallel^u/L_\perp^u$, and the evolution of $\langle u_\parallel^2 \rangle/\langle u_\perp^2 \rangle$. The time is normalized by the initial large-eddy turnover time T and $T = 1/k_p$. In run 1 $(N = 0)$, $L_\parallel^u/L_\perp^u$ and $\langle u_\parallel^2 \rangle/\langle u_\perp^2 \rangle$ are close to unity. On the other hand, the ratio $L_\parallel^u/L_\perp^u$ grows with time in runs 2, 3, and 4 $(N \neq 0)$, which implies

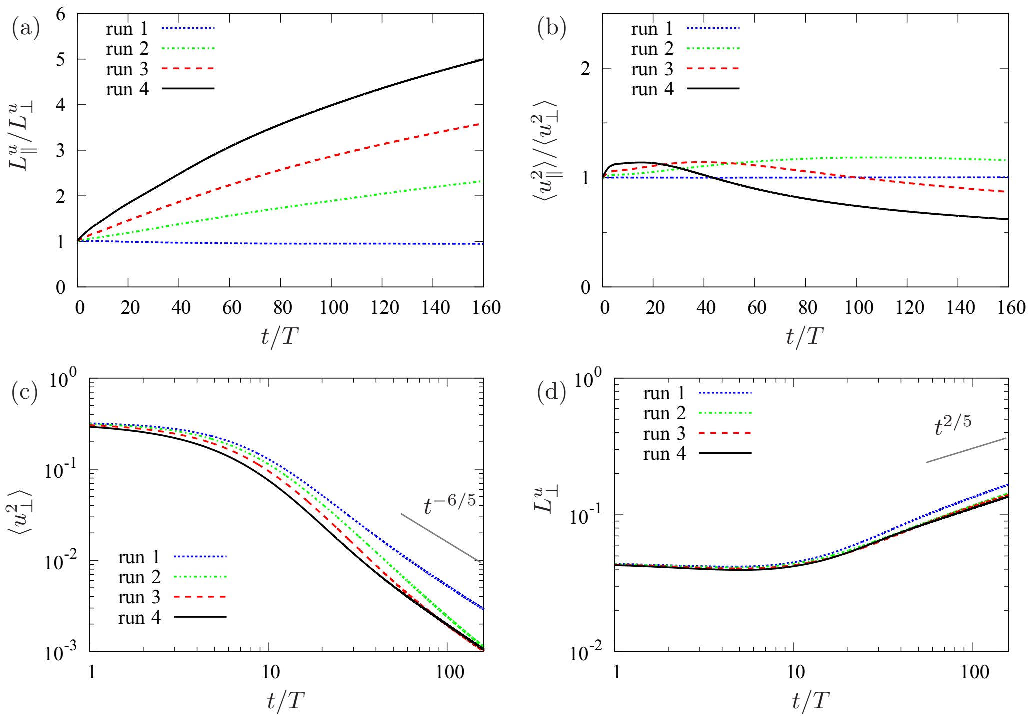

FIG. 1. Evolution of (a) $L^u_\parallel/L^u_\perp$, (b) $\langle u^2_\parallel\rangle/\langle u^2_\perp\rangle$, (c) $\langle u^2_\perp\rangle$, and (d) $L^u_\perp$ in run 1 ($N = 0$), run 2 ($N = 0.1$), run 3 ($N = 0.25$), and run 4 ($N = 0.5$).

the anisotropy growing with time. It can be also seen in Fig. 1(b) that for $t/T \gtrsim 50$, $\langle u^2_\parallel\rangle/\langle u^2_\perp\rangle$ is nearly constant in run 2 whereas $\langle u^2_\parallel\rangle/\langle u^2_\perp\rangle$ decays in runs 3 and 4. The latter is compatible with previous DNSs [17,18]. Figures 1(a)–1(d) show that in run 1, the decay of the kinetic energy and the growth of the integral length scales for $t/T \gtrsim 50$, in the fully developed state of the velocity field, well agree with Saffman's decay laws [19]: $\langle u^2_\parallel\rangle = \langle u^2_\perp\rangle \propto t^{-6/5}$ and $L^u_\parallel = L^u_\perp \propto t^{2/5}$. These power-law-like behaviors were observed in DNSs [15,16] and laboratory experiments [20].

B. Anisotropy freezing of the scalar field

Figure 2(a) shows the ratio $L^\theta_\parallel/L^\theta_\perp$ as a function of time t/T in all the runs. It can be seen that $L^\theta_\parallel/L^\theta_\perp$ is nearly time independent for $t/T \gtrsim 50$, i.e., in the fully developed states of the passive scalar turbulence. This independence contrasts with the substantial growth of $L^u_\parallel/L^u_\perp$ in runs 2, 3, and 4 shown in Fig. 1(a). This contrast implies that the anisotropy of the scalar field represented by $L^\theta_\parallel/L^\theta_\perp$ is frozen, in spite of the growing anisotropy of the velocity field represented by $L^u_\parallel/L^u_\perp$. The ratio $L^\theta_\parallel/L^\theta_\perp$ depends weakly on N. The scalar fields in runs 2, 3, and 4 become anisotropic, owing to the anisotropy of the velocity field.

In Fig. 2(b), it can be seen that $\langle\theta^2\rangle(L^\theta_\perp)^2L^\theta_\parallel$ is nearly time independent in the fully developed states, irrespective of the decay rates of $\langle\theta^2\rangle$ shown in Fig. 2(c). The time independence of $\langle\theta^2\rangle(L^\theta_\perp)^2L^\theta_\parallel$ and that of $L^\theta_\parallel/L^\theta_\perp$ are necessary conditions for the self-similarity of the large-scale scalar evolution and the invariance of C in Eq. (1) [10]. The time independence of $\langle\theta^2\rangle\ell^3$ was suggested in laboratory experiments for fully developed passive scalar turbulence [21], where ℓ is an integral length scale of the scalar field. Figures 2(a), 2(c), and 2(d) show that in run 1, $\langle\theta^2\rangle$ decays approximately as $t^{-6/5}$ while $L^\theta_\perp$ and $L^\theta_\parallel$ grow like $t^{2/5}$ for $t/T \gtrsim 50$. These power-law-like behaviors well agree with the prediction by dimensional analysis [5,6,10]. The agreement

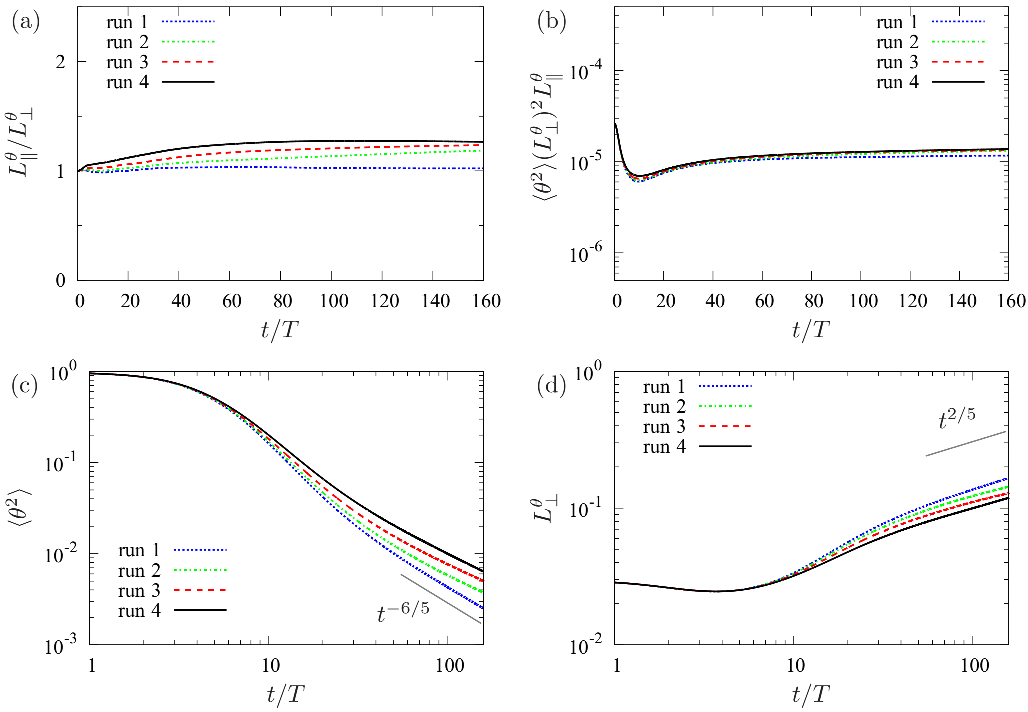

FIG. 2. Evolution of (a) $L_{\parallel}^{\theta}/L_{\perp}^{\theta}$, (b) $\langle\theta^2\rangle(L_{\perp}^{\theta})^2L_{\parallel}^{\theta}$, (c) $\langle\theta^2\rangle$, and (d) $L_{\perp}^{\theta}$ in run 1 ($N=0$), run 2 ($N=0.1$), run 3 ($N=0.25$), and run 4 ($N=0.5$).

suggests that the nonlinear term $(\boldsymbol{u}\cdot\nabla)\theta$ plays significant roles in the decay of $\langle\theta^2\rangle$. Note that $\langle|\boldsymbol{u}(\boldsymbol{x},t)|^2\rangle/\langle|\boldsymbol{u}(\boldsymbol{x},0)|^2\rangle$ is comparable to $\langle\theta^2(\boldsymbol{x},t)\rangle/\langle\theta^2(\boldsymbol{x},0)\rangle$, as shown by Figs. 1(b), 1(c), and 2(c). Here, $\langle|\boldsymbol{u}(\boldsymbol{x},0)|^2\rangle=\langle\theta^2(\boldsymbol{x},0)\rangle=1$. If a homogeneous passive scalar field is well governed by the equation that $\partial_t\theta=\kappa\nabla^2\theta$ together with its initial scalar spectrum satisfying Eq. (1) at $k\to0$, then $\langle\theta^2\rangle\propto t^{-3/2}$ for sufficiently large t [22].

C. Invariance of C and self-similarity

Figure 3 shows the spherically averaged scalar spectra $E_{\text{ave}}^{\theta}(k)$, given by $E^{\theta}(k)/(4\pi k^2)$, at different time instants in run 1 ($N=0$) and run 3 ($N=0.25$). It can be observed that at each N,

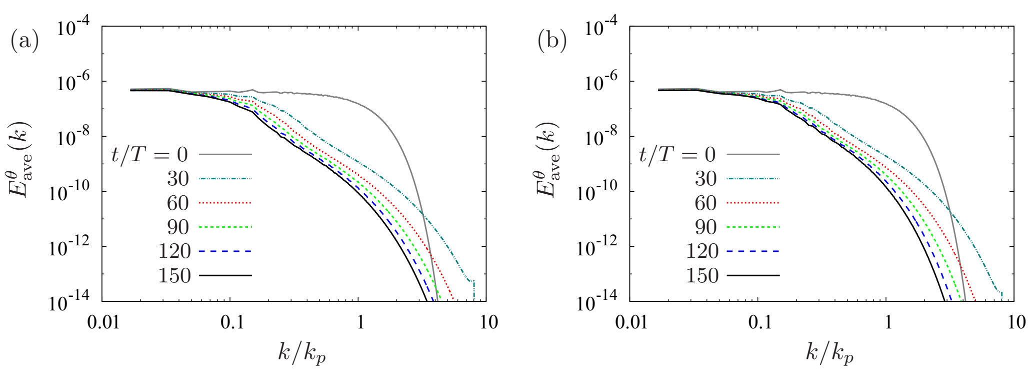

FIG. 3. Scalar spectra in (a) run 1 ($N=0$) and (b) run 3 ($N=0.25$): $E_{\text{ave}}^{\theta}(k)$ vs k/k_p.

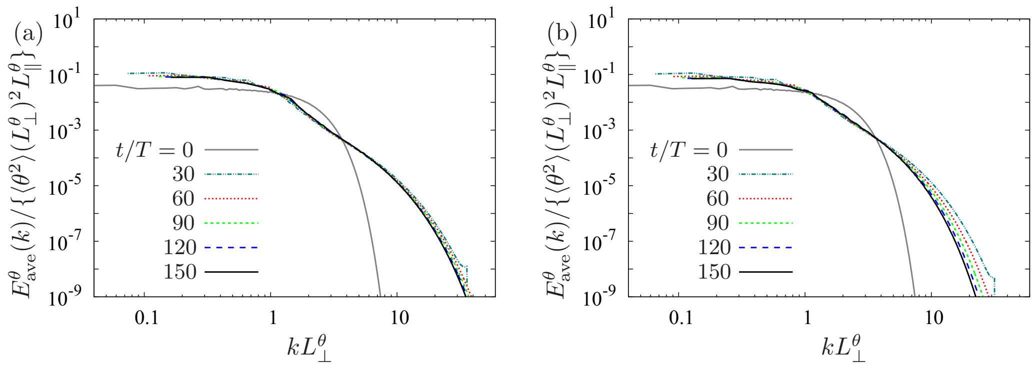

FIG. 4. Normalized scalar spectra in run 1 ($N = 0$) and (b) run 3 ($N = 0.25$): $E^{\theta}_{\rm ave}(k)/\{\langle\theta^2\rangle(L^{\theta}_{\perp})^2L^{\theta}_{\parallel}\}$ vs $kL^{\theta}_{\perp}$.

$E^{\theta}_{\rm ave}(k)$ remains nearly flat at sufficiently low k/k_p range and is nearly independent of time. This observation supports the invariance of C.

On the basis of Eqs. (2) and (A1), the self-similar evolution of the scalar field implies that the normalized spectrum $E^{\theta}_{\rm ave}(k)/\{\langle\theta^2\rangle(L^{\theta}_{\perp})^2L^{\theta}_{\parallel}\}$ is a function only of $kL^{\theta}_{\perp}$ in the wave-number range where k is smaller than or comparable to $1/L^{\theta}_{\perp}$ and in the fully developed states of the turbulent passive scalar fields. Figure 4 shows the $kL^{\theta}_{\perp}$ dependence of the normalized spectra, where $\langle\theta^2\rangle(L^{\theta}_{\perp})^2L^{\theta}_{\parallel}$ is almost constant in the fully developed states. The normalized spectra at different time instants for $t/T \gtrsim 60$ well collapse in the small $kL^{\theta}_{\perp}$ range, $kL^{\theta}_{\perp} \lesssim 10$, which verifies the self-similarity assumption of the large-scale scalar evolution at least approximately. It can be seen in Fig. 4(b) that in $kL^{\theta}_{\perp} \gtrsim 10$, the normalized spectra do not overlap, which implies that the self-similarity does not hold in the diffusive range. The lack of the self-similarity in the range is attributed to the multiscale properties of turbulence and the passive scalar fields. Small-scale statistics of the scalar can behave independently of its large-scale statistics. In contrast, the small-scale anisotropy of the scalar field can reflect that of the velocity field, as suggested by Ref. [23] for QS MHD turbulence at moderate (not high) Reynolds number in which the Lorentz force can make small-scale velocity fields substantially anisotropic. The invariance of C and the collapse of the normalized spectra are likewise observed in the other runs (figures omitted for brevity). Large eddy simulation suggested the invariance of C and the self-similarity for freely decaying isotropic passive scalar turbulence [6].

IV. CONCLUSION

We studied the large-scale structure of passive scalar fields without any scalar source in two kinds of incompressible homogeneous turbulence, i.e., the freely decaying HD turbulence and the decaying QS MHD turbulence. The initial scalar spectrum $E^{\theta}(k)$ at $k \to 0$ is given by Eq. (1). We carried out the DNSs of the passive scalar turbulence in which $L^{\theta}_{j}/L^{u}_{j} \sim 1$ at the initial time and the Schmidt number is unity. Through the DNSs, we found that the large-scale anisotropy of the scalar field is frozen for the fully developed passive scalar turbulence, regardless of the growth of the counterpart of the velocity field. In other words, $L^{\theta}_{\parallel}/L^{\theta}_{\perp}$, the measure of the scalar field anisotropy, is almost independent of time for the fully developed turbulence, even though $L^{u}_{\parallel}/L^{u}_{\perp}$, the measure of the velocity field anisotropy, grows with time. This independence is in contrast to the decay of the scalar variance $\langle\theta^2\rangle$ which depends on the velocity field via the advection $(\boldsymbol{u}\cdot\nabla)\theta$. Moreover, the DNSs showed that C in Eq. (1) is almost time independent and that the fully developed passive scalar turbulence well satisfies the self-similarity at the large scales where k is smaller than or comparable to $1/L^{\theta}_{j}$.

These DNS results are consistent with the theory [10]. Therefore, the anisotropy freezing of the scalar field is due to the invariance and the self-similarity. The self-similarity relates the invariance, which arises from a property of the leading order term of $\hat{\Theta}(\boldsymbol{k}, t)$ at $\boldsymbol{k} \to \boldsymbol{0}$, to the scalar evolution at the large scales, and the invariance partly restricts the evolution. It would be interesting to examine whether such a type of invariance and large-scale self-similarity hold in other kinds of turbulence.

Strictly speaking, the DNS results show small departure from the theoretical prediction [10]. The small discrepancy from the prediction may be due to the influences of the Reynolds number, the Péclet number, the box size, small-scale resolution, and the number of the realizations in taking the ensemble average of the DNS results. DNS study about the influences might be interesting, but would require very high computational cost. Such DNS study is therefore beyond the scope of this work.

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APPENDIX: SELF-SIMILARITY OF THE PASSIVE SCALAR FIELD

We shortly summarize the self-similarity assumption about the evolution of the passive scalar field. For the details, see Ref. [10]. We assume that $k^2\hat{\Theta}(\boldsymbol{k}, t)$ evolves in accordance with the self-similar form

$$k^2\hat{\Theta}(\boldsymbol{k}, t) = d(t) f^{\theta}(\boldsymbol{\zeta}^{\theta}) \tag{A1}$$

in the wave-number range where k is smaller than or comparable to $1/L_j^{\theta}$, and in a certain appropriate time range when the scalar field is turbulent and fully developed. Here, $\boldsymbol{\zeta}^{\theta}$ is the self-similar variable defined as $\boldsymbol{\zeta}^{\theta} = (k_1\ell_1^{\theta}, k_2\ell_2^{\theta}, k_3\ell_3^{\theta})$, $f^{\theta}(\boldsymbol{\zeta}^{\theta})$ is a dimensionless function, and $\ell_j^{\theta}(t)$ is a length scale of the passive scalar in the jth Cartesian direction. It was shown in Ref. [10] that $\ell_i^{\theta}(t)/\ell_j^{\theta}(t)$ and $d(t)\ell_i^{\theta}(t)\ell_j^{\theta}(t)$ are time independent, based on the invariance of the leading zeroth order term in k of $\hat{\Theta}(\boldsymbol{k}, t)$ at $\boldsymbol{k} \to \boldsymbol{0}$. Furthermore, we assume that the right-hand side of Eq. (A1) has dominant contribution to integrals such as $\int_{\mathbb{R}^3} \hat{\Theta}(\boldsymbol{k}, t) d\boldsymbol{k}$. After a little algebra, it was found that $\langle\theta^2\rangle L_1^{\theta} L_2^{\theta} L_3^{\theta} \simeq$ constant and that $L_j^{\theta}(t) \simeq \gamma_j^{\theta}\ell_j^{\theta}(t)$ [10], where γ_j^{θ} is constant. The time independence of $\ell_i^{\theta}/\ell_j^{\theta}$ therefore implies Eq. (2).

The passive scalar field can be anisotropic, even if the leading zeroth order term in k of $\hat{\Theta}(\boldsymbol{k})$ at $\boldsymbol{k} \to \boldsymbol{0}$ is isotropic. The anisotropy of the velocity field causes the anisotropy of the scalar field. Let the scalar field be isotropic at an initial time $t = t_0$ so that $L_1^{\theta}(t_0) = L_2^{\theta}(t_0) = L_3^{\theta}(t_0)$. Since the self-similarity assumption may not hold in an initial transient or premature stage, $L_i^{\theta}(t)/L_j^{\theta}(t)$ $(i \neq j)$ may evolve with t during the transient time period. Therefore, $L_i^{\theta}(t)/L_j^{\theta}(t)$ $(i \neq j)$ does not need to be unity in the fully developed states of the passive scalar turbulence. The anisotropy of the scalar field means that $\hat{\Theta}(\boldsymbol{k})$ is anisotropic in the range where $kL_j^{\theta} \sim 1$, even if the zeroth order term in k is isotropic.

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