

2019 Doctor's Thesis

**Intergenerational Conflicts over Welfare and
Growth: Policy Effects of Education and Public
Pensions in OLG Frameworks**

Graduate School of Economics, Nagoya University

Academic Advisor: Professor YANAGIHARA Mitsuyoshi

Name: HU Weizhen

Contents

ACKNOWLEDGEMENTS.....	3
CHAPTER 1 INTRODUCTION	4
1.1 BACKGROUND.....	4
1.1.1 Global aging and pensions	4
1.1.2 Education and economic growth.....	6
1.2 RESEARCH PURPOSES.....	8
1.3 LITERATURE REVIEW.....	9
1.3.1 Dynamic analyses	9
1.3.2 Public pensions.....	11
1.3.3 Education and human capital accumulation.....	12
1.3.4 Capital tax competition.....	17
CHAPTER 2 POLICY EFFECTS ON TRANSITIONAL WELFARE IN AN OLG MODEL: A PAYG PENSION RECONSIDERED	20
2.1 THE ANALYTICAL FRAMEWORK	21
2.1.1 Benchmark solutions.....	21
2.1.2 Transitional dynamics.....	22
2.1.3 Monotonic welfare change.....	25
2.1.4 Oscillatory welfare change	26
2.2 PAYG PENSION SYSTEM	30
2.2.1 The basic model.....	31
2.2.2 Dynamically efficient economy	33
2.2.3 Dynamically inefficient economy	36
2.2.4 Comparison: a fully funded pension system.....	37
2.3 CONCLUDING REMARKS	39
APPENDIX 2.A PROOF OF PROPOSITION 2.1.....	39
APPENDIX 2.B PROOF OF PROPOSITION 2.3.....	40
CHAPTER 3 SELF-EDUCATION, FULLY FUNDED PENSION AND ECONOMIC GROWTH.....	41

3.1	THE MODEL	42
3.1.1	Basic framework	42
3.1.2	Equilibrium	44
3.1.3	Fully funded pension	45
3.2	FULLY FUNDED PENSION AND ECONOMIC GROWTH.....	46
3.3	EFFECTS OF INTEREST RATE ON GROWTH.....	49
3.4	CONCLUDING REMARKS	50
APPENDIX 3.A	PROOF OF (3.12)	50
APPENDIX 3.B	PROOF OF PROPOSITION 3.1.....	50
APPENDIX 3.C	PROOF OF PROPOSITION 3.2.....	51
CHAPTER 4	CAPITAL TAX COMPETITION AND PUBLIC EDUCATION	52
4.1	THE MODEL	52
4.1.1	Firms	52
4.1.2	Individuals.....	53
4.1.3	Government.....	53
4.1.4	Formation of human capital.....	54
4.1.5	Physical capital market equilibrium.....	54
4.1.6	Nash policy equilibrium.....	55
4.2	OPTIMAL POLICY RULES	55
4.3	WELFARE EFFECTS OF A COORDINATED TAX REFORM	56
4.4	DISCUSSION	59
APPENDIX 4.A	PROOF OF THE LOCAL STABILITY.....	62
APPENDIX 4.B	PROOF OF PROPOSITION 4.1.....	63
CHAPTER 5	CONCLUSIONS.....	64
REFERENCES	67

Acknowledgements

I would like to express my appreciation and thanks to my advisor Professor Dr. Mitsuyoshi Yanagihara. He has provided me with continuous support and instructions on my research since my master's period and his guidance helped me during all my time researching and writing this dissertation.

I also wish to acknowledge both Professor Dr. Toshiki Tamai, my vice supervisor, and Professor Dr. Noritaka Kutoh. They offered me many invaluable comments and suggestions for my writing.

In particular, I am grateful to Professor Dr. Hikaru Ogawa. It was he who provided the opportunity for me to study at Nagoya University and who gave me priceless suggestions and help in becoming a researcher.

Last but not least, I want to thank all other professors and experts who have helped me in writing this dissertation, and to thank my parents and friends for their support and encouragements. Without their assistance and company, I would not have had the courage and power to be a better me.

Chapter 1 Introduction

1.1 Background

Pensions and education, as two representative issues that reflect intergenerational conflicts, that is, social and economic conflicts, between young and old generations, are tightly connected to the two great social themes of current global economics: population aging and economic growth.

1.1.1 Global aging and pensions

In recent decades, population aging has become a significant global issue. In 2018, those aged 65 years and above of the total population was 19.93% in the European Union, 15.81% in the United States and 27.58% in Japan. Although the world average was 8.87% for that year, more developing countries have also started to experience aging population. For instance, China's ratio of those aged 65 years to the total has rapidly increased from 7.82% in 2008 to 10.92% in 2018 (Figure 1.1).

"The Chinese Academy of Social Sciences, the country's chief think tank, predicts China's pension surplus will turn into a deficit by 2023. By 2050, it predicts, the cumulative deficit will be \$118 trillion barring significant policy changes."

——*The Wall Street Journal* (Nov 3, 2016)

In response to the global aging problem, more and more countries and regions have begun to establish or complete the implementation of pension systems. At present, the pay-as-you-go (PAYG) pension has become one of the main styles of pensions that has been introduced (totally or partially) in many countries, including the United States, the United Kingdom, Japan, and Germany.¹ However, despite the declining birthrate and

¹ Source: Ministry of Health, Labour and Welfare (<https://www.mhlw.go.jp/stf/seisakunitsuite/bunya/nenkin/nenkin/shogaikoku.html>, accessed on November 15, 2019).

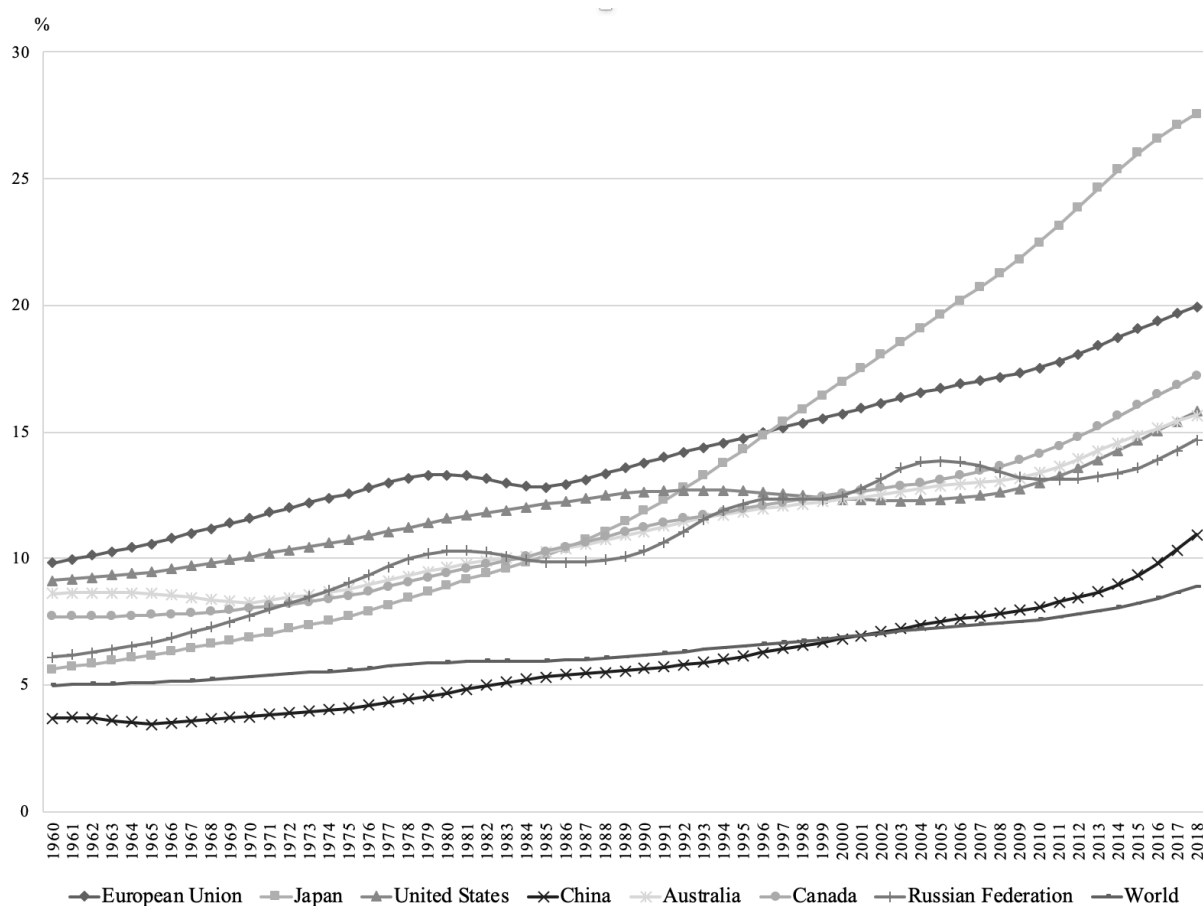


Figure 1.1 Population aged 65 years and above (% of total population)

This figure shows the population aged 65 years and above of the total population in representative developed countries (region) and China in 1960–2018. Data source: The World Bank.

(<https://data.worldbank.org/indicator/sp.pop.65up.to.zs?contextual=default&end=2018&start=1960&view=chart>, accessed on November 15, 2019)

aging population, a PAYG pension system exacerbates governments' fiscal burden and many countries have started to consider pension reform. For instance, the Chinese government made a policy announcement to establish a system that combines the social pooling and personal accounts, which, in theory, is a combination of the two main types of pension systems: PAYG pension systems and fully funded pension systems.

1.1.2 Education and economic growth

Ever since Schultz (1960, 1961), the role of human capital in economic growth has been widely studied by economists, based on the fact that in reality, the recent economic growth not only relies on physical capital as in the past, but is also driven by human capital. As the main source of human capital, education—public education or private education, intergenerational education or self-education—has been attributed great importance for both individuals and countries.

In particular, besides the very general intergenerational education investments (investments in children made by parents or governments), educational expenditures on people for themselves are also increasing in a significant way. Since the 1970s, the OECD has advocated “recurrent education” as a strategy for its “lifelong learning for all” policy. It declared that a recurrent education offers “an alternative to unlimited further expansion of the formal and youth-oriented educational system”. As for the emerging imbalance between low supply and high demand of highly qualified human resources and the rapid speed of knowledge expansion, recurrent education is a solution to the education crisis (Kallen and Bengtsson, 1973). Recently, this proposal has been adopted in many countries around the world. China made “lifelong learning” a policy in its Outline of Educational Reform and Development in 1993 and added legislation on adult education as an annual priority in 1995.

Furthermore, as a member of the OECD, the Japanese government has adopted a similar concept of lifelong learning or recurrent education. It states that lifelong learning is necessary for individuals’ adaptation to social and economic changes, and enriches their spiritual demands as well as increases their lifelong earnings (MEXT, 2006). Given these social and economic contexts, an increasing number of people, especially in developed countries, tend to improve their skills or accumulate their own human capital. In fact, they often choose to undertake further education after university or professional training, rather than educate their children. This, in Japan, has pushed a policy of promoting lifelong learning, including the Program for Education Promotion in Response to the Re-learning Needs of Adults.² As a result, more and more new graduate students

²This program was made “for each and every person to enjoy a fulfilling life and for the continued growth of the country” under a background of Japan’s rapid decline in birthrate and its aged population. It contains a series of reforms, including an integrated reform on high school education, university education, and

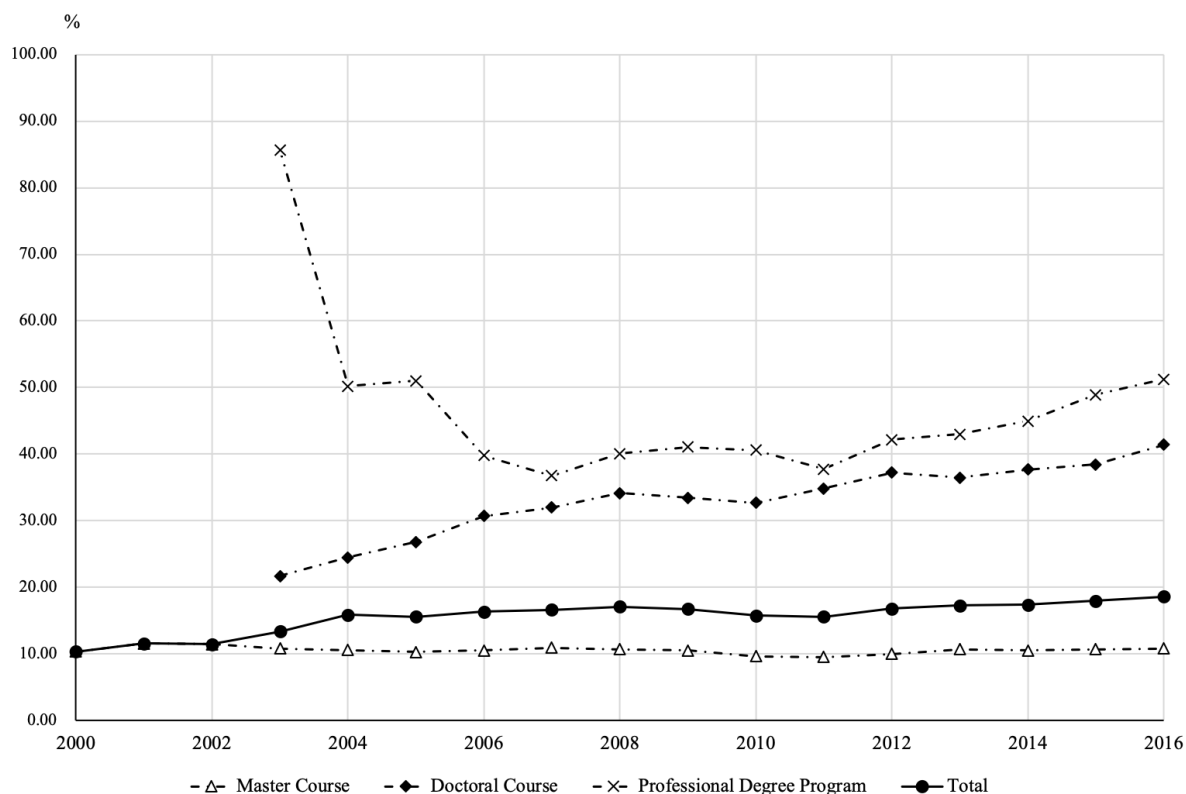


Figure 1.2 Proportion of on-the-job postgraduates in Japan

This figure shows the proportion of new on-the-job postgraduates to aggregate enrollments in Japan's graduate schools. Source: created by editing "School Basic Survey" (MEXT).

(http://www.mext.go.jp/b_menu/toukei/chousa01/kihon/1267995.htm, accessed on November 15, 2019)

return to study from the workforce: they return to college or university after several years of working, to study either as on-the-job students or even full-time students (Figure 1.2). In old age, many individuals find a new job or return to work through reemployment after retirement. As policymakers, governments that are concerned about economic growth and social welfare must therefore consider both the intergenerational education and the education on people themselves in both the short run and the long run.

selection of university entrants as well as a reform on the system of the boards of education. Specifically, it proposed a policy of enhancing relearning for adults. (MEXT)

(http://www.mext.go.jp/b_menu/hakusho/html/hpab201401/detail/1376942.htm, accessed on November 22, 2019)

1.2 Research purposes

This dissertation therefore concentrates on the two representative issues of intergenerational conflicts around pensions and education in the framework of an overlapping generations (OLG) model. We study their effects on welfare or economic growth from different aspects in various situations. We consider three topics: 1) to reconsider the PAYG pension from the perspective of transitional effects, 2) to evaluate the policy effects of a fully funded pension when individuals make a trade-off between educating themselves and educating their children, and 3) to analyze the effects of a coordinated capital tax reform across countries where the capital taxes are financed to provide public education.

Specifically, in Chapter 2, we discuss the short-, medium- and long-run effects of a PAYG pension system on social welfare. As is well-known (e.g., Blanchard and Fischer, 1989), a PAYG pension system is not neutral in terms of either welfare or economic growth. It improves utilities only when the population growth rate is higher than the interest rate. Moreover, as will be shown in Section 1.3.2, most of the existing research that uses an OLG framework to examine pensions has tended to focus only on the welfare effects in the steady state, paying little attention to the welfare effects in transition, i.e., those caused by a change in or the introduction of the PAYG pension. In other words, the short- and medium-run effects have not been considered fully. Under very basic settings, the introduction of a PAYG pension system has a positive (negative) effect on the steady-state welfare when the economy is dynamically inefficient (efficient). This leads to a conclusion: a policy evaluation that considers only the welfare effect in the steady state would conclude that the pension system should (not) be introduced in the case of dynamic inefficiency (efficiency). However, in Chapter 2, we expand the analysis to examine a PAYG pension system on a transition path and therefore revisit its policy feasibility.

In Chapter 3, we consider the education for individuals themselves and the education for their children in the same framework. Generally, the existing literature has only considered one type of human capital accumulation as one of the main resources of economic growth: either their children's human capital or their own human capital.

However, people in fact often face a trade-off between these two types of human capital accumulation. The former, which we name intergenerational education, occurs when parents with altruism invest in their children's education and obtain utility from the human capital or wage income of their children in the future. The latter, which, in this dissertation, we call "self-education", occurs when people prepare for their old age by investing in themselves in order to earn more in old age. Therefore, in this chapter, we investigate whether the parents' trade-off between educating themselves and educating their children affects efficiency from the viewpoint of economic growth. Then, if the answer is yes, we propose investigating whether a fully funded pension system can be used as a policy tool to accelerate the growth.

In Chapter 4, we turn our attention to public education. For an open economy, we discuss a public education policy that is determined by an international capital tax competition and how its policy effects will be affected by coordinated tax reforms. In many previous static theories, a tax competition gives rise to the fiscal externality that provision of public goods is insufficient. Thus, it is generally believed that if governments in all countries make an agreement to coordinately raise their capital tax rate, social welfare should be improved. However, considering a dynamic framework where intergenerational conflicts exist, this rule may not be supported. In this chapter, we consider an economy where the (physical) capital taxes are used to provide public education, instead of public goods, which can be seen in the static literature. We investigate whether a coordinated tax reform necessarily increases the human capital level of each country and thus whether it is necessarily Pareto-improving. We also try to give specific conditions under which the coordinated reform improves welfare.

1.3 Literature review

In this section, we review the literature related to the following four topics: research on dynamic analyses, research tackling public pension problems, research focusing on the relationship between human capital accumulation (education) and economic growth, and research on capital tax competition.

1.3.1 Dynamic analyses

Since the OLG model was constructed by Diamond (1965), many researchers have used this model to investigate the effects of various fiscal policies on social welfare from a dynamic perspective. In particular, because it presumes that both young and old generations exist in the same period, the OLG model is used to analyze intergenerational conflicts brought about by policy changes. However, most of the existing analyses focus only on steady-state effects, excluding transitional-state effects (e.g., Bewley, 2009). The transitional effects have not been examined because of the complexity of the calculations, which leads to difficulties in determining the economic implications. In short, mathematical inconvenience has led to the dynamical analysis of transitions being ignored.

Two main methods were devised to tackle this problem. One is by Matsuyama (1991), who first studied transitional effects in an OLG model by applying a diagrammatic approach. Matsuyama (1991) superimposed indifference curves on the plane containing the factor–price frontier and the capital market equilibrium, both of which had been given by Diamond (1965), and showed the transitional effects by “connecting” the original and new steady-state equilibria arising from a policy change. The other method is by Cremers and Sen (2008), who examined the effects of international transfers algebraically. They examined not only a steady-state generation, but also transitional generations in a two-country OLG model, and showed that “a strong form of the transfer paradox in which the donor country experiences a welfare gain while the recipient country experiences a welfare loss may occur both in and out of steady state”.³ Following Cremers and Sen (2008), Kuhle (2014) extended the algebraic analysis of the effect of international transfers to encompass an investigation of the effects of general fiscal policies. Kuhle (2014) developed a method to Pareto-evaluate the dynamics of utility along the transition path between two competitive steady states before and after a one-time permanent policy change. However, Kuhle (2014)’s focus was on the case of monotonic convergence and he only briefly mentioned what would occur when the dynamics of capital stock are oscillatory, essentially as a component of his application of

³ Another research study that analyzes the transitional welfare effects was done by Hamada and Yanagihara (2016), who investigated the effect of international transfers on welfare in a two-country OLG model, in which individuals have altruistic utility, by applying the procedure of Cremers and Sen (2008).

the model to a public debt problem, rather than attempting to analyze this case in a general manner.

1.3.2 Public pensions

In recent decades, along with population aging becoming an important issue in an increasing number of countries and regions, the relationship between public pensions and economic growth has been in focus. Most researchers in this area have paid attention to the PAYG pension or to a comparison of PAYG and fully funded pension systems (e.g., Kaganovich and Zilcha, 1999, 2012).

For example, Jimeno, Rojas, and Puente (2008) investigated several different approaches to studying the effects of population aging on social security expenditures along the transition path by calibrating the case of Spain, providing an analysis that is quantitative rather than qualitative. Hamada et al. (2017) investigated the effects of international transfers on welfare levels when two countries introduce PAYG pension systems. Chen and Fang (2013) studied the effects of population aging and international migration on economic growth in an OLG framework with endogenous fertility, where social security is financed by an income tax. Kemnitz and Wigger (2000) studied the growth and efficiency effects of a PAYG social security system when human capital is the main source of economic growth and found that it leads to higher growth than a fully funded social security system. Other research studies on PAYG pensions are Zhang (1995), Kaganovich and Meier (2012), Fehr and Uhde (2014), Ono and Uchida (2016), Tran (2016), and Bishnu and Wang (2017), among others.

Besides the theoretical research, empirical studies investigated the effects of PAYG pension systems on capital accumulation in real-world economies. Granville and Mallick (2004) evaluated the effects on savings of pension reforms in the United Kingdom that began in the 1980s. They found that the privatization of the state PAYG pension system could ultimately raise the level of national savings, even though the increase in the occupational pension savings could be offset by a decrease in other forms of savings. This was because the greater liquidity and capitalization arising from the private pension funds would promote economic growth through more efficient resource allocation. Michailidis et al. (2019) examined the effect of population aging on pension spending,

public education, and the interaction between them by showing empirical evidence for OECD countries. In particular, their results suggested that, with projected population aging, the structure of the PAYG pension system provides incentives to the working-age generation to support educational transfers toward the young generation even when there is no altruism. Euwals (2000) investigated the case of the Netherlands and tested the effect of the Dutch mandatory pensions on discretionary household savings. The results showed that while the impact of the public part of the Dutch pension system is not well identified, the occupational part of the pension has a significant negative impact on savings motives with respect to old age. Pereira and Andr  z (2012) empirically estimated the effects of the Portuguese PAYG social security system using data spanning 1970–2007. They found that the growing social security spending has had negative effects on all of the private sector variables under consideration. This suggested the existence of sizable inefficiencies, which highlighted the necessity for pension reforms. Brinkman et al. (2018) studied the determinants of municipal pension funding and its implications for intergenerational redistribution. They showed that under perfect capital markets, pension funding choices are fully capitalized into land prices, which, however, fails when agents face a binding down-payment constraint in the land market. Their empirical analysis showed that the correlations in the data of the United States are broadly consistent with this prediction.

As for the other common type of pension systems, the fully funded pension, it is clarified in basic economic textbooks (see, e.g., Blanchard and Fischer, 1989) that when the pension contributions do not exceed the amount of savings, a fully funded pension has no effect on total savings or capital accumulation, and thus, no effect on social welfare. However, as an example, Karni and Zilcha (1989) showed that fully funded social security may have a negative impact on economic growth. In their OLG economy with bequests where labor supply is determined endogenously and lifespan is uncertain, a fully funded social security system reduces aggregate output through a decrease in the capital stock.

1.3.3 Education and human capital accumulation

Generally, human capital is accumulated through education. From the perspective of financial resources, education can be classified into public education and private education. From the perspective of the education target, education then can be classified

into intergenerational education and self-education. Most of the existing research on public education problems investigated fiscal expenditures on intergenerational education, based on real-world policies. Therefore, in this subsection, we organize the literature in three parts: studies on public education systems, studies on self-education, and studies on private, intergenerational education.

- ***Public education***

Since governments control the scale of education to promote higher economic growth and/or greater social welfare, as a policy tool, the design and effects of public education have been widely studied.

Glomm and Ravikumar (1992) compared the effects of private and public education in an OLG model where human capital investment through formal schooling by heterogeneous individuals is the engine of growth, and found that income inequality declines faster under public education than under private education. Glomm and Ravikumar (2001) took the maximization of social welfare as the policy goal. In their OLG model, individuals accumulate human capital through formal schooling, and the public sector collects taxes from households, provides inputs to the learning technology, and endogenously decides the expenditure on schools. They found that their results qualitatively match the observations under plausible restrictions. Aguiar-Conraria (2005) presented an OLG model close to that of Glomm and Ravikumar (1992) but showed that, compared with a private education, public education may stimulate economic growth more. In this model, individuals face a choice to work or to be educated during their childhood.

Other similar research studies that focused on the role of public education in improving social welfare or promoting economic growth are Shirai (1990), Galor and Moav (2006), Azarnert (2014), among others.

- ***Self-education***

There are two main types of individuals' educational investment in themselves: pecuniary input and time input.

An example of research assuming that intergenerational human capital accumulation depends on pecuniary inputs, i.e., money, is that by Lord and Rangazas (1998). They considered a model with life cycle precautionary saving and human capital investment and showed that income taxation could have a positive effect on human capital accumulation because of an insurance effect of taxing uncertain returns.⁴

Different from Lord and Rangazas (1998) where human capital is non-inherited, Chakraborty and Das (2005) and Del Rey and Lopez-Garcia (2013, 2016) assumed that individuals partially inherit human capital from their parents. Chakraborty and Das (2005) considered a formation of individuals' human capital that is inherited from their parents by focusing on health capital. Del Rey and Lopez-Garcia (2013) studied the Golden Rule of both physical and human capital accumulations in an endogenous growth model and characterized the optimal policy to decentralize the Golden Rule balanced growth path when there are no constraints to education investments. Del Rey and Lopez-Garcia (2016) analyzed the welfare effects of intergenerational transfers and education subsidies in an endogenous growth model.

Moreover, Le Garrec (2012) assumed that individuals' human capital accumulation relates not to their parents' human capital level, but to the average level of the whole previous generation. They showed that an actuarially fair PAYG pension system can both reduce lifetime income inequality and enhance economic growth.

In addition, some research also considered a situation where individuals have to borrow money for self-education. Galor and Zeira (1993) showed that if credit markets are imperfect and the investment in human capital is indivisible, the initial distribution of wealth would have an effect on aggregate output as well as on investment in both the short run and the long run. Docquier et al. (2007) is another example. They considered a three-period overlapping generations model in which human capital is the engine of economic growth. In their model, children borrow money to accumulate their human capital. In Bhattacharya et al. (2016), individuals have to borrow to finance human capital investments and would be punished if they defaulted.

Second, there are also many research studies that consider a formation of human capital depending on educational time input. Ben-Porath (1967) first studied individuals'

⁴ See Fernandez and Rogerson (1995) for other similar research.

allocation of time between accumulating human capital and working. Following Ben-Porath (1967), Azariadis and Drazen (1990) formulated this problem in an OLG model. Castelló-Climent and Doménech (2008) found that if investment of time in education is related to individuals' life expectancy, inequality would affect per capita income and there could exist multiple steady states depending on the initial distribution of education. Other examples are Blankenau and Camera (2009), Chen (2010), Galor (2011), De la Croix and Licandro (2012), and Cipriani (2015).

- ***Intergenerational education***

We focus initially on pecuniary inputs in the process of human capital accumulation. Chanda (2008) assumed that children's human capital accumulation does not depend on parents' human capital level. That study evaluated the effects of rising returns to human capital investment and found that a rise in the return to education raises the education spending ratio as well as the return to capital due to the complementarity between physical and human capital.

In contrast, Lambrecht et al. (2005), Cremer and Pestieau (2006), Emerson and Knabb (2007), among others, assumed an intergenerational inheritance of human capital. Lambrecht et al. (2005) found that unfunded social security systems promote growth when families face liquidity constraints that prevent them from investing optimally in their children's education. Cremer and Pestieau (2006) studied the design of education policies and Emerson and Knabb (2007) built a model with child labor in which parents need to decide whether to educate their children or to make them work.

Furthermore, there are also studies assuming that children partially inherit the average human capital of the whole previous generation: for example, Azarnert (2010), Kitaura and Yakita (2010), McDonald and Zhang (2011).

In addition, Ferreda and Tapia (2010) introduced an education market with heterogeneous private schools in an OLG model, and simulated the effects of taxation on growth, intergenerational mobility, inequality, as well as welfare. Again, other examples are Zilcha (2003) and Alonso-carrera et al. (2012).

		Self-education	Intergenerational education	Self-education (the same as intergenerational education)
Pecuniary input	Non-inherited	Galor and Zeira (1993), Fernandez and Rogerson (1995), Lord and Rangazas (1998), Bhattacharya et al. (2016), etc.	Zilcha (2003), Chanda (2008), Alonso-carrera et al. (2012), etc.	
	Inherited from parents	Chakraborty and Das (2005), Aguiar-Conraria (2005), Docquier et al. (2007), Del Rey and Lopez-Garcia (2013, 2016), etc.	Glomm and Ravikumar (1992), Glomm and Kaganovich (2003), Lambrecht et al. (2005), Cremer and Pestieau (2006), Emerson and Knabb (2007), Ferreda and Tapia (2010), etc.	
	Inherited from the whole society	Garrec (2012), etc.	Azarnert (2010), Kitaura and Yakita (2010), McDonald and Zhang (2011), etc.	
Time input	Non-inherited	Ben-Porath (1967), Castelló-Climent and Doménech (2008), Chen (2010), Blankenau and Camera (2004), Galor (2011), De la Croix and Licandro (2012), Cipriani (2015), etc.	Futagami and Yanagihara (2008), Chu et al. (2016), etc.	
	Inherited from parents	Azariadis and Drazen (1990), etc.	Glomm and Kaganovich (2003), Tamura (2006), etc.	De Gregorio (1996), De Gregorio and Kim (2000), De la Croix (2001), Yakita (2003), Valente (2005), etc.
	Inherited from the whole society	Kemnitz and Wigger (2000), etc.	Tamura (2006), etc.	

Table 1.1 Literature on private education

Second, some studies focused on the role of parents' time input in intergenerational education. For instance, Tamura (2006) developed a general equilibrium model of fertility and human capital investment with young adult mortality. That study showed that due to the negative relationship between young adult mortality and average young adult human capital, human capital accumulation lowers mortality, which leads to demographic transition and industrial revolution. Futagami and Yanagihara (2008) and Chu et al. (2016) also paid attention to parents' time spent educating the next generation.

Third, some studies considered the role of both financial input and time input. One example is Glomm and Kaganovich (2003). They considered an OLG economy with heterogeneous individuals and both public and private education. In their model, parents' decisions are of heterogeneity and lead to heterogeneous incomes. Their results indicated that an increase in spending on public education may result in higher inequality.

- ***Other related research***

In some research, individuals' human capital would be inherited by their offspring just as it is. In this case, from the view of growth maximization, self-education is essentially the same as intergenerational education.

De Gregorio (1996), as an example, discussed the negative effects of borrowing constraints, by reducing human capital accumulation, on growth in an OLG model with endogenous growth. De Gregorio and Kim (2000) presented an endogenous growth model with credit markets that affect the time allocation between studying and working of individuals with different abilities. Yakita (2003) studied the growth effects of wage and interest income taxation in an endogenous growth model with diminishing returns in human capital accumulation. Moreover, Valente (2005) examined the effects of distortionary taxes and public investment in an endogenous growth model with knowledge transmission. See also De la Croix (2001), among others, for other similar studies.

1.3.4 Capital tax competition

Following the discussion of Oates (1972), Zodrow and Mieszkowski (1986) and Wilson (1986) started a formal analysis of physical tax competition.⁵ According to their results, a tax competition leads to the underprovision of public goods. In a symmetric framework, because each regional government takes the welfare of only its own citizens and not those in other regions into consideration, it makes an inefficient policy decision that the tax rate is excessively low. Consequently, this fiscal externality causes an inefficiency of resource allocation. Therefore, in the settings of a static framework, it is generally believed that a coordinated increase in capital tax rate alleviates this kind of externality and improves social welfare.

Many researchers made efforts to challenge this notion and the most representative ones are from Leviathan models where the objective of every jurisdiction is to maximize its own tax revenues (Brennan and Buchanan, 1980). Rauscher (1996) introduced an international factor mobility into a simple model of interjurisdictional competition and found that the effects of increased factor mobility on the efficiency of the public sector are ambiguous. Wilson (2005) constructed such a model whereby tax competition is welfare-improving but leads to a greater size of government based on the Zodrow–Mieszkowski model, with a self-motivated government providing public input instead of public good (consumption).^{6, 7}

Contrasting with these static analyses, Batina (2009, 2012) provided a possibility for the underprovision of public goods from the dynamical perspective. Batina (2009) extended the static horizontal capital tax competition model to an OLG economy and studied the effects of a coordinated reform that capital tax rates across all countries are increased, which is aimed at alleviating the fiscal externality. The study showed that this coordinated tax reform has an ambiguous effect on welfare and does not necessarily lead to a Pareto improvement. By applying Batina (2009), Batina (2012) comprised a PAYG social security funded by the taxation on wage income and a public good funded by the taxation on physical capital. That study found that a coordinated capital tax rise creates an endogenous funding crisis for the social security program. It provided sufficient

⁵ See, for example, Hoyt (1991) and Wilson (1999) for other related studies.

⁶ See Rauscher (1998), among others, for other related studies.

⁷ See Zodrow (2010), Wilson and Wildasin (2004), and Baskaran and Lopes da Fonseca (2013) for surveys of the literature.

conditions under which all current and future generations are better off after the reform and showed that social security may reduce the gain to capital tax reform.

Empirical studies on real-world economies provided evidence for tax competition. Buettner (2001) tested the case of Germany and confirmed the existence of local tax competition, particularly showing that large jurisdictions set higher tax rates in interjurisdictional competition. Similarly, Brueckner and Saavedra (2001) investigated the case of Boston in the United States; Leprince et al. (2007) and Charlot and Paty (2007) the case of France; and Feld and Reulier (2009) the case of Switzerland.

Besides local tax competition, studies also provided evidence on international tax competition. Devereux et al. (2008) and Egger et al. (2007) tested the case of OECD countries, and Altshuler and Goodspeed (2015) tested the case of a competition between European countries and the United States.

The remainder of this dissertation is as follows: Chapter 2 re-considers the PAYG pension from the perspective of transitional effects by applying an analytical framework for dynamical analysis inspired by Kuhle (2014). Chapter 3 evaluates the policy effects of a fully funded pension system when individuals make a trade-off between intergenerational education and self-education. Chapter 4 analyzes the effects of a coordinated capital tax reform across countries where governments provide public education instead of public goods. Chapter 5 gives conclusions and further prospects.

Chapter 2 Policy effects on transitional welfare in an OLG model: a PAYG pension reconsidered

Most of the previous studies on PAYG and fully funded pensions limited their focuses to the steady-state or the balanced growth path. In contrast to Hamada et al. (2017) and Chen and Fang (2013), this chapter analyzes the effects of introducing a PAYG pension system on the welfare of both transitional and steady-state generations in an OLG model.

We show that the total effects of a PAYG pension on utility can be decomposed into three parts: a direct effect on utility, a cumulative effect that occurs through capital accumulation, and an intertemporal effect brought about by a change in the interest rate faced in old age.⁸ Because the cumulative effect through capital accumulation becomes stronger as time goes by while the direct effect remains unchanged, the direction of the total welfare effects will mainly depend on the direct effect in the short run, and on the cumulative effect at the steady state. Therefore, the pension affects welfare differently, even oppositely, in the short run and the long run. Specifically, we show that in the short and medium runs: (1) even when the economy is dynamically efficient, a PAYG pension can be welfare-improving; and (2) when the economy is dynamically inefficient, a PAYG pension can reduce social welfare. In contrast, when we derive the effects of a fully funded pension on welfare, its neutrality holds not only at the steady states but also along the whole transition path.

To conduct the above analyses, we first provide a mathematical analytical framework. We introduce a simpler and clearer method for obtaining the solution of the difference equation system or the values of variables in transitional states. We show that the effects of a policy change on utility along the transition path are a weighted average of the marginal change in utility of the initial period and that of the steady state. Further, we generally examine the local dynamics of utility in the case where the steady-state

⁸ Besides those empirical studies on PAYG pension systems introduced in Chapter 1, the effects of other policy instruments have also been empirically analyzed. For example, Muradoglu and Taskin (1996) examined the differences in the determinants of household savings between developing and industrial countries. They found that to increase the level of household savings, developing countries should improve their financial markets and adopt new instruments, rather than introducing conventional policies, such as lowering real returns and the inflation rate, which are only effective in industrial countries.

equilibrium is oscillatory in a stable manner. By doing this, we show how a policy change affects social welfare in the short and long runs, and demonstrate that there could be a turning period along the transition path before and after which generations experience opposite utility changes. Therefore, the government should judge and weigh the short- and long-run effects when making policy decisions.

This chapter is organized as follows. In Section 2.1, we provide a mathematical preparation and show the benchmark solutions for a neoclassical OLG model. We clarify the dynamics of utility in the cases of both monotonic and oscillatory convergence. Section 2.2 analyzes the effects of a basic PAYG pension on transitional welfare and compares it with a fully funded pension.

2.1 The analytical framework

2.1.1 Benchmark solutions

First, we briefly introduce the OLG model developed by Diamond (1965) and then describe the dynamics of utility following Kuhle (2014).

Our economy begins in period 0 (in an “original” steady state) with a level of capital stock k_0 . In an economy with a constant (net) population growth rate of n , individuals live for two periods: initially as the young and then as the old. In each period, the young and the old overlap. In period t , the young, generation t , inelastically provide one unit of labor to earn a wage income w_t and they spend it either to consume or to save for consumption in their old age. In period $t + 1$, when they have become old, they dissave for their retirement consumption. Thus, the budget constraints of generation t in the young and the old periods can be written as:

$$w_t = c_t + s_t, \quad (2.1)$$

$$d_{t+1} = (1 + r_{t+1})s_t, \quad (2.2)$$

respectively. Here, c_t and d_t denote young consumption in period t and old consumption in period $t + 1$, respectively; s_t is the savings in period t and r_{t+1} is the interest rate on savings in period $t + 1$. Hence, the first-order condition for maximizing their utilities, $U_t = U(c_t, d_{t+1})$, under the above budget constraints (2.1) and (2.2), can be given by:

$$\frac{U_c}{U_d} = 1 + r_{t+1}, \quad (2.3)$$

as long as $c_t > 0$, $d_{t+1} > 0$ holds.⁹ This gives a savings function of $s_t = s(w_t, r_{t+1})$.

The aggregate output in period t produced by capital stock K_t and labor L_t is characterized by $Y_t = F(K_t, L_t)$, where the production function $F(.,.)$ exhibits constant returns to scale. This feature leads to the following per capita production function:

$$y_t = f(k_t), \quad k \equiv \frac{K_t}{L_t}, \quad f'(k_t) > 0 \quad \text{and} \quad f''(k_t) < 0,$$

with Inada conditions. Maximizing the profits of firms implies:

$$r_t = f'(k_t), \quad w_t = f(k_t) - f'(k_t)k_t. \quad (2.4)$$

When the capital market clears, the following condition holds:

$$(1 + n)k_{t+1} = s(w_t, r_{t+1}). \quad (2.5)$$

Thus, equation (2.5) describes the intertemporal equilibrium of the economy. The above conditions define sequences of prices $\{w_t, r_t\}_{t=1}^{\infty}$, consumption $\{c_t, d_t\}_{t=1}^{\infty}$, and per capita capital stock $\{k_t\}_{t=1}^{\infty}$.

2.1.2 Transitional dynamics

According to (2.4) and (2.5), we can express the intertemporal equilibrium only in terms of per capita capital stock k_t and k_{t+1} , as follows:

$$(1 + n)k_{t+1} = s(w(k_t), r(k_{t+1})),$$

which leads to $k_{t+1} = \psi(k_t)$. Therefore, the local stability conditions for monotonic and oscillatory convergences can be written, respectively, as:

$$0 < \frac{dk_{t+1}}{dk_t} = \psi_k < 1, \quad (2.6a)$$

$$-1 < \psi_k < 0. \quad (2.6b)$$

For now, we examine the case where the equilibrium is monotonically convergent and consider a policy change in the initial period, 0, as shown in Kuhle (2014). To investigate

⁹ U_t satisfies the neoclassical assumptions because the utility function is concave and twice continuously differentiable. In addition, the subscripts represent the derivatives if there is no confusion.

the marginal effects of an exogenously given policy parameter, b , on the utility of generation t , we incorporate the policy parameter into the above basic model following Kuhle (2014):

$$k_{t+1} = \psi(k_t; b), \quad 0 < \frac{dk_{t+1}}{dk_t} = \psi_k < 1, \quad (2.7)$$

$$U = U(w(k_t) - s(w(k_t), r(k_{t+1}); b), (1 + r(k_{t+1}))s(w(k_t), r(k_{t+1}); b)) = U(k_t, k_{t+1}; b). \quad (2.8)$$

Here, the change in b is a “quantitative”, rather than a “qualitative” change in the policy system. It should be noted that the change in the policy parameter not only affects the individuals’ utility directly, as seen in (2.8), but also the evolution of the capital stock, as seen in (2.10) below. The latter effect results in indirect effects on utilities in the following periods.

Following Kuhle (2014), in an economy starting from a monotonically stable steady state, the effect of a marginal change in a policy parameter, b , on utility can be expressed as follows:

$$\frac{dU_{-1}}{db} = \frac{\partial U_{-1}}{\partial b}, \quad (2.9)$$

$$\frac{dU_t}{db} = \frac{\partial U}{\partial k_t} \frac{dk_t}{db} + \frac{\partial U}{\partial k_{t+1}} \frac{dk_{t+1}}{db} + \frac{\partial U}{\partial b}. \quad (2.10)$$

The direct effect on the utility of generation t can be seen in the third term on the right-hand side of (2.10). In contrast, the first term describes a cumulative effect, which is an indirect effect of a policy change on utility that occurs through the change in the capital level in the present t -th period. This shows an impact on utility arising from the accumulated changes in capital levels, which captures the changes in both wages and interest rates that occurred throughout the prior periods. Similarly, the second term represents the effect on utility that arises through the change in the capital level in the next $(t + 1)$ -th period, k_{t+1} . In other words, this term only covers the effect brought about by a change in the interest rate faced in old age.

As $\frac{dk_t}{db} = \sum_{i=0}^{t-1} \psi_k(k; b)^i \psi_b(k; b) = \frac{1 - \psi_k^t(k; b)}{1 - \psi_k(k; b)} \psi_b(k; b)$ holds, substituting this into (2.10) yields:

$$\frac{dU_t}{db} = \frac{dU}{dk} \frac{1 - \psi_k^t(k; b)}{1 - \psi_k(k; b)} \psi_b(k; b) + \frac{\partial U}{\partial k_{t+1}} \psi_k(k; b)^t \psi_b(k; b) + \frac{\partial U}{\partial b}, \quad (2.11)$$

where $\frac{dU}{dk} = \frac{\partial U}{\partial k_t} + \frac{\partial U}{\partial k_{t+1}}$.¹⁰ Using this, we obtain the effects of the policy change on the utility level in each period. First, for the young in period 0 and the generation in the new steady state, we have:

$$\frac{dU_0}{db} = \frac{\partial U}{\partial k_0} \frac{dk_0}{db} + \frac{\partial U}{\partial k_1} \frac{dk_1}{db} + \frac{\partial U}{\partial b} = \frac{\partial U}{\partial k_1} \psi_b(k; b) + \frac{\partial U}{\partial b}, \quad (2.12)$$

$$\begin{aligned} \frac{dU_{ss}}{db} &= \lim_{t \rightarrow \infty} \frac{dU_t}{db} = \lim_{t \rightarrow \infty} \left(\frac{\partial U}{\partial k_t} \frac{dk_t}{db} + \frac{\partial U}{\partial k_{t+1}} \frac{dk_{t+1}}{db} + \frac{\partial U}{\partial b} \right) \\ &= \left(\frac{\partial U}{\partial k_t} + \frac{\partial U}{\partial k_{t+1}} \right) \frac{1}{1 - \psi_k(k; b)} \psi_b(k; b) + \frac{dU}{db} \\ &= \frac{dU}{dk} \frac{1}{1 - \psi_k(k; b)} \psi_b(k; b) + \frac{dU}{db}. \end{aligned} \quad (2.13)$$

It is obvious from (2.12) that the effect on utility brought about by capital accumulation in previous periods cannot be determined in the initial period because $\frac{dk_0}{db} = 0$. In contrast, from (2.13), in the (new) steady state, the cumulative effect emerges.

It is worth noting that, by using (2.12) and (2.13), (2.10) can be rewritten in a concise form as:

$$\frac{dU_t}{db} = \psi_k^t \frac{dU_0}{db} + (1 - \psi_k^t) \frac{dU_{ss}}{db}. \quad (2.14)$$

As this shows, $\frac{dU_t}{db}$ can be represented using the weighted averages of $\frac{dU_0}{db}$ and $\frac{dU_{ss}}{db}$, and the weights are ψ_k^t and $(1 - \psi_k^t)$, respectively. Therefore, to determine the welfare change in any transitional period, we need only find $\frac{dU_0}{db}$ and $\frac{dU_{ss}}{db}$, given ψ_k . This can be summarized as the following proposition.

Proposition 2.1

The effects of a policy change on utility along the transition path are a weighted average of the marginal change in utility of the initial period and that of the steady state, that is, $\frac{dU_t}{db} =$

$$\psi_k^t \frac{dU_0}{db} + (1 - \psi_k^t) \frac{dU_{ss}}{db}.$$

¹⁰ For the detailed calculation of $\frac{dk_t}{db}$, see Kuhle (2014).

Proof.

See Appendix 2.A. □

Using this weighted average form, the utility change along the transition path can be investigated easily. We consider the cases of monotonic and oscillatory welfare changes.

2.1.3 Monotonic welfare change

First, we consider the case of monotonic convergence. Below, we show that by determining the relationship among the welfare changes in the initial period, the steady state, and period t , we can further investigate how welfare changes along the transitional path and in what direction welfare changes.

Proposition 2.2

Consider the case of monotonic convergence, (2.6a). At a locally stable steady state, the effect of a marginal change in policy b on utility levels for all generations along the transition path born in $t \geq 0$ is as follows:

(i) *If $\frac{dU_0}{db} > \frac{dU_{ss}}{db}$, then $\frac{dU_0}{db} > \frac{dU_1}{db} > \dots > \frac{dU_t}{db} > \dots > \frac{dU_{ss}}{db}$. That is, utility becomes lower as time goes by.*

(ii) *If $\frac{dU_0}{db} < \frac{dU_{ss}}{db}$, then $\frac{dU_0}{db} < \frac{dU_1}{db} < \dots < \frac{dU_t}{db} < \dots < \frac{dU_{ss}}{db}$. That is, utility becomes higher as time goes by.*

Proof.

Applying (2.14), it can be easily verified that if $\frac{dU_0}{db} > \frac{dU_{ss}}{db}$, then $\frac{dU_t}{db}$ is decreasing over time.

The reverse also holds. □

By using the weighted average expression, it is clarified more explicitly that the marginal change in utility caused by an exogenous policy change shows a monotonic

convergence along the transition path and that it monotonically converges on the new steady-state level. Kuhle (2014) implied this in his proof but did not examine it in detail. To put it more concretely, when $\frac{dU_0}{db} < \frac{dU_{ss}}{db}$, the policy effect on utility is magnified over time; however, the volume of the increase becomes smaller and smaller. In contrast, when $\frac{dU_0}{db} > \frac{dU_{ss}}{db}$, the policy effect on utility shrinks over time, whereas the volume of the decrease becomes larger. Here, it is worthwhile to consider the case where both $\frac{dU_0}{db}$ and $\frac{dU_{ss}}{db}$ are greater (less) than zero. In this case, all generations along the transition path are better (worse) off.¹¹ However, when $\frac{dU_0}{db} < (>) 0$ and $\frac{dU_{ss}}{db} > (<) 0$, there exists a turning period t , where the direction of the change in utility is reversed. In the OLG model literature, it is often seen that the qualitative effects of a policy change are divergent between the initial and steady states in comparative dynamic analysis. In addition to such results, our analysis further clarifies the transitional effects through the algebraic description using the weighted sum of the effects in the initial and steady states.

2.1.4 Oscillatory welfare change

The above discussion for the case of monotonic convergence can be applied to the case of oscillatory convergence. In the case where (2.6b) holds, the effect of a policy change on utility can be obtained as the following proposition.

Proposition 2.3

Consider the case of oscillatory convergence, (2.6b). At a locally stable steady state, the effect of a marginal change in policy b on utility levels for all generations along the transition path born in $t \geq 0$ is as follows:

- (i) *if $\frac{dU_0}{db} > 0$, $\frac{dU_1}{db} > 0$ and $\frac{dU_{ss}}{db} > 0$, then $\frac{dU_t}{db} > 0$; if $\frac{dU_0}{db} < 0$, $\frac{dU_1}{db} < 0$ and $\frac{dU_{ss}}{db} < 0$, then $\frac{dU_t}{db} < 0$ for $t = 0, 1, \dots$,*
- (ii) *if $\frac{dU_0}{db} > (<) 0$, $\frac{dU_1}{db} < (>) 0$ and $\frac{dU_{ss}}{db} > (<) 0$, then*

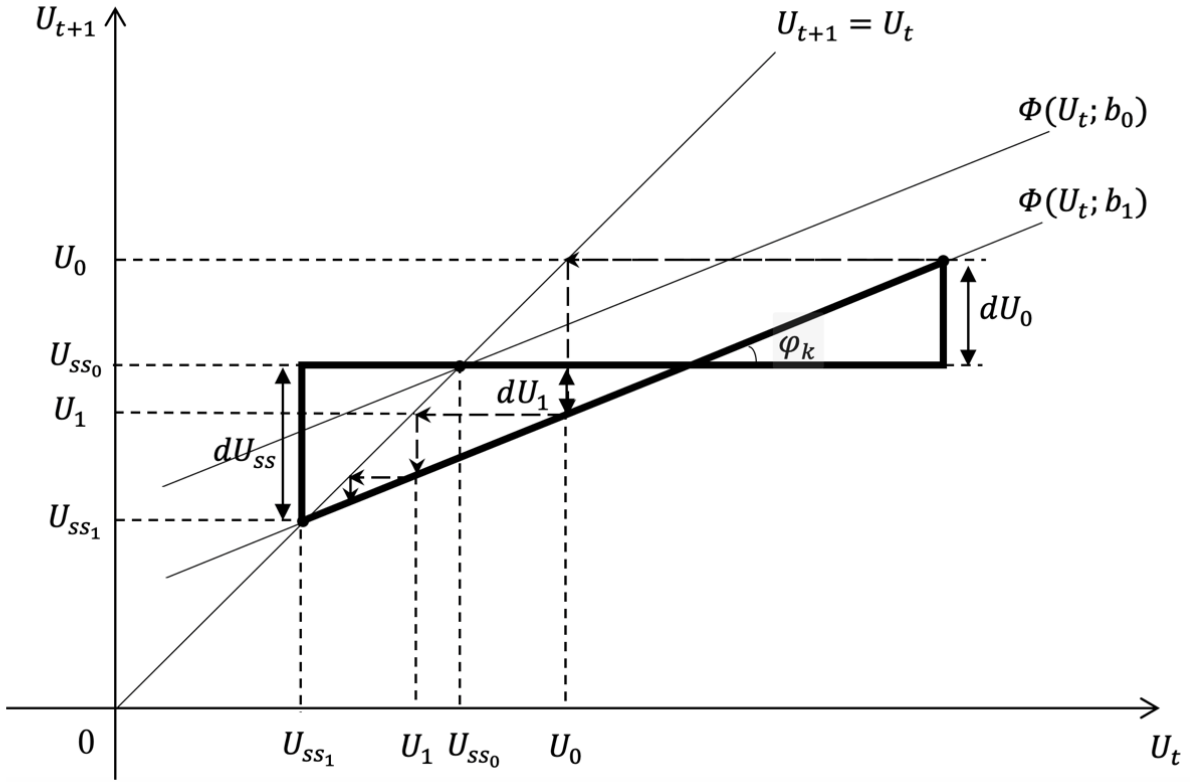
¹¹ See Proposition 2-1 in Kuhle (2014).

- (a) $\frac{dU_t}{db} > (<) 0$, for $t = 2i, i = 0, 1, \dots$,
- (b) $\frac{dU_t}{db} \leq (\geq) 0$, for $t = 2i + 1, i = 0, 1, \dots, \frac{t^*-1}{2}$, and $\frac{dU_t}{db} > (<) 0$, for $t = 2i + 1, i = \frac{t^*+1}{2}, \frac{t^*+3}{2}, \dots$, where $\frac{dU_{t^*}}{db} \leq 0 < \frac{dU_{t^*+2}}{db}$ ($\frac{dU_{t^*}}{db} \geq 0 > \frac{dU_{t^*+2}}{db}$);
- (iii) if $\frac{dU_0}{db} > (<) 0, \frac{dU_1}{db} < (>) 0$ and $\frac{dU_{ss}}{db} < (>) 0$, then
- (a) $\frac{dU_t}{db} \geq (\leq) 0$, for $t = 2i, i = 0, 1, \dots, \frac{t^*}{2}$, and $\frac{dU_t}{db} < (>) 0$, for $t = 2i, i = \frac{t^*+2}{2}, \frac{t^*+4}{2}, \dots$, where $\frac{dU_{t^*}}{db} \geq 0 > \frac{dU_{t^*+2}}{db}$ ($\frac{dU_{t^*}}{db} \leq 0 < \frac{dU_{t^*+2}}{db}$),
- (b) $\frac{dU_t}{db} < (>) 0$, for $t = 2i + 1, i = 0, 1, \dots$ ¹²

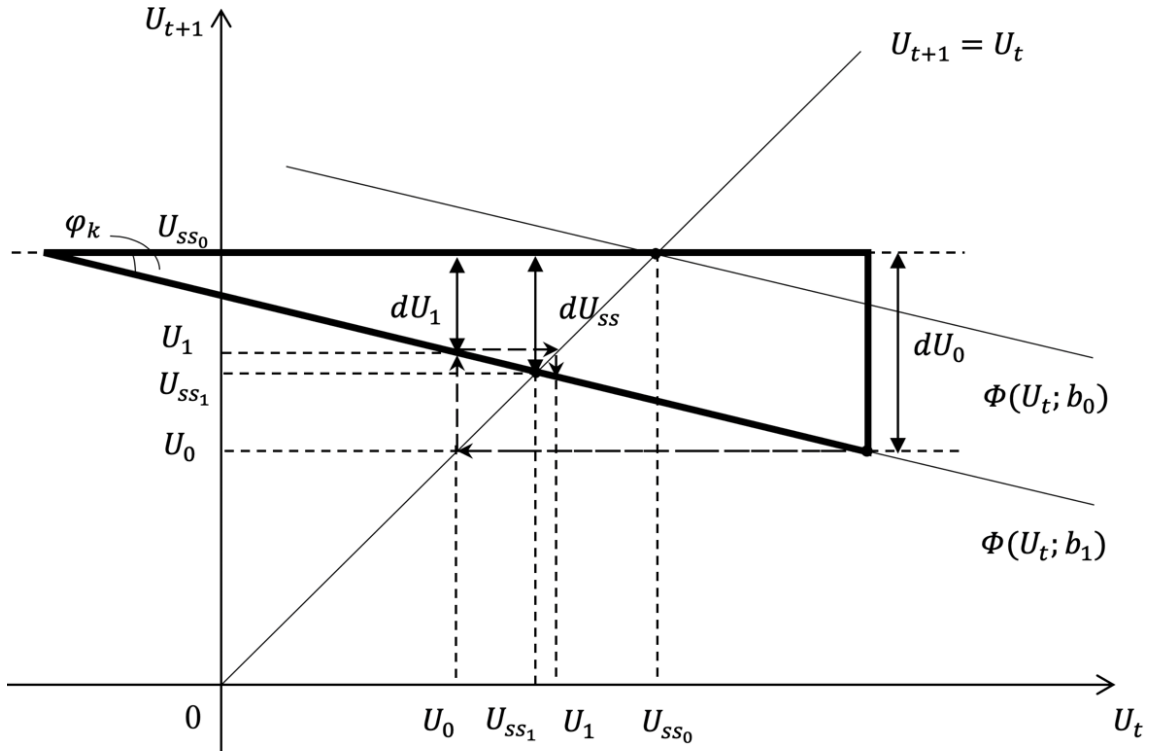
In the case of monotonic convergence, according to Proposition 2.2, the direction of effects on utility can be reversed no more than once during a transition. However, as shown in Proposition 2.3 parts (ii) and (iii), the transitional effects in the case of oscillatory convergence can be more complex: i.e., the reversal of direction continues up to a certain period. This occurs when the initial and steady-state effects on utility differ in direction from those in the second period (in the above argument, “period 1”). Therefore, both the effects in these two periods are crucial for the determination of the transitional effects in the case of oscillatory convergence. This means that policy makers should also pay attention to the welfare effect in the second period in transitional states when evaluating the welfare effects in the following transitional states. It should be noted that these results can be attributed to the assumption of (oscillatory) convergence: dynamic stability, ψ_k .

From Propositions 2.2 and 2.3, the most important point is that, regardless of the cases of monotonic or oscillatory convergences, the transitional effects on welfare can be expressed as a weighted average of the welfare effects in the initial and the steady states. In addition, note that the transitional effects consist of the indirect effect, which occurs through the change in capital stock, and the direct effect on utility of the policy change,

¹² See Appendix 2.B for the proof.



$$(A) \frac{dU_0}{db} > 0 > \frac{dU_{ss}}{db}$$



$$(B) 0 > \frac{dU_1}{db} > \frac{dU_{ss}}{db} > \frac{dU_0}{db}$$

Figure 2.1 The dynamics of utility

as discussed in (2.10). These two effects are acknowledged in the first two terms and in the last term of (2.10), respectively. When the direct effect of a policy change on capital stock is opposite to the direct effect, the change in utility may have a different sign, by turns, as time goes by. In contrast, if the direct effect is strong enough, the direction of the effects on utility on the whole transition path would be the same as at the steady state.

This situation can be explained more intuitively by a diagram. In Figure 2.1, $\phi(U_t; b_i), i = 0, 1, b_0 < b_1$ represents the dynamics of U_t along the transition path in the neighborhood of the steady states. First, we show the case of $\frac{dU_0}{db} > 0 > \frac{dU_{ss}}{db}$ in Figure 2.1-(A). Assume that U_{ss_0} is the original steady-state level of utility, corresponding to the original level of the policy parameter, b_0 . The dynamics of U_t in its neighborhood can be determined as $\phi(U_t; b_0)$. Here, we assume that the government changes the policy parameter from b_0 to b_1 . This alters utility at the original steady state through direct and indirect effects that occur through the change in the level of capital stock only in the next period, which is given by (2.12). Therefore, U_{ss_0} “jumps” to some point on the new dynamic path of $\phi(U_t; b_1)$, which we denote as U_0 . In other words, the original $\phi(U_t; b_0)$ shifts downward to $\phi(U_t; b_1)$ on which U_0 must be, and the economy shifts to a new transition path leading to a new steady state U_{ss_1} .

It should be noted that, in this figure, we assume that the direct effect $\frac{\partial U}{\partial b}$ is negative and small, but the indirect effect $\frac{\partial U}{\partial k_1} \psi_k(k; b)$ is positive and large, and that only the generation born in the initial period is made better off by the increase in b .¹³ After the increase in b , U_0 must be located on the upper-right side of U_{ss_0} . As the negative effects that occur through the change in the capital stock accumulate, the utilities of all following generations along the new transition path are decreased compared with the original steady-state U_{ss_0} , and these utilities become smaller as time passes. Moreover, we can see that all the values of the utility change dU_t fall into the range of $[dU_{ss}, dU_0]$, and that this range can be displayed by the two bold triangles. The heights of the triangles indicate dU_0

¹³ Here, we assume that U_{ss_0} jumps to U_0 when b_0 becomes b_1 because it is unclear where U_0 is located. This can be seen in the example given by Kuhle (2014). That is, when the amount of the government bond increases, the interest rate faced by the initial young generation rises under an unchanged wage rate. Thus, the utility level of the initial old generation necessarily rises. The point here is that the effect of the change in the policy parameter on the utility of the initial young generation appears to be different from the effect on the descendent generations.

and dU_{ss} , and utility can only vary within the ranges indicated by the areas of the triangles. In an intuitive way, this indicates what we have emphasized: e.g., $\frac{dU_t}{db}$ can be expressed as a weighted average of $\frac{dU_0}{db}$ and $\frac{dU_{ss}}{db}$.

Figure 2.1-(B) illustrates one of the oscillatory cases in Proposition 2.2 (ii): $0 > \frac{dU_0}{db} > \frac{dU_{ss}}{db} > \frac{dU_0}{db}$. Here, we assume that the direct effect of a change from b_0 to b_1 , i.e., $\frac{\partial U}{\partial b}$, is negative and strong, and that the indirect effect $\frac{\partial U}{\partial k_1} \psi_k(k; b)$ is of an uncertain sign but relatively weak. Similar to the case in Figure 2.1-(A), the original steady state U_{ss_0} “jumps” to U_0 , located on the lower-right side. Due to the oscillation of capital accumulation, utility changes in an oscillatory way. However, all generations along the transition path become worse off. We can describe the variation range of the utility change in the bold triangle, the height of which implies a change boundary.

2.2 PAYG pension system

In this section, based on the above analytical method for evaluating welfare effects in transition, we examine a PAYG pension system. We investigate how a change in the pension policy affects welfare in transition, as well as in the steady state. We can easily confirm that the welfare changes in transitional states can be represented by the weighted average of the welfare effects in the initial state and in the steady state. Further, when it becomes possible to consider the welfare effects of a PAYG pension in transition, these should be investigated, not only in the case of monotonic convergence, but also in the case of oscillatory convergence. In the latter case, positive and negative welfare effects appear in turn, over time, during the transition phase. Therefore, we can clarify which generations gain or lose from the introduction of a change in the PAYG pension system.

We apply Propositions 2.1–2.3 to the cases of dynamic efficiency and dynamic inefficiency, considering, respectively, situations where the steady-state equilibrium is monotonically and oscillatory convergent. More concretely, in Section 2.2.2, we show that, along the transition path, a PAYG pension can be Pareto-improving even when it is

dynamically efficient with monotonic convergence.¹⁴ Then, in Section 2.2.3, we show that the pension can reduce welfare in transition when the economy is dynamically inefficient with oscillatory convergence.¹⁵

2.2.1 The basic model

Under a PAYG pension, the government collects a pension contribution in a lump-sum form from the young and transfers it to the old as a pension payment. Therefore, the budget constraints of generation t , (2.1) and (2.2), can be modified as follows:

$$w_t = c_t + s_t + T, \quad (2.15)$$

$$d_{t+1} = (1 + r_{t+1})s_t + (1 + n)T, \quad (2.16)$$

where T is the pension contribution made by the young and $(1 + n)T$ is the pension payment received in old age. With the same first-order condition (2.3), we can rewrite the market-clearing condition (2.5) as:

$$(1 + n)k_{t+1} = s(w_t - T, (1 + n)T, r_{t+1}), \quad (2.17)$$

which implies that $k_{t+1} = \psi(k_t; T)$.¹⁶ Differentiating (17) and the utility function, $U_t = U(c_t, d_{t+1}) = U(w_t - s_t - T, s_t(1 + r_{t+1}) + T(1 + n)) = U(f(k_t) - f'(k_t)k_t - s_t - T, s_t(1 + f'(k_{t+1})) + T(1 + n))$, and evaluating them at the steady state, we have:

$$\begin{aligned} \psi_k(k; T) &= \frac{dk_{t+1}}{dk_t} = - \frac{s_w f'' k}{1 + n - s_r f'' - \frac{1+n}{(1+r)^2} (1-s_w) f'' T}, \\ \psi_T(k; T) &= \frac{dk_{t+1}}{dT} = - \frac{1}{1+r} \frac{(1+r)s_w + (1+n)(1-s_w)}{1 + n - s_r f'' - \frac{1+n}{(1+r)^2} (1-s_w) f'' T}, \\ \frac{dU}{dk} &= \frac{\partial U}{\partial k_t} + \frac{\partial U}{\partial k_{t+1}} = U_c f'' k \frac{n-r}{1+r}, \\ \frac{\partial U}{\partial k_{t+1}} &= U_c f'' k \frac{1+n}{1+r}, \end{aligned}$$

¹⁴ In fact, we can obtain the same result when the dynamics of capital are oscillatory. However, for simplicity, we present only the case of monotonic convergence.

¹⁵ When capital stock converges monotonically, all steady-state generations and all generations along the transition path are made better off by an increase in the pension contribution (benefit). This can be easily proved by a similar calculation.

¹⁶ Here, we follow Chapter 3 of De La Croix and Michel (2002).

$$\frac{\partial U}{\partial T} = U_c \frac{n-r}{1+r}.$$

Recalling (2.9), (2.12), and (2.13), we obtain the effect of a marginal change in the pension contribution T on the utility levels of the first old generation, generation -1 , the first young generation, generation 0 , and those generations born at the steady state:

$$\frac{dU_{-1}}{dT} = \frac{U_c(1+n)}{1+r}, \quad (2.18)$$

$$\frac{dU_0}{dT} = \frac{U_c}{1+r} (1+n)\psi_k M + \frac{U_c}{1+r} (n-r), \quad (2.19)$$

$$\frac{dU_{ss}}{dT} = \frac{U_c}{1+r} (n-r) \frac{\psi_k}{1-\psi_k} M + \frac{U_c}{1+r} (n-r), \quad (2.20)$$

where $M = \frac{(1+r)s_w + (1+n)(1-s_w)}{(1+r)s_w} > 1$.

Equation (2.18) is the utility change of the first old generation, generation -1 . When the pension contribution T increases, they are better off because their pension payment increases. Next, (2.19) represents the utility change of the first young generation, generation 0 . The introduction of the pension reduces their disposable income when they are young but increases their income in old age. This influences their saving behavior and their utility level, which is expressed by the first term on the right-hand side. When $n < (>)r$, the second term is a direct utility loss (gain) caused by the dynamic (in)efficiency. Equation (2.20) shows the utility change of the steady-state generations. The first term on the right-hand side shows the cumulative effects on utility that occur through the accumulation of capital stock. Again, the second term is a direct utility loss (gain) arising from the dynamic (in)efficiency. As the values of (2.19) and (2.20) depend on whether the economy converges monotonically or in an oscillatory manner, and on the sign of $n - r$, they can have opposite signs.

So far, we have focused on the utility changes in the initial period and the steady state. However, it is significant here to determine the effects of the policy change in the transitional states. Therefore, we substitute (2.19) and (2.20) into (2.14) to obtain the utility change along the transition path:

$$\begin{aligned} \frac{dU_t}{dT} &= \frac{U_c}{1+r} \left\{ \psi_k^t [(1+n)M\psi_k + n-r] + (1-\psi_k^t)(n-r) \left(\frac{\psi_k}{1-\psi_k} M + 1 \right) \right\} \\ &= \psi_k^t \frac{dU_0}{dT} + (1-\psi_k^t) \frac{dU_{ss}}{dT}. \end{aligned} \quad (2.21)$$

Again, this is expressed by a weighted average of (2.19) and (2.20).

As there can be qualitative differences in the utility effects in the short run and long run, i.e., for the first young generation and the steady-state generations, respectively, there might exist a turning period before and after which the change in the PAYG pension tends to have opposing effects on utilities. Defining $t^* > 0$ as the (possible) turning period for which we are searching, it then satisfies:

$$\begin{aligned}\frac{dU_{t^*}}{dT} &= \frac{U_c}{1+r} \left\{ \psi_k^{t^*} [(1+n)M\psi_k + n - r] + (1 - \psi_k^{t^*})(n - r) \left(\frac{\psi_k}{1-\psi_k} M + 1 \right) \right\} \\ &= \psi_k^{t^*} \frac{dU_0}{dT} + (1 - \psi_k^{t^*}) \frac{dU_{ss}}{dT} \\ &= 0.\end{aligned}\tag{2.22}$$

In the following subsections, we first examine the circumstances when the economy is dynamically efficient and then when it is dynamically inefficient.

2.2.2 Dynamically efficient economy

In this subsection, we consider a case of monotonic convergence in a dynamically efficient economy. Applying Propositions 2.1 and 2.2 obtained in the general case to the analysis of the PAYG pension, we have the following proposition.

Proposition 2.4

Consider the case of monotonic convergence, (2.6a). Suppose that $r > n$. When the pension contribution is marginally increased, $\frac{dU_0}{dT} > 0$ and $\frac{dU_{ss}}{dT} < 0$. Utility levels along the transition path become lower as time goes by and there exists a turning period t^ , defined in (2.22), such that generations born before this period are better off but those born after this period are worse off.*

Proof.

Applying (i) in Proposition 2.2, we can obtain the above results straightforwardly. \square

When the economy is dynamically efficient, the introduction of a pension scheme is beneficial to the initial old generation. However, all the following generations experience

a direct loss in utility when they are young. This leads to a lower saving level in each period (and thus, a lower capital level in the next period), which accumulates through the capital stock dynamics along the transition path. Therefore, utility is decreased in the long run.

On the one hand, in the very short run, there are no, or relatively weak, cumulative effects. On the other hand, the decrease in the capital stock in the next period brings about an increase in the interest rate, which increases consumption in old age; therefore, it has a positive effect on utility. As a result, generations may still be better off.

The above approach can be illustrated intuitively in Figure 2.2-(A). When introducing a PAYG pension, because of the weak cumulative effect and the strong increase in the old age consumption, utility jumps from the original steady-state level U_{ss_0} to a higher level, U_0 , which is the utility of generation 0 (the initial young). The dynamics of utility $\phi(U_t; 0)$ shifts down to $\phi(U_t; T)$ and the utility of the following generations decreases but may still be higher than the original steady-state level. As the cumulative effects become stronger and finally exceed the increase in old age consumption, after some Pareto-improved generations in the short run, the economy converges to a new steady state, at which the utility level U_{ss_1} is lower than U_{ss_0} .

This may explain why most countries and regions in the real world totally or partially adopt a PAYG pension system instead of a fully funded one. In most previous studies, the PAYG pension is regarded as having a negative effect on social welfare when the economy is dynamically efficient at the steady states. However, as we show, even though the introduction of a PAYG pension reduces social welfare in the long run, it could be an effective policy tool in the short or medium run. More concretely, until the turning point period is reached, a PAYG pension will improve welfare.

2.2.3 Dynamically inefficient economy

Next, we consider a case of oscillatory convergence in an economy that is dynamically inefficient.¹⁷ Similar to the previous subsection, applying Propositions 2.1 and 2.3, we obtain the following proposition.

Proposition 2.5

Consider the case of oscillatory convergence, (2.6b). Suppose that $r < n$. If the speed of convergence of the capital stock is low, such that $r > n + (1+n)\psi_k \frac{(1+r)s_w + (1+n)(1-s_w)}{(1+r)s_w}$, then, when the pension contribution is marginally increased, $\frac{dU_0}{dT} < 0 < \frac{dU_{ss}}{dT} < \frac{dU_1}{dT}$. All generations born in odd periods, i.e., generations 1, 3, and 5, ..., become better off and their utility becomes higher as time goes by. However, for generations born in even periods, i.e., generations 0, 2, and 4, ..., utility becomes lower as time goes by. Moreover, for those born in the even periods, there exists a turning period t^ that is defined in (2.22), such that generations born before this period are worse off, but those born after this period are better off.*

Proof.

Similarly, we can obtain the above results by applying (iii) in Proposition 2.3. \square

¹⁷ It is valuable to check whether an oscillatory convergence can appear or not. In fact, we can give an example of oscillatory convergence when there is a PAYG pension system. Following Hamada et al. (2018), we consider a utility function that $u(c_t, d_{t+1}) = \frac{c_t^{1-\sigma}-1}{1-\sigma} + \beta \frac{d_{t+1}^{1-\sigma}-1}{1-\sigma}$. Here, $1/\sigma$ is the elasticity of substitution. Maximizing the utility subject to (2.15) and (2.16), we can obtain the savings function:

$$s_t = \frac{\beta^{\frac{1}{\sigma}(1+r_{t+1})} w_t - \left[1 + \beta^{\frac{1}{\sigma}(1+r_{t+1})} T\right]}{1+r_{t+1} + \beta^{\frac{1}{\sigma}(1+r_{t+1})} T}.$$

It is easy to obtain: $s_r = \frac{\beta^{\frac{1}{\sigma}(1+r_{t+1})} T^{-1}}{\left[1+r_{t+1} + \beta^{\frac{1}{\sigma}(1+r_{t+1})} T\right]^2} \left[\left(\frac{1}{\sigma} - 1\right)(1+r_{t+1})(w-T) + \left(1 + \frac{1}{\sigma}\right)T\right]$.

When $\frac{1}{\sigma} < 1$ holds and T is small enough, $s_r < 0$ holds. Then, it is possible that $-1 < \frac{dk_{t+1}}{dk_t} = -\frac{s_w f'' k}{1+n-s_r f'' - \frac{1+n}{(1+r)^2} (1-s_w) f'' T} < 0$ holds since $s_w > 0$. That is to say, when the elasticity of substitution is small, which means the substitutive (income) effect of an increase in interest rate is small (great), the dynamics of capital could be oscillatory. (When $T = 0$, it is the very general case in Section 2.1.)

It is worth pointing out that even when the economy is dynamically inefficient, the introduction of a PAYG pension can still negatively affect the utilities of some generations born in even periods. Before reaching the turning period, the pension has a negative or positive effect on generations born in even and odd periods by turns, which can be considered as creating divergence between old and young generations.

Similarly, Figure 2.2-(B) describes the above dynamics. The utility level jumps from the original steady state U_{ss_0} to U_0 as a result of the PAYG pension. Because capital accumulation moves in an oscillatory manner, $k_1 < k_0$ holds. The utility of the first young generation (generation 0) decreases, that is, $U_0 < U_{ss_0}$. However, $\phi(U_t, 0)$ shifts up to $\phi(U_t, T)$ and the economy converges to a new steady state, at which the utility level is U_{ss_1} . As the economy is under dynamical inefficiency, utility becomes higher in the long run. U_{ss_1} is located above U_{ss_0} . The welfare of all odd generations is improved on the transition path, but again, some generations born in even periods, such as generation 0, are worse off in the short run because of the oscillation.

Therefore, it is imperative for the government to realize that the introduction of a PAYG pension does not necessarily increase social welfare in the short or medium run, even when the population growth rate is higher than the interest rate.

2.2.4 Comparison: a fully funded pension system

For governments, a fully funded pension system might be an alternative to the PAYG pension system. Therefore, it is reasonable to compare the welfare effects of the two. Here, we consider the welfare effects of a standard fully funded pension along the transition path. Then, we indirectly compare it with the PAYG pension by comparing it with the original steady state.

Starting from a steady state, the government collects the pension contribution in a lump-sum form from the young and pays it back (with interest) as a pension payment to the old in the next period.¹⁸ Thus, we can modify the budget constraints as follows:

¹⁸ The introduction of a fully funded pension implies that the initial old generation is not affected.

$$w_t = c_t + s_t + P, \quad (2.23)$$

$$d_{t+1} = (1 + r_{t+1})(s_t + P), \quad (2.24)$$

where P represents the pension contribution, which is assumed to be less than the original steady-state level of savings. Again, we obtain the same first-order condition (2.3) and we can rewrite the market-clearing condition (2.5) as:

$$(1 + n)k_{t+1} = s(w_t - T, (1 + r_{t+1})P, r_{t+1}) + P, \quad (2.25)$$

which implies that $k_{t+1} = \psi(k_t; P)$. Similarly, we differentiate (2.25) and the utility function, $U_t = U(c_t, d_{t+1}) = U(w_t - s_t - P, (1 + r_{t+1})(s_t + P)) = U(f(k_t) - f'(k_t)k_t - s_t - P, (1 + f'(k_{t+1}))(s_t + P))$, and evaluate them at the steady state to obtain:

$$\psi_k(k; P) = \frac{dk_{t+1}}{dk_t} = -\frac{s_w f'' k}{1 + n - s_r f''}, \quad (2.26)$$

$$\psi_P(k; P) = \frac{dk_{t+1}}{dP} = 0, \quad (2.27)$$

$$\frac{dU}{dk} = \frac{\partial U}{\partial k_t} + \frac{\partial U}{\partial k_{t+1}} = U_c f'' k \frac{n-r}{1+r}, \quad (2.28)$$

$$\frac{\partial U}{\partial k_{t+1}} = U_c f'' k \frac{1+n}{1+r}, \quad (2.29)$$

$$\frac{\partial U}{\partial P} = 0. \quad (2.30)$$

Equations (2.27) and (2.30) imply that a fully funded pension affects neither capital accumulation nor utility.

Then, substituting (2.26)–(2.30) into (2.12) and (2.13), we have $\frac{dU_0}{dT} = 0$ and $\frac{dU_{ss}}{dT} = 0$. From Proposition 2.1, it can be straightforwardly obtained that:

$$\frac{dU_t}{dP} = 0.$$

Therefore, we can confirm that a fully funded pension is neutral to utility, that is, a fully funded pension has no effect on utilities in all initial, transitional, and steady states, as Blanchard and Fischer (1989) pointed out.

The above argument can also be confirmed by Figure 2.2. In both (A) and (B), because the capital accumulation has not been changed, the dynamic path of $\phi(U_t; 0)$ would not shift. As a result, the utility levels of the initial, transitional, and steady-state generations, U_0 , U_t , and U_{ss_0} , would not change either. Therefore, it can be verified both algebraically

and graphically that the introduction of a fully funded pension is neutral to welfare, which is different to the case of introducing a PAYG pension in the short, medium, and long runs.

From the view of policy making, a fully funded pension might not be useful as a policy tool because its introduction has no effect on the welfare of transitional generations. By contrast, the introduction of a PAYG pension is more useful as a policy tool. This is because it brings about different patterns of welfare effects in transition under various conditions or circumstances, even if the economies may converge to the same steady state.

2.3 Concluding remarks

In Sections 2.2.2 and 2.2.3, we have shown two representative cases of introducing the PAYG pension: a case of monotonic convergence under dynamic efficiency and a case of oscillatory convergence under dynamic inefficiency. In addition, there are two other combinations: a case of monotonic convergence under dynamic inefficiency and a case of oscillatory convergence under dynamic efficiency. Similar to the former two cases, it can be predicted that the welfare effects will be opposite in the short and long runs in these two additional cases. This implies that a government can set an appropriate policy to achieve a specific target. Therefore, as a redistribution policy, the PAYG pension system has been commonly applied in many countries.

As a mathematical preparation, we highlighted the contribution of Kuhle (2014) using a weighted average method and showed how the welfare effects in transitional states change. We clarified that in the case of monotonic convergence, the welfare effects, which can be expressed as a weighted average of the welfare effects for the initial and steady-state generations, also change monotonically. However, in the case of oscillatory convergence, the change in the transitional welfare effects follows the rule determined not only by the initial and steady-state effects, but also by effects in the second period.

Appendix 2.A Proof of Proposition 2.1

Defining $A \equiv \frac{dU}{dk} \psi_b$, $B \equiv \frac{\partial U}{\partial k_{t+1}} \psi_b$ and $C \equiv \frac{\partial U}{\partial b}$, and rewriting equation (2.11) gives:

$$\begin{aligned}
\frac{dU_t}{db} &= \frac{dU}{dk} \left(\sum_{i=0}^{t-1} \psi_k(k; b)^i \right) \psi_b(k; b) + \frac{\partial U}{\partial k_{t+1}} \psi_k(k; b)^t \psi_b(k; b) + \frac{\partial U}{\partial b} \\
&= A \sum_{i=0}^{t-1} \psi_k(k; b)^i + B \psi_k(k; b)^t + C \\
&= A \frac{1-\psi_k^t}{1-\psi_k} + B \psi_k^t + C.
\end{aligned} \tag{2.A1}$$

So far, the proof is the same as that in Proposition 2.2 in Kuhle (2014).

To reveal the essence of this expression more clearly, we simply delete and then add $C\psi_k^t$ from the right-hand side of (2.A1) as follows:

$$\begin{aligned}
\frac{dU_t}{db} &= A \frac{1-\psi_k^t}{1-\psi_k} + C - C\psi_k^t + C\psi_k^t + B\psi_k^t \\
&= A \frac{1-\psi_k^t}{1-\psi_k} + C(1 - \psi_k^t) + C\psi_k^t + B\psi_k^t \\
&= \psi_k^t(B + C) + (1 - \psi_k^t) \left(\frac{A}{1-\psi_k} + C \right) \\
&= \psi_k^t \frac{dU_0}{db} + (1 - \psi_k^t) \frac{dU_{ss}}{db},
\end{aligned}$$

using (2.12) and (2.13).

Appendix 2.B Proof of Proposition 2.3

Recalling (2.14), $\frac{dU_t}{db} = \psi_k^t \frac{dU_0}{db} + (1 - \psi_k^t) \frac{dU_{ss}}{db}$. Further, ψ_k^t and ψ_k^{t+1} , $t = 2i, i = 1, 2, \dots, \frac{t^*}{2}$ hold because $-1 < \psi_k < 0$. We can easily verify that if $\frac{dU_0}{db} > \frac{dU_{ss}}{db}$, $\frac{dU_t}{db}$ decreases and $\frac{dU_{t+1}}{db}$ increases. Similarly, if $\frac{dU_0}{db} < \frac{dU_{ss}}{db}$, then $\frac{dU_t}{db}$ increases and $\frac{dU_{t+1}}{db}$ decreases.

Because of the monotonicity, it is straightforward to prove that (i) holds. For other cases, it is sufficient to prove the case of $\frac{dU_0}{db} > 0$, $\frac{dU_1}{db} < 0$, and $\frac{dU_{ss}}{db} > 0$ in (ii).

Because $\frac{dU_1}{db} = \psi_k \left(\frac{dU_0}{db} - \frac{dU_{ss}}{db} \right) + \frac{dU_{ss}}{db}$, $\frac{dU_0}{db} > \frac{dU_{ss}}{db} > 0$ must hold. Therefore, $\frac{dU_0}{db} > \frac{dU_1}{db} > \dots > \frac{dU_{2i}}{db} > \dots > \frac{dU_{ss}}{db} > 0$, for $t = 2i, i = 0, 1, \dots$; and $\frac{dU_1}{db} < \frac{dU_3}{db} < \dots < \frac{dU_{2i+1}}{db} < \dots < \frac{dU_{t^*}}{db} \leq 0 < \frac{dU_{t^*+1}}{db} < \dots < \frac{dU_{ss}}{db}$, for $t = 2i + 1, i = 0, 1, \dots, \frac{t^*}{2} - 1$, where $\frac{dU_{t^*}}{db} \leq 0 < \frac{dU_{t^*+1}}{db}$.

We can prove the other cases in (ii) and (iii) by following the same procedure.

Chapter 3 **Self-education, fully funded pension and economic growth**

This chapter formulates the behaviors of individuals in a small open economy model in which there exists this kind of trade-off between self-education and intergenerational education. Our main research interest is how the negative effect of individuals' self-education on economic growth can be mitigated, which has not been discussed in previous studies; we examine which policy can eliminate such a negative effect. We also investigate the possibility that a fully funded pension system could reduce the negative effects and bring about welfare improvement.

In this chapter, we first consider how individuals' self-education can contribute to raising earning power. In previous studies, any type of human capital accumulation was found to necessarily promote economic growth. However, we also consider the possibility that self-education or human capital accumulation of individuals has the opposite effect on economic growth. This reflects the fact that education of older individuals neither plays a role in accumulating the human capital of the next generation nor affects the human capital accumulation inherited by their descendants. If individuals invest in their children's education rather than in their own education, the economy would achieve faster growth. Therefore, in order to remove this kind of negative effect, the government must lower individuals' incentive for self-education and encourage them to educate their children.

Here, we show that the introduction of a fully funded pension system is one of the possible policy tools that can be used to achieve this goal.¹⁹ In our setting, a fully funded pension system is no longer neutral. Furthermore, there exists a unique optimal scale of fully funded pension that maximizes the economic growth rate. The fully funded pension guarantees a higher income in the retirement period, which discourages individuals from

¹⁹ The literature has focused on the effects of fully funded social security on economic growth in OLG models with human capital accumulation. Docquier and Paddison (2003) investigated the effects of social security in a closed economy in which education is the engine of growth, and found that growth can only be stimulated under a fully funded social security system based on a partial earnings history. Kunze (2012) showed that when individuals face a trade-off between educating their children and leaving bequests, a fully funded social security system may depress economic growth.

investing in their own self-education and instead encourages them to invest more in intergenerational education. As a result, economic growth is improved. We also find that in a small open economy, when the interest rate is increased exogenously, the government should reduce the amount of a fully funded pension in order to promote economic growth.

The organization of the rest of this chapter is as follows. Section 3.1 introduces our model and the intertemporal equilibrium. Section 3.2 evaluates the effects of a fully funded pension on economic growth. Section 3.3 analyzes how an increase in the interest rate in a small open economy model affects pension policy and thus economic growth.

3.1 The model

3.1.1 Basic framework

Based on an OLG model inspired by Glomm and Ravikumar (1992), we consider a small open economy in discrete time that lasts forever. Defining the population of young individuals in period t (denoted as generation t) as N_t , the net population growth rate can be expressed as $n \equiv \frac{N_{t+1}-N_t}{N_t}$, which we assume is constant over time.

Goods in period t are produced using only human capital and the aggregate production function is given by:

$$Y_t = H_t, \quad (3.1)$$

where Y_t is the amount of goods in aggregate terms and H_t is the aggregate level of human capital. As the produced goods are totally distributed to the workers, the wage rate is equal to 1.

Individuals live for three periods, childhood, youth, and old age. They work in the second and third of these two periods but make no decisions on economic activities during their childhood. In period t , young people (generation t) provide their human capital h_{yt} to earn a wage income that is equivalent to their human capital. They spend their income only to either educate themselves (self-education) or to educate their children (intergenerational education). To focus on this trade-off, we assume that individuals neither consume nor save while they are young. In period $t + 1$, when becoming old, individuals use their developed human capital h_{ot+1} , which is obtained by

self-education, to work, and use up the wage income to consume in their old age. Therefore, the budget constraints of generation t in the young and the old periods can be written as:

$$h_{yt} = (1 + n)e_t + \lambda_t, \quad (3.2)$$

and

$$d_{t+1} = h_{ot+1}, \quad (3.3)$$

where e_t and λ_t denote the expenditure on education for their children (intergenerational education) and for themselves (self-education) in period t , and d_{t+1} is old age consumption in period $t + 1$, respectively.

We assume that individuals have an additively separable utility function, following Glomm and Ravikumar (1992). Generation t 's lifetime utility is defined by the following log-linear function:

$$U(d_{t+1}, h_{yt+1}) = \alpha \ln d_{t+1} + (1 - \alpha) \ln h_{yt+1}, \quad 0 < \alpha < 1. \quad (3.4)$$

Individuals care about their old age consumption as well as their children's human capital; α and $1 - \alpha$ represent the degree of these kinds of concerns. This utility function exhibits the feature that individuals are altruistic as well as egoistic and thus have the incentive to educate their children and to educate themselves. These two kinds of human capital accumulate depending both on the human capital level in the young period and on the expenditure for education:

$$h_{yt+1} = E e_t^\gamma h_{yt}^{1-\gamma}, \quad 0 < \gamma < 1, E > 0 \quad (3.5)$$

and

$$h_{ot+1} = D \lambda_t^\delta h_{yt}^{1-\delta}, \quad 0 < \delta < 1, D > 0. \quad (3.6)$$

Equation (3.5) depicts intergenerational human capital accumulation through self-education. Here, E and D represent total factor productivity and γ and δ the intensity rate of education expenditure on human capital accumulation for each kind of education, respectively.

In each period, both the labor market and goods market clear:

$$H_t = N_t h_{yt} + N_{t-1} h_{ot}, \quad (3.7)$$

and

$$Y_t = N_t[(1+n)e_t + \lambda_t] + N_{t-1}d_t \quad (3.8)$$

hold. Equation (3.7) can also be conceived as a measure of aggregate human capital.

3.1.2 Equilibrium

From the first-order conditions of utility maximization by individuals, we obtain:

$$\frac{d_{t+1}}{(1+n)e_t} = \frac{\alpha\delta D\lambda_t^{\delta-1}h_{yt}^{1-\delta}}{\gamma(1-\alpha)}. \quad (3.9)$$

By defining the self-education spending ratio as $m_t \equiv \frac{\lambda_t}{h_{yt}}$, we have $\frac{e_t}{h_{yt}} = \frac{1-m_t}{1+n}$ as the intergenerational education spending ratio. Thus, from (3.2), (3.3), and (3.9), we have:

$$m_t = \frac{A}{1+A}, \quad (3.10)$$

where $A \equiv \frac{\delta\alpha}{\gamma(1-\alpha)}$. From (3.10), we see that m_t is constant over time. Hence, we have $m_t = m_{t-1} = m$ and $\frac{e_t}{h_{yt}} = \frac{1-m}{1+n}$. This indicates that individuals of all generations always spend on self-education in proportion to their wage income.

The growth rate of the economy is defined by $g_{t+1} \equiv \frac{Y_{t+1}}{Y_t}$. From (3.1), (3.5), (3.6), and (3.7), we obtain:

$$g_{t+1} = \frac{Y_{t+1}}{Y_t} = \frac{H_{t+1}}{H_t} = (1+n)^{1-\gamma} E(1-m)^\gamma. \quad (3.11)$$

From (3.11), it can be easily verified that $g_{t+1} = g_t = g$, which implies the growth rate also remains constant over time. It can also be acknowledged that the growth rate decreases as the self-education spending ratio increases. If individuals spend all of their wage income on their children's education, i.e., $m = 0$, the growth rate is at a maximum. In other words, educating children is the unique source of economic growth, while educating themselves reduces economic growth indirectly, even though it benefits them.

This is because, in our formulation of human capital, the old no longer participate in their children's education, which does not lead to human capital accumulation among

²⁰ See Appendix 3.A.

their descendants. Therefore, in order to promote economic growth, intergenerational education, rather than self-education, is crucial.

3.1.3 Fully funded pension

Here, we show that when the government introduces a fully funded pension system in the economy, the economic growth rate may increase. In period t , the government collects taxes from the young and invests the funds outside of the economy. Then, in period $t + 1$, the returns from the fund are used to finance the pension payment P_{t+1} to the generation t that is old. The budget constraint of the government is:

$$(1 + r)\tau h_{yt} = P_{t+1}, \quad (3.12)$$

where r denotes the interest rate and is exogenously given in the world capital market, and τ represents the wage tax rate, $0 \leq \tau \leq 1$.

When this pension is introduced, the goods market clearance condition (3.8) can be modified as:

$$Y_t = N_t[(1 + n)e_t + \lambda_t + \tau h_{yt}] + N_{t-1}[d_t - (1 + r)\tau h_{yt-1}]. \quad (3.8')$$

The labor market clearing condition is the same as (3.7).

Individuals' utility maximization problem can be formulated as:

$$\begin{aligned} \max_{d_{t+1}} U(d_{t+1}, h_{yt+1}) &= \alpha \ln d_{t+1} + (1 - \alpha) \ln h_{yt+1} \\ \text{s. t. } (1 - \tau)h_{yt} &= (1 + n)e_t + \lambda_t \end{aligned} \quad (3.2')$$

$$d_{t+1} = h_{ot+1} + P_{t+1}. \quad (3.3')$$

Again, from the first-order conditions, we can obtain (3.9).

With (3.2'), (3.3'), and (3.9), the relationship between the self-education spending ratio m_t and the wage tax rate τ can be represented as:

$$(1 - \tau)A = (1 + r)\frac{\tau}{\delta}m_t^{1-\delta} + (1 + A)m_t. \quad (3.10')$$

It can be easily seen that m_t remains constant over time, which is the same as the case without a pension. In fact, when $\tau = 0$, (3.10') returns to (3.10). Equation (3.10') gives

the relationship between m_t and τ as an implicit function of $m = m(\tau)$. Totally differentiating (3.10') gives:

$$\left[(1 - \delta)(1 + r) \frac{\tau}{D} m^{-\delta} + 1 + A \right] dm = - \left[A + \frac{1}{D} (1 + r) m^{1-\delta} \right] d\tau,$$

which implies $m'(\tau) < 0$ and $m''(\tau) > 0$.²¹ Namely, as τ increases, m decreases and the degree of m 's decrease becomes smaller. That is, if the scale of the fully funded pension system is small, individuals will substantially reduce their investment in self-education. This means the policy effect is very strong when there is no or limited pension.

To conclude, when the government raises the pension payment by increasing the wage tax rate, individuals decrease the ratio of self-education spending to total wage income. There are two reasons for this: first, individuals' disposable income decreases directly because of the increase in the wage tax rate and there is a negative direct "income" effect on the investment in self-education; second, the increase in the wage tax rate means that individuals can obtain larger pensions when they are old, and this also reduces their incentive to spend on self-education for their lives of old age. This indirect "incentive" effect enforces the above direct "income" effect.

3.2 Fully funded pension and economic growth

In this section, we evaluate the effects on economic growth of fully funded pension policy.

Under the fully funded pension system, the economic growth rate, (3.11), can be rewritten as:

$$\begin{aligned} g_{t+1} &= \frac{Y_{t+1}}{Y_t} = \frac{H_{t+1}}{H_t} = \frac{N_{t+1}h_{yt+1} + N_t h_{ot+1}}{N_t h_{yt} + N_{t-1} h_{ot}} \\ &= (1 + n)^{1-\gamma} E(1 - \tau - m(\tau))^\gamma. \end{aligned} \quad (3.13)$$

Equation (3.13) implies that the economic growth rate is constant and depends on the wage tax rate: $g_t = g_{t+1} = g(\tau)$.

The first-order condition of economic growth maximization is given by:

²¹ We can easily obtain: $m'(\tau) < 0$ and $m''(\tau) = - \frac{1}{\left[(1 - \delta)(1 + r) \frac{\tau}{D} m^{-\delta} + 1 + A \right]^2} \left\{ (1 - \delta) \frac{\delta}{D} (1 + r) m^{-\delta} m'(\tau) \left[(1 - \delta)(1 + r) \frac{\tau}{D} m^{-\delta} + 1 + A \right] - (1 - \delta)(1 + r) \frac{1}{D} m^{-\delta} [1 - \tau \delta m^{-1} m'(\tau)] \left[A + \frac{1}{D} (1 + r) m^{1-\delta} \right] \right\} > 0$.

$$\begin{aligned}
g'(\tau) &= -E\delta(1+n)^{1-\gamma}(1-\tau-m(\tau))^{\gamma-1}(1+m'(\tau)) \\
&= -E\delta(1+n)^{1-\gamma}(1-\tau-m(\tau))^{\gamma-1} \left\{ 1 - \frac{A+\frac{1}{D}(1+r)(m(\tau))^{1-\delta}}{(1-\delta)(1+r)\frac{\tau}{D}(m(\tau))^{-\delta}+1+A} \right\} \\
&= 0.
\end{aligned}$$

By solving this, we obtain:

$$(1+r)\frac{\delta}{D}(m(\tau))^{1-\delta} = (1-\delta)(1+r)\frac{\tau\delta}{D}(m(\tau))^{-\delta} + 1. \quad (3.14)$$

For analytical convenience, we assume that:

$$\frac{1}{D}(1+r)\left(\frac{1}{1+A}\right)^{\delta} \geq 1. \quad (3.15)$$

This assumption implies that the interest rate r or the dependence level of intergenerational human capital accumulation on pecuniary investment γ is relatively high, while the return to self-education D is relatively low.²²

Under this assumption, the optimal tax rate that maximizes the economic growth rate is characterized by the following proposition.

Proposition 3.1

Under the assumption of (3.15), there exists a unique wage tax rate $\tau^ \in [0,1)$ that maximizes the economic growth rate. This growth-maximizing tax rate is given by (3.11') and (3.14).*

Proof.

See Appendix 3.B. □

From Proposition 3.1, we can directly obtain the following corollary.

²² A detailed explanation of this assumption is given below.

Corollary 3.1

Under the assumption of (3.15), the growth-maximizing wage tax rate τ^ also maximizes individuals' intergenerational education spending ratio.*

Proof.

Recall (12), $g = (1 + n)E\left(\frac{e_t}{h_{yt}}\right)^\gamma$. It is obvious that when $g'(\tau) = 0$, $\frac{d\left(\frac{e_t}{h_{yt}}\right)}{d\tau} = 0$ and $\frac{d^2\left(\frac{e_t}{h_{yt}}\right)}{d\tau^2} < 0$ all hold. □

In our model, production does not depend on physical capital but only on human capital. This indicates that, essentially, economic growth is perfectly determined by the human capital accumulation of the young in each period (and thus by the investment in intergenerational education, $\frac{e_t}{h_{yt}}$). Therefore, self-education would have a negative effect on growth indirectly. One of the available policy tools to reduce this negative effect is a fully funded pension.

Under a fully funded pension system, the increase in the tax rate on the one hand decreases individuals' net income and makes individuals invest less in their children, while on the other hand, as mentioned above, it guarantees the increase in income when older, which leads the individuals to invest less in themselves but invest more in their children. The former represents a negative "income" effect on economic growth, while the latter brings about a positive "distributive" effect. Therefore, there could exist a unique optimal tax rate at which economic growth is maximized. In fact, the introduction of the fully funded pension system in this model creates a short-run effect on individuals' behavior. However, this policy effect lasts in all subsequent periods through the human capital accumulation, so that the new economic growth rate can be achieved in the long run.

It should be noted that the results in Proposition 1 and Corollary 1 hold under the assumption of (3.15). If the interest rate is relatively low or the return to self-education is large enough, i.e., (3.15) does not hold, the optimal wage tax rate is zero or even negative. In this case, the distributive effect is too small to reduce individuals' incentive

for self-education. Regardless of the pension amount, individuals will always invest substantially in themselves. In contrast, an increase in the wage tax rate brings about a large negative income effect, which leads to a decrease in intergenerational education and a fall in economic growth. As a result, it is better to refrain from implementing the policy of a fully funded pension when assumption (3.16) does not hold.

3.3 Effects of interest rate on growth

In this section, we discuss the effects of the interest rate on the optimal tax rate as well as on economic growth. The following proposition characterizes the effects of the interest rate on the optimal scale of pension with regard to maximizing the economic growth rate in a small open economy.

Proposition 3.2

In a small open economy, if (3.15) is satisfied, an exogenous increase in the interest rate reduces the optimal amount of a fully funded pension for maximizing economic growth; it also raises the economic growth rate in the steady-state growth path.

Proof.

See Appendix 3.C. □

The interpretation of Proposition 3.2 is straightforward. First, the increase in the interest rate attracts individuals' attention to the return from pensions in their old age. Thus, individuals' incentives for self-education are weakened and they spend more of their income educating their children. This affects economic growth positively.

Then, in order to increase the economic growth rate, the government must lower the wage tax rate. As the interest rate is high, the government will set a lower wage tax rate but can still maintain a high level of pension payments. This allows the young to spend more money educating their children. Therefore, there exists a lower optimal tax rate, which could induce individuals to invest more in intergenerational education.

3.4 Concluding remarks

Generally, in OLG models, an increase in the interest rate does not affect the policy decision on fully funded pensions, because a fully funded pension is equivalent to private savings, which are perfectly substitutable. However, in the model presented in this chapter, a fully funded pension is no longer neutral with respect to the policy decision. The reason for having this policy implication, which is quite different from the ones obtained in the previous literature, can be attributed to the existence of self-education. Reflecting the real economic circumstances whereby more and more self-education has been playing an important role in people's lives, the government should choose appropriate policy instruments. In this chapter, because we have seen that changes in the interest rate alter individuals' choices, the government must adjust the policy of fully funded social security accordingly.

Appendix 3.A Proof of (3.12)

Recalling the definition of the economic growth rate, it is easy to verify:

$$\begin{aligned}
 g_{t+1} &= \frac{Y_{t+1}}{Y_t} = \frac{H_{t+1}}{H_t} = \frac{N_{t+1}h_{yt+1} + N_t h_{ot+1}}{N_t h_{yt} + N_{t-1} h_{ot}} = (1+n) \frac{(1+n)h_{yt+1} + h_{ot+1}}{(1+n)h_{yt} + h_{ot}} \\
 &= (1+n) \frac{(1+n)Ee_t^\gamma h_{yt}^{1-\gamma} + D\lambda_t^\delta h_{yt}^{1-\delta}}{(1+n)Ee_{t-1}^\gamma h_{yt-1}^{1-\gamma} + D\lambda_{t-1}^\delta h_{yt-1}^{1-\delta}} \\
 &= (1+n) \frac{h_{yt}}{h_{yt-1}} = (1+n) \frac{Ee_t^\gamma h_{yt}^{1-\gamma}}{h_{yt-1}} = (1+n)E \left(\frac{e_{t-1}}{h_{yt-1}} \right)^\gamma.
 \end{aligned}$$

Appendix 3.B Proof of Proposition 3.1

When $\tau = 0$, $m(0) = \frac{A}{1+A}$ and $m'(0) = -\frac{A+\frac{1}{D}(1+r)(\frac{1}{1+A})^\delta}{1+A} \leq -1$ hold under (3.15).

Therefore, $g(0) = (1+n)^{1-\gamma} \left(1 - \frac{A}{1+A}\right)^\gamma > 0$ and $g'(0) = -E\delta(1+n)^{1-\gamma} \left(1 - \frac{A}{1+A}\right)^{\gamma-1} \left(1 - \frac{A+\frac{1}{D}(1+r)(\frac{1}{1+A})^\delta}{1+A}\right) \geq 0$ can be verified.

When $\tau = 1$, $m(1) = 0$ and $m'(1) = -\frac{A}{1+A} > -1$. Therefore, we can obtain $g(1) = 0$ and $\lim_{\tau \rightarrow 1} g'(\tau) = -\infty$.

Because $m''(\tau) > 0$, $g''(\tau) = -E\delta(1+n)^{1-\gamma}(1-\tau-m(\tau))^{\gamma-1}m''(\tau) < 0$ holds for $0 \leq \tau \leq 1$. Therefore, there must exist a unique, growth-maximizing wage tax rate $\tau^* \in [0,1)$.

Appendix 3.C Proof of Proposition 3.2

Recall (3.10'), (3.13), and (3.14). Then, totally differentiating (3.10') and (3.14) gives:

$$dm = -\frac{\frac{\tau}{D}m^{1-\delta}dr + \left[(1+r)\frac{1}{D}m^{1-\delta} + A\right]d\tau}{\frac{\tau}{D}(1+r)(1-\delta)m^{-\delta} + 1 + A} \text{ and } \left[(1-\delta)(1+r)\frac{1}{D}m^{-\delta} + \frac{\tau\delta}{D}(1+r)(1-\delta)m^{-\delta-1}\right]dm = (1-\delta)(1+r)\frac{1}{D}m^{-\delta}d\tau + \frac{\tau}{D}(1-\delta)m^{-\delta}dr.$$

Combining these two equations, we have $-\left[\frac{[(1+r)\frac{1}{D}m^{-\delta} + A]M}{\frac{\tau^*}{D}(1+r)(1-\delta)m^{-\delta} + 1 + A} + (1-\delta)(1+r)\frac{1}{D}m^{-\delta}\right]d\tau^* = \left[\frac{\frac{\tau^*}{D}m^{1-\delta}M}{\frac{\tau^*}{D}(1+r)(1-\delta)m^{-\delta} + 1 + A} + (1-\delta)\frac{\delta\tau^*}{D}m^{-\delta}\right]dr$, where $M = \left[(1-\delta)(1+r)\frac{1}{D}m^{-\delta} + \frac{\tau^*\delta}{D}(1+r)(1-\delta)m^{-\delta-1}\right] > 0$. Therefore, $\frac{d\tau^*}{dr} < 0$ can be verified.

Next, totally differentiating (3.13) gives:

$$\begin{aligned} dg &= -(1+n)^{1-\gamma}E\gamma(1-\tau-m)^{\gamma-1}[d\tau + dm] \\ &= -(1+n)^{1-\gamma}E\gamma(1-\tau-m)^{\gamma-1}\left[d\tau - \frac{\frac{\tau}{D}m^{1-\delta}dr + \left[(1+r)\frac{1}{D}m^{1-\delta} + A\right]d\tau}{\frac{\tau}{D}(1+r)(1-\delta)m^{-\delta} + 1 + A}\right] \\ &= -(1+n)^{1-\gamma}E\gamma(1-\tau-m)^{\gamma-1}\left[\frac{\left[\frac{\tau}{D}(1+r)(1-\delta)m^{-\delta} + 1 - (1+r)\frac{1}{D}m^{1-\delta}\right]d\tau - \frac{\tau}{D}m^{1-\delta}dr}{\frac{\tau}{D}(1+r)(1-\delta)m^{-\delta} + 1 + A}\right]. \end{aligned}$$

Thus, we have $\frac{dg}{dr} > 0$.

Chapter 4 **Capital tax competition and public education**

This chapter investigates the effects of a coordinated capital tax reform across countries in an OLG economy where capital tax is collected to provide public education.

In theory, while a tax competition within one period only brings a static fiscal externality, the existence of human capital determines the welfare of the next (following) generation and thus causes an intertemporal externality. Therefore, compared with Batina (2009), who only considered the accumulation of physical capital, this chapter evaluates the policy effect in an economy where both physical capital and human capital are the main sources of determining social welfare. Above all, we give sufficient conditions under which a coordinated reform on capital tax increases the level of human capital and is Pareto-improving.

In the following parts, Section 4.1 introduces the framework, Section 4.2 derives the optimal policy rules, Section 4.3 evaluates the welfare effects of the coordinated capital tax reform, and Section 4.4 provides further discussion.

4.1 The model

Time is discrete and the economy lasts forever, $t = 1, 2, \dots$. In this economy, there are $J > 1$ symmetric countries. The population of each generation in each country is normalized to unity. Physical capital moves among all countries but individuals cannot. Individuals are homogeneous and live for three periods: first as children, then as adults, and finally as the old. In childhood, individuals are children in period t (denoted as generation t) and receive a public education to accumulate their human capital level, h_t , but do not take part in economic activities.

4.1.1 Firms

Firms are owned by the old and behave competitively in each country. We take them to be identical and to adhere to constant returns to scale. Firms use both physical and human capital to produce the private good. In each period, they maximize the profit per

human capital $f(\kappa_t) - (r_t + \tau_t)\kappa_t - w_t$. Here, κ_t is the capital per human capital, r_t is the real interest rate determined in the real-world capital market, τ_t is the source-based capital tax rate, and w_t is the local wage per human capital. The location subscript has been omitted for brevity. From the first-order conditions of profit maximization by firms, we obtain

$$\frac{df}{d\kappa_t} = f_\kappa(\kappa_t) = r_t + \tau_t = r_{nt}, \quad (4.1)$$

where $f_\kappa(\kappa_t)$ is the marginal product of capital per human capital in each country and r_{nt} is the net cost of physical capital. From (4.1), we can have the demand for capital per human capital, $\kappa_t = \kappa(r_{nt})$, with $\kappa_r = d\kappa_t/dr_{nt} = \kappa_\tau = d\kappa_t/d\tau_t = 1/f_{\kappa\kappa} < 0$, where $1/f_{\kappa\kappa} < 0$ is the second derivative. The wage function is then obtained by $w_t = f(\kappa(r_{nt})) - r_{nt}\kappa(r_{nt}) = w(\kappa(r_{nt}))$, with $w_r = dw_t/dr_{nt} = dw_t/d\tau_t = -\kappa_t$ by the envelope theorem.

4.1.2 Individuals

In period t , the adults devote themselves to the production process by providing their human capital. They thus earn a wage income and use it all either to consume a private good or to save. When gold, i.e., in period $t+1$, they pay for the private good with their savings and interest returns. For simplicity, we assume that the utility function is additively separable. Therefore, individuals maximize their utility as:

$$\max_{c_t, d_{t+1}} U^t(c_t, d_{t+1}) = u(c_t) + \beta u(d_{t+1})$$

$$s. t. \quad w_t h_t = c_t + s_t,$$

$$(1 + r_{t+1})s_t = d_{t+1}.$$

Here, a subutility function $u(\cdot)$ is assumed to be concave, twice continuously differentiable, as well as satisfy the Inada conditions. c_t , s_t and d_{t+1} represent the private good consumption of their adulthood in period t , their savings in period t , and the private good consumption of their old age in period $t+1$, respectively. β is the discount factor.

4.1.3 Government

The government in each country levies tax on physical capital to finance a public education system. The general budget constraint of each government in period t is given by

$$\tau_t \kappa(r_t + \tau_t) h_t = e_t, \quad (4.2)$$

where e_t is the investment in public education per capita.

In period t , the indirect utility function of individuals can be obtained as:

$$V^t = u((1 - T_t)w_t h_t - s(\cdot)) + \beta u((1 + r_{t+1})s(\cdot)).$$

In each country, the social welfare function is defined as a sum of all generations' utility from period 1 to infinity weighted by a discount rate of individuals:

$$SW = u((1 + r_1)s_0) + \sum_{t=1}^{\infty} \beta^{t-1} [u((1 - T_t)w_t h_t - s(\cdot)) + \beta u((1 + r_{t+1})s(\cdot))]. \quad (4.3)$$

The government chooses the infinite policy sequence $\{\tau_t, e_t\}$ to maximize (4.3) subject to (4.2). All countries make their policy decisions simultaneously under the same conditions and these decisions are public to the whole economy.

4.1.4 Formation of human capital

The human capital generation at $t+1$ is formulated by the both the public education investment and the human capital level of their parents h_t :

$$h_{t+1} = h(e_t, h_t). \quad (4.4)$$

We assume this human capital accumulation function is decreasing returns to scale and we have that $h_e = dh_{t+1}/de_t > 0$, $h_h = \frac{dh_{t+1}}{dh_t} > 0$, $h_{ee}, h_{hh} < 0$, and $h_{eh} = h_{he} > 0$ hold.

4.1.5 Physical capital market equilibrium

Equilibrium in the physical capital market in period $t+1$ satisfies

$$\sum_{j=1}^J K_{j,t+1} = \sum_{j=1}^J S_{j,t}, \quad (4.5)$$

where the sum is indexed across countries by j . In a symmetric, steady-state equilibrium, this market equilibrium condition can be rewritten as

$$K_{t+1} \equiv h_{t+1}\kappa_{t+1} = s_t. \quad (4.6)$$

4.1.6 Nash policy equilibrium

A symmetric, steady-state, *Nash policy equilibrium* (NPE) is composed of an interest rate and a policy for the representative public education authority, (τ, e) such that:

- i. individuals behave optimally as described above
- ii. firms behave optimally as described above
- iii. governments choose their policies optimally as described above
- iv. human capital accumulation equation (4.4) is satisfied.

Totally differentiating (4.4) and (4.6) gives

$$\begin{bmatrix} k_{t+1} & h_{t+1}\kappa_r - s_r \\ 1 & 0 \end{bmatrix} \begin{bmatrix} dh_{t+1} \\ dr_{t+1} \end{bmatrix} + \begin{bmatrix} h_{t+1}\kappa_\tau \\ 0 \end{bmatrix} d\tau_{t+1} = \begin{bmatrix} s_w w_t & s_w w_r h_t \\ \tau_t \kappa_t h_e + h_h & \tau_t h_t h_e \kappa_r \end{bmatrix} \begin{bmatrix} dh_t \\ dr_t \end{bmatrix} + \begin{bmatrix} -s_w w_r h_t \\ \kappa_t h_t h_e + \tau_t h_t h_e \kappa_\tau \end{bmatrix} d\tau_t. \quad (4.7)$$

From (4.7), we can obtain the conditions of local stability at the steady-state NPE:²³

$$\left\{ \begin{array}{l} \left| T w h_e + h_h - \frac{1}{\frac{h}{f_{kk}} - s_r} \left[s_w \kappa h + \frac{s_w w \tau h h_e}{f_{\kappa\kappa}} + \frac{\tau h h_e \kappa (1 - \tau \kappa h_e)}{f_{\kappa\kappa}} \right] \right| < 1 - h_h s_w \kappa h, \\ 0 < \frac{1}{s_r - \frac{h}{f_{kk}}} \left[s_w \kappa h h_h + \frac{1}{f_{\kappa\kappa}} (s_w w - \tau h h_e \kappa) \right] < 1. \end{array} \right. \quad (4.8)$$

4.2 Optimal policy rules

The policy rules chosen by each government that constitute part (iii) in 4.1.6 can be obtained as:

²³ See Appendix 4.A.

$$\beta w_t h_e = \frac{1}{1 + \frac{\tau_t}{\kappa_t f_{\kappa\kappa}}} - \frac{\beta \tau_{t+1} \kappa_{t+1} h_e}{1 + \frac{\tau_{t+1}}{\kappa_{t+1} f_{\kappa\kappa}}}, \quad (4.9)$$

and we have the following proposition.

Proposition 4.1

In a locally stable, symmetric steady-state NPE, the optimal policy rules are determined by

$$\frac{\beta w h_e}{1 - \beta \tau \kappa h_e} = \frac{1}{1 + \frac{\tau}{\kappa f_{\kappa\kappa}}}. \quad (4.10)$$

Proof.

See Appendix 4.B. □

This policy rule (4.10) is the modified Samuelson rule for local public education. The usual Samuelson rule $U_p/U_c = 1/[1 + \tau/(\kappa f_{\kappa\kappa})]$ (Zodrow and Mieszkowski, 1986) implies the marginal rate of transformation between private and public goods is greater than one, indicating that the local public services are underprovided at the margin. Similarly, the left-hand side of (4.10) is the marginal rate of transformation between private consumption and public education (human capital). $\beta w h_e$ is the current value of a marginal increase in human capital level. $\beta \tau \kappa h_e$ in the denominator is a “benefit” for taxes through the increase in human capital level, so that $1 - \beta \tau \kappa h_e$ represents the “real value of the reduction” in private consumption. The right-hand side then implies the marginal rate of transformation, which is greater than one. Therefore, in a locally stable, symmetric steady-state NPE, a fiscal externality occurs and public education is underprovided. On this occasion, it might be natural to think that a coordinated tax reform should raise the social welfare level.

4.3 Welfare effects of a coordinated tax reform

In this section, we evaluate the effects on social welfare of a coordinated tax reform to reduce, or internalize, the externality from the horizontal capital tax competition across countries in the steady-state NPE: following Batina (2012), we consider the situation where all countries make an agreement to increase the capital tax rate permanently, and

this agreement is publicly announced and implemented simultaneously. All individuals believe the announcement and change their expectations accordingly.

In the steady state, (4.7) can be rewritten as:

$$\begin{bmatrix} k - s_w w & h\kappa_r - s_r - s_w w_r h \\ 1 - \tau\kappa h_e - h_h & -\tau h h_e \kappa_r \end{bmatrix} \begin{bmatrix} dh \\ dr \end{bmatrix} = \begin{bmatrix} (s_w w_r \kappa + \kappa_\tau) h \\ (\kappa + \tau\kappa_\tau) h h_e \end{bmatrix} d\tau_t. \quad (4.11)$$

From (4.11), we obtain

$$\begin{aligned} \frac{dh}{d\tau} &= \frac{h h_e}{M} [s_r(\kappa + \tau\kappa_\tau) + s_w w_r \kappa h + \kappa h \kappa_\tau] \\ &= \frac{\kappa h h_e}{M} \left[s_r \left(1 + \frac{\tau}{\kappa f_{\kappa\kappa}} \right) - \left(s_w \kappa + \frac{1}{f_{\kappa\kappa}} \right) h \right] \end{aligned} \quad (4.12)$$

and

$$\begin{aligned} \frac{dr_n}{d\tau} &= \frac{dr}{d\tau} + 1 = \frac{1}{M} \{ \kappa^2 h h_e + [h h_e s_w w w_r + s_r(1 - \tau\kappa h_e - h_h)] \} \\ &= \frac{1}{M} [\kappa h h_e (\kappa - s_w w) + s_r(1 - \tau\kappa h_e - h_h)], \end{aligned} \quad (4.13)$$

where $M = -(\kappa - s_w w)(\tau h h_e / f_{\kappa\kappa}) - (h / f_{\kappa\kappa} - s_r + s_w \kappa h)(1 - \tau\kappa h_e - h_h)$.

Equations (4.12) and (4.13) represent the effects of a tax increase on the level of human capital and on the interest rate, respectively. As for (4.12), $\kappa h^2 \kappa_\tau h_e / M$ represents the direct effect of the coordinated tax increase through the change in the budget constraint of each country's government. $[s_r(\kappa + \tau\kappa_\tau) + s_w w_r \kappa h] h h_e / M$ is the indirect effect through individuals' savings behavior: an increase in τ decreases the wage, which changes savings and thus the level of physical capital, leading to a reduction in the budget of public education and thus in the level of human capital.

Similarly, in (4.13), $\kappa^2 h h_e / M$ is the direct effect through the change in the budget constraint and $[h h_e s_w w w_r + s_r(1 - \tau\kappa h_e - h_h)] / M$ represents the indirect effect through the change in savings.²⁴

²⁴ Comparing the effect on interest rate in Batina (2009), $\frac{dr}{d\tau} + 1 = \frac{s_r}{s_r - (1+n)\kappa_r - \kappa s_w}$, our results include effects caused by the existence of human capital: a change in the level of human capital would straightforwardly lead to a change in governments' budget constraints and individuals' wage income, shown as a direct effect and an indirect income effect as mentioned above.

To evaluate the welfare effects, we consider a representative individual in the steady state. His or her utility function can be written as $U(c, d) = u(c) + \beta u(d)$. Differentiating U by τ gives

$$\frac{dU}{d\tau} = u_c \left[h \frac{dw}{d\tau} + w \frac{dh}{d\tau} - \frac{ds}{d\tau} \right] + \beta u_d \left[s \frac{dr}{d\tau} + (1+r) \frac{ds}{d\tau} \right].$$

From the first-order condition of an individual's utility maximization, $u_c = \beta(1+r)u_d$ holds, and from the physical capital market equilibrium at the steady state, $s = \kappa h$ holds. Therefore, we have

$$\frac{dU}{d\tau} = u_c \left(-\kappa h + w \frac{dh}{d\tau} + \frac{\kappa h}{1+r} \frac{dr}{d\tau} \right) = u_c \left(-\kappa h + w \frac{dh}{d\tau} + \frac{\kappa h}{1+r} \frac{dr}{d\tau} \right). \quad (4.14)$$

Here, $-\kappa h u_c$ is the direct effect of the tax reform, indicating a decrease in utility because of the reduction in wage income. $u_c \left(w \frac{dh}{d\tau} + \frac{\kappa h}{1+r} \frac{dr}{d\tau} \right)$ are the indirect effects, which contain an effect through the change in physical capital, $u_c w \frac{dh}{d\tau}$, and an effect through the change in human capital, $u_c \frac{\kappa h}{1+r} \frac{dr}{d\tau}$.

We define the elasticity of savings to wage income as $\eta_{sw} \equiv s_w w h / s$. The following proposition captures the welfare effects of the coordinated tax reform.

Proposition 4.2

Consider a coordinated capital tax reform such that $d\tau_j = d\tau > 0$ at the steady state. If $\eta_{sw} < 1$ and $s_r(1 + \tau/\kappa f_{\kappa\kappa}) - (s_w \kappa + 1/f_{\kappa\kappa})h > 0$ (s_r is large enough) hold, a coordinated capital tax increase:

- (i) raises the level of human capital, h*
- (ii) raises the net cost of physical capital, $r_n = r + \tau$*
- (iii) is Pareto-improving.*

Proof.

When $\eta_{sw} < 1$, at the steady state, we have $s = \kappa h$ and thus $\kappa - s_w w > 0$. With $s_r(1 + \tau/\kappa f_{\kappa\kappa}) - (s_w \kappa + 1/f_{\kappa\kappa})h > 0$, it is obvious that $M > 0$ holds.²⁵ Therefore, $dh/d\tau > 0$ and $dr_n/d\tau = dr/d\tau + 1 > 0$ hold.

From (4.12) and (4.13), (4.14) can be rewritten as:

$$\frac{dU}{d\tau} = \frac{u_c \kappa h}{M} \left\{ \left[\frac{\kappa h h_e}{1+r} \left(1 + \frac{\tau}{\kappa f_{\kappa\kappa}} \right) - \frac{\tau h h_e}{f_{\kappa\kappa}} \right] (k - s_w w) + (1 - \tau \kappa h_e - h_h) \left[s_r - \frac{r}{1+r} \left(s_w \kappa + \frac{1}{f_{\kappa\kappa}} \right) h \right] + w h_e \left[s_r \left(1 + \frac{\tau}{\kappa f_{\kappa\kappa}} \right) - \left(s_w \kappa + \frac{1}{f_{\kappa\kappa}} \right) h \right] \right\} > 0. \quad \square$$

When the elasticity of savings to wage income is small, a change in wage income causes little in savings (physical capital), which has a relatively weak effect on the interest rate. Considering the increase in the capital tax rate, the total effects on the net cost of physical capital, r_n , would be positive.

If s_r is large enough, when the capital tax rate is coordinately increased, the level of physical capital would be decreased, which will raise the interest rate. This leads to a raise on savings through the physical capital market. Therefore, the indirect effect would dominate the direct effect on the level of human capital, increasing that level in the steady-state NPE.

As for welfare, when $\eta_{sw} < 1$ holds and s_r is large enough, the indirect effects through both the changes in physical capital and human capital are positive and will dominate the direct effect. The coordinated tax reform is thus Pareto-improving.

4.4 Discussion

In the static Zodrow–Mieszkowski model, tax competition brings a fiscal externality. Local governments set a relatively low capital tax rate to attract physical capital, which leads the supply of public goods to be too low and thus a coordination is considered to be Pareto-improving. In this static framework, an autarkic economy and an open economy share the same first-best rule: the marginal rate of transformation between

²⁵ From the assumption of DRS, it can be easily shown that $1 - \tau \kappa h_e - h_h > 0$ holds.

	Autarky	Open economy (in a symmetric steady-state NPE)
ZM model (Static)	First-Best: $\frac{U_c}{U_d} = 1$	First-Best: $\frac{U_c}{U_d} = 1$ Under tax competition: $\frac{U_c}{U_d} > 1$ Fiscal externality occurs. A coordination improves welfare.
OLG framework (Dynamic)	First-Best: $\frac{U_3}{U_1} + \frac{1}{1+n} \frac{U_4}{U_2} = 1$ Decentralized: $\frac{U_3}{U_1} + \frac{1}{1+r} \frac{U_4}{U_2} = 1$ $\frac{U_1}{U_2} = 1 + r$ $f'(k) = r + \delta$	First-Best: $\frac{U_3}{U_1} + \frac{1}{1+n} \frac{U_4}{U_2} = 1$ (first-best rule) $\frac{U_1}{U_2} = 1 + n$ (biological interest rate) $f'(k) = n + \delta$ (golden rule) Under tax competition: $\frac{U_3}{U_1} + \frac{1}{1+r} \frac{U_4}{U_2} = \frac{1}{1+\frac{\tau}{k}K_r} > 1$ $\frac{U_1}{U_2} = 1 + r$ $f'(k) = r + \tau + \delta$ A coordination only alleviates the fiscal externality brought about by tax competition but cannot eliminate the distortion from the OLG feature. The total welfare effects depend on the relationship between n and r as well.

Table 4.1 Equilibrium conditions under static and dynamic frameworks²⁶

private and public goods is one, $U_p/U_c = 1$. In the open economy, when tax competition occurs, the marginal rate of transformation between private and public goods becomes greater than one ($U_p/U_c > 1$), implying there is a distortion from the first-best rule that public goods are underprovisioned. Therefore, a coordinated tax reform is considered to be Pareto-improving.

However, in an OLG economy, the above findings cannot be acknowledged. One feature of the OLG model is that because individuals live in the short run while social planners are concerned about the long run, the fundamental theorems of welfare economics no

²⁶ Here, U_c and U_1 are the marginal utility of consumption on private goods in the young age, while U_d and U_3 are the marginal utility of consumption on private goods in the old age. U_2 and U_4 are the marginal utility of public goods in the young and ages, respectively. δ is the depreciation rate for physical capital.

longer hold. Only when the interest rate is equal to the population growth rate ($n = r$) does the economy evolve along the golden rule path. Otherwise, it is possible to accumulate too much (little) physical capital (Batina, 2009).

Take a provision of public goods into consideration in an OLG framework. Individuals' utility function is modified as $U_t = u(c_t, d_{t+1}, g_t, g_{t+1})$, where g_t and g_{t+1} are the consumption of public goods in their young and old ages, respectively. The government taxes physical capital to provide public goods, which indicates its budget constraint is $\tau_t K(\tau_t + r_t) = g_t$, where $K(\tau_t + r_t)$, the aggregate level of physical capital, is a function of $\tau_t + r_t$.

Under autarky, the first-order condition of the social planner's problem (the first-best) can be easily derived as:

$$\frac{U_3}{U_1} + \frac{1}{1+n} \frac{U_4}{U_2} = 1. \quad (4.15)$$

When the economy is decentralized,

$$\frac{U_3}{U_1} + \frac{1}{1+r} \frac{U_4}{U_2} = 1, \quad (4.16a)$$

$$\frac{U_1}{U_2} = 1 + r, \quad (4.16b)$$

and

$$f'(k) = r + \delta \quad (4.16c)$$

hold in equilibrium. Compared with the economy under the first-best rule, there is only a distortion, which implies $n \neq r$.

In an open economy, the conditions of the first-best rule would be modified as:

$$\frac{U_3}{U_1} + \frac{1}{1+n} \frac{U_4}{U_2} = 1, \quad (4.17a)$$

$$\frac{U_1}{U_2} = 1 + n, \quad (4.17b)$$

$$f'(k) = n + \delta. \quad (4.17c)$$

Equation (4.17a) is the first-best rule, (4.17b) represents Samuelson's biological interest rate, and (4.17c) is the golden rule (Batina, 2009).

When tax competition occurs, at the steady-state NPE, we have:

$$\frac{U_3}{U_1} + \frac{1}{1+r} \frac{U_4}{U_2} = \frac{1}{1+\frac{\tau}{k}Kr} > 1, \quad (4.18a)$$

$$\frac{U_1}{U_2} = 1 + r, \quad (4.18b)$$

$$f'(k) = r + \tau + \delta. \quad (4.18c)$$

Comparing (4.18a–c) with (4.17a–c), it is obvious that when tax competition occurs in an OLG economy, the distortions double: there are both the fiscal externality and the distortion seen in the classical OLG model. Under an OLG framework, even though the local governments coordinate, the coordination alleviates the fiscal externality brought about by tax competition, but cannot eliminate the one seen in the classical OLG model.

Besides the provision of public goods, whether a coordinated tax increase is Pareto-improving or not is independent of the situation where the economy is under dynamic efficiency. Coordination is no longer necessarily seen to be an effective behavior.

In Sections 4.1–4.3, we evaluated the effects of a coordinated tax reform in an OLG economy. Different to Batina (2009, 2012), we take both physical capital and human capital into consideration, and instead of public good provision, in our model the taxes are used to provide public education. Moreover, human capital is both a public input and an intergenerational conflict that exacerbates the distortion from first-best. Similarly, the coordinated tax reform could have a negative effect on welfare. We could provide sufficient conditions under which the reform is welfare-improving.

Appendix 4.A Proof of the local stability

With $d\tau_t = 0$, we can obtain the the dynamics of h_t and r_t from (4.7):

$$\begin{bmatrix} dh_{t+1} \\ dr_{t+1} \end{bmatrix} = \begin{bmatrix} h_h & \frac{\tau_t h_t h_e}{f_{\kappa\kappa}} \\ \frac{s_w w - (\tau_t h_t h_e + h_h) \kappa_t}{\frac{h_{t+1}}{f_{\kappa\kappa}} - s_r} & - \left(s_w h_t + \frac{\tau_t h_t h_e}{f_{\kappa\kappa}} \right) \kappa_t \end{bmatrix} \begin{bmatrix} dh_t \\ dr_t \end{bmatrix},$$

which can be denoted as $[dh_{t+1}, dr_{t+1}]^T = \mathbf{A}[dh_t, dr_t]^T$. To ensure the local stability, $1 \pm tr(\mathbf{A}) + det(\mathbf{A}) > 0$ and $0 < det(\mathbf{A}) < 1$ should be satisfied, which means (4.8) holds.

Appendix 4.B Proof of Proposition 4.1

Recall the social welfare function (4.3):

$$SW = u((1 + r_1)s_0) + \sum_{t=1}^{\infty} \beta^{t-1} [u((1 - T_t)w_t h_t - s(\cdot)) + \beta u((1 + r_{t+1})s(\cdot))].$$

Each government chooses an infinite sequence $\{\tau_t, e_t\}$ for an optimal policy rule of public education authority. It maximizes (4.3) subject to (4.2). The first-order conditions for the optimal policy sequence at period t are given by

$$\kappa_t u_c - \lambda_t (\kappa_t + \tau_t \kappa_\tau) = 0$$

and

$$\beta u_c w_t h_e + \lambda_{t+1} \beta \tau_{t+1} \kappa_{\tau+1} h_e - \lambda_t = 0,$$

where λ_t and λ_{t+1} are the Lagrange multipliers. Thus, at the steady state, $\beta w h_e / [1 - \beta \tau \kappa h_e] = 1 / [1 + \tau / \kappa f_{\kappa\kappa}]$ holds, which is known as (4.10).

Chapter 5 **Conclusions**

We give conclusions about the results obtained throughout the dissertation and discuss any remaining issues as well as the possibilities for extension of Chapters 2–4.

The main purpose of Chapter 2 was to complete a general analysis of the effect of the introduction of a PAYG pension system on utility in the short, medium, and long runs. In our work, we clarified the effects in the cases of monotonic and oscillatory convergences for both steady-state and transitional generations.

We analyzed the PAYG pension problem and clarified effects that have received little or no attention in the literature to date. Specifically, we found that a PAYG pension could be Pareto-improving in the short or medium run, even if the interest rate is higher than the population growth rate. By contrast, we gave an alternative proof that the introduction of a fully funded pension system is neutral to capital accumulation, and thus, to welfare. This assists in explaining, especially from the viewpoint of a real economy, why the PAYG pension is a useful policy tool in the short or medium run even though it reduces welfare at the steady state, whereas a fully funded pension does not. Therefore, governments can and should evaluate their pension policies with reference to their current situations.

As a mathematical preparation, we highlighted the contribution of Kuhle (2014) using a weighted average method for evaluating the changes in welfare in changing transitional states. We clarified that in the case of monotonic convergence, the welfare effects, which can be expressed as a weighted average of the welfare effects for the initial and steady-state generations, also change monotonically. However, in the case of oscillatory convergence, the change in the transitional welfare effects follows the rule determined not only by the initial and steady-state effects, but also by effects in the second period.

Chapter 3 investigated the effects of fully funded pension systems on economic growth in a small open economy with two kinds of human capital accumulation and education: intergenerational education and self-education. We found that the introduction of a fully funded pension system can accelerate economic growth through the “intergenerational” human capital accumulation. This is because intergenerational education has a positive effect on the human capital accumulation of the next generation, but self-education only

affects individuals' own human capital accumulation, which cannot be inherited by subsequent generations. A fully funded pension system then increases individuals' income in their old age and therefore reduces their incentives for self-education. This leads individuals to invest more in educating their children instead, and this may stimulate economic growth. Moreover, when the exogenously determined interest rate is high enough, there could exist an optimal scale of pension that maximizes the economic growth rate.

As is known, because a fully funded pension system operates as forced savings, it generally does not affect either social welfare or economic growth. It should be noted, however, that in our framework that considers both self-education and intergenerational education, the fully funded pension both increases social welfare in the short run and promotes economic growth through intergenerational human capital accumulation in the long run.

In our model, the fully funded pension is introduced as a policy tool to reduce the incentives for self-education. In a world with non-inherited old age human capital, in order to stimulate economic growth or to improve social welfare, the government needs to lead individuals to invest in themselves less and to invest in their children more. Therefore, any policy that increases individuals' old age income or the returns to intergenerational education could be important.

Research has so far only focused on a single kind of education: either intergenerational education or self-education. We, however, studied both types of education in the same framework for the first time. In our model, although the two types of education both lead to human capital accumulation, they have different properties: intergenerational education accelerates growth, whereas self-education does not. This indicates the possibility for a new form of intergenerational conflict that is different from the traditional ones such as the contradiction between savings and bequests. In this sense, this chapter provides a new perspective on evaluating the effects of education on economic growth or social welfare on various outcomes in an OLG model, and this approach should be extended in future studies.

In Chapter 4, we evaluate the effects of a coordinated capital tax reform among symmetric countries in an OLG model where public education brings an intertemporal effect on social welfare. We show that in an open economy where governments tax

physical capital to provide public education instead of public goods, a coordinated tax reform is no longer necessarily Pareto-improving. In particular, when the elasticity of savings to wage income is small and s_r is great enough, a lower capital tax rate will call for a higher level of human capital accumulation.

Therefore, since human capital plays a role that is as important as physical capital in promoting economic growth, when the government of each country tries to attract physical capital through a tax competition or apply to coordinately undertake a tax reform, it should take the policy effects on human capital as well as those on physical capital into consideration. These kinds of extensions would be valuable for providing concrete policy tools that could lessen generational conflicts in a real economy.

As representatives of intergenerational conflicts, pensions and education problems have been and should still be attached importance. In future studies, on the one hand, we should consider the reforms of the education system and public pension systems from more diverse aspects. On the other hand, we should also address other issues that reflect the conflicts between different generations, such as fertility choices, bequests, other fiscal transfers between the young and the old, and so on. Furthermore, we could apply the mathematical framework we improved. In this manner, we would be able to shed light on other generational conflicts by determining not only the long-run effects but also the short- and medium-run effects and the welfare gains and losses along the transition path.

References

- [1] Aguiar-Conraria, L. (2005). Public vs private schooling in an endogenous growth model. *Economics Bulletin*, 9(10), 1-6.
- [2] Alonso-Carrera, J., Caballé, J., & Raurich, X. (2012). Fiscal policy, composition of intergenerational transfers, and income distribution. *Journal of Economic Behavior & Organization*, 84(1), 62-84.
- [3] Altshuler, R., & Goodspeed, T. J. (2015). Follow the leader? Evidence on European and US tax competition. *Public Finance Review*, 43(4), 485-504.
- [4] Azariadis, C., & Drazen, A. (1990). Threshold externalities in economic development. *The Quarterly Journal of Economics*, 105(2), 501-526.
- [5] Azarnert, L. V. (2010). Free education, fertility and human capital accumulation. *Journal of Population Economics*, 23(2), 449-468.
- [6] Azarnert, L. V. (2014). Integrated public education, fertility and human capital. *Education Economics*, 22(2), 166-180.
- [7] Baskaran, T., & Lopes da Fonseca, M. (2013). The economics and empirics of tax competition: A survey. *CEGE Center for European, Governance and Economic Development Research Discussion Paper*, (163).
- [8] Batina, R. G. (2009). Local capital tax competition and coordinated tax reform in an overlapping generations economy. *Regional Science and Urban Economics*, 39(4), 472-478.
- [9] Batina, R. G. (2012). Capital tax competition and social security. *International Tax and Public Finance*, 19(6), 819-843.
- [10] Ben-Porath, Y. (1967). The production of human capital and the life cycle of earnings. *Journal of political economy*, 75(4, Part 1), 352-365.
- [11] Bewley, T. F. (2009). *General equilibrium, overlapping generations models, and optimal growth theory*. Harvard University Press.
- [12] Bhattacharya, J., Qiao, X., & Wang, M. (2016). Endogenous borrowing constraints and wealth inequality. *Macroeconomic Dynamics*, 20(6), 1413-1431.
- [13] Bishnu, M., & Wang, M. (2017). The political intergenerational welfare state. *Journal of Economic Dynamics and Control*, 77, 93-110.

- [14] Blanchard, O. J., & Fischer, S. (1989). *Lectures on macroeconomics*. MIT press.
- [15] Blankenau, W., & Camera, G. (2009). Public spending on education and the incentives for student achievement. *Economica*, 76(303), 505-527.
- [16] Brennan, G., & Buchanan, J. M. (1980). *The power to tax: Analytic foundations of a fiscal constitution*. Cambridge University Press.
- [17] Brinkman, J., Coen-Pirani, D., & Sieg, H. (2018). The political economy of municipal pension funding. *American Economic Journal: Macroeconomics*, 10(3), 215-46.
- [18] Brueckner, J. K., & Saavedra, L. A. (2001). Do local governments engage in strategic property—tax competition?. *National Tax Journal*, 54(2), 203-229.
- [19] Buettner, T. (2001). Local business taxation and competition for capital: the choice of the tax rate. *Regional Science and Urban Economics*, 31(2-3), 215-245.
- [20] Castelló-Climent, A., & Doménech, R. (2008). Human capital inequality, life expectancy and economic growth. *The Economic Journal*, 118(528), 653-677.
- [21] Chakraborty, S., & Das, M. (2005). Mortality, human capital and persistent inequality. *Journal of Economic growth*, 10(2), 159-192.
- [22] Chanda, A. (2008). The rise in returns to education and the decline in household savings. *Journal of Economic Dynamics and Control*, 32(2), 436-469.
- [23] Charlot, S., & Paty, S. (2007). Market access effect and local tax setting: evidence from French panel data. *Journal of Economic Geography*, 7(3), 247-263.
- [24] Chen, H. J. (2010). Life expectancy, fertility, and educational investment. *Journal of Population Economics*, 23(1), 37-56.
- [25] Chen, H. J., & Fang, I. H. (2013). Migration, social security, and economic growth. *Economic Modelling*, 32, 386-399.
- [26] Chu, A. C., Furukawa, Y., & Zhu, D. (2016). Growth and parental preference for education in China. *Journal of Macroeconomics*, 49, 192-202.
- [27] Cipriani, G. P. (2015). Child labour, human capital and life expectancy. *Economics Bulletin*, 35(2), 978-985.
- [28] Cremer, H., & Pestieau, P. (2006). Intergenerational transfer of human capital and optimal education policy. *Journal of Public Economic Theory*, 8(4), 529-545.
- [29] Cremers, E. T., & Sen, P. (2008). The transfer paradox in a one-sector overlapping generations model. *Journal of Economic Dynamics and Control*, 32(6), 1995-2012.

- [30] De Gregorio, J. (1996). Borrowing constraints, human capital accumulation, and growth. *Journal of Monetary Economics*, 37(1), 49-71.
- [31] De Gregorio, J., & Kim, S. J. (2000). Credit markets with differences in abilities: education, distribution, and growth. *International Economic Review*, 41(3), 579-607.
- [32] De la Croix, D. (2001). Growth dynamics and education spending: The role of inherited tastes and abilities. *European Economic Review*, 45(8), 1415-1438.
- [33] De la Croix, D., & Licandro, O. (2012). The child is father of the man: Implications for the demographic transition. *The Economic Journal*, 123(567), 236-261.
- [34] De la Croix, D., & Michel, P. (2002). *A theory of economic growth: dynamics and policy in overlapping generations*. Cambridge University Press.
- [35] Del Rey, E., & Lopez-Garcia, M. A. (2013). Optimal education and pensions in an endogenous growth model. *Journal of Economic Theory*, 148(4), 1737-1750.
- [36] Del Rey, E., & Lopez-Garcia, M. A. (2016). Endogenous growth and welfare effects of education subsidies and intergenerational transfers. *Economic Modelling*, 52, 531-539.
- [37] Devereux, M. P., Lockwood, B., & Redoano, M. (2008). Do countries compete over corporate tax rates?. *Journal of Public Economics*, 92(5-6), 1210-1235.
- [38] Diamond, P. A. (1965). National debt in a neoclassical growth model. *The American Economic Review*, 55(5), 1126-1150.
- [39] Docquier, F., & Paddison, O. (2003). Social security benefit rules, growth and inequality. *Journal of Macroeconomics*, 25(1), 47-71.
- [40] Docquier, F., Paddison, O., & Pestieau, P. (2007). Optimal accumulation in an endogenous growth setting with human capital. *Journal of Economic Theory*, 134(1), 361-378.
- [41] Egger, P., Pfaffermayr, M., & Winner, H. (2007, April). Competition in corporate and personal income taxation. In *meeting of the European Tax Policy Forum, London*.
- [42] Emerson, P. M., & Knabb, S. D. (2007). Fiscal policy, expectation traps, and child labor. *Economic Inquiry*, 45(3), 453-469.
- [43] Euwals, R. (2000). Do mandatory pensions decrease household savings? Evidence for the Netherlands. *De Economist*, 148(5), 643-670.
- [44] Fehr, H., & Uhde, J. (2014). Means-testing and economic efficiency in pension design. *Economic Modelling*, 44, S57-S67.
- [45] Feld, L. P., & Reulier, E. (2009). Strategic tax competition in Switzerland: Evidence from a panel of the Swiss cantons. *German Economic Review*, 10(1), 91-114.

- [46] Fernandez, R., & Rogerson, R. (1995). On the political economy of education subsidies. *The Review of Economic Studies*, 62(2), 249-262.
- [47] Ferreda, C., & Tapia, M. (2010). *Redistributive Taxation, Incentives, and the Intertemporal Evolution of Human Capital* (No. 390).
- [48] Futagami, K., & Yanagihara, M. (2008). Private and public education: Human capital accumulation under parental teaching. *The Japanese Economic Review*, 59(3), 275-291.
- [49] Galor, O. (2011). *Unified growth theory*. Princeton University Press.
- [50] Galor, O., & Moav, O. (2006). Das human-kapital: A theory of the demise of the class structure. *The Review of Economic Studies*, 73(1), 85-117.
- [51] Galor, O., & Zeira, J. (1993). Income distribution and macroeconomics. *The review of economic studies*, 60(1), 35-52.
- [52] Glomm, G., & Kaganovich, M. (2003). Distributional effects of public education in an economy with public pensions. *International Economic Review*, 44(3), 917-937.
- [53] Glomm, G., & Ravikumar, B. (1992). Public versus private investment in human capital: endogenous growth and income inequality. *Journal of political economy*, 100(4), 818-834.
- [54] Glomm, G., & Ravikumar, B. (2001). Human capital accumulation and endogenous public expenditures. *Canadian Journal of Economics*, 34(3), 807-826.
- [55] Granville, B., & Mallick, S. (2004). Pension reforms and saving gains in the United Kingdom. *The Journal of Policy Reform*, 7(2), 123-136.
- [56] Hamada, K., Kaneko, A., & Yanagihara, M. (2017). The transfer paradox in a pay-as-you-go pension system. *International Economics and Economic Policy*, 14(2), 221-238.
- [57] Hamada, K., Kaneko, A., & Yanagihara, M. (2018). Oligopolistic competition in the banking market and economic growth. *Economic Modelling*, 68, 239-248.
- [58] Hamada, K., & Yanagihara, M. (2016). Intergenerational altruism and the transfer paradox in an overlapping generations model. *The Quarterly Review of Economics and Finance*, 59, 161-167.
- [59] Hoyt, W. H. (1991). Property taxation, Nash equilibrium, and market power. *Journal of Urban Economics*, 30(1), 123-131.

- [60] Jimeno, J. F., Rojas, J. A., & Puente, S. (2008). Modelling the impact of aging on social security expenditures. *Economic Modelling*, 25(2), 201-224.
- [61] Kaganovich, M., & Meier, V. (2012). Social security systems, human capital, and growth in a small open economy. *Journal of Public Economic Theory*, 14(4), 573-600.
- [62] Kaganovich, M., & Zilcha, I. (1999). Education, social security, and growth. *Journal of public economics*, 71(2), 289-309.
- [63] Kaganovich, M., & Zilcha, I. (2012). Pay-as-you-go or funded social security? A general equilibrium comparison. *Journal of Economic Dynamics and Control*, 36(4), 455-467.
- [64] Kallen, D., & Bengtsson, J. (1973). Recurrent Education: A Strategy for Lifelong Learning.
- [65] Karni, E., & Zilcha, I. (1989). Aggregate and distributional effects of fair social security. *Journal of Public Economics*, 40(1), 37-56.
- [66] Kemnitz, A., & Wigger, B. U. (2000). Growth and social security: the role of human capital. *European Journal of Political Economy*, 16(4), 673-683.
- [67] Kitaura, K., & Yakita, A. (2010). School Education, Learning-by-Doing, and Fertility in Economic Development. *Review of Development Economics*, 14(4), 736-749.
- [68] Kuhle, W. (2014). The dynamics of utility in the neoclassical OLG model. *Journal of Mathematical Economics*, 52, 81-86.
- [69] Kunze, L. (2012). Funded social security and economic growth. *Economics Letters*, 115(2), 180-183.
- [70] Lambrecht, S., Michel, P., & Vidal, J. P. (2005). Public pensions and growth. *European Economic Review*, 49(5), 1261-1281.
- [71] Le Garrec, G. (2012). Social security, income inequality and growth. *Journal of Pension Economics & Finance*, 11(1), 53-70.
- [72] Leprince, M., Madies, T., & Paty, S. (2007). Business tax interactions among local governments: an empirical analysis of the French case. *Journal of Regional Science*, 47(3), 603-621.
- [73] Lord, W., & Rangazas, P. (1998). Capital accumulation and taxation in a general equilibrium model with risky human capital. *Journal of Macroeconomics*, 20(3), 509-531.
- [74] Matsuyama, K. (1991). Immiserizing growth in Diamond's overlapping generations model: A geometrical exposition. *International Economic Review*, 251-262.

- [75] McDonald, S., & Zhang, J. (2012). Income inequality and economic growth with altruistic bequests and human capital investment. *Macroeconomic Dynamics*, 16(S3), 331-354.
- [76] Michailidis, G., Patxot, C., & Solé, M. (2019). Do pensions foster education? An empirical perspective. *Applied Economics*, 1-24.
- [77] Ministry of Health, Labour and Welfare. "International comparison of pension plans".
<https://www.mhlw.go.jp/stf/seisakunitsuite/bunya/nenkin/nenkin/shogaikoku.html>,
 (2018). (accessed on December 20, 2018)
- [78] Muradoglu, G., & Taskin, F. (1996). Differences in household savings behavior: evidence from industrial and developing countries. *The Developing Economies*, 34(2), 138-153.
- [79] Oates, W.E. (1972). Fiscal Federalism. Harcourt Brace Jovanovich, New York.
- [80] Oded, G. (2011). Inequality, human capital formation, and the process of development. In *Handbook of the Economics of Education* (Vol. 4, pp. 441-493). Elsevier.
- [81] Ono, T., & Uchida, Y. (2016). Pensions, education, and growth: A positive analysis. *Journal of Macroeconomics*, 48, 127-143.
- [82] Pereira, A. M., & Andraz, J. M. (2012). Social security and economic performance in Portugal: after all that has been said and done how much has actually changed?. *Portuguese Economic Journal*, 11(2), 83-100.
- [83] Rauscher, M. (1996). Interjurisdictional competition and the efficiency of the public sector: the triumph of the market over the state? (No. 732). Kiel Working Paper.
- [84] Rauscher, M. (1998). Leviathan and competition among jurisdictions: the case of benefit taxation. *Journal of Urban Economics*, 44(1), 59-67.
- [85] Schultz, T. (1960). Capital Formation by Education. *Journal of Political Economy*, 68(6), 571-583. Retrieved from www.jstor.org/stable/1829945
- [86] Schultz, T. (1961). Investment in Human Capital. *The American Economic Review*, 51(1), 1-17.
- [87] Shirai, M. (1990). The investment rule for public education. *Economics of Education Review*, 9(1), 25-30.

- [88] Tamura, R. (2006). Human capital and economic development. *Journal of Development Economics*, 79(1), 26-72.
- [89] Tran, C. (2016). Fiscal policy as a temptation control device: Savings subsidy and social security. *Economic Modelling*, 55, 254-268.
- [90] Valente, S. (2005). Tax policy and human capital formation with public investment in education. *Journal of Economics*, 86(3), 229-258.
- [91] Wilson, J. D. (1986). A theory of interregional tax competition. *Journal of urban Economics*, 19(3), 296-315.
- [92] Wilson, J. D. (1999). Theories of tax competition. *National tax journal*, 269-304.
- [93] Wilson, J. D. (2005). Welfare-improving competition for mobile capital. *Journal of Urban Economics*, 57(1), 1-18.
- [94] Wilson, J. D., & Wildasin, D. E. (2004). Capital tax competition: bane or boon. *Journal of public economics*, 88(6), 1065-1091.
- [95] Yakita, A. (2003). Taxation and growth with overlapping generations. *Journal of Public Economics*, 87(3-4), 467-487.
- [96] Zhang, J. (1995). Social security and endogenous growth. *Journal of Public Economics*, 58(2), 185-213.
- [97] Zilcha, I. (2003). Intergenerational transfers, production and income distribution. *Journal of Public Economics*, 87(3-4), 489-513.
- [98] Zodrow, G. R. (2010). Capital mobility and capital tax competition. *National Tax Journal*, 63(4), 865-901.
- [99] Zodrow, G. R., & Mieszkowski, P. (1986). Pigou, Tiebout, property taxation, and the underprovision of local public goods. *Journal of urban economics*, 19(3), 356-370.