ECONOMIC RESEARCH CENTER
DISCUSSION PAPER

E-Series

No.E20-1

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by

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February 2020

ECONOMIC RESEARCH CENTER
GRADUATE SCHOOL OF ECONOMICS
NAGOYA UNIVERSITY

Economic Growth, Equilibrium Welfare, and Public Goods Provision with Intergenerational Altruism

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February 14, 2020

Abstract

The present study examines the government policy of public good provision and its effects on economic growth and welfare under intergenerational altruism. We consider an endogenous growth model with altruistic overlapping generations. The preferences of the current youth exhibit a future bias and so democratically elected governments are subject to this future bias. The optimal rule of public good supply under future bias differs from the original Samuelson rule. Unlike the standard growth model without any bias, the equilibrium growth rate is not independent of government size under the optimal rule. Future bias gives young generations the dynamic incentives to invest more. With future bias, intergenerational redistributive effects of public good stimulates such incentives. Hence, the government size affects economic growth via intertemporal changes in their resource allocations. The growth effect of the government size provides nontrivial outcomes of welfare analysis. Our numerical analyses show the growth and welfare superiority of the democratic but future-biased economy to the economy with nonbiased social planner.

Keywords: Intergenerational altruism; Future bias; Economic growth; Public goods

JEL classification: D71; D72; H40; O40

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1 Introduction

The present study examines the government policy of public good provision under intergenerational altruism. The study also addresses the effects of the existence of public goods on capital accumulation and economic growth as well as welfare. The optimal provision of public goods requires that the sum of marginal rates of substitutions between the public and private good must be equal to the marginal rate of transformation between the two goods (Samuelson 1954). This is widely known as the Samuelson rule in a static model. In contrast, dynamic setting engenders different outcomes. The overlapping generations model proposed by Diamond (1965) has the potential inefficiency in a dynamic sense. The economy evolves along which growth paths evolve even though the Samuelson rule in the overlapping generations economy is parallel to static models (Batina 1990). The economy on the growth path that deviated from the modified golden rule path will be inefficient if it satisfies only the Samuelson rule.

Operative bequests such as intergenerational transfers neutralize the public intergenerational transfers on dynamic resource allocation even though each generation finitely lives (Barro 1974).[1] In other words, the intergenerational links enable different generations with finite lifetime to be infinitly-lived households. Then, the issue for the inefficiency is settled down by the bequests. The bequest behavior is analyzed by numerous studies, in particular, pioneered by Strotz (1956) and developed by Phelps and Pollak (1968) and others.[2] These approaches have adopted specifications in intergenerational altruism. Some of them prove or suggest the existence of the time inconsistency and present bias. Intergenerational altruism may affect the optimal conditions for investment decision and publicly provided goods through emerging some types of bias. Consequently, capital accumulation and economic growth should be influenced by governments' self-control problems for fiscal policy investigated in Phelps and Pollak (1968) and Krussel et al. (2002).

To address these issues, the present study constructs an endogenous growth model with public goods and intergenerational altruistic preference. In particular, the study includes public goods and the Arrow-Romer knowledge spillover into the altruistic overlapping generations model (Kimball 1987; Hori and Kanaya 1989). Gonzalez et al. (2018) proved that the intergenerationally altruistic preferences of overlapping generations exhibit future bias. The present young generation discounts the utility from the present consumption of its parent generation in comparison with the utility from the future consumption of its own. Hence, the young generation is more willing to postpone consumption at the time than at any future time (Gonzalez et al. 2018). The future bias significantly affects the political demands for intergenerational redistribution. Takeuchi (2011, 2012) found future bias through experiments. The experimental evidences suggest the importance of studying the implication of framing government policy under future bias.

The present study considers a government as a provider of public goods. One-period lived governments are obliged to supply public goods to maximize their democratic objectives. Such democratic governments are delegates of the coexisting generations such as the young and old. Since young generations have future bias while old generations has none, the governments are essentially future biased. The present study derives the extended Samuelson rule from the dynamic model with the future-biased governments. It reveals that the sum of marginal rates of substitutions between the public and private good must be weighted by the preference parameters that represent the degree of altruism for its ancestors and descendants. Public good provided in each period benefits all existing generations during the period, while its cost is financed by a tax burden on the young generation. The public good serves the intergenerational redistribution. If the young generation can control the tax or government expenditure level through political decision, it will be an effective device to reallocate the intergenerational resources. Thus, the public good as the device of redistribution and investment decision with future bias mutually affect each other.

Future bias motivates people to invest more in capital. Depending on the degree of future bias, a preference for the public good influences the investment level through the effective rate of return on the

[1] Becker (1974) found a similar result, which is known as Rotten Kid Theorem. Andreoni (1989, 1990) demonstrates that the Ricardian equivalence does not hold under impure altruism (warm glow).

[2] For instance, Kohlberg (1976), Goldman (1979), Harris (1985), Ray (1987), Kimball (1987), Hori and Kanaya (1989).

investment. Hence, the equilibrium growth rate is not independent of the government expenditure level related to the preference for the public good even if the public good is nonproductive and it is financed by the lump-sum tax. Surprisingly, in this case, it is likely that the government size affects the log-run growth rate positively under future bias. Therefore, the overall implication of the presence of public goods under future bias has also nontrivial consequences of the welfare properties. The present study numerically shows that the welfare levels of the planned economy with future-biased governments and of the decentralized economy are superior to the welfare level of the planned economy with non-biased planner. These welfare properties are parallel to those of Krusell et al. (2002) even though their model demonstrates present bias, whereas the proposed model exhibits future bias.

The present study is organized as follows. Section 2 discusses the related literatures. Section 3 describes the settings of our theoretical framework. Section 4 derives the equilibrium policy determined by future-biased authorities and characterizes the equilibrium outcome. Section 5 deals with some extensions such as decentralized economy and private provision of public goods. Finally, Section 6 provides conclusions.

2 Related literature

Numerous studies examine the issues of the provision of public goods. Bergstrom et al. (1986) developed a generalized static model of public goods and characterized its equilibrium properties. Atkinson and Stern (1974), Christiansen (1981), and Boadway and Keen (1993) studied optimal provision of public goods when the supply costs are financed by distorting taxation. The dynamically extended Samuelson rules are derived from various dynamic settings of providing public goods. For instance, Myles (1997) showed that one of the key elements is a degree of intergenerational altruism with durable public goods. In the model, perfect depreciation leads to the original form of Samuelson rule. Pirttila and Tuomala (2001) derived the modified Samuelson rule and optimal nonlinear income tax in an overlapping generations model with durable public goods.

The modified Samuelson rule derived in this study is parallel to those shown by the extant literatures, in the sense that the intertemporal preference parameters weight the marginal benefit of public goods. A planner is future biased since the planner coincides with a democratically elected government by future-biased individuals. The optimal rule for supplying public goods differs from the Samuelson rule in the original sense. In particular, the welfare weight of the old to the young in the current period and that during the times after the period are important to determine the optimal supply of public goods.

Turnovsky (1996) and Tamai (2010) clarified the theoretical interaction between public goods and economic growth in endogenous growth models. These studies demonstrated that the government expenditure does not affect the equilibrium growth rate when the public good is optimally supplied by its cost-financing coming from nondistortionary tax.[3] According to our theoretical findings, the equilibrium growth rate is affected by the preference for the public good and therefore the equilibrium government expenditure level. The government size can be associated with economic growth either positively or negatively, depending on the degree of altruism to ancestors and descendants. The result provides theoretical explanation of the controversial empirical findings.[4]

The present study is also relevant to the studies on time inconsistency of preferences (e.g., Strotz 1956; Phelps and Pollak 1968; Laibson 1997). Jackson and Yariv (2014) proved that utilitarian aggregation exhibits present bias if there is some heterogeneity in the population. They showed that three-quarters of "social planners" exhibited present biases and less than 2% were time consistent in

[3]Distortionary tax financing and tax deduction (subsidy) influence the equilibrium growth rate (e.g. Turnovsky 1996; Tamai 2010). Unsuprisingly, productive public goods affects economic growth rate. For example, Barro (1990) considered productive government expenditure as the public input, and Futagami et al. (1993) formulated public capital as the production factor.

[4]See Bergh and Henrekson (2011) for a general review on the empirical relationship between government size and economic growth.

laboratory experiments. Galperti and Strulovici (2017) developed an axiomatic theory of pure and direct altruism with nonoverlapping generations setting. Their study mainly focused on forward-looking preferences as well as treated backward-looking preferences.[5] They showed that pure altruism across generations makes it difficult to construct a welfare criterion that includes the preferences of all generations and renders a planner time consistent, excluding Phelps and Pollack's (1968) quasi-hyperbolic model.

Gonzalez et al. (2018) showed that two-sided altruistic preference of overlapping generations leads to future bias, which was experimentally suggested by Takeuchi (2011, 2012). They also demonstrated that the governments having future bias have incentives to legislate and sustain a pay-as-you-go pension system through self-enforcing commitment mechanism to increase future old-age benefits. Considering the intergenerational redistributive effects of public goods, the government policy of public good provision should be investigated under future bias. The present analysis treats the issues differently from Gonzalez et al. (2018) even though the present view is similar. The present study contributes to clarify how the governments should optimally provide the public goods.

Numerous studies consider fiscal policy if the government has time-inconsistent preferences with present bias. For instance, Krusell et al. (2002) consider a representative-agent equilibrium model with the consumer who has quasi-geometric discounting and cannot commit to future actions. They compare two different economic outcomes: one derived from the competitive economy and the other from a planning problem. The planner is a consumer representative who, without commitment but in a time-consistent way, maximizes the present-value utility subject to resource constraints. Their analysis showed that the welfare in the competitive equilibrium is strictly higher than the welfare of the planning problem whenever the discounting is not geometric.

Aronsson and Granlund (2011) investigated a provision of durable public good in an overlapping generations model with two types of consumers under present bias, and showed the formula derived by Pirttila and Tuomala (2001), including the self-control problem. Halac and Yared (2014) revealed that the ex ante optimal rule such as the simple form of a renegotiated debt limit is not sequentially optimal if the shocks to the economy are persistent. Then, the ex ante optimal rule exhibits history dependence and brings about impecunious misery by debt accumulation.[6] As a complementary to the all existing literatures, the present study covers the issues of the government policy concerning public goods provision in the growing economy under future bias.

3 The basic model

Consider a closed economy with altruistic overlapping generations who live for two periods. The population of each generation is normalized to unity. In the first (young) period, each generation works with and earns the labor income. Furthermore, the young generation receives the bequest from the parent generation. In the second (old) period, the people retire. Disposable income in the young period is allocated among the private consumptions in the first and second periods with an operative bequest. Hence, the budget equations in the young and old period are as follows:

$$\begin{aligned} c_t^y &= (1-\tau_t)\, w_t + b_t - h_t - s_t, \\ c_{t+1}^o &= (1+r_{t+1})\, s_t - b_{t+1}, \end{aligned}$$

where c_t^y is the private consumption in the young period, c_{t+1}^o the private consumption in the old period, w_t the wage rate, r_{t+1} the interest rate, τ_t the labor income tax rate, h_t the lump-sum tax, s_t the saving, b_t the bequest from their parent, and b_{t+1} the bequest for their children.

Each generation has two-sided altruism for the ancestors and descendants and benefits from own consumption on private and public goods. Assuming the time additive separability, the utility function is

$$U_t = u(c_t^y, g_t) + \beta u(c_{t+1}^o, g_{t+1}) + \mu U_{t-1} + \lambda U_{t+1},$$

[5] See Bergstrom (1999) for the study on backward-looking preferences under specific preferences.

[6] The political economy of fiscal policy also investigates this issue (e.g., Song et al. 2012; Arai et al. 2018).

where g_t is the public good in period t, β is the weight of the period-t generation's utility in the old period ($\beta > 0$), μ is the degree of altruism for their ancestors ($\mu > 0$), and λ is the degree of altruism for their descendants ($\lambda > 0$). In addition, $u(c_t^i, g_t)$ is specified as

$$u(c_t^i, g_t) = \begin{cases} \frac{\left(c_t^i\right)^{1-\sigma} + \gamma g_t^{1-\sigma}}{1-\sigma} \text{ for } \sigma \neq 1 \text{ and } \sigma > 0, \\ \log c_t^i + \gamma \log g_t \text{ for } \sigma = 1, \end{cases}$$

where $i = (y, o)$ and $\gamma > 0$.

As shown by Kimball (1987) and Hori and Kanaya (1989), $\mu + \lambda < 1$ leads to

$$\begin{aligned} U_t = & \sum_{s=1}^{\infty} \theta^s \left[u(c_{t-s}^y, g_{t-s}) + \beta u(c_{t-s+1}^o, g_{t-s+1}) \right] \\ & + u(c_t^y, g_t) + \beta u(c_{t+1}^o, g_{t+1}) + \sum_{s=1}^{\infty} \delta^s \left[u(c_{t+s}^y, g_{t+s}) + \beta u(c_{t+s+1}^o, g_{t+s+1}) \right], \end{aligned} \tag{1}$$

where

$$\theta = \frac{1 - \sqrt{1 - 4\mu\lambda}}{2\lambda} \in (0, 1)\,, \delta = \frac{1 - \sqrt{1 - 4\mu\lambda}}{2\mu} \in (0, 1)\,.$$

Total differentiation of ψ and ϕ yields

$$\frac{\partial \theta}{\partial \lambda} < 0, \frac{\partial \theta}{\partial \mu} > 0, \frac{\partial \delta}{\partial \lambda} > 0, \text{ and } \frac{\partial \delta}{\partial \mu} < 0.$$

The discount factor of the ancestors θ (of the descendants δ) is positively associated with the weight for the parents' utility μ (for the kids' utility λ) while the discount factor of the descendants δ (of the ancestors) is negatively associated with the weight for the parents' utility μ (for the kids' utility λ).

During the period t, there are two different generations: young and old generation. Omitting the term of dead ancestors, equation (1) leads to the utility function for the young generation during the period t as

$$\begin{aligned} U_t & \simeq \theta\beta u(c_t^o, g_t) + u(c_t^y, g_t) + \beta u(c_{t+1}^o, g_{t+1}) + \sum_{s=1}^{\infty} \delta^s \left[u(c_{t+s}^y, g_{t+s}) + \beta u(c_{t+s+1}^o, g_{t+s+1}) \right] \\ & = -\left(\delta^{-1} - \theta\right) \beta u(c_t^o, g_t) + \sum_{s=0}^{\infty} \delta^s \left[u(c_{t+s}^y, g_{t+s}) + \delta^{-1} \beta u(c_{t+s}^o, g_{t+s}) \right]. \end{aligned} \tag{2}$$

Similarly, the utility function for the old generation in the period t becomes

$$\begin{aligned} U_{t-1} & \simeq \beta u(c_t^o, g_t) + \sum_{s=1}^{\infty} \delta^s \left[u(c_{t+s-1}^y, g_{t+s-1}) + \beta u(c_{t+s}^o, g_{t+s}) \right] \\ & = \delta \sum_{s=0}^{\infty} \delta^s \left[u(c_{t+s}^y, g_{t+s}) + \delta^{-1} \beta u(c_{t+s}^o, g_{t+s}) \right]. \end{aligned} \tag{3}$$

Assume that the production technology takes the form of the Cobb-Douglas production function with the knowledge spillover a la Romer (1986). Formally, the production function is

$$y_t = A k_t^{\alpha} (\overline{k}_t l_t)^{1-\alpha},$$

where y_t is the output, k_t the capital input, $\overline{k}_t$ the average capital stock, l_t the labor input, $A > 0$, and $0 < \alpha \leq 1$.[7] Each firm chooses the capital and labor inputs to maximize the profit, taking the average capital stock as given. Incorporating $k_t = \overline{k}_t$ and $l_t = 1$, the production function becomes

$$y_t = A k_t. \tag{4}$$

[7] With $\alpha = 1$, the externality is perfectly internalized.

The interest rate and wage rate with $k_t = \overline{k}_t$ and $l_t = 1$ are

$$r_t = \alpha A \text{ and } w_t = (1-\alpha) A k_t. \tag{5}$$

Following Gonzalez et al. (2018), we assume that there exists a sequence of a one-period lived governments, which is responsible for supplying a public good. The government taxes the labor income or imposes the lump-sum tax to finance the government expenditure for providing public goods. We assume that the marginal rate of transformation between private and public good is equal to unity. The government's budget equation is

$$g_t = \begin{cases} \tau_t w_t > 0 \text{ and } h_t = 0 \text{ if } w_t > 0, \\ h_t > 0 \text{ and } \tau_t w_t = 0 \text{ if } w_t = 0. \end{cases} \tag{6}$$

The period-t government has the objective function is

$$W_t = U_{t-1} + \eta U_t, \tag{7}$$

where $\eta > 0$.[8] Equation (7) can be explained as a probabilistic voting model (Lindbeck and Weibull, 1987; Grossman and Helpman, 1998). Each generation t disregards dead ancestors by the period t. Hence, using equations (2) and (3), equation (7) can be reduced to

$$\begin{aligned} W_t &\simeq \delta \sum_{s=0}^{\infty} \delta^s \left[u(c_{t+s}^y, g_{t+s}) + \delta^{-1} \beta u(c_{t+s}^o, g_{t+s}) \right] \\ &+ \eta \left\{ \sum_{s=0}^{\infty} \delta^s \left[u(c_{t+s}^y, g_{t+s}) + \delta^{-1} \beta u(c_{t+s}^o, g_{t+s}) \right] - \left(\delta^{-1} - \theta \right) \beta u(c_t^o, g_t) \right\} \\ &= (\delta + \eta) \left\{ u(c_t^y, g_t) + \psi u(c_t^o, g_t) + \sum_{s=1}^{\infty} \delta^s \left[u(c_{t+s}^y, g_{t+s}) + \phi u(c_{t+s}^o, g_{t+s}) \right] \right\}, \end{aligned} \tag{8}$$

where

$$\psi \equiv \frac{(1+\eta\theta)\beta}{\delta + \eta} \text{ and } \phi \equiv \frac{\beta}{\delta}.$$

In equation (8), ψ is the relative weight of the old to the young during the period t and ϕ is the relative weight of the old to the young in the periods after the period t. The period-t government faces the different weights of the old to the young. Hence, there is a time inconsistency (Krusell et al. 2002). Furthermore, the intergenerational altruism leads to future-biased preferences (Gonzalez et al. 2018).

Since there is only one type of asset, the following equation holds:

$$s_t = k_{t+1} = a_{t+1}, \tag{9}$$

where a_{t+1} denotes the asset in the period $t+1$. Equation (9) is equivalent to the clearing condition of the asset market. This equation and the budget equations in the young and old yield

$$a_{t+1} = (1+r) a_t + (1-\tau_t) w_t - h_t - c_t. \tag{10}$$

Since $a_t \equiv k_t$, equations (5), (6), (9), and (10) lead to the following resource constraint:

$$A k_t = c_t^y + c_t^o + g_t + k_{t+1} - k_t = c_t + g_t + i_t, \tag{11}$$

where

$$c_t \equiv c_t^y + c_t^o \text{ and } i_t \equiv k_{t+1} - k_t.$$

[8] Equation (7) is functionally equivalent to the populational welfare function presented by Hori (1997). See also Aoki and Nishimura (2017) for the formulation.

4 Equilibrium government policy

This section considers that the policy making by the period-t government can actualize the desired period-t allocation. Furthermore, the study focuses on the growth and welfare properties of the planning economy. The planning problem is based on the setting developed by Krusell et al. (2002) and Gonzalez et al. (2018). The governments directly choose both the allocation of the aggregate resources between consumption and investment and of the aggregate consumption between the young and the old during the period t. As mentioned earlier, such governments can be regarded as democratically elected planners. In the present study, this economy is referred to as *a democratically planned economy.*

The static problem to choose the consumption allocation in the period t is formulated as[9]

$$\max_{0\leq\pi\leq 1}\left[u(\pi c, g)+\psi u((1-\pi)\,c, g)\right],$$

where

$$\pi \equiv \frac{c^y}{c}.$$

The subscript t is omitted from the notation hereafter (i.e., c stands for c_t). Furthermore, the prime is used to represent the variables one period later; c' is used for c_{t+1}. Solving the optimization problem, we obtain

$$\pi^* \equiv \frac{1}{1+\psi^{\frac{1}{\sigma}}}.$$

The young's share of private consumption decreases with the relative utility weight of the old period.

The following optimization problem represents the decision-making of the period-t government:[10]

$$V_0(k)=\max_{k',g}\left\{q\left(\pi,\psi\right)\frac{c^{1-\sigma}}{1-\sigma}+\Gamma\left(\psi\right)\frac{g^{1-\sigma}}{1-\sigma}+\delta V\left(k'\right)\right\}, \tag{12}$$

with

$$V(k)=q\left(\hat{\pi},\phi\right)\frac{c^{1-\sigma}}{1-\sigma}+\Gamma\left(\phi\right)\frac{g^{1-\sigma}}{1-\sigma}+\delta V(k'), \tag{13}$$

where

$$c=(A+1)\,k-g-k', \phi\equiv\frac{\beta}{\delta}, q\left(\pi,z\right)\equiv\pi^{1-\sigma}+z\left(1-\pi\right)^{1-\sigma}, \text{ and } \Gamma\left(z\right)\equiv\left(1+z\right)\gamma \text{ for } z=\phi,\psi.$$

The parameters ψ and ϕ are important to determine whether the period-t government has a present or future bias. From the restrictions of parameters, we have $\delta\theta<1$. Hence, as shown in Gonzalez et al. (2018), the period-t government has a future bias and $\psi<\phi$ holds.[11]

The first-order conditions are

$$g: -q\left(\pi,\psi\right)c^{-\sigma}+\Gamma\left(\psi\right)g^{-\sigma}=0, \tag{14}$$

$$k': -q\left(\pi,\psi\right)c^{-\sigma}+\delta\frac{\partial V(k')}{\partial k'}=0. \tag{15}$$

Equation (14) leads to

$$\left[\frac{q\left(\pi,\psi\right)}{\Gamma\left(\psi\right)}\right]^{\frac{1}{\sigma}}\frac{g}{c}=1. \tag{16}$$

[9] See Hori (1997) and Aoki and Nishimura (2017) for solving the maximization problem.

[10] Note that the value function (12) is a reduced form: $W(k)=(\eta+\delta)\,V_0(k)$.

[11] Since $\delta\theta<1$, the difference between ψ and ϕ is

$$\psi-\phi=\frac{(1+\eta\theta)\,\beta}{\delta+\eta}-\frac{\beta}{\delta}=\left[\frac{\delta\theta-1}{(\delta+\eta)\,\delta}\right]\beta\eta<0.$$

The basic premise of equation (16) is equating the sum of marginal benefits to the marginal cost, which is equal to unity. Note that the marginal benefit is measured by its weighted value. The weight in equation (16) depends on the parameters related to the intertemporal concerns for ancestors and descendants, intertemporal substitution, and so on.

Equation (16) corresponds to the extended Samuelson rule, in the sense that it optimally determines the allocation between private and public good during the period t with ψ.[12] The extended Samuelson rule will not be the best for the future periods because of the difference between ϕ and ψ. Even though the future provision of the public goods is determined by the future governments, the governments face identical decision making for the current government. Therefore, this rule is carried over to the governments. On the other hand, the deviation between ϕ and ψ brings about welfare loss by consumption misallocation. It should be compensated by more investment or more consumption. This economic response influences the equilibrium growth rate through such an investment change.

Definition 1. *A Markov strategy of the period-t government is a triplet of $\{c_t(k_t), i_t(k_t), g_t(k_t)\}$. A Markov perfect equilibrium is a set of sequences $\{c_t(k_t), i_t(k_t), g_t(k_t)\}_{t=0}^{\infty}$, satisfying equations (11), (12)–(15), and $\{c_t(k_t), i_t(k_t), g_t(k_t)\} = \{c(k_t), i(k_t), g(k_t)\}\, \forall t$.*

To derive the equilibrium government policy, we assume that the period-t government has a linear strategy and anticipates future government's policy as

$$k' = \begin{cases} \kappa k \text{ for period } t, \\ \widehat{\kappa} k \text{ for the periods after period } t. \end{cases} \tag{17}$$

The definition of investment function and equation (17) derive

$$i(k) \equiv \begin{cases} (\kappa - 1)\, k \text{ for period } t, \\ (\widehat{\kappa} - 1)\, k \text{ for the periods after period } t. \end{cases}$$

Using equations (13), (15), (16) and (17), we obtain

$$\frac{q(\pi,\psi)}{\delta}\left(\frac{\kappa}{A+1-\kappa}\right)^{\sigma}(A+1-\widehat{\kappa})^{\sigma} = \frac{\left[q(\widehat{\pi},\phi) + \Gamma(\phi)\left(\frac{\Gamma(\psi)}{q(\pi,\psi)}\right)^{\frac{1-\sigma}{\sigma}}\right](A+1-\widehat{\kappa})}{1+\left(\frac{\Gamma(\psi)}{q(\pi,\psi)}\right)^{\frac{1}{\sigma}}} + q(\pi,\psi)\,\widehat{\kappa}. \tag{18}$$

Equation (18) is the best response of the period-t government for future governments.

Following the standard growth models, we impose the following assumption.

Assumption 1. $(1+A)^{-1} < \delta < (1+A)^{\sigma-1}$.

Regarding existence and uniqueness of a Markov perfect equilibrium, equations (11), (16), (17), and (18) with $\pi = \pi^* = \widehat{\pi}$ and $\kappa = \widehat{\kappa}$ provide the following proposition (See Appendix A for the proof of Proposition 1):

Proposition 1. *In a democratically planned economy, there exists a unique Markov perfect equilibrium in linear strategies with*

$$\begin{aligned} i^*(k) &= (\kappa^* - 1)\, k > 0, \\ c^*(k) &= (A+1-\kappa^*)\, \Pi^* k > 0, \\ g^*(k) &= (A+1-\kappa^*)\,(1-\Pi^*)\, k > 0, \end{aligned}$$

[12] Myles (1997) derived a similar condition by considering a durable public good and showed that the degree of intertemporal concern (i.e., discounting based on intergenerational altruism) and long-lived nature of the public good weight the marginal benefit of public goods. He showed that the Samuelson rule is independent of the discounting rate under perfect depreciation. In contrast, the degree of intertemporal concern for ancestors and descendants is reflected in equation (16).

where

$$\Pi^* \equiv \frac{1}{1+\chi^*}, \chi^* \equiv \left[\frac{\Gamma(\psi)}{q(\pi^*,\psi)}\right]^{\frac{1}{\sigma}},$$

and κ^* *is given by the solution of equation*

$$\frac{\kappa^\sigma}{\delta} = \left\{\frac{q(\pi^*,\phi)}{q(\pi^*,\psi)} + \frac{\Gamma(\phi)}{\Gamma(\psi)}\chi^*\right\}\frac{A+1-\kappa}{1+\chi^*} + \kappa \equiv D(\kappa).$$

The last equation in Proposition 1 corresponds to the consumption Euler equation in the equilibrium. If there is no difference between ϕ and ψ ($\phi = \psi$), the right-hand side of the consumption Euler equation becomes $D(\kappa) = 1 + A$. Hence, $D(\kappa)$ captures the deviation from the gross rate of return on investment $(1 + A)$ by a future bias. The left-hand side of the consumption Euler equation represents the marginal rate of substitution (MRS) of future consumption for the current consumption. Figure 1 illustrates the degree of the deviation, $\delta D(\kappa)$, and the MRS, κ^σ, in the consumption Euler equation. The bias affects the benefit of investment (i.e. increasing future consumption). Hence, $\delta D(\kappa)$ depends on investment κ. The utility weight of the first period in the planning horizon is less than that of the period afterward. On the one hand, higher weight of future utility motivates the agent to invest. On the other hand, the current cost of investment (reducing consumption) is not sufficiently compensated by higher future consumption. Therefore, $\delta D(\kappa)$ is decreasing in κ, which indicates the downward curve in Figure 1. Since κ^σ is the upward curve, these two graphs reveal that there exists a unique intersection point E.

To verify the properties of the dynamic equilibrium shown in Proposition 1, we consider the planning economy that the period-t government can commit to all allocations from period $t + 1$ onward as a benchmark case (Gonzalez et al., 2018). The equilibrium in the planned economy with a nonbiased dictator (for instance, the elderly) can be derived from a standard dynamic optimization problem.[13] This benchmark economy is referred to as *an elderly planned economy*. The problem of choosing the sharing rule is rewritten as

$$\max_{0\le\pi\le1} \left[u(\pi c, g) + \phi u((1-\pi)c, g)\right].$$

Hence, the sharing rule $\pi^\dagger$ and the optimal growth factor become

$$\pi^\dagger = \frac{1}{1+\phi^{\frac{1}{\sigma}}} \text{ and } \kappa^\dagger = \delta^{\frac{1}{\sigma}}(1+A)^{\frac{1}{\sigma}}.$$

Furthermore, the consumption and investment functions are

$$i^\dagger(k) = \left(\kappa^\dagger - 1\right)k, c^\dagger(k) = \left(A+1-\kappa^\dagger\right)\Pi^\dagger k, \text{ and } g^\dagger(k) = \left(A+1-\kappa^\dagger\right)\left(1-\Pi^\dagger\right)k,$$

where

$$\Pi^\dagger \equiv \frac{1}{1+\chi^\dagger} \text{ and } \chi^\dagger \equiv \left[\frac{\Gamma(\phi)}{q(\pi^\dagger,\phi)}\right]^{\frac{1}{\sigma}}.$$

Comparisons between the democratically planned (future-biased) case and the elderly planned case (nonbiased) characterize the properties of the Markov equilibrium derived in Proposition 1. First, the following results concerning private and public consumptions are obtained (See Appendix B for the proof of Proposition 2):

Proposition 2. *The sharing rules of private consumptions in the two planned economies complies with* $\pi^* > \pi^\dagger$. *The ratios of public goods consumption to private goods consumption in the two planned economies satisfy the following properties:*

$$\begin{aligned} \chi^* &< \chi^\dagger \text{ for } 1 < \psi < \phi, \\ \chi^* &\gtreqless \chi^\dagger \text{ for } \psi < 1 < \phi, \\ \chi^* &> \chi^\dagger \text{ for } \psi < \phi < 1. \end{aligned}$$

[13] The derivations of key equations in the planned economy are derived from the results of Proposition 1 by substituting ϕ for ψ.

The first result of Proposition 2 is identical as that of Gonzalez et al. (2018). A present bias induces higher share of the young consumption to aggregate consumption. The second result shows that the size relation of χ^* and χ^o depends on the size of ψ and ϕ. We have

$$\begin{aligned}\Gamma(\psi) &= (1+\psi)\gamma < (1+\phi)\gamma = \Gamma(\phi),\\ q(\pi^*,\psi) &= \left(1+\psi^{\frac{1}{\sigma}}\right)^{\sigma} < \left(1+\phi^{\frac{1}{\sigma}}\right)^{\sigma} = q\left(\pi^{\dagger},\phi\right).\end{aligned}$$

Then, σ plays a key role of determining the size of χ^* and $\chi^{\dagger}$. Since the private good is rival in consumption, the intergenerational allocation of the private consumption is affected by the utility weight ψ depending on the elasticity of intertemporal substitution σ. In contrast to the private good, the public good is nonrival in consumption. The supply of the public good is not affected by the intertemporal factors.

Considering $\sigma > 1$ as an example,[14] if $\psi > 1$, the impact of a rise in ψ on the utility weight of private consumption is smaller than the impact on $\Gamma(\psi)$, because

$$\frac{\psi}{\Gamma(\psi)}\frac{d\Gamma(\psi)}{d\psi} - \frac{\psi}{q(\pi^*,\psi)}\frac{dq(\pi^*,\psi)}{d\psi} = \frac{\psi-\psi^{\frac{1}{\sigma}}}{(1+\psi)\left(1+\psi^{\frac{1}{\sigma}}\right)} > 0.$$

Hence, $\psi > 1$ leads to $\chi^* < \chi^{\dagger}$. When $\phi < 1$, $\chi^* > \chi^{\dagger}$ holds through the opposite mechanism. $\phi > 1 > \psi$ is the intermediate situation of the former two cases. Depending on the size of ψ and ϕ, each of $\chi^* < \chi^{\dagger}$ and $\chi^* > \chi^{\dagger}$ is possible.

Further, exploring the growth properties, the comparison between κ^* and $\kappa^{\dagger}$ and the partial derivative of κ^* with respect to γ lead to the following proposition (See Appendix C for the proof of Proposition 3):

Proposition 3. *The growth properties in the two planned economies are*

$$\kappa^* > \kappa^{\dagger} \text{ and } \frac{\partial\kappa^*}{\partial\gamma} \gtreqless 0 \Leftrightarrow \psi^{\frac{\sigma-1}{\sigma}} \gtreqless 1.$$

Figure 1 explains the results of Proposition 3. The equilibrium in the elderly planned economy is given by the point F, while the equilibrium in the democratically planned economy is the point E in Figure 1. As explained earlier, the degree of deviation, $D(\kappa)$, is larger than $(1+A)$ for any value of $\kappa \in [0, 1+A)$. Future bias boosts investment in private capital. This implies that the growth rate in the democratically planned economy exceeds that in the elderly planned economy.

Proposition 3 shows that a preference for public goods γ affects the equilibrium growth rate. Without any bias, the optimal growth rate is independent of γ (e.g., Turnovsky 1996; Tamai 2010).[15] In particular, γ affects $D(\kappa)$ through a change in χ if there is a future bias. The impact depends on the relative weight of the old to the young ψ and the elasticity of intertemporal substitution $1/\sigma$. A rise in γ decreases the marginal cost of public fund by increasing the weighted marginal benefit of public goods. It changes the allocation between private and public goods. Consequently, χ rises. Depending on the size of $\psi^{\frac{\sigma-1}{\sigma}}$, a rise in χ increases or decreases the effective rate of return on investment, which induces more or less investment. The background of the mechanism is originated in nonrivalness of the public good in consumption.

Finally, the welfare properties are examined to make the following proposition (See Appendix for the proof of Proposition 4):

[14]If $\sigma < 1$, the opposite effects work. When $\sigma = 1$, there is no difference between χ^* and $\chi^{\dagger}$. The terms of the utility weights are offset each other.

[15]Turnovsky (1996) also showed that the financing methods of public goods supply affect the growth rate because of the distortionary taxes.

Proposition 4. *The value functions in the two planned economies are*

$$\begin{aligned} W^* &= \Delta^* v(k,\kappa^*), \\ W^\dagger &= \Delta^\dagger v(k,\kappa^\dagger), \end{aligned}$$

where

$$\begin{aligned} v(k,\kappa) &\equiv \frac{(\delta+\eta)(A+1-\kappa)^{1-\sigma}k^{1-\sigma}}{(1-\delta\kappa^{1-\sigma})(1-\sigma)}, \\ \Delta^* &\equiv \left\{\left[1-\delta(\kappa^*)^{1-\sigma}\right]q(\pi^*,\psi)+\delta(\kappa^*)^{1-\sigma}q(\pi^*,\phi)\right\}(\Pi^*)^{1-\sigma} \\ &\quad +\left\{\left[1-\delta(\kappa^*)^{1-\sigma}\right]\Gamma(\psi)+\delta(\kappa^*)^{1-\sigma}\Gamma(\phi)\right\}(1-\Pi^*)^{1-\sigma}, \\ \Delta^\dagger &\equiv q\left(\pi^\dagger,\phi\right)\left(\Pi^\dagger\right)^{1-\sigma}+\Gamma(\phi)\left(1-\Pi^\dagger\right)^{1-\sigma}. \end{aligned}$$

In the present value of utility W, the coefficient Δ is integrated weights of private and public consumptions and $v(k,\kappa)$ is the utility level measured by a unit of integrated consumptions. For a given k, the value of κ to maximize $v(k,\kappa)$ is $\kappa=\kappa^\dagger$.[16] Hence, $v(k,\kappa^*)<v(k,\kappa^\dagger)<0$ holds. Ignoring the difference between ϕ and ψ, $\kappa=\kappa^\dagger$ is the best solution of maximizing the welfare, similar to the standard AK growth model with geometrical discounting. In other words, the excess investment reduces the social welfare. However, there is a difference of the utility weights ϕ and ψ, so that Δ^* and $\Delta^\dagger$. Furthermore, κ^* affects the level of Δ^*. Given that the size relation of Δ^* and $\Delta^\dagger$ is ambiguous, it is analytically hard to compare the welfare level. Alternatively, quantitative analysis provides obvious numerical examples.

The parameters and the initial capital stock are specified as $\beta=0.8$, $\eta=0.8$, $\sigma=2$, $A=2.25$, and $k_0=1$. Figure 2 illustrates two curves W^* and $W^\dagger$ with respect to $\gamma\in[0,2]$ for $\mu=0.4$ and $\lambda=0.5$. It demonstrates that the welfare in the democratically planned economy is larger than that in the elderly planned economy. Furthermore, the welfare difference $\left(W^*-W^\dagger\right)$ increases with γ. Setting $\gamma=0.5$, the robustness to changes in μ and λ is examined. Figure 3 reveals that the welfare difference is positive on the domain of μ and λ. These results imply that the democratically planned economy exhibits the welfare dominance to the elderly planned economy.

5 Extensive analysis and applications

In the previous section, we have assumed that the period-t government has the policy instruments to execute its desired period-t allocation. The assumption allows clarification of the policy insights. However, we also should consider the market economy in which the government policy decision is separated from personal decision making as an example of fiscal policy with some limitations. This section characterizes the decentralized dynamic equilibrium by using the results in the previous section as an analytical basis. Furthermore, the equilibrium outcomes with voluntary provision of international/interregional public goods are investigated.

5.1 Competitive equilibrium

The analysis developed in the previous section is applicable to the market economy with public provision of public goods if the representative chooses the consumption level subject to equation (16) with the resource constraint. Then, the outcome in the market economy is given by Proposition 1.

[16] Partial differentiation of $v(k,\kappa)$ with respect to κ yields

$$\frac{\partial v}{\partial\kappa}=\frac{(A+1-\kappa)^{-\sigma}\left[(1+A)\delta\kappa^{-\sigma}-1\right]}{(1-\delta\kappa^{1-\sigma})^2}\gtreqless 0\Leftrightarrow\kappa\lesseqgtr\delta^{\frac{1}{\sigma}}(1+A)^{\frac{1}{\sigma}}=\kappa^\dagger.$$

However, the decentralized economy should be considered in the sense that the determination of the government policy is separated from the decision making of the representatives. Then, the household's optimization problem is formulated as

$$V_0(a,\omega|\Omega) = \max_{a'} \left\{ q(\pi,\psi)\frac{c^{1-\sigma}}{1-\sigma} + \Gamma(\psi)\frac{g^{1-\sigma}}{1-\sigma} + \delta V(a',\omega'|\Omega) \right\}, \tag{19}$$

with

$$V(a,\omega|\Omega) = q(\hat{\pi},\phi)\frac{c^{1-\sigma}}{1-\sigma} + \Gamma(\phi)\frac{g^{1-\sigma}}{1-\sigma} + \delta V(a',\omega'|\Omega). \tag{20}$$

In equations (19) and (20), Ω denotes the information set of the current and future government policies.

The representative chooses the investment level to maximize the value function for the given government policies. Subsequently, the government determines the level of ω to maximize the value function subject to the decision rule derived by the representative. The detail of Ω is essential to obtain the closed-form solution of the problem. The period-t generation anticipates the tax-related variable and public goods,

$$\omega' = \widehat{\kappa}\omega \text{ and } g' = \widehat{\kappa}g, \tag{21}$$

where $\widehat{\kappa}$ is the predicted growth rate and

$$\omega \equiv \begin{cases} (1-\tau)\, w \text{ if } w > 0, \\ -h \text{ if } w = 0. \end{cases}$$

The first-order condition of the household's optimization problem is

$$a' : -q(\pi,\psi)\, c^{-\sigma} + \delta\frac{\partial V(a',\omega')}{\partial a'} = 0. \tag{22}$$

As described in the previous section, if the period-t generation adopts a linear strategy and the person prospects investment policies of future generations as

$$a' = \begin{cases} \kappa a \text{ for period } t, \\ \widehat{\kappa} a \text{ for the periods after period } t. \end{cases} \tag{23}$$

The concept of our dynamic competitive equilibrium is defined as follows:

Definition 2. *A dynamic competitive equilibrium is a set of sequences* $\{c_t, \pi_t, g_t, k_t, \omega_t\}_{t=0}^{\infty}$ *satisfying equations (5), (6), (9), (10), (19)–(23),* $\pi_t = \pi^*$*, and* $\arg\max_{\omega_t} V_0$*, given* k_0*.*

In addition to Assumption 1, we impose the following assumption.

Assumption 2. $(1+\alpha A)^{-1} < \delta$.

Then, equations (19)–(23) provide the following results (See Appendix E for the proof of Proposition 5):

Proposition 5. *There exists a unique dynamic competitive equilibrium in linear strategies with*

$$\begin{aligned} i^\star(k) &= (\kappa^\star - 1)\, k > 0, \\ c^\star(k) &= (A + 1 - \kappa^\star)\, \Pi^\star k > 0, \\ g^\star(k) &= (A + 1 - \kappa^\star)(1 - \Pi^\star)\, k > 0, \end{aligned}$$

where

$$\Pi^\star \equiv \frac{1}{1+\chi^\star}, \chi^\star \equiv \left\{ \frac{\left[1-\delta(\kappa^\star)^{1-\sigma}\right]\Gamma(\psi) + \delta(\kappa^\star)^{1-\sigma}\Gamma(\phi)}{\left[1-\delta(\kappa^\star)^{1-\sigma}\right] q(\pi^*,\psi) + \delta(\kappa^\star)^{1-\sigma} q(\pi^*,\phi)} \right\}^{\frac{1}{\sigma}},$$

and $\kappa^\star$ *is given as the solution of the equation*

$$\frac{\kappa^\sigma}{\delta} = \frac{q(\pi^*,\phi)(\alpha A + 1 - \kappa)}{q(\pi^*,\psi)} + \kappa \equiv F(\kappa).$$

The interpretation of Proposition 5 is similar to that of Proposition 1. The differences between the results in the two propositions are the impacts of an externality α and of the separate decision making. Unlike the planned economy, the externality through knowledge spillover is not removed in the competitive economy without an instrument to reduce it. The allocation between private and public consumption is separated from the investment decision making by the household. Hence, the equilibrium growth rate in the dynamic competitive equilibrium is independent of a taste for public good γ.

To characterize the dynamic competitive equilibrium, we compare the outcomes of Propositions 1 and 5. The young's share of private consumption in the dynamic competitive equilibrium is equal to π^*. Hence, we obtain $\pi^* > \pi^\dagger$. Considering to the properties of the dynamic competitive equilibrium, we have the following proposition (See Appendix F for the proof of Proposition 6):

Proposition 6. *Private and public consumptions in the dynamic competitive equilibrium satisfies*

$$\frac{g^\star}{c^\star} \gtreqless \frac{g^*}{c^*} \Leftrightarrow \psi^{\frac{\sigma-1}{\sigma}} \gtreqless 1.$$

The growth rate in the dynamic competitive equilibrium exhibits $\kappa^\star \leq \kappa^*$ if $\psi^{\frac{\sigma-1}{\sigma}} \geq 1$. If $\psi^{\frac{\sigma-1}{\sigma}} < 1$, the growth rate satisfies $\kappa^\star \gtreqless \kappa^* \Leftrightarrow \alpha \gtreqless \alpha^*$ with

$$\frac{\partial \alpha^*}{\partial \gamma} < 0.$$

Proposition 6 shows that $\psi^{\frac{\sigma-1}{\sigma}}$ is the key element of characterizing the public good supply and growth properties. The intuitions of Propositions 2 and 3 explain its basic mechanism. Focusing on the growth properties, when $\psi^{\frac{\sigma-1}{\sigma}} \geq 1$, the growth rate in the dynamic competitive equilibrium is less than the growth rate in the democratically planned economy. The capital (positive) externality leads to less investment because it decreases the net return on investment. Future bias increases the net return on investment and induces more investment. Furthermore, κ^* increases with γ when $\psi^{\frac{\sigma-1}{\sigma}} \geq 1$ (Proposition 3). Therefore, we have $\kappa^\star \leq \kappa^*$ if $\psi^{\frac{\sigma-1}{\sigma}} \geq 1$. In contrast, there is a critical value of α satisfying $\kappa^\star = \kappa^*$ when $\psi^{\frac{\sigma-1}{\sigma}} < 1$. It implies that $\kappa^\star > \kappa^*$ occurs if γ is sufficiently large. Given that κ^* decreases with γ when $\psi^{\frac{\sigma-1}{\sigma}} < 1$ (Proposition 3), the net return on investment in the dynamic competitive economy dominates the net return in the democratically planned economy for a sufficiently low γ. Hence, $\kappa^\star > \kappa^*$ if $\alpha > \alpha^*$. In all the cases, note that there is the possibility of $\kappa^\star > \kappa^\dagger$. Future bias boosts the equilibrium growth.

As an example of the second result in Proposition 6, the growth properties are quantitatively assessed. The same values of the parameters of β, η, σ, and A, and the initial capital stock are used in the previous section. The value of α varies from 0 to 1. Focusing on $\psi^{\frac{\sigma-1}{\sigma}} < 1$, we set $\mu = 0.1$ and $\lambda = 0.2$; $\psi^{\frac{\sigma-1}{\sigma}} \approx 0.928$ with $\beta = 0.8$ and $\eta = 0.8$. Figure 4a displays two curves of $(\kappa^\star - \kappa^*)$ with two different values of γ on $\alpha \in [0.98, 1]$. It shows that there exists the value of equating $\kappa^\star$ and κ^*. Since the critical values are nearly equal to unity, $\kappa^\star < \kappa^*$ is plausible within realistic values of $\alpha \in (0.3, 0.6)$. On the other hand, Figure 4b shows that $\kappa^\star > \kappa^\dagger$ ($\kappa^\star < \kappa^\dagger$) holds for sufficiently large (small) α.[17] Hence, $\kappa^\star > \kappa^\dagger$ is a realistic situation.

The welfare level in the competitive economy depends on the size of spillover effect. It complicates the welfare analysis. Numerical analysis enables direct comparison of the two different welfare outcomes under different equilibria. Figure 5a illustrates the two curves of $(W^\star - W^*)$ with two different values of γ on $\alpha \in [0.85, 1]$. $W^\star$ is larger than W^* for $\alpha \in (0.86, 0.99)$. This result shows that there is a possibility of welfare dominance of the decentralized economy in comparison with the elderly planned economy, even though the values of α are far from the realistic value range in this case. In contrast, $W^\star$ dominates $W^\dagger$ for the values of α around 0.5 in Figure 5b. These numerical results demonstrate the welfare superiority of the decentralized economy as shown in Krusell et al. (2002).

[17] The critical value is calculated as $\alpha \approx 0.317$.

5.2 Voluntary provision

The basic model developed in Sections 3 and 4 can be extended to the model with voluntary provision of international/interregional public goods. The setup is in line with Tamai (2010) without a redistribution policy except for the altruistic agents with future bias. Consider that the economy consists of n symmetric regions. All regions contribute to providing interregional public goods. Let G be the supply of the interregional public goods and g be the contribution by each region. Then, $G = ng$ holds. All the other setups except for the public goods in the previous sections are unchanged. Note that all outcomes are identical to those of Section 3 and 4 when $n = 1$. Under these settings, the optimization problem for the period-t regional government is

$$V_0(k) = \max_{k',g} \left\{ q(\pi, \psi) \frac{c^{1-\sigma}}{1-\sigma} + \Gamma(\psi) \frac{G^{1-\sigma}}{1-\sigma} + \delta V(k') \right\},$$

with

$$V(k) = q(\hat{\pi}, \phi) \frac{c^{1-\sigma}}{1-\sigma} + \Gamma(\phi) \frac{G^{1-\sigma}}{1-\sigma} + \delta V(k').$$

Solving the optimization problem, we obtain

$$\frac{G}{c} = n^{\frac{1}{\sigma}} \chi^* \text{ and } \frac{g}{c} = n^{\frac{1-\sigma}{\sigma}} \chi^*. \tag{24}$$

When $n = 1$, equation (24) becomes equation (16). Equation (24) indicates that the contribution by each region decreases with the number of the regions, while the supply of public goods increases with the number of the regions (e.g., Andoreoni 1988).[18]

Defined $\widetilde{\chi}$ as $n^{\frac{1-\sigma}{\sigma}} \chi^*$, the investment and consumption functions are obtained as follows:

$$\begin{aligned}
\widetilde{i}(k) &= (\widetilde{\kappa} - 1) k > 0, \\
\widetilde{c}(k) &= (A + 1 - \widetilde{\kappa}) \widetilde{\Pi} k > 0, \\
\widetilde{g}(k) &= (A + 1 - \widetilde{\kappa}) \left(1 - \widetilde{\Pi}\right) k > 0,
\end{aligned}$$

where

$$\widetilde{\Pi} \equiv \frac{1}{1 + \widetilde{\chi}}.$$

Note that $\widetilde{\kappa}$ is derived from

$$\frac{\kappa^\sigma}{\delta} = \left\{ \frac{q(\pi^*, \phi)}{q(\pi^*, \psi)} + \frac{\Gamma(\phi)}{\Gamma(\psi)} \widetilde{\chi} \right\} \frac{A + 1 - \kappa}{1 + \widetilde{\chi}} + \kappa \equiv H(\kappa, n).$$

These equations shows that Proposition 1 still holds with the values of κ, χ, and Π which are different from its their originals. Therefore, the basic properties of the equilibrium are similar to those of the basic model. Given that $\widetilde{\chi}$ depends on n, a change in n affects the equilibrium outcome.

The study then focuses on the growth property concerning a change in the number of regions n. Differentiating $H(\kappa, n)$ with respect to n leads to

$$\frac{\partial H(\kappa, n)}{\partial n} = \frac{\frac{\Gamma(\phi)}{\Gamma(\psi)} - \frac{q(\pi^*,\phi)}{q(\pi^*,\psi)}}{(1 + \widetilde{\chi})^2} \left(\frac{1-\sigma}{\sigma} \right) \frac{\widetilde{\chi}}{n}. \tag{25}$$

From equation (25), the following result is derived.

[18] Chamberlin (1974) and McGuire (1974) studied how group size affects contributions to public goods, and Andreoni (1988) generalized their results. Using a repeated game, Pecorino (1999) proved the existence of admissible values of the discount parameter such that cooperation may be maintained in the limit.

Proposition 7. *For $\sigma > 1$, an increase in the number of regions affects the equilibrium growth rate in accordance with*

$$\frac{\partial \widetilde{\kappa}}{\partial n} \gtreqless 0 \Leftrightarrow \psi^{\frac{\sigma-1}{\sigma}} \lesseqgtr 1.$$

The number of regions n determines the value of $\widetilde{\chi}$. Hence, the similar (but opposite) mechanism in the case of γ works. Tamai (2010) also showed that the number of agents affects the equilibrium growth rate under a redistributionary taxation even though such an effect does not exist without the redistribution. In the model, redistributional taxation is the key to derive the interdependency between population and growth. Therefore, the importance of the future bias is demonstrated, which originates from a two-sided altruism. Proposition 7 indicates that such altruistic behavior should be considered in the dynamic analysis of public goods provision.

6 Conclusion

The present study examined the government policy of the public good provision and its effects on the economic growth and welfare in an endogenous growth model with altruistic overlapping generations. In the model, the democratically elected government is subject to future bias, which has been inherited from the existing individuals. The future bias influences the equilibrium government policy and economic performance of the equilibrium. Without any bias, the government policy of the public good provision does not affect the equilibrium resource allocation and therefore the economic growth. However, the economic growth is not independent of the government policy with future bias. This growth effect of the government policy provides nontrivial outcomes in welfare analysis. In particular, the welfare in the democratically planned economy dominates that in the elderly planned economy.

Some extensions were developed to confirm the robustness of the results derived from the basic analysis. In reality, the government might not have enough instruments to control the resource allocation of the economy. The in-depth analysis considered that the determination of the government policy is separated from personal decision making. Even if separate decision making is assumed, the government and individuals are still subject to future bias. Given that an externality resulting from knowledge spillover occurs in the decentralized economy, the equilibrium growth rate and welfare in the decentralized economy can be larger or smaller than those in the elderly planned economy depending on the degree of the knowledge spillover. Quantitative analysis shows the superiority of the decentralized economy to the elderly planned economy in the aspects of both growth and welfare.

The analysis on voluntary provision of public goods was also examined and applied to the regional government policy of public good provision. We supposed that there are multiple regions with the regional governments. The regional governments are elected in a similar manner to the democratically planned economy. Hence, the difference between the planned economy and multiregions economy is only the scale effect caused by the nonrivalness in consumption and the presence of the multiregions. The number of the regions has significant effects on economic growth and welfare in the multiregions economy. Furthermore, the underlying mechanisms of the impacts are based on those of the basic model. Therefore, the in-depth analysis indicates that the results from the basic model are robust.

Finally, future directions of this research should be described. First, incorporating distortionary-tax financing with labor-leisure choice into our model is a natural way to extend our analysis. As mentioned earlier, some studies address the similar issues with present bias. The extension of our analysis will give a different policy insight, which is important to consider the policy with intergenerational conflicts. Second, it is interesting to consider the public good in the production and allocation between two public goods in the utility and production function. In relation to this extension, a public capital as durable public goods will be worthwhile to investigate. These analyses will lead to important policy implications under the democratic determination of policy and its effects on economic growth and welfare. The present study provides an analytical basis of these future studies.

Appendix

A. Proof of Proposition 1

Differentiating (13) with respect to k and using equations (15) and (16) yield the following:

$$\begin{aligned}\frac{\partial V(k)}{\partial k} &= \left\{ q\left(\hat{\pi},\phi\right) + \Gamma\left(\phi\right)\left[\frac{\Gamma\left(\psi\right)}{q\left(\pi,\psi\right)}\right]^{\frac{1-\sigma}{\sigma}} \right\} c^{-\sigma}\frac{\partial c}{\partial k} + \delta \frac{\partial V(k')}{\partial k'}\frac{\partial k'}{\partial k} \\ &= \left\{ q\left(\hat{\pi},\phi\right) + \Gamma\left(\phi\right)\left[\frac{\Gamma\left(\psi\right)}{q\left(\pi,\psi\right)}\right]^{\frac{1-\sigma}{\sigma}} \right\} c^{-\sigma}\frac{1+A-\frac{\partial k'}{\partial k}}{1+\left[\frac{\Gamma(\psi)}{q(\pi,\psi)}\right]^{\frac{1}{\sigma}}} + q\left(\pi,\psi\right)c^{-\sigma}\frac{\partial k'}{\partial k},\end{aligned} \tag{A1}$$

where

$$\frac{\partial c}{\partial k} = \frac{A+1-\frac{\partial k'}{\partial k}}{1+\left[\frac{\Gamma(\psi)}{q(\pi^*,\psi)}\right]^{\frac{1}{\sigma}}}.$$

Inserting $\pi = \pi^* = \hat{\pi}$ and $\kappa = \widehat{\kappa}$ into equation (18) leads to

$$\frac{\kappa^\sigma}{\delta} = \frac{\frac{q(\pi^*,\phi)}{q(\pi^*,\psi)} + \frac{\Gamma(\phi)}{\Gamma(\psi)}\chi^*}{1+\chi^*}\left(A+1-\kappa\right)+\kappa. \tag{A2}$$

The left-hand side of this equation, κ^σ/δ, monotonically increases with κ. Furthermore, we have $\kappa^\sigma/\delta = 0$ ($\kappa^\sigma/\delta \to \infty$) as $\kappa = 0$ ($\kappa \to \infty$). The right-hand side of the equation exhibits the following properties:

$$\begin{aligned}\frac{dD(\kappa)}{d\kappa} &= 1 - \frac{\frac{q(\pi^*,\phi)}{q(\pi^*,\psi)} + \frac{\Gamma(\phi)}{\Gamma(\psi)}\chi^*}{1+\chi^*} < 0, \\ \Gamma(0) &= \frac{\frac{q(\pi^*,\phi)}{q(\pi^*,\psi)} + \frac{\Gamma(\phi)}{\Gamma(\psi)}\chi^*}{1+\chi^*}\left(1+A\right) > 0, \\ \Gamma(1+A) &= 1+A.\end{aligned}$$

These results show that there exists a unique value of κ satisfies equation (A2).

Using $\kappa = \kappa^*$, equation (17) and the definition of investment function provide

$$i^*(k) = (\kappa^* - 1)k.$$

$\kappa^* > 1$ must hold to be $i^*(k) > 0$. By Assumption 1, we have $\delta\left(1+A\right) > 1$. Then, $\kappa^* > 1$ holds. Equations (11) and (16) yield

$$c^*(k) = \frac{Ak - i^*(k)}{1+\chi^*} \text{ and } g^*(k) = \frac{\chi^*\left[Ak - i^*(k)\right]}{1+\chi^*}.$$

To ensure positive consumptions, $Ak - i^*(k) > 0$ is needed; $\kappa^* < 1 + A$. We have

$$\kappa^* < 1 + A \Leftrightarrow \frac{\left(1+A\right)^\sigma}{\delta} > 1 + A \Leftrightarrow \delta < \left(1+A\right)^{\sigma-1}.$$

Assumption 1 shows that the above condition holds. Under Assumption 1, we have

$$\delta\left(\kappa^*\right)^{1-\sigma} < \delta\left(1+A\right)^{1-\sigma} < 1. \tag{A3}$$

Using equations (12) and (13), we obtain

$$V^* = \left\{ \frac{\left[1-\delta\left(\kappa^*\right)^{1-\sigma}\right] q\left(\pi^*, \psi\right)+\delta\left(\kappa^*\right)^{1-\sigma} q\left(\pi^*, \phi\right)}{1-\delta\left(\kappa^*\right)^{1-\sigma}}\left(\Pi^*\right)^{1-\sigma} \right.$$
$$\left. +\frac{\left[1-\delta\left(\kappa^*\right)^{1-\sigma}\right] \Gamma(\psi)+\delta\left(\kappa^*\right)^{1-\sigma} \Gamma(\phi)}{1-\delta\left(\kappa^*\right)^{1-\sigma}}\left(1-\Pi^*\right)^{1-\sigma}\right\} \frac{\left(A+1-\kappa^*\right)^{1-\sigma} k^{1-\sigma}}{1-\sigma} \tag{A4}$$

for $\sigma \neq 1$. Furthermore, the transversality condition holds if $\delta\left(\kappa^*\right)^{1-\sigma}<1$. Equation (A3) is sufficient to ensure the bounded lifetime utility and transversality condition.

B. Proof of Proposition 2

Differentiating π with respect to z, we obtain

$$\frac{z}{\pi} \frac{d\pi}{dz}=-\frac{z^{\frac{1}{\sigma}}}{\left(1+z^{\frac{1}{\sigma}}\right) \sigma}<0. \tag{A5}$$

By $\pi'(z)<0$, $\pi^*=\pi(\psi)>\pi(\phi)=\pi^{\dagger}$ holds.

Using (A5), we have

$$\frac{d}{dz}\left[\frac{\Gamma(z)}{q(\pi, z)}\right]=\frac{\theta \pi(z)^{1-\sigma}}{q(\pi, z)^2}\left(1-z^{\frac{1-\sigma}{\sigma}}\right) \gtreqless 0 \Leftrightarrow z^{\frac{\sigma-1}{\sigma}} \gtreqless 1. \tag{A6}$$

For $1<\psi<\phi$, equation (A6) has a positive sign. Hence, we obtain

$$\frac{g^*}{c^*}=\chi^*<\chi^{\dagger}=\frac{g^{\dagger}}{c^{\dagger}}.$$

In contrast, equation (A6) is negative for $\psi<\phi<1$. This result leads to

$$\frac{g^*}{c^*}=\chi^*>\chi^{\dagger}=\frac{g^{\dagger}}{c^{\dagger}}.$$

For $\psi<1<\phi$, the sign of equation (A6) changes from negative to positive. Therefore, the magnitude relation between g^*/c^* and $g^{\dagger}/c^{\dagger}$ is undetermined.

C. Proof of Proposition 3

Partial differentiation of $q(\pi^*, z)$ with respect to z is

$$\frac{\partial q(\pi^*, z)}{\partial z}=\left(1-\pi^*\right)^{1-\sigma}>0.$$

This equation shows that $q(\pi^*, \phi)>q(\pi^*, \psi)$ holds. By the definition of $\Gamma(z)$, $\Gamma(\phi)>\Gamma(\psi)$ is obtained. These two inequalities yield

$$\frac{\frac{q(\pi^*, \phi)}{q(\pi^*, \psi)}+\frac{\Gamma(\phi)}{\Gamma(\psi)} \chi^*}{1+\chi^*}>1.$$

Hence, we have

$$D(0)=\frac{\frac{q(\pi^*, \phi)}{q(\pi^*, \psi)}+\frac{\Gamma(\phi)}{\Gamma(\psi)} \chi^*}{1+\chi^*}(1+A)>1+A.$$

Since $D'(\kappa) < 0$ and $D(1+A) = 1+A$, $D(\kappa) > 1 + A = (\kappa^o)^\sigma/\delta$ holds for $\kappa \in [0, 1+A)$. Using $1 < \kappa^* < 1 + A$, we arrive at

$$D(\kappa^*) = \frac{(\kappa^*)^\sigma}{\delta} > 1 + A = \frac{(\kappa^o)^\sigma}{\delta} \Rightarrow \kappa^* > \kappa^o.$$

Partial differentiation of $D(\kappa)$ with respect to γ is

$$\begin{aligned}
\frac{\partial D(\kappa)}{\partial \gamma} &= \frac{\left[\frac{\Gamma(\phi)}{\Gamma(\psi)} - \frac{q(\pi^*,\phi)}{q(\pi^*,\psi)}\right](A+1-\kappa)}{(1+\chi^*)^2} \frac{\partial \chi^*}{\partial \gamma} \\
&= \frac{\left[\frac{\Gamma(\phi)}{\Gamma(\psi)} - \frac{q(\pi^*,\phi)}{q(\pi^*,\psi)}\right](A+1-\kappa)\chi^*}{(1+\chi^*)^2 \gamma\sigma} \\
&= \frac{(\phi-\psi)\left(1-\psi^{\frac{1-\sigma}{\sigma}}\right)(A+1-\kappa)\chi^*}{(1+\psi)\left(1+\psi^{\frac{1}{\sigma}}\right)^\sigma (1+\chi^*)^2\gamma\sigma} \gtreqless 0 \Leftrightarrow \psi^{\frac{\sigma-1}{\sigma}} \gtreqless 1.
\end{aligned}$$

Using this equation, we obtain

$$\frac{\partial \kappa^*}{\partial \gamma} \gtreqless 0 \Leftrightarrow \frac{\partial D(\kappa)}{\partial \gamma} \gtreqless 0 \Leftrightarrow \psi^{\frac{\sigma-1}{\sigma}} \gtreqless 1.$$

D. Proof of Proposition 4

The value function in the elderly planned economy is

$$V^\dagger = \frac{q\left(\pi^\dagger,\phi\right)\left(\Pi^\dagger\right)^{1-\sigma} + \Gamma(\phi)\left(1-\Pi^\dagger\right)^{1-\sigma}}{1-\delta\left(\kappa^\dagger\right)^{1-\sigma}} \frac{\left(A+1-\kappa^\dagger\right)^{1-\sigma} k^{1-\sigma}}{1-\sigma},$$

where

$$\Pi^\dagger \equiv \frac{1}{1+\chi^\dagger}.$$

We have

$$W^* = (\delta+\eta)V^* \text{ and } W^\dagger = (\delta+\eta)V^\dagger.$$

Dividing W^* by $W^\dagger$ leads to

$$\frac{W^*}{W^\dagger} = \frac{\Delta^* v(k,\kappa^*)}{\Delta^\dagger v(k,\kappa^\dagger)}.$$

Note that $v(k,\kappa) < 0$ ($W^* < 0$ and $W^\dagger < 0$) for $\sigma > 1$. Hence,

$$\frac{\Delta^* v(k,\kappa^*)}{\Delta^\dagger v(k,\kappa^\dagger)} \gtreqless 1 \Leftrightarrow \frac{W^*}{W^\dagger} \gtreqless 1 \Leftrightarrow W^* \lesseqgtr W^\dagger.$$

E. Proof of Proposition 5

Differentiating equation (20) with respect to a and inserting equation (22) into it yields

$$\begin{aligned}
\frac{\partial V(a,\omega)}{\partial a} &= q(\hat{\pi},\phi)c^{-\sigma}\frac{\partial c}{\partial a} + \delta\frac{\partial V(a',\omega')}{\partial a'}\frac{\partial a'}{\partial a} \\
&= \left\{q(\hat{\pi},\phi)R + \left[q(\pi,\psi) - q(\hat{\pi},\phi)\right]\frac{\partial a'}{\partial a}\right\}c^{-\sigma},
\end{aligned} \tag{A7}$$

where $R \equiv 1 + r$. With a linear strategy $a' = \kappa a$, we have

$$\frac{\kappa^{\sigma}}{\delta} = \frac{q(\pi^*, \phi)(1 + \alpha A - \kappa)}{q(\pi^*, \psi)} + \kappa. \tag{A8}$$

The right-hand side $F(\kappa)$ is monotonically decreasing in κ ($F'(\kappa) < 0$). Furthermore, we have

$$F(0) = \frac{q(\pi^*, \phi)(1 + \alpha A)}{q(\pi^*, \psi)} \text{ and } F(1 + \alpha A) = 1 + \alpha A < 1 + A.$$

These properties show that there exists a unique solution of (A8), $\kappa^{\star}$.

The value function becomes

$$V_0 = \frac{\left[1 - \delta(\kappa^{\star})^{1-\sigma}\right] q(\pi^*, \psi) + \delta(\kappa^{\star})^{1-\sigma} q(\pi^*, \phi)}{1 - \delta(\kappa^{\star})^{1-\sigma}} \frac{[(R - \kappa^{\star})k + \omega]^{1-\sigma}}{1-\sigma} + \frac{\left[1 - \delta(\kappa^{\star})^{1-\sigma}\right] \Gamma(\psi) + \delta(\kappa^{\star})^{1-\sigma} \Gamma(\phi)}{1 - \delta(\kappa^{\star})^{1-\sigma}} \frac{[(1-\alpha)Ak - \omega]^{1-\sigma}}{1-\sigma}.$$

The first-order condition for choosing ω is

$$\frac{\partial V_0}{\partial \omega} = \frac{\left\{\left[1 - \delta(\kappa^{\star})^{1-\sigma}\right] q(\pi^*, \psi) + \delta(\kappa^{\star})^{1-\sigma} q(\pi^*, \phi)\right\} [(R - \kappa^{\star})k + \omega]^{-\sigma}}{1 - \delta(\kappa^{\star})^{1-\sigma}} - \frac{\left\{\left[1 - \delta(\kappa^{\star})^{1-\sigma}\right] \Gamma(\psi) + \delta(\kappa^{\star})^{1-\sigma} \Gamma(\phi)\right\} [(1-\alpha)Ak - \omega]^{-\sigma}}{1 - \delta(\kappa^{\star})^{1-\sigma}} = 0.$$

This equation leads to

$$\frac{g}{c} = \left\{ \frac{\left[1 - \delta(\kappa^{\star})^{1-\sigma}\right] \Gamma(\psi) + \delta(\kappa^{\star})^{1-\sigma} \Gamma(\phi)}{\left[1 - \delta(\kappa^{\star})^{1-\sigma}\right] q(\pi^*, \psi) + \delta(\kappa^{\star})^{1-\sigma} q(\pi^*, \phi)} \right\}^{\frac{1}{\sigma}} = \chi^{\star}.$$

In the similar way as the proof of Proposition 1, $i^{\star}(k)$, $c^{\star}(k)$, and $g^{\star}(k)$ are derived from the definition of investment and equations (9) and (11). Using these consumption and investment functions, the value function can be rewritten as

$$V^{\star} = \left\{ \frac{\left[1 - \delta(\kappa^{\star})^{1-\sigma}\right] q(\pi^*, \psi) + \delta(\kappa^{\star})^{1-\sigma} q(\pi^*, \phi)}{1 - \delta(\kappa^{\star})^{1-\sigma}} (\Pi^{\star})^{1-\sigma} + \frac{\left[1 - \delta(\kappa^{\star})^{1-\sigma}\right] \Gamma(\psi) + \delta(\kappa^{\star})^{1-\sigma} \Gamma(\phi)}{1 - \delta(\kappa^{\star})^{1-\sigma}} (1 - \Pi^{\star})^{1-\sigma} \right\} \frac{(A + 1 - \kappa^{\star})^{1-\sigma} k^{1-\sigma}}{1-\sigma},$$

where

$$\Pi^{\star} \equiv \frac{1}{1 + \chi^{\star}}.$$

$1 < \kappa^{\star} < 1 + \alpha A$ is needed to ensure positive values of growth rate and consumptions. The bounded lifetime utility and transversality condition hold because $\delta(\kappa^{\star})^{1-\sigma} < \delta(1 + \alpha A)^{1-\sigma} < \delta(1 + A)^{1-\sigma} < 1$ under Assumption 1. Furthermore, Assumption 2 leads to $\kappa^{\star} > 1$.

F. Proof of Proposition 6

We have

$$\begin{aligned}
(\chi^*)^\sigma - (\chi^\star)^\sigma &= \frac{\Gamma(\psi)}{q(\pi^*,\psi)} - \frac{\left[1-\delta(\kappa^\star)^{1-\sigma}\right]\Gamma(\psi) + \delta(\kappa^\star)^{1-\sigma}\Gamma(\phi)}{\left[1-\delta(\kappa^\star)^{1-\sigma}\right]q(\pi^*,\psi) + \delta(\kappa^\star)^{1-\sigma}q(\pi^*,\phi)} \\
&= \frac{\delta(\kappa^\star)^{1-\sigma}\left[\Gamma(\psi)q(\pi^*,\phi) - q(\pi^*,\psi)\Gamma(\phi)\right]}{q(\pi^*,\psi)\left\{\left[1-\delta(\kappa^\star)^{1-\sigma}\right]q(\pi^*,\psi) + \delta(\kappa^\star)^{1-\sigma}q(\pi^*,\phi)\right\}} \gtreqless 0 \\
&\Leftrightarrow \frac{q(\pi^*,\phi)}{q(\pi^*,\psi)} \gtreqless \frac{\Gamma(\phi)}{\Gamma(\psi)} \Leftrightarrow \psi^{\frac{\sigma-1}{\sigma}} \lesseqgtr 1.
\end{aligned}$$

Therefore, we obtain

$$\frac{g^*}{c^*} = \chi^* \gtreqless \chi^\star = \frac{g^\star}{c^\star} \Leftrightarrow \psi^{\frac{\sigma-1}{\sigma}} \lesseqgtr 1.$$

Equations (A2) and (A8) yield

$$F(\kappa) - D(\kappa) = \frac{q(\pi^*,\phi)(1+\alpha A-\kappa)}{q(\pi^*,\psi)} - \frac{\left\{\frac{q(\pi^*,\phi)}{q(\pi^*,\psi)} + \frac{\Gamma(\phi)}{\Gamma(\psi)}\left[\frac{\Gamma(\psi)}{q(\pi^*,\psi)}\right]^{\frac{1}{\sigma}}\right\}(1+A-\kappa)}{1+\left[\frac{\Gamma(\psi)}{q(\pi^*,\psi)}\right]^{\frac{1}{\sigma}}}$$

with

$$F(0) - D(0) = \frac{\left[\frac{q(\pi^*,\phi)}{q(\pi^*,\psi)} - \frac{\Gamma(\phi)}{\Gamma(\psi)}\right]\chi^* q(\pi^*,\psi) - q(\pi^*,\phi)(1+\chi^*)(1-\alpha)A}{q(\pi^*,\psi)(1+\chi^*)} \gtreqless 0$$

and

$$F(1+\alpha A) - D(1+\alpha A) = -\frac{\left[\frac{q(\pi^*,\phi)}{q(\pi^*,\psi)} + \frac{\Gamma(\phi)}{\Gamma(\psi)}\chi^*\right](1-\alpha)A}{1+\chi^*} < 0.$$

Differentiating $[F(\kappa) - D(\kappa)]$ with respect to κ leads to

$$\begin{aligned}
\frac{d\left[F(\kappa) - D(\kappa)\right]}{d\kappa} &= -\frac{q(\pi^*,\phi)}{q(\pi^*,\psi)} + \frac{\frac{q(\pi^*,\phi)}{q(\pi^*,\psi)} + \frac{\Gamma(\phi)}{\Gamma(\psi)}\chi^*}{1+\chi^*} \\
&= \frac{\left[\frac{\Gamma(\phi)}{\Gamma(\psi)} - \frac{q(\pi^*,\phi)}{q(\pi^*,\psi)}\right]\chi^*}{1+\chi^*} \gtreqless 0 \Leftrightarrow \psi^{\frac{\sigma-1}{\sigma}} \gtreqless 1. \qquad \text{(A9)}
\end{aligned}$$

When $\alpha = 1$, we have

$$\begin{aligned}
F(0) - D(0) &= \frac{\left[\frac{q(\pi^*,\phi)}{q(\pi^*,\psi)} - \frac{\Gamma(\phi)}{\Gamma(\psi)}\right]\chi^*}{1+\chi^*} \gtreqless 0 \Leftrightarrow \psi^{\frac{\sigma-1}{\sigma}} \lesseqgtr 1, \\
F(1+A) - D(1+A) &= 0.
\end{aligned}$$

From these results and equation (A9), we obtain $\kappa^\star \gtreqless \kappa^* \Leftrightarrow \psi^{\frac{\sigma-1}{\sigma}} \lesseqgtr 1$ when $\alpha = 1$. We have

$$\frac{\partial F(\kappa)}{\partial \alpha} > 0.$$

A decrease in α reduces $[F(\kappa) - D(\kappa)]$. Hence, if $\psi^{\frac{\sigma-1}{\sigma}} < 1$, there exist the critical value of α that change the magnitude relationship between $\kappa^\star$ and κ^*. We have

$$\begin{aligned}
\kappa^\star &\gtreqless \kappa^* \Leftrightarrow \alpha \gtreqless \alpha^* \text{ for } \psi^{\frac{\sigma-1}{\sigma}} < 1, \\
\kappa^\star &\leq \kappa^* \text{ for } \psi^{\frac{\sigma-1}{\sigma}} \geq 1.
\end{aligned}$$

On the other hand, we obtain

$$\frac{\partial\left[F(\kappa)-D(\kappa)\right]}{\partial\gamma}=-\frac{\partial D(\kappa)}{\partial\gamma}\lesseqgtr 0\Leftrightarrow\psi^{\frac{\sigma-1}{\sigma}}\gtreqless 1.$$

When $\psi^{\frac{\sigma-1}{\sigma}}\geq 1$, an increase in γ reduces $[F(\kappa)-D(\kappa)]$. Since $F(\kappa)<D(\kappa)$, it means that $\kappa^{\star}\leq\kappa^{*}$ holds for $\gamma>0$. If $\psi^{\frac{\sigma-1}{\sigma}}<1$, a raise in γ increases $[F(\kappa)-D(\kappa)]$ for given α. Initially, $\alpha=\alpha^{*}$ leads to $\kappa^{\star}=\kappa^{*}$. Depending on γ, the critical value of α should be changed: large γ makes α^{*} small. Thus, we have

$$\frac{\partial\alpha^{*}}{\partial\gamma}<0.$$

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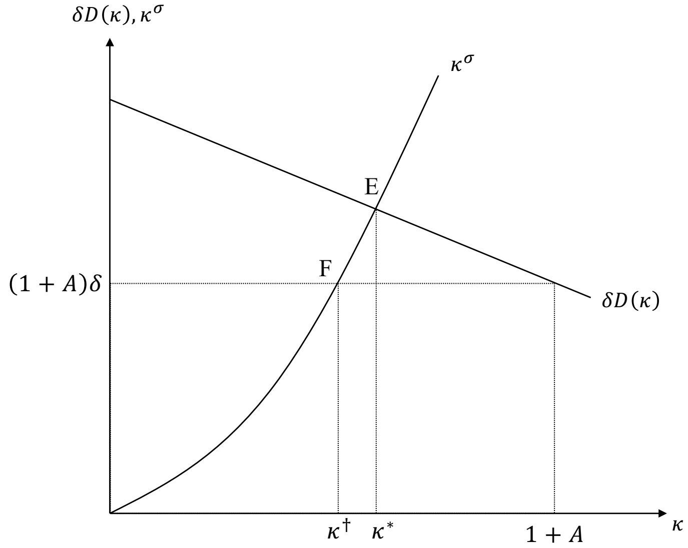

Figure 1. Equilibrium

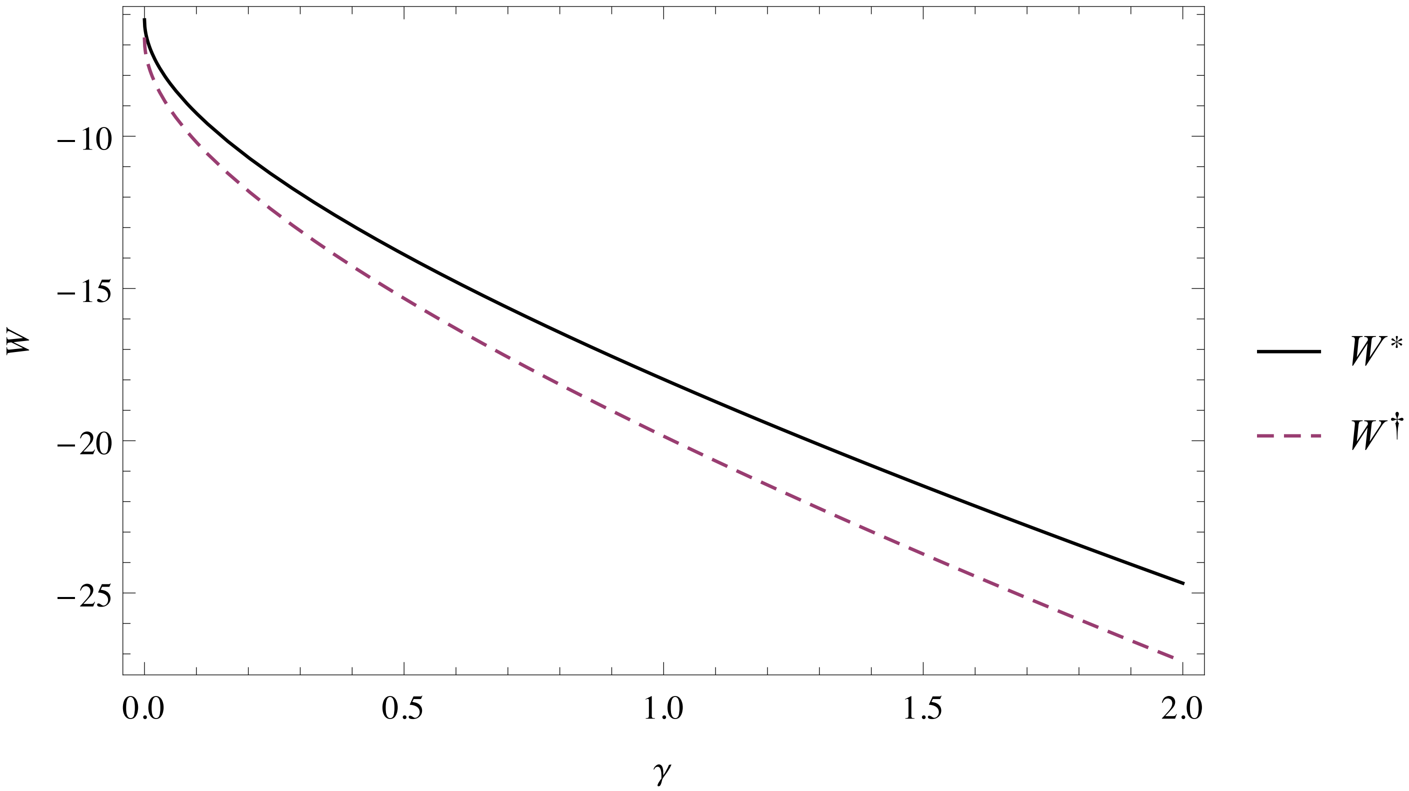

Figure 2. The preference for public goods and welfare level

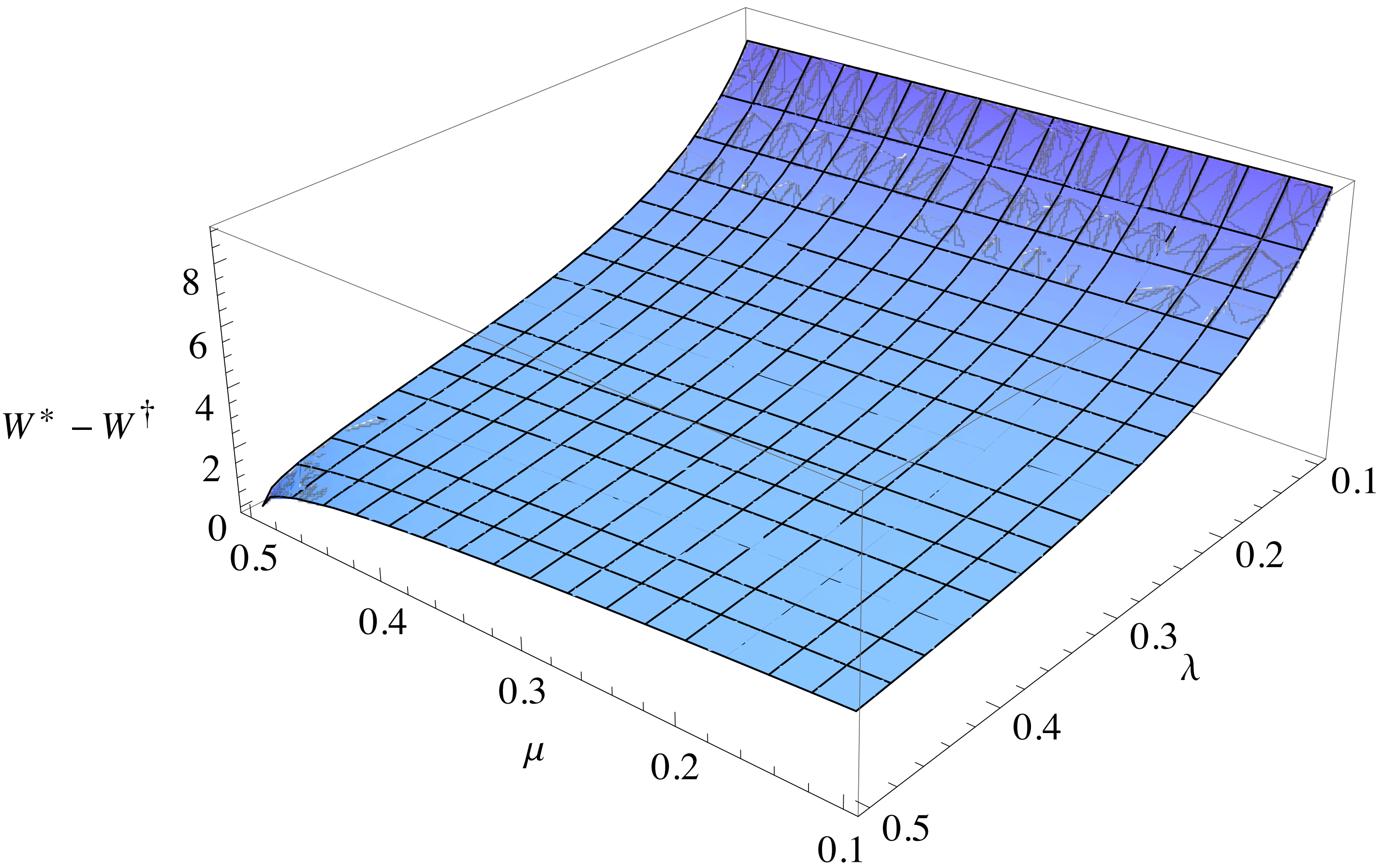

Figure 3. The degree of altruism and welfare deference

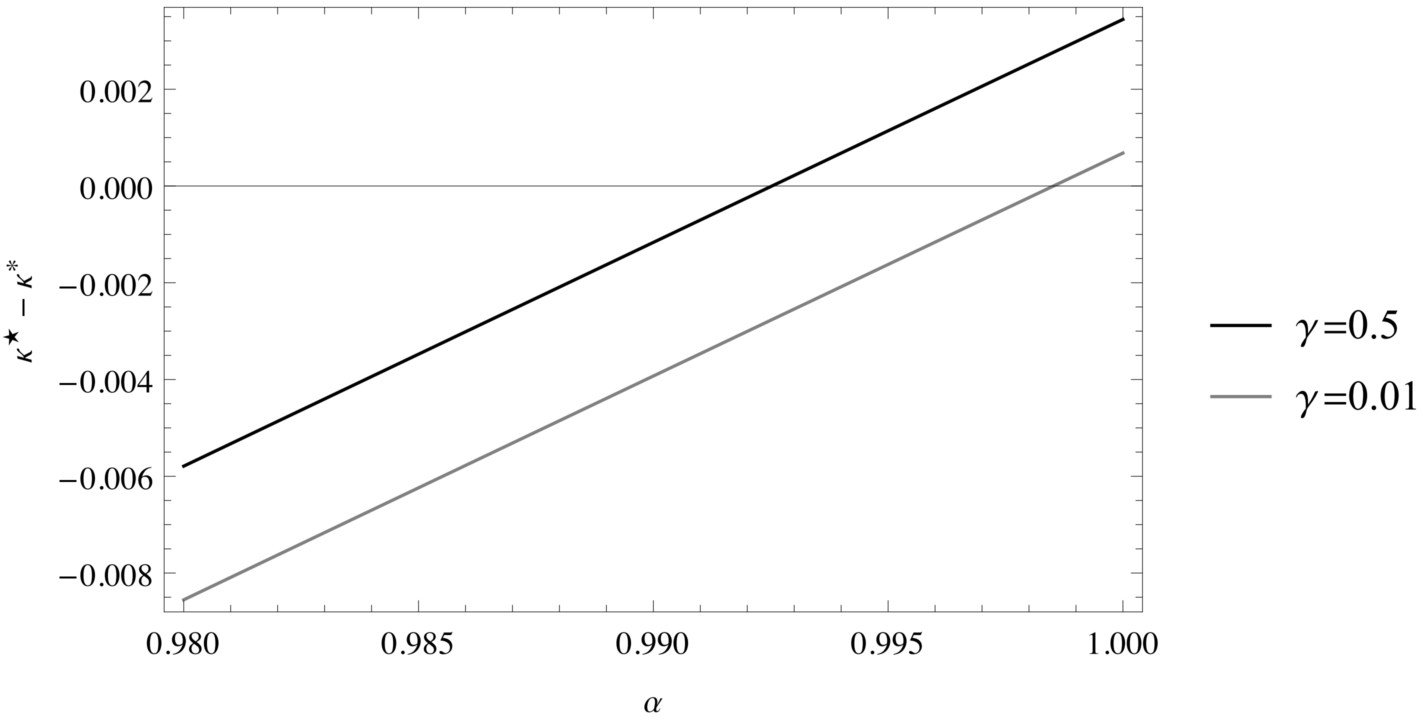

Figure 4a. Knowledge spillover and growth difference

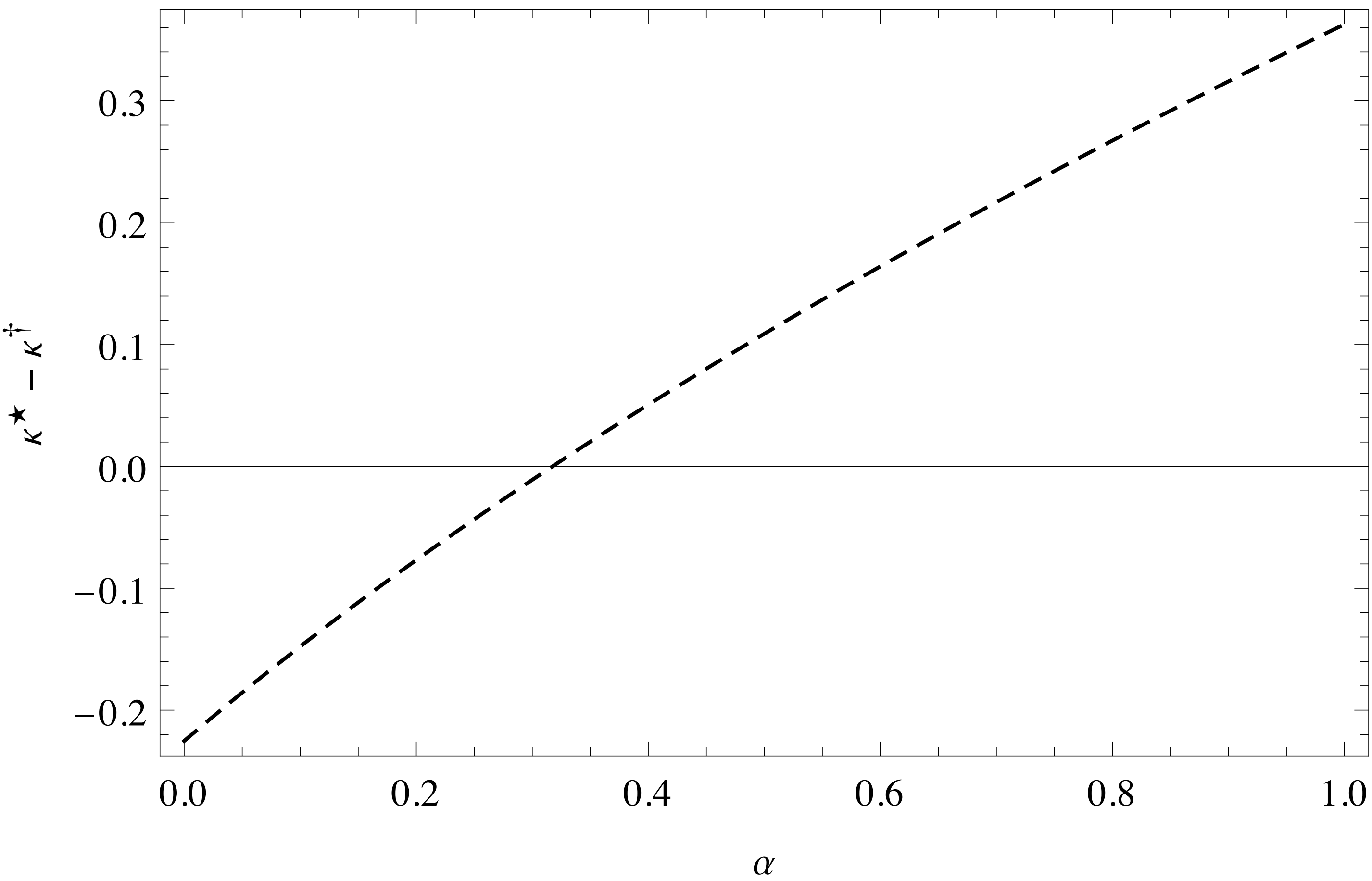

Figure 4b. Knowledge spillover and growth difference

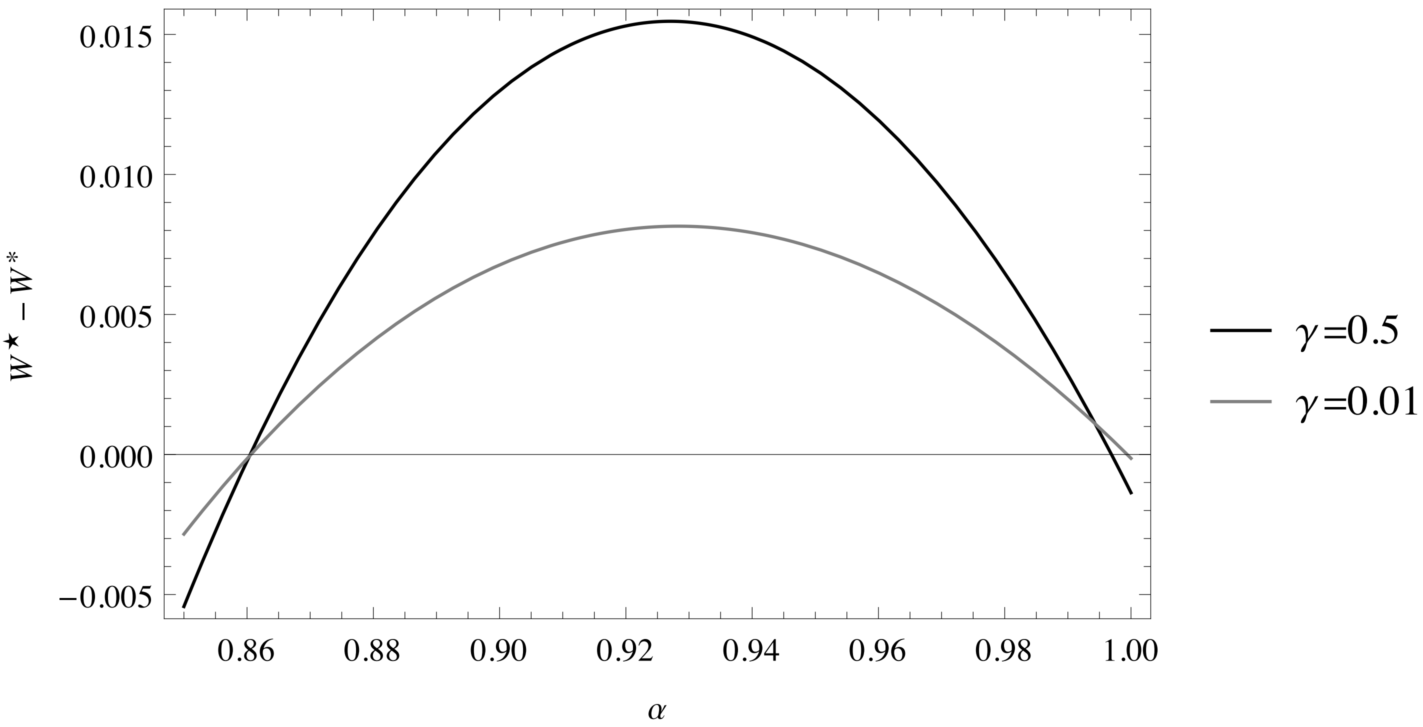

Figure 5a. Knowledge spillover and welfare difference

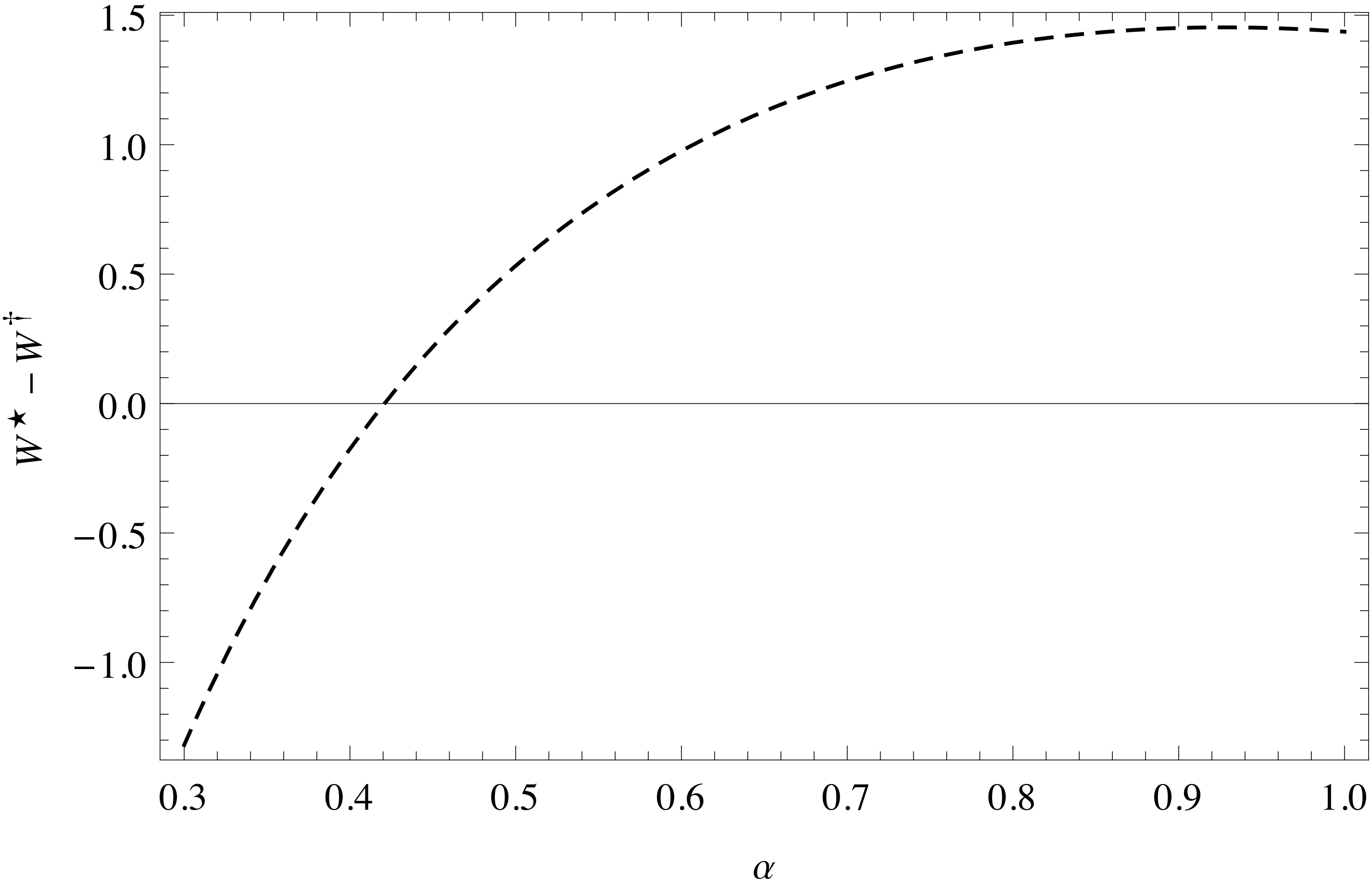

Figure 5b. Knowledge spillover and welfare difference