2019 Doctor's Thesis

Two-Sided Platform Competition with Heterogeneous Characteristics of Potential Users

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Acknowledgment

First, I am indebted to Takanori Adachi and Makoto Hanazono for their detailed advice and guidance while studying as a graduate student at Nagoya University. Professor Adachi provided me with opportunities to learn a lot of expertise knowledge from coursework topics to advanced topics and from theoretical studies to empirical studies, and with many invaluable comments that significantly improved my research projects and fostered my attitude toward becoming a researcher. Professor Hanazono provided me with an opportunity to talk on my studies at his academic seminar and with a lot of constructive comments on my research projects, which especially included those while preparing presentations and when I submitted a paper of mine to a peer-reviewed journal. I do not believe that I could complete this dissertation without their extensive support.

I was benefited from various support provided by other faculty members and the alumni association as well. Noritaka Kudoh, Jiro Nemoto, and Akihiko Yanase served as members of the dissertation committee during the process to evaluate my dissertation. In particular, the comments from Professor Kudoh and Professor Nemoto at my pre-submission presentations helped me improve this dissertation. Hiroshi Tateishi gratefully had a considerable number of talks with me that enhanced my knowledge in various fields. Moreover, this dissertation contains the content financially supported by Kitan-kai, the alumni association from School of Economics and Graduate School of Economics, Nagoya University.

This dissertation includes research projects that received comments and personal communications, and that appeared at scholarly events. I thank Alvaro Domínguez, Maria Martin-Rodríguez, Masayoshi Maruyama, Noriaki Matsushima, and Lijun Pan for their beneficial personal communications on part of this dissertation. Chapter 2 is based on a paper that has been accepted in a peer-reviewed journal; I am grateful for the anonymous referee's helpful comments. Earlier versions of papers corresponding to chapters in this dissertation appeared at the 2017 Japanese Economic Association Spring Meeting and a seminar at Nanzan University (the paper corresponding to chapter 2), and at International Conference on Business, Economics, and Information Technology (ICBEIT) 2016 (the paper corresponding to chapter 4 although substantial changes were made after that conference).

Lastly, I acknowledge that many other people provided indirect but non-negligible support for me. Those people include (but are not limited to) my friends, seniors, juniors, and family. I must keep their support in mind forever.

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Chapter 1

Introduction

1.1 Background

There are a number of goods and services that behave as intermediaries between two distinct groups of economic agents. For instance, newspapers, TV channels, social networking services, and other types of media often sell advertising slots to third-party firms. Firms that purchase advertising slots from a medium can easily attract subscribers to that medium. Advertising-supported media in this sense play a role as intermediaries between producers and consumers. Apps for sharing economies, including Uber and Airbnb, are another example: this type of app matches individuals who provide services (e.g., transportation) and those who use the services so that they can make transactions. Other examples include operating systems (app developers and end users), video-streaming services (content providers and subscribers), and shopping malls (sellers and buyers of a good). These intermediaries are known as platforms in a two-sided market, or just two-sided platforms.

The most important feature of a two-sided market is that agents on each side of the market are potential partners of those on the opposite side in most cases. This feature implies that potential platform users care about how many agents each platform attracts on the opposite side while making platform choices. For instance, advertising firms in general have higher valuations of a medium as the medium attracts a larger number of consumers. This logic applies to valuations of most other platforms, which include operating systems, payment services, sharing-economy apps, and shopping malls. The literature calls this phenomenon an indirect network externality, in the sense that a platform's users exert a technological externality on the same platform's opposite-side users. Note that indirect network externalities are not always positive in any market. For example, advertisers in a medium might exert a negative indirect network externality on subscribers to that medium because those subscribers have chosen the medium to enjoy its content and do not necessarily appriciate seeing (or watching) advertisements (see the next section for a discussion). This dissertation thus allows for the possibilities that an indirect externality is exerted positively and negatively, depending on the market structure in question.

I should also remark that the existence of indirect network externalities often makes platform-choice behavior complicated. The above paragraph argues that potential users care about the number of agents who use each platform on the opposite side. However, potential users tend to be unable to observe the exact number of platform users at the time of their choices because that number is realized as a consequence of each opposite-side agent's platform adoption. Potential users must thus *expect* the number of each platform's users on the opposite side and choose platforms to join based on the respective expected payoff functions. To see the impacts of agents' expectations, suppose that two platforms (labeled 1 and 2) between firms and consumers have just entered the market and announced their prices, and define the processes of each firm's and each consumer's platform choices as a game given the announced prices. If both platforms choose moderate prices and platform 1 is expected to be an only platform that attracts all consumers, all firms have incentives to participate in platform 1 and rarely use platform 2. If platform 1 is also expected to be an only platform that attracts all firms, all consumers exhibit similar platform-choice behavior. All firms and all consumers correctly expect consumer behavior and firm behavior, respectively, and maximize the respective payoffs; thus, the bundle of their choices (given the platforms' prices) is a Nash equilibrium. Notice that there can also exist an equilibrium in which platform 2 attracts all firms and all consumers, both platforms obtain sufficient market shares, or neither platform attracts a number of firms nor consumers if such an expectation is formed and realized, in which sense expectations in a two-sided market play a decisive role in the market outcome.

This dissertation consists of three different studies on two-sided platform competition when potential users have heterogeneous characteristics in indirect network externality and expectation, two of which are concerned with two different possibilities of heterogeneous externalities. The first is that potential users have heterogeneous *tastes* for interactions with those on the opposite side. Consider, for instance, a market for advertising-supported media. On the subscription side, consumers are likely to incur disutility when they see advertisements although some consumers may in total enjoy advertisements in the sense that they can obtain opportunities to purchase the advertised products. On the advertising side, some firms might face limited capacities (e.g., individual professionals and family-owned restaurants) and desire to advertise their businesses in media with small audiences in order to avoid addressing demand that exceeds their capacities although firms can in general enhance their expected payoffs by choosing media with a large number of subscribers. It is thus a possible situation that positive and negative indirect network externalities coexist on both (e.g., the firm and consumer) sides of a market. The second type of heterogeneous indirect network externalities is that potential users play heterogeneous *roles* in exerting externalities. Consider, here, a market for broadcast (e.g., TV) channels and alternative streaming services. Third-party firms generally have opportunities to participate as content providers (i.e., "production companies") on the one hand and advertisers on the other hand in this type of market. Consumers prefer third-party contents to advertisements (and may even incur disutility by seeing advertisements), which encourages each medium to place higher priority on stimulating third-party content provision than advertising. However, the medium also has an incentive to attract advertising firms because it can earn an additional profit by selling its advertising slots. The balance between third-party contents and advertisements thus matters in each medium's profit maximization and its implications from a welfare viewpoint. These two possibilities of heterogeneous indirect network externalities deserve to be addressed in this dissertation because, as discussed in the next section, there is no existing paper that explicitly addresses either possibility of heterogeneity in externality although the literature proposes frameworks that might apply.

The third study in this dissertation focuses on heterogeneity in expectation. Consider, for example, the situation in which two companies (or company groups) run incompatible platforms of a new type that can be considered to belong to a new market. Little market information is available in this case because the market is new. Some potential users may have special knowledge (e.g., experts or market analysts in that field) or be informed of such a specialist's opinion from a medium and appropriately expect each platform's final market share. The other potential users might, however, form expectations affected by their idiosyncratic biases such that a particular platform must attract a larger number of opposite-side potential users than its rival due to lack of necessary information to judge the correctness of their expectations. As reviewed in the next section, the literature usually assumes all potential users to form rational expectations such that they can (i) correctly calculate the number of platform users, (ii) hold expectations to be fulfilled, or (iii) form expectations *ex post* consistent with the actual market structure. The assumption of rational expectations enables one to define the consequence of platform competition as a Nash equilibrium in the sense that all players maximize the respective payoffs with correct expectations about the other players but cannot explain the impacts of biased expectations introduced above. Two-sided platform competition should therefore be revisited in this dissertation allowing for biased expectations.

The rest of this chapter is organized by two sections to show the roadmap of this dissertation. The next section reviews related existing papers and clarifies this dissertation's standpoint. The last section explains the structure of this dissertation with a summary of each study in the dissertation.

1.2 Literature Review

This section discusses the related literature to this dissertation. In particular, the section focuses on the standard framework proposed in seminal papers and reviews how existing papers treat the three types of agent heterogeneity introduced in the preceding section. One can find a review of papers specifically related to each study (e.g., those who develop close models) in the corresponding chapter.

1.2.1 Price Competition in a Two-Sided Market

Following seminal papers (Caillaud and Jullien 2001, 2003; Rochet and Tirole 2003; Armstrong 2006; White and Weyl 2016), I introduce the standard framework to describe two-sided platform competition.

There exists a platform market with two sides A and B. Two platforms, labeled 1 and 2, offer intermediation services on both sides of the market. Each platform maximizes its profit with respect to its pair of prices. Platforms may incur strictly positive production costs although the costs are usually assumed to be low enough for neither platform to earn a strictly negative profit. A unit mass of economic agents exist on each side as potential platform users and are formalized as discrete-choice players who have quasi-linear utility functions. Each of the agents obtains utility (which also includes disutility or a technological cost) by joining a platform mainly in two ways: (i) transactions or other types of interactions with opposite-side users of the platform (i.e., receiving the indirect network externality exerted by those users) and (ii) the stand-alone part of the platform.[1] Potential users may have heterogeneous or identical tastes for cross-side interactions and for the stand-alone part of each platform, depending on the nature of the platform market. Each agent then chooses one option that maximizes his/her utility among using platform 1 only, platform 2 only, and neither, and both if possible.[2]

[1]Stand-alone parts include original articles or programs of advertising-supported media and basic applications (e.g., desktop environments and text editors) for operating systems in the case of positive utility and may include sources of disutility as a consequence of horizontal product differentiation or technological adjustment in the case of negative utility.

[2]The literature often calls joining at most one platform singlehoming and joining multiple platforms multihoming. An example for singlehoming agents is consumers in the market for periodicals of a specific genre, most of whom do not seem to subscribe to multiple newspapers or magazines of the same genre.

Platforms and potential users on both sides are supposed to play a one-shot game. The timing of the game is as follows: each platform simultaneously determines its pair of prices and announces it at the beginning, and then each agent on each side simultaneously makes a platform choice. Combining the existence of indirect network externalities and the simultaneity of platform choices, one should note that potential users make decisions based on their expectations of the opposite-side allocation. If they can calculate the number of opposite-side platform users with all agent and price information (which the aforementioned papers assume), the market demand for each platform on each side is defined as a function of both platforms' prices on the same and other sides. This formulation of market demand implies that the demand for a platform increases or decreases as the platform reduces its price on the opposite side, which enables each platform to take a flexible price strategy such that the platform pays monetary benefits on one side to attract a sufficient number of agents there and charges relatively high fees on the other side to earn profits.[3] The price of each platform and the number of its users on each side constitute a Nash equilibrium.

One can also consider social-welfare maximization in this type of market. Social welfare consists of total side-A agent surplus, total side-B agent surplus, and platform surplus. It also equals the sum of total surplus, which consists of the total utility and cost, with regard to each side because the total payment from agents corresponds to the total revenue for platforms and (if any) the total payment from platforms constitutes part of the total utility. Notice that total side-A agent surplus and total side-B agent surplus include the number of platform users on side B and the number of platform users on side A, respectively, because indirect network externalities exist. Social welfare tends to be maximized if all agents on both sides are allocated to a single platform because the total benefit from the indirect network externality on each side becomes the highest, except in

An example for multihoming agents is third-party firms that produce contents and sell the licenses on the contents to TV channels. Note that whether potential users can multihome are whether they face emotional or technological limitations on the number of platforms to use. Multihoming agents can thus choose using one or neither platform if such choices are the best for them. See Doganoglu and Wright (2006, section 3) and Armstrong and Wright (2007) for detailed analyses of multihoming behavior.

[3] For instance, pay-TV channels and online video-streaming services (e.g., Netflix) pay fees to production companies and offer various third-party programs to consumers on the one hand, and receive subscription fees on the other hand.

two cases. The first case is that such concentration yields non-negligible costs or disutility as a consequence of, say, product differentiation and cost inefficiency. The second case is that a sufficient number of agents incur a negative indirect network externality.

1.2.2 Heterogeneous Tastes for Cross-Side Interactions

The above framework can be extended by allowing for the possibility of heterogeneous tastes for cross-side interactions such that some potential users positively value such interactions and the others negatively value them, as observed in advertising-supported media markets. Caillaud and Jullien (2001, 2003), Armstrong (2006), and subsequent works of theirs assume that agents on the same side have identical tastes for cross-side interactions. Some seminal papers (Rochet and Tirole 2003, 2006; Weyl 2010; White and Weyl 2016) allow for the situation in which potential users on each side may have either positive or negative valuations of cross-side interactions; however, they aim to propose general models and do not obtain an explicit result that suits this situation. Existing theoretical papers on advertising-supported platforms also abstract the possibility that positive and negative valuations of cross-side interactions coexist on both sides. Consumers are usually assumed to have negative valuations of seeing advertisements (e.g., Anderson and Coate 2005; Carroni and Paolini 2019) or obtain zero benefit from advertisements (e.g., Gabszewicz, Laussel, and Sonnac 2001; von Ehrlich and Greiner 2013). To the best of my knowledge, no existing paper incorporates the possibility that some advertising firms incur a negative indirect network externality.

This type of heterogeneity is notable because some potential users do not appriciate receiving an intense indirect network externality. Platforms in some cases might possess incentives to attract a small number of agents on one (or each) side. The efficient allocation on each side is no longer a trivial question.

1.2.3 Heterogeneous Roles of Potential Users

The standard framework can also be extended by allowing for the possibility that potential users may be heterogeneous in the roles that they play, such as content-providing and advertising firms in a medium. All papers cited in the first subsection abstract this type of possibility. Weyl (2010) develops a general framework of multi-sided markets with an interpretation as a special type of two-sided markets such that all sides of a multi-sided market belong to either of two side groups and agents on a side of a side group discriminate those on each side of the opposite side group. If the media industry is the case, for instance, (i) third-party firms constitute the content-provision and advertising sides of the firm-side group, and (ii) consumers have different valuations of interactions with firms on different sides. Weyl's (2010) interpretation implies that all papers in which a market consists of more than two agent sides (e.g., Tan and Zhou 2019) are related. Some papers explicitly incorporate third-party content provision for advertising-supported platforms. Weeds (2014) and D'Annunzio (2017) consider TV competition such that a (monopolistic) third-party firm produces a premium content and provides it for one or both channel(s). Carroni and Paolini (2017, 2019) model the behavior exhibited by a monopolistic freemium platform that receives contents from third-party firms.[4]

The idea of heterogeneous roles is especially notable if each platform faces a constraint on the total number of third-party firms that the platform can attract. For instance, a TV channel by nature needs to broadcast specific programs and specific advertisements at specific periods of time. Platforms in this case tend to face tradeoffs between attracting content-providing and advertising firms. The welfare consequence of each platform's choice should be examined because platforms and consumers may have different interests. The above papers cannot explain this situation.

1.2.4 Heterogeneity in Expectation

The last extension of the standard framework is to allow for the coexistence of different allocation expectations. The majority of existing papers formulate agents' expectations under the concept of rational expectation. All works mentioned in the first subsection and, to the best of my knowledge, most other papers assume that potential users can

[4] Freemium is a business concept under which service providers, whether they are platforms or not, simultaneously offer free-of-charge and charged options so that consumers can choose either option depending on their tastes.

correctly calculate the opposite-side market allocation with the information of all market demand functions and all prices. Some papers (e.g., Gabszewicz and Wauthy 2004, 2014) adopt a different rational-expectation concept such that (i) agents form some expectations and (ii) their expectations are fulfilled in equilibrium, following Katz and Shapiro's (1985) concept of fulfilled expectation.[5] Hagiu and Hałaburda (2014) propose their concept of heterogeneous expectation by mixing the preceding two expectation concepts yet rely on the assumption of rational expectations. Some existing papers explicitly consider different expectations. Jullien and Pavan (2019) apply the framework of global games and describe two-sided markets in which potential users form expectations based on their own tastes for the stand-alone part of each platform; however, they adopt an equilibrium concept with incomplete information and thus do not mean to allow *ex post* inconsistent expectations. Hossain and Morgan (2013) develop a model in which agents face different cognitive levels and cannot recognize the decisions made by those at strictly higher cognitive levels, regarding their situation as a matter of boundedly rational behavior. In sum, no existing paper models the situation in which some potential users form biased expectations.[6]

If some potential users cannot correctly expect the opposite-side market allocation but form, say, biased expectations toward a particular platform, platforms need to incorporate both the fact that biased expectations exist and the impacts of such expectations on agents who can form rational expectations to their price strategies. How does the platform with an advantage from biased expectations utilize its advantage to enhance its profit, and does the advantageous platform have an incentive to attract all potential users on both sides of the market? How do such biases affect the welfare consequence of platform competition? This dissertation addresses these questions.

[5] In this dissertation, chapter 2 adopts this expectation concept.

[6] An exception is that some papers using dynamic models (e.g., Sun and Tse 2007) do not rely on the concept of rational expectation; however, to the best of my knowledge, none of those papers allows agents to form biased expectations.

1.3 Structure of this Dissertation

This dissertation consists of five chapters, including this chapter. The next three chapters contain studies on platform competition with various types of heterogeneity among potential users, each of which is summarized below. The last chapter concludes by reviewing each study and discussing remaining problems in this dissertation.

Chapter 2

Chapter 2 ("Two-Sided Platforms, Heterogeneous Tastes, and Coordination") is concerned with heterogeneity in externality such that potential users have heterogeneous tastes for interactions with those on the opposite side, based on a paper that has been accepted in *Economics Bulletin*. To incorporate such heterogeneity, the chapter models duopolistic two-sided platform competition with positive and negative indirect network externalities exerted on both sides in the following way. The potential users have heterogeneous valuations of cross-side interactions. On each side, the indirect network externality is positive for some agents and negative for the others. The chapter conducts equilibrium and welfare analyses.

The findings from the chapter are twofold. The equilibrium analysis shows two market-outcome configurations. If the proportion of agents who incur a negative indirect network externality is small, a particular platform attracts a larger number of agents on both sides. If the proportion of agents who incur a negative indirect network externality is large, each platform attracts a larger number of potential users on one side and a smaller number of those on the other side. These equilibrium configurations can be interpreted as coordination such that the positive indirect network externality is enhanced and the negative indirect network externality is mitigated on each side. Social welfare is maximized if and only if a certain platform attracts all agents on one side and a different number of agents on the other side according to the proportion of agents who incur a negative indirect network externality. The number of agents that the platform should attract on the latter side decreases as a larger proportion of agents incur a negative indirect network externality, which mitigates the negative indirect network externalities. The above equilibrium configurations do not maximize social welfare.

Chapter 3

Chapter 3 (“Content Provision, Advertising, and Capacity-Constrained Platforms”) is concerned with heterogeneity in externality such that potential users play heterogeneous roles in externality exerted on the opposite side. To highlight such heterogeneity, I develop a model of a duopolistic two-sided market in which third-party firms behave as content-providers and advertisers, and each platform cares about not just the number of firms to attract but its proportion of third-party contents and advertisements because the platform faces a capacity constraint on the total amount of slots that can be allocated to firms. The chapter conducts equilibrium and welfare analyses.

The findings from the chapter are summarized as below. The equilibrium analysis shows three different market-outcome configurations according to the market structure. Two of the configurations are symmetric: each platform in equilibrium allocates all of its slots to firms of the same type (i.e., either content-providing or advertising firms). The other configuration is that a platform fills all of its slots with third-party contents and the other platform sells all of its slots to advertising firms. The latter configuration describes each platform’s vertical differentiation by which type of firms they attract in the sense that consumers can obtain higher payoffs from an amount of third-party contents than the same amount of advertisements. However, I find that the platform with third-party contents does not always earn a higher profit. The welfare analysis establishes that the efficient outcome has similar properties to the equilibrium outcome. Social welfare is, however, not necessarily maximized in equilibrium. In particular, an asymmetric equilibrium never maximizes social welfare because the equilibrium and efficient consumer allocations are different.

Chapter 4

Chapter 4 ("Two-Sided Platform Competition with Biased Expectations") is concerned with the possibility that potential users cannot necessarily form correct expectations of the opposite-side market allocation. Specifically, the chapter models a duopolistic two-sided market in which some potential users form biased expectations toward a certain platform. Biased expectations may be inconsistent with the actual allocation on the opposite side, which means that the market outcome cannot necessarily be expressed as a Nash equilibrium. The chapter therefore adopts a solution concept that relaxes Nash equilibrium and allows for such inconsistent expectations. The chapter conducts an analysis of the competition game and a welfare analysis.

The competition-game analysis establishes that the platform with an advantage from biased expectations attracts a larger number of potential users than its rival on each side, and that the former platform utilizes its advantage in different ways according to the impacts of the indirect network externality exerted on each side and the proportion of agents with biased expectations. If an indirect network externality is not strongly exerted on each side, the advantageous platform attracts the groups of potential users who exhibit relatively high willingness to pay for that platform on both sides by exploiting its advantage, which enables the disadvantageous platform to obtain market shares as well. If the indirect network externality exerted on each side is sufficiently intense, the market outcome depends on the number of agents who form biased expectations. The platform-coexistence configuration discussed above arises when the number of potential users with biased expectations is *large* enough. On the other hand, the advantageous platform dominates both sides of the market when a sufficiently small number of agents hold biased expectations. The latter market outcome becomes more likely to arise as the number of potential users with biased expectations increases because platforms engage in more severe price reduction and thus the resulting prices are low enough for the advantageous platform to dominate the entire market. The outcome of the advantageous platform's market dominance constitutes a Nash equilibrium in that biased expectations are consistent, which describes a platform's market-dominance behavior driven by some agents' biased expectations.

The findings from the welfare analysis can be summarized as follows. First, the market outcome is efficient if the advantageous platform dominates the entire market. Second, the competitive outcome yields an inefficient allocation on each side if the two platforms coexist. Biased expectations tend to play a negative role in the welfare consequence of the market outcome in the sense that biased expectations reduce the possibility of market dominance, which is a necessity to maximize social welfare.

Chapter 2

Two-Sided Platforms, Heterogeneous Tastes, and Coordination

2.1 Introduction

Advertising-supported media are two-sided platforms in that advertising slots help firms reach consumers, but the consequence of advertising is not simple. Consumers tend to incur disutility each time they see an advertisement. Some empirical studies, however, claim that consumers may obtain utility that increases in the number of advertisements (e.g., Kaiser and Song 2009).[1] It is thus relevant that some consumers obtain higher utility while others incur greater disutility from a medium with a larger number of advertisements. Firms in general can earn higher expected revenues by showing their advertisements to a larger number of consumers. Regarding profits, nevertheless, firms with limited capacities (e.g., individual professionals and family-owned restaurants) might face excess demand and incur additional costs. This fact implies that some firms might exhibit higher willingness to pay for media with smaller audiences (e.g., local newspapers) because those media yield higher expected profits to the firms. The consideration above suggests that an indirect network externality (simply called an "externality" hereafter)

[1]Kaiser and Song (2009) conduct an empirical analysis of the German magazine industry and obtain the following results. First, consumer utility tends to increase in the number of advertising pages divided by that of content pages. Second, simulated models with consumer heterogeneity suggest the existence of both consumers who enjoy advertisements and who do not in some magazine segments.

is positive for some economic agents and negative for others on both sides of a media market. To the best of my knowledge, the literature seldom investigates this situation.[2] An exception is Sokullu (2016a, 2016b), who empirically shows that the market demand functions are not monotone in opposite-side demand on both sides of the U.S. newspaper and German magazine industries. Sokullu (2016a, 2016b), however, constructs a model for a monopolistic medium.[3] This chapter studies price competition when positive and negative externalities coexist on each side of a duopolistic two-sided market.

I find that the pattern of the equilibrium configuration varies according to the proportion of potential users who incur a negative externality. If the proportion is smaller than half, one platform exceeds the rival platform in market share and price on both sides. This equilibrium configuration replicates the pattern in Gabszewicz and Wauthy (2004, 2014), who analyze the case of a positive externality exerted on each potential user and interpret the configuration as vertical differentiation in terms of market share. The configuration in this chapter is notable in that the lower market share is an advantage for the rival to attract agents who incur a negative externality, in which sense each platform engages in horizontal differentiation. If the externality is negative for the majority of potential users, then each platform attracts a larger number of agents on one side than the rival but fewer agents on the other side. Ambrus and Argenziano (2009) analyze the case of a positive externality exerted on each potential user and apply the concept of coalitional rationalizability proposed by Ambrus (2006),[4] and they show that a similar user allocation may arise in equilibrium. The difference is that each platform charges higher fees on its side with a larger number of users (incurring a weaker negative externality) in this chapter, whereas each platform chooses a lower price on such a side (where a weaker positive externality is exerted) in Ambrus and Argenziano (2009). Platforms avoid fierce competition in my model because each platform mildly competes on its side with fewer users. In sum, two different patterns of user allocations arise under

[2] Rochet and Tirole (2003, 2006), Weyl (2010), and White and Weyl (2016) develop general models that allow for this situation but do not explicitly discuss the competitive outcome in this situation.

[3] Sokullu (2016b) calculates the relative prices of the magazines to account for competition but does not explicitly model price competition among magazines.

[4] Coalitional rationalizability rules out any bundle of strategies that are never optimal for an arbitrary group of players given other players' strategies.

a standard equilibrium concept in a single model with the coexistence of positive and negative externalities.

This chapter shows that social welfare is maximized only if one platform attracts all agents on one side (say, side A). If the externality is positive for a sufficiently large number of agents, all agents on the other side (side B) should join the platform. As the proportion of agents who incur a negative externality grows, the number of side-B agents who should choose the platform decreases because the welfare impact of the negative externality cannot be ignored. In particular, the other platform should attract all agents on side B if the proportion is sufficiently high. This result arises only in the case in which positive and negative externalities coexist.

2.2 Model

This section develops a duopoly model for a two-sided market á la Gabszewicz and Wauthy (2004, 2014) but with two differences. First, the externality is positive for some potential users and negative for others on each of the two sides. Second, both sides of the market are assumed to be fully covered.

There are two different groups of unit-mass agents on sides A and B of a platform market. Platforms 1 and 2 are symmetric firms that provide agents on both sides with their services, charging participation fees. Each platform consists of stand-alone and intermediation services. A stand-alone service has an agent-, platform-, and side-common intrinsic value, which is denoted by $v \in \mathbb{R}_{++}$ and is high enough for any potential user to enjoy a strictly positive payoff from either platform. An intermediation service connects agents on side A with those on side B, which causes the side-A users of a platform to exert an externality on the side-B users of the platform and *vice versa*. The agents on each side are assumed to have different valuations of intermediation services, in that the externality is exerted positively on some of them and negatively on others, and that the impact of the externality depends on each agent's type.[5] The types are uniformly distributed on a

[5]This formulation can apply to the advertising side of a media market. Firms generally obtain higher benefits from a medium with a larger audience, which is the case of a positive externality. Some firms, however, possibly run their businesses with too small staffs to accept a large number of consumers (e.g.,

unit interval $[-\alpha, 1-\alpha]$, where $\alpha \in (0,1)$ is an exogenous side-common parameter that indicates the proportion of agents who incur a negative externality.[6] Potential users on each side simultaneously choose one platform after each platform determines its prices. Provided that platform 1 attracts $n_1^A \in [0,1]$ agents on side A and charges $p_1^B \in \mathbb{R}$ on side B, an agent of type $\theta \in [-\alpha, 1-\alpha]$ on side B receives a payoff of

$$u_1^B\left(p_1^B, n_1^A; \theta\right) \equiv v + \theta n_1^A - p_1^B$$

from that platform. Let $u_2^B(p_2^B, n_2^A; \theta)$ denote the payoff obtained by the agent from platform 2, where the notations of p_2^B and n_2^A are analogous. Define $u_1^A(p_1^A, n_1^B; \theta)$, $u_2^A(p_2^A, n_2^B; \theta)$, p_1^A, p_2^A, n_1^B, and n_2^B similarly. Notice here that θ is the coefficient on n_1^A or n_2^A and that each market share is a consequence of platform choices on side A. Thus, the relation between each side-B agent's expectations of n_1^A and n_2^A plays a crucial role in determining the configuration of the allocation on side B. The rest of this chapter assumes potential users to (rationally) expect that

$$n_1^{Ae} > n_2^{Ae} \quad \text{and} \quad n_1^{Be} \neq n_2^{Be}, \tag{2.1}$$

where $n_1^{Ae} \in [0,1]$, $n_2^{Ae} \in [0,1]$, $n_1^{Be} \in [0,1]$, and $n_2^{Be} \in [0,1]$ denote their expectations of the respective market shares. Note that this expression represents all cases in which $n_1^{Ae} \neq n_2^{Ae}$ and $n_1^{Be} \neq n_2^{Be}$ because the market is symmetrically formulated.[7] Define

individual professionals and family-owned firms). These firms might incur higher costs if showing their advertisements to a larger audience because they need to address demand that exceeds their capacities. In this sense, a negative externality may be exerted on some firms.

The formulation can also apply to the subscription side. Although some consumers might enjoy advertisements *per se*, it is a natural assumption that consumers are likely to incur disutility by seeing advertisements. Nevertheless, the latter consumers can also obtain benefits if they see matched advertisements and purchase the advertised products. The sign of such a consumer's payoff from advertisements is determined by the relation between the total disutility of seeing them and the total utility from his/her purchase(s). In sum, positive and negative externalities plausibly coexist on the side.

[6]Appendix 2.B shows that the main results are robust if the parameter α is side-specific as long as the side-A and side-B parameters are not substantially different.

[7]See footnote 14 for the derivation of an equilibrium when $n_1^{Ae} = n_2^{Ae}$ and/or $n_1^{Be} = n_2^{Be}$, which is unstable in that it may arise only if both platforms attract exactly the same number of agents on one or both side(s).

$\widetilde{\theta}^B(p_1^B, p_2^B; n_1^{Ae}, n_2^{Ae})$ as a type such that

$$
\begin{aligned}
& u_1^B \left[p_1^B, n_1^{Ae}; \widetilde{\theta}^B \left(p_1^B, p_2^B; n_1^{Ae}, n_2^{Ae} \right) \right] = u_2^B \left[p_2^B, n_2^{Ae}; \widetilde{\theta}^B \left(p_1^B, p_2^B; n_1^{Ae}, n_2^{Ae} \right) \right] \\
\Longleftrightarrow\ & \widetilde{\theta}^B \left(p_1^B, p_2^B; n_1^{Ae}, n_2^{Ae} \right) = \frac{p_1^B - p_2^B}{n_1^{Ae} - n_2^{Ae}}. \qquad (2.2)
\end{aligned}
$$

If $n_1^{Ae} > n_2^{Ae}$ and under the full-coverage assumption,[8] platform 1 (which attracts a larger number of side-A agents) is chosen by the side-B potential users of type θ such that

$$
\begin{aligned}
& u_1^B \left(p_1^B, n_1^{Ae}; \theta \right) \geq u_2^B \left(p_2^B, n_2^{Ae}; \theta \right) \\
\Longleftrightarrow\ & \theta \geq \widetilde{\theta}^B \left(p_1^B, p_2^B; n_1^{Ae}, n_2^{Ae} \right) \quad \text{and} \quad -\alpha \leq \theta \leq 1 - \alpha,
\end{aligned}
$$

and platform 2 is chosen by those of type θ such that

$$
\begin{aligned}
& u_1^B \left(p_1^B, n_1^{Ae}; \theta \right) < u_2^B \left(p_2^B, n_2^{Ae}; \theta \right) \\
\Longleftrightarrow\ & \theta < \widetilde{\theta}^B \left(p_1^B, p_2^B; n_1^{Ae}, n_2^{Ae} \right) \quad \text{and} \quad -\alpha \leq \theta \leq 1 - \alpha.
\end{aligned}
$$

Let $D_1^B(p_1^B, p_2^B; n_1^{Ae}, n_2^{Ae})$ and $D_2^B(p_2^B, p_1^B; n_2^{Ae}, n_1^{Ae})$ denote the market demand functions for platforms 1 and 2 on side B, respectively:

$$
\begin{aligned}
& D_1^B \left(p_1^B, p_2^B; n_1^{Ae}, n_2^{Ae} \right) = \begin{cases} 1 - \alpha - \widetilde{\theta}^B \left(p_1^B, p_2^B; n_1^{Ae}, n_2^{Ae} \right) & \text{if } \widetilde{\theta}^B (\cdot) \in [-\alpha, 1 - \alpha] \\ 1 & \text{if } \widetilde{\theta}^B (\cdot) < -\alpha \\ 0 & \text{if } \widetilde{\theta}^B (\cdot) > 1 - \alpha \end{cases} \\
& D_2^B \left(p_2^B, p_1^B; n_2^{Ae}, n_1^{Ae} \right) = 1 - D_1^B \left(p_1^B, p_2^B; n_1^{Ae}, n_2^{Ae} \right).
\end{aligned}
$$

The market demand for platforms 1 and 2 on side A, denoted by $D_1^A(p_1^A, p_2^A; n_1^{Be}, n_2^{Be})$ and $D_2^A(p_2^A, p_1^A; n_2^{Be}, n_1^{Be})$, respectively, is analogously obtained.

Before proceeding to profit maximization, I formulate the process to form market-share expectations. This chapter assumes that potential users expect the opposite-side

[8]Side B is fully covered if agents of the lowest type ($\theta = -\alpha$) eventually obtain weakly positive payoffs, where $v \geq \alpha n_2^A + p_2^B$.

market shares independently of the opposite-side prices and that their expectations are fulfilled in equilibrium.[9] Under this formulation, the market demand functions are defined as those of the own-side prices and the expected opposite-side market shares only.

Platforms 1 and 2 maximize their own profits with respect to their participation fees, given one another's price strategy and the potential users' expectations of the market shares. Specifically, platform 1 chooses (p_1^A, p_1^B) that maximizes

$$\pi_1\left(p_1^A, p_1^B; p_2^A, p_2^B, n_1^{Ae}, n_1^{Be}, n_2^{Ae}, n_2^{Be}\right) \equiv p_1^A D_1^A\left(p_1^A, p_2^A; n_1^{Be}, n_2^{Be}\right) + p_1^B D_1^B\left(p_1^B, p_2^B; n_1^{Ae}, n_2^{Ae}\right)$$

given (p_2^A, p_2^B) and $(n_1^{Ae}, n_1^{Be}; n_2^{Ae}, n_2^{Be})$. Platform 2's profit maximization is symmetrically formulated, where $\pi_2(p_2^A, p_2^B; p_1^A, p_1^B, n_2^{Ae}, n_2^{Be}, n_1^{Ae}, n_1^{Be})$ denotes the platform's profit. Note that the marginal and fixed costs of production are normalized to zero for both platforms. Platform 1's optimal side-B price, denoted by $p_1^B(p_2^B; n_1^{Ae}, n_2^{Ae})$, follows from the first-order condition:

$$\frac{\partial \pi_1\left(p_1^A, p_1^B; \cdot\right)}{\partial p_1^B} = 0 \iff p_1^B\left(p_2^B; n_1^{Ae}, n_2^{Ae}\right) = \frac{p_2^B + (1-\alpha)\left(n_1^{Ae} - n_2^{Ae}\right)}{2}$$

because the second-order condition that $\partial^2 \pi_1(p_1^A, p_1^B; \cdot)/\partial(p_1^B)^2 < 0$ holds for any p_1^B.[10] Platform 2's optimal side-B price is analogous:

$$\frac{\partial \pi_2\left(p_2^A, p_2^B; \cdot\right)}{\partial p_2^B} = 0 \iff p_2^B\left(p_1^B; n_2^{Ae}, n_1^{Ae}\right) = \frac{p_1^B + \left(n_1^{Ae} - n_2^{Ae}\right)\alpha}{2},$$

and $\partial^2 \pi_2(p_2^A, p_2^B; \cdot)/\partial(p_2^B)^2 < 0$ for any p_2^B. One can similarly derive each platform's price strategy on side A.

An equilibrium consists of the $(p_1^{A*}, p_1^{B*}; p_2^{A*}, p_2^{B*})$ and $(n_1^{A*}, n_2^{A*}; n_1^{B*}, n_2^{B*})$ that solve

[9] This formulation follows Gabszewicz and Wauthy (2004, 2014), who adapt Katz and Shapiro's (1985) fulfilled-expectation concept to the context of a two-sided market. There is another expectation concept (see Hagiu and Hałaburda 2014 for a discussion), employed for instance by Armstrong (2006), that allows for expectation dependent on the opposite-side prices. Appendix 2.B discusses the robustness of the main results if the latter concept applies.

[10] The derivatives of $\pi_1(p_1^A, p_1^B; \cdot)$ contain no term derived from the respective other sides because (i) platform choices are made independently of the opposite-side prices and (ii) the platform incurs zero production cost. The platform therefore maximizes its side-A and side-B profits separately. In particular, this separation causes the platform to charge a positive price on each side. This discussion would also apply if the market were partially covered.

the following equation system:

$$p_1^{A*} = p_1^A\left(p_2^{A*}; n_1^{B*}, n_2^{B*}\right) \qquad p_1^{B*} = p_1^B\left(p_2^{B*}; n_1^{A*}, n_2^{A*}\right) \tag{2.3}$$

$$p_2^{A*} = p_2^A\left(p_1^{A*}; n_2^{B*}, n_1^{B*}\right) \qquad p_2^{B*} = p_2^B\left(p_1^{B*}; n_2^{A*}, n_1^{A*}\right) \tag{2.4}$$

$$n_1^{A*} = D_1^A\left(p_1^{A*}, p_2^{A*}; n_1^{B*}, n_2^{B*}\right) \qquad n_2^{A*} = D_2^A\left(p_2^{A*}, p_1^{A*}; n_2^{B*}, n_1^{B*}\right) \tag{2.5}$$

$$n_1^{B*} = D_1^B\left(p_1^{B*}, p_2^{B*}; n_1^{A*}, n_2^{A*}\right) \qquad n_2^{B*} = D_2^B\left(p_2^{B*}, p_1^{B*}; n_2^{A*}, n_1^{A*}\right). \tag{2.6}$$

In equations (2.3) and (2.4), each platform's profit-maximizing prices are consistent with the competitor's expectation of them. In equations (2.5) and (2.6), the potential users' expectations of each platform's market share are fulfilled.

The policymaker is interested in a welfare-maximizing user allocation. In this chapter, social welfare is the sum of the total benefits on sides A and B because the costs are zero. Consider first the total benefit on side B, denoted by $W^B(n_1^B, n_2^B; n_1^A, n_2^A)$.[11] The agents of higher types should join the platform with a larger number of opposite-side users. Recall that the types are uniformly distributed on a unit interval. Similar to the equilibrium analysis, hereafter, assume that

$$n_1^A > n_2^A \quad \text{and} \quad n_1^B \neq n_2^B \tag{2.7}$$

and focus on the full-coverage case.[12] Then,

$$W^B\left(n_1^B, n_2^B; n_1^A, n_2^A\right) = v + n_1^A \int_{1-\alpha-n_1^B}^{1-\alpha} \theta \mathrm{d}\theta + n_2^A \int_{-\alpha}^{n_2^B-\alpha} \theta \mathrm{d}\theta.$$

The total benefit on side A, denoted by $W^A(n_1^A, n_2^A; n_1^B, n_2^B)$, is analogous, but one should

[11] Here, the total benefit is defined as an expression of (n_1^A, n_2^A) instead of (n_1^{Ae}, n_2^{Ae}) because agents, regardless of how they form expectations, *ex post* obtain benefits evaluated with the actual market shares. Appendix 2.B formulates the benchmark problem to maximize social welfare given each agent's expectation and finds the solution to be inefficient.

[12] Appendix 2.B shows that full coverage is efficient because social welfare decreases as any agent on each side exits from the market.

note that its functional form depends on the relation between n_1^B and n_2^B:[13]

$$W^A\left(n_1^A, n_2^A; n_1^B, n_2^B\right) = v + \begin{cases} n_1^B \displaystyle\int_{1-\alpha-n_1^A}^{1-\alpha} \theta \mathrm{d}\theta + n_2^B \int_{-\alpha}^{n_2^A-\alpha} \theta \mathrm{d}\theta & \text{if } n_1^B > n_2^B \\ n_1^B \displaystyle\int_{-\alpha}^{n_1^A-\alpha} \theta \mathrm{d}\theta + n_2^B \int_{1-\alpha-n_2^A}^{1-\alpha} \theta \mathrm{d}\theta & \text{if } n_1^B < n_2^B. \end{cases}$$

Social welfare is therefore

$$W\left(n_1^A, n_2^A, n_1^B, n_2^B\right) \equiv W^A\left(n_1^A, n_2^A; n_1^B, n_2^B\right) + W^B\left(n_1^B, n_2^B; n_1^A, n_2^A\right).$$

Let $(n_1^{A**}, n_2^{A**}, n_1^{B**}, n_2^{B**})$ denote $(n_1^A, n_2^A, n_1^B, n_2^B)$ that maximizes $W(n_1^A, n_2^A, n_1^B, n_2^B)$.

2.3 Equilibrium and Its Welfare Consequence

This section discusses the equilibrium and welfare maximization. I show that each platform in equilibrium chooses a different price strategy according to the proportion of agents who incur a negative externality. The section establishes that the efficient allocation pattern also depends on the proportion and differs from the equilibrium configuration (except when $\alpha = 1/2$). Moreover, I consider the welfare implications of the results for advertising-supported media. See Appendix 2.A for proofs of the propositions.

2.3.1 Competitive Outcome

The following proposition states the equilibrium configurations.[14]

Proposition 2.1. The equilibrium under condition (2.1) is characterized as follows.

1. If $\alpha \in (0, 1/2)$, $p_1^{A*} = p_1^{B*} = (1-2\alpha)(2-\alpha)/9$, $p_2^{A*} = p_2^{B*} = (1-2\alpha)(1+\alpha)/9$, $n_1^{A*} = n_1^{B*} = (2-\alpha)/3$, and $n_2^{A*} = n_2^{B*} = (1+\alpha)/3$.

[13] If $n_1^B < n_2^B$, for instance, the side-A agents of higher types should use platform 2.

[14] There also exist equilibria in which both platforms are expected on, say, side B to attract the same number of side-A agents. The platforms face Bertrand competition on side A, and the allocation on that side is determined by the expectation formed on side B. If the side-B expectation is that $n_1^{Ae} > n_2^{Ae}$, $p_1^{A*} = p_1^{B*} = p_2^{A*} = p_2^{B*} = 0$, $0 \leq n_2^{A*} < n_1^{A*} \leq 1$, and $n_1^{B*} = n_2^{B*} = 1/2$, which arises only when $\alpha = 1/2$. If the expectation is that $n_1^{Ae} = n_2^{Ae}$, $n_1^{A*} = n_2^{A*} = n_1^{B*} = n_2^{B*} = 1/2$ and $p_1^{A*} = p_2^{A*} = p_1^{B*} = p_2^{B*} = 0$ for all α, as in Gabszewicz and Wauthy (2004, 2014).

Table 2.1: Equilibrium Configuration If $0 < \alpha < 1/2$ in Proposition 2.1

	Side A	Side B
Platform 1	n^*: high / p^*: high types: +	n^*: high / p^*: high types: +
Platform 2	n^*: low / p^*: low types: + and −	n^*: low / p^*: low types: + and −

Note: The sub- and superscripts of n^* and p^* are omitted, which applies to Table 2.2.

2. If $\alpha \in (1/2, 1)$, $p_1^{A*} = p_2^{B*} = (2\alpha - 1)(1 + \alpha)/9$, $p_2^{A*} = p_1^{B*} = (2\alpha - 1)(2 - \alpha)/9$, $n_1^{A*} = n_2^{B*} = (1 + \alpha)/3$, and $n_2^{A*} = n_1^{B*} = (2 - \alpha)/3$.

Table 2.1 summarizes the properties of the equilibrium configuration when the externality is positive for the majority of potential users. Platform 1 obtains larger market shares on both sides and attracts agents of higher types under higher participation fees. Platform 2 forms smaller networks on both sides, where agents of lower types participate and pay lower fees. Gabszewicz and Wauthy (2004, 2014) demonstrate a similar configuration in the absence of a negative externality and regard the configuration as the occurrence of vertical differentiation in opposite-side market share. The configuration when $0 < \alpha < 1/2$ in this chapter, on the other hand, exhibits horizontal differentiation due to the coexistence of positive and negative externalities. Platform 1 attracts only agents of positive (and higher) types, who choose the platform because it yields higher benefits to them. Platform 2 attracts all of the agents incurring a negative externality, who can mitigate their disutilities by choosing the platform. One can clarify this property by altering the type distribution. Suppose, in addition to the agents of types $\theta \in [-\alpha, 1 - \alpha]$, that there is a small mass of potential users whose type is $\delta < -\alpha$ on each side. Once δ decreases enough, platform 2 can improve its profit by attracting only the type-δ agents ($n_1^{A*} > n_2^{A*}$ and $n_1^{B*} > n_2^{B*}$) under participation fees higher than those of platform 1 ($p_2^{A*} > p_1^{A*}$ and $p_2^{B*} > p_1^{B*}$),[15] some of whose users incur a weak negative externality. This example supports the possibility of the platform with the lower market shares charging higher fees as a consequence of horizontal differentiation.

[15] Weyl (2010) and White and Weyl (2016) propose pricing that enables the platform to attract a desired number of agents only.

Table 2.2: Equilibrium Configuration If $1/2 < \alpha < 1$ in Proposition 2.1

	Side A	Side B
Platform 1	n^*: high / p^*: high types: $-$	n^*: low / p^*: low types: $+$ and $-$
Platform 2	n^*: low / p^*: low types: $+$ and $-$	n^*: high / p^*: high types: $-$

Table 2.2 shows the equilibrium configuration when the externality is negative for the majority of potential users. Each platform has a side with a larger market share (called its larger side) occupied totally by negative types of agents and a side with a smaller market share, which enables each platform to mitigate the negative externality incurred by the platform's users on its larger side and to charge a higher participation fee there. Each platform also makes price competition less severe because a platform obtains a larger market share on a side if the other platform attracts fewer agents on that side. Note that this configuration displays a similar user allocation to Ambrus and Argenziano's (2009). However, the characteristics of Ambrus and Argenziano's (2009) configuration are that each user on a larger side enjoys a small benefit and each platform cannot charge a high fee on its larger side. This difference occurs because this chapter allows for the coexistence of positive and negative externalities.

2.3.2 Welfare Maximization

The following proposition shows the efficient allocation pattern for each α.

Proposition 2.2. Under condition (2.7), social welfare is maximized if and only if the agents are allocated as follows.[16]

1. If $\alpha \in (0, 1/4]$, $n_1^{A**} = n_1^{B**} = 1$ and $n_2^{A**} = n_2^{B**} = 0$.
2. If $\alpha \in (1/4, 3/4)$, $n_1^{A**} = 1$, $n_1^{B**} = (3 - 4\alpha)/2$, $n_2^{A**} = 0$, and $n_2^{B**} = (4\alpha - 1)/2$.
3. If $\alpha \in [3/4, 1)$, $n_1^{A**} = 1$, $n_1^{B**} = 0$, $n_2^{A**} = 0$, and $n_2^{B**} = 1$.

Tables 2.3 and 2.4 display the efficient allocation configuration according to the proportion of agents who incur a negative externality. When the externality is positive for a

[16]This statement holds whenever $n_1^A \not< n_2^A$ because $W(1/2, 1/2, 1/2, 1/2) = \lim_{(n_1^A, n_1^B) \to (1/2, 1/2)} W(\cdot)$.

Table 2.3: Welfare-Maximizing Allocation Configuration If $0 < \alpha < 1/2$ in Proposition 2.2

	Market Share on Side A	Market Share on Side B
Platform 1	1	1 ($0 < \alpha \leq 1/4$) high ($1/4 < \alpha < 1/2$)
Platform 2	0	0 ($0 < \alpha \leq 1/4$) low ($1/4 < \alpha < 1/2$)

Table 2.4: Welfare-Maximizing Allocation Configuration If $1/2 < \alpha < 1$ in Proposition 2.2

	Market Share on Side A	Market Share on Side B
Platform 1	1	0 ($3/4 \leq \alpha < 1$) low ($1/2 < \alpha < 3/4$)
Platform 2	0	1 ($3/4 \leq \alpha < 1$) high ($0 < \alpha \leq 1/4$)

sufficiently large number of agents, social welfare is maximized if platform 1 gathers all agents on both sides because most agents have high enough types to enjoy the highest benefits from the platform. When the externality is negative for a sufficiently large number of agents, social welfare is maximized if platform 1 attracts all agents on side A but none on side B because most agents have sufficiently low types and that allocation minimizes their disutilities. When the proportion of agents who incur a negative externality is moderate, social welfare is maximized if platform 1 attracts all agents on side A and some agents on side B. The platform's efficient side-B market share decreases in the proportion of agents who incur a negative externality. The configurations discussed above significantly differ from those in Gabszewicz and Wauthy (2004, 2014) and Ambrus and Argenziano (2009), where social welfare is maximized if one platform attracts all agents on both sides. This difference arises because agents who incur a negative externality play important roles in reducing social welfare in this chapter.

2.3.3 Welfare Implications for Advertising-Supported Media

In the media industry, the flexibility of advertising plays a role in determining the value of α, the proportions of firms and consumers incurring negative externalities (see footnote 5 for how the model suits the industry). First, online platforms often charge firms advertising fees per interaction and support advertising individualized to visitor characteristics. This advertising method helps firms control to whom and how many times their advertisements appear, which might raise the proportion of matched advertisements for each consumer. Markets for online platforms thus seem to be where α is relatively low. On the other hand, print media have difficulties adopting this advertising method. It is plausible in this case that firms with limited capacities face too high demand to address and that consumers see a small proportion of matched advertisements. This property may especially matter in markets for local newspapers, where typical advertisers are small firms and consumers tend to have an interest in neiborhood firms' advertisements only. Thus, α might be relatively high in these markets.

I apply the preceding discussion to evaluate acquisitions between advertising-supported media from a policy perspective. Suppose first that α is relatively small. Under the assumption of duopoly, Proposition 2.2 states that social welfare is maximized if one platform attracts all agents on both sides. This statement may affirm an acquisition such that the acquirer integrates its own and acquired platforms.[17] Suppose next that α is relatively high.[18] The proposition states that social welfare is maximized if platform 1 gathers all agents on one side while platform 2 attracts all agents on the other side. From a welfare perspective, this result might suggest that a publisher that acquires a local newspaper maintain the acquired newspaper as it is.[19] Therefore, the flexibility of advertising and the acquirer's post-acquisition treatment of the acquired platforms deserve consideration in judging an acquisition.

[17] An example is the acquisition of YouTube by Google, in which Google integrated its own video-sharing platform, Google Video, into YouTube.

[18] See, for example, Fan (2013) for an empirical analysis of mergers among local newspapers in the U.S.

[19] For instance, Tribune Publishing is a U.S.-based media company that has acquired several local newspapers and maintained them after the acquisitions.

2.4 Conclusion

This chapter studies the equilibrium and efficient outcomes in a two-sided market where positive and negative externalities coexist on both sides. Each potential user's expectation of opposite-side market demand differentiates the platforms such that a positive externality is enhanced and a negative externality is mitigated. Social welfare is maximized only if a platform attracts all agents on one side, in which the platform's efficient market share on the other side weakly decreases as the proportion of agents who incur a negative externality grows. The equilibrium and efficient outcomes almost always differ.

Appendicies

2.A Proofs

This section contains proofs of the propositions established in the main text.

2.A.1 Proof of Proposition 2.1

Consider the equilibrium allocation and prices on side B. Equations (2.3) and (2.4) yield each platform's side-B price:

$$p_1^{B*} = \frac{\frac{p_1^{B*}+\left(n_1^{A*}-n_2^{A*}\right)\alpha}{2} + (1-\alpha)\left(n_1^{A*}-n_2^{A*}\right)}{2} \iff p_1^{B*} = \frac{(2-\alpha)}{3}\left(n_1^{A*}-n_2^{A*}\right) \tag{2.8}$$

$$p_2^{B*} = \frac{\frac{(2-\alpha)\left(n_1^{A*}-n_2^{A*}\right)}{3} + \left(n_1^{A*}-n_2^{A*}\right)\alpha}{2} = \frac{(1+\alpha)}{3}\left(n_1^{A*}-n_2^{A*}\right). \tag{2.9}$$

The price difference on side B is

$$p_1^{B*} - p_2^{B*} = \frac{(1-2\alpha)\left(n_1^{A*}-n_2^{A*}\right)}{3}.$$

Equation (2.6) yields the equilibrium side-B demand for each platform:

$$n_1^{B*} = 1-\alpha-\frac{\frac{(1-2\alpha)}{3}\left(n_1^{A*}-n_2^{A*}\right)}{n_1^{A*}-n_2^{A*}} = \frac{2-\alpha}{3} \tag{2.10}$$

$$n_2^{B*} = \frac{\frac{(1-2\alpha)}{3}\left(n_1^{A*}-n_2^{A*}\right)}{n_2^{A*}-n_2^{A*}} + \alpha = \frac{1+\alpha}{3}. \tag{2.11}$$

Therefore, $n_1^{B*} > n_2^{B*}$ if $\alpha \in (0, 1/2)$, and $n_1^{B*} < n_2^{B*}$ if $\alpha \in (1/2, 1)$. The difference in market share on side B is

$$n_1^{B*} - n_2^{B*} = \frac{1-2\alpha}{3}.$$

The equilibrium prices and side-A allocation are obtained as follows. If $\alpha \in (0, 1/2)$, the derivations of p_1^{A*}, p_2^{A*}, n_1^{A*}, and n_2^{A*} are analogous to those of expressions (2.8) to

(2.11), respectively. The equilibrium prices are

$$p_1^{B*} = \frac{2-\alpha}{3} \cdot \frac{1-2\alpha}{3} = p_1^{A*} \qquad p_2^{B*} = \frac{1+\alpha}{3} \cdot \frac{1-2\alpha}{3} = p_2^{A*}.$$

If $\alpha \in (1/2, 1)$, the same discussion applies except that (p_1^{A*}, n_1^{A*}) and (p_2^{A*}, n_2^{A*}) replace one another.

2.A.2 Proof of Proposition 2.2

This proof consists of two parts. The first part shows that the first-order conditions for the welfare-maximization problem violate one of the second-order conditions, which implies that the welfare-maximizing outcomes are corner solutions. The second part obtains the welfare-maximizing allocation under condition (2.7).

Second-Order Conditions for Welfare Maximization

Suppose that $n_1^B > n_2^B$. Note that $n_2^A = 1 - n_1^A$ and $n_2^B = 1 - n_1^B$, which implies that $\mathrm{d}n_2^A/\mathrm{d}n_1^A = \mathrm{d}n_2^B/\mathrm{d}n_1^B = -1$. I have the following:

$$\begin{aligned}
\frac{\partial W^A\left(n_1^A, n_2^A; n_1^B, n_2^B\right)}{\partial n_1^A} &= n_1^B \cdot \left(1-\alpha-n_1^A\right) - \left(n_2^A - \alpha\right) n_2^B \\
&= n_1^B \cdot \left(1-\alpha-n_1^A\right) - \left(1 - n_1^A - \alpha\right)\left(1-n_1^B\right) \\
&= n_1^A - 2n_1^A n_1^B + 2\left(1-\alpha\right) n_1^B - \left(1-\alpha\right) \\
\frac{\partial W^B\left(n_1^B, n_2^B; n_1^A, n_2^A\right)}{\partial n_1^A} &= \left(1-\alpha\right) n_1^B - \frac{\left(n_1^B\right)^2}{2} - \frac{\left(n_2^B\right)^2}{2} + \alpha n_2^B \\
&= \left(1-\alpha\right) n_1^B - \frac{\left(n_1^B\right)^2}{2} - \frac{\left(1-n_1^B\right)^2}{2} + \left(1-n_1^B\right)\alpha \\
&= -\left(n_1^B\right)^2 + 2\left(1-\alpha\right) n_1^B - \frac{1}{2} + \alpha.
\end{aligned}$$

The first-order condition with respect to n_1^A is that

$$\begin{aligned}
\frac{\partial W\left(n_1^A, n_2^A, n_1^B, n_2^B\right)}{\partial n_1^A} &= \frac{\partial}{\partial n_1^A}\left[W^A\left(n_1^A, n_2^A; n_1^B, n_2^B\right) + W^B\left(n_1^B, n_2^B; n_1^A, n_2^A\right)\right] \\
&= n_1^A - 2n_1^A n_1^B - \left(n_1^B\right)^2 + 4\left(1-\alpha\right) n_1^B + 2\alpha - \frac{3}{2} = 0.
\end{aligned}$$

Analogously, the first-order condition with respect to n_1^B is that

$$\frac{\partial W\left(n_1^A, n_2^A, n_1^B, n_2^B\right)}{\partial n_1^B} = -\left(n_1^A\right)^2 + 4\left(1-\alpha\right)n_1^A - 2n_1^A n_1^B + n_1^B + 2\alpha - \frac{3}{2} = 0.$$

Extracting the condition on side B from that on side A yields the following:

$$\begin{aligned}
& n_1^A - n_1^B - \left(n_1^B\right)^2 + \left(n_1^A\right)^2 + 4\left(1-\alpha\right)\left(n_1^B - n_1^A\right) = 0 \\
\Longleftrightarrow\ & \left(n_1^A - n_1^B\right)\left(n_1^A + n_1^B + 4\alpha - 3\right) = 0 \\
\Longleftrightarrow\ & n_1^A = n_1^B \quad \text{or} \quad n_1^A + n_1^B = 3 - 4\alpha.
\end{aligned}$$

If the latter equality is the case (which holds only if $1/4 < \alpha < 1/2$), the first-order condition with respect to n_1^A is rewritten as

$$\begin{aligned}
\frac{\partial W\left(n_1^A, n_2^A, n_1^B, n_2^B\right)}{\partial n_1^A} &= n_1^A \underbrace{-\left(n_1^A + n_1^B\right)^2 + \left(n_1^A\right)^2}_{=-2n_1^A n_1^B - \left(n_1^B\right)^2} + 4\left(1-\alpha\right)n_1^B + 2\alpha - \frac{3}{2} \\
&= n_1^A + \left[-\left(3-4\alpha\right)^2 + \left(n_1^A\right)^2\right] + 4\left(1-\alpha\right)\underbrace{\left(3 - 4\alpha - n_1^A\right)}_{=n_1^B} + 2\alpha - \frac{3}{2} \\
&= \underbrace{\left(n_1^A\right)^2 - \left(3-4\alpha\right)n_1^A + \frac{1}{2}\left(3-4\alpha\right)}_{\equiv f\left(n_1^A\right)} = 0.
\end{aligned}$$

The function $f(n_1^A)$ is minimized if

$$\frac{\mathrm{d}f\left(n_1^A\right)}{\mathrm{d}n_1^A} = 0 \Longleftrightarrow n_1^A = \frac{\left(3-4\alpha\right)}{2}.$$

The first-order condition with respect to n_1^A does not hold in this case because

$$\begin{aligned}
f\left(\frac{3-4\alpha}{2}\right) &= \frac{\left(3-4\alpha\right)^2}{4} - \frac{\left(3-4\alpha\right)^2}{2} + \frac{1}{2}\left(3-4\alpha\right) \\
&= \frac{2\left(3-4\alpha\right) - \left(3-4\alpha\right)^2}{4} \\
&= \frac{-16\alpha^2 - 3 + 16\alpha}{4} \\
&= \frac{-16\left(\alpha - \frac{1}{2}\right)^2 + 1}{4} > 0
\end{aligned}$$

as long as $1/4 < \alpha < 1/2$ (the value is close to 0 as $\alpha \to 1/4$). Therefore, $n_1^A = n_1^B \equiv n_1 \in (1/2, 1)$ if n_1^A and n_1^B solve the first-order conditions. One can derive the following single condition from the original first-order condition with respect to n_1^A multiplied by 2:

$$-6n_1^2 + 2(5 - 4\alpha) n_1 + 4\alpha - 3 = 0 \tag{2.12}$$

for all α. The second-order partial derivatives of $W(n_1^A, n_2^A, n_1^B, n_2^B)$ regarding platform 1 are

$$\begin{aligned}
\frac{\partial^2 W\left(n_1^A, n_2^A, n_1^B, n_2^B\right)}{\partial\left(n_1^A\right)^2} &= -2n_1^B + 1 < 0 \\
\frac{\partial^2 W\left(n_1^A, n_2^A, n_1^B, n_2^B\right)}{\partial\left(n_1^B\right)^2} &= -2n_1^A + 1 < 0 \\
\frac{\partial^2 W\left(n_1^A, n_2^A, n_1^B, n_2^B\right)}{\partial n_1^A \partial n_1^B} &= -2\left(n_1^A + n_1^B\right) + 4(1-\alpha),
\end{aligned}$$

which is simplified as follows:

$$\begin{aligned}
\left.\frac{\partial^2 W\left(n_1^A, n_2^A, n_1^B, n_2^B\right)}{\partial\left(n_1^A\right)^2}\right|_{n_1^A = n_1^B = n_1} &= \left.\frac{\partial^2 W\left(n_1^A, n_2^A, n_1^B, n_2^B\right)}{\partial\left(n_1^B\right)^2}\right|_{n_1^A = n_1^B = n_1} = -2n_1 + 1 \\
\left.\frac{\partial^2 W\left(n_1^A, n_2^A, n_1^B, n_2^B\right)}{\partial n_1^A \partial n_1^B}\right|_{n_1^A = n_1^B = n_1} &= -4\left[n_1 - (1-\alpha)\right].
\end{aligned}$$

The determinant of the Hessian matrix is a function of n_1:

$$\begin{aligned}
H(n_1) &\equiv \left(4n_1^2 - 4n_1 + 1\right) - \left[16n_1^2 - 32(1-\alpha) n_1 + 16(1-\alpha)^2\right] \\
&= -12n_1^2 + 4(7 - 8\alpha) n_1 - 16\alpha^2 + 32\alpha - 15.
\end{aligned}$$

Multiplying equation (2.12) by 2 and solving the equation for $H(n_1)$ yields

$$\begin{aligned}
& -12n_1^2 + 4(5 - 4\alpha) n_1 + 8\alpha - 6 = 0 \\
\iff & -12n_1^2 + 4(7 - 8\alpha) n_1 - 4(2 - 4\alpha) n_1 + \left(-16\alpha^2 + 32\alpha - 15\right) \\
& + 16\alpha^2 - 24\alpha + 9 = 0
\end{aligned}$$

$$\Longleftrightarrow H\left(n_1\right) = 8\left(1-2\alpha\right)n_1 - 16\alpha^2 + 24\alpha - 9,$$

which is linear in n_1. One can obtain the supremum of $H(n_1)$ as follows:

$$\begin{cases} \lim\limits_{n_1 \to 1} H\left(n_1\right) = -16\alpha^2 + 8\alpha - 1 = -16\left(\alpha - \dfrac{1}{4}\right)^2 \leq 0 & \text{if } 0 < \alpha < \dfrac{1}{2} \\ H\left(n_1\right) = 0 \cdot n_1 - 16 \cdot \left(\dfrac{1}{2}\right)^2 + 24 \cdot \dfrac{1}{2} - 9 = -1 < 0 & \text{if } \alpha = \dfrac{1}{2} \\ \lim\limits_{n_1 \to \frac{1}{2}} H\left(n_1\right) = -16\alpha^2 + 16\alpha - 5 = -16\left(\alpha - \dfrac{1}{2}\right)^2 - 1 < 0 & \text{if } \dfrac{1}{2} < \alpha < 1. \end{cases}$$

The determinant of the Hessian matrix cannot be strictly positive if the first-order conditions hold. There is no interior welfare-maximizing allocation such that $n_1^B > n_2^B$ for any α.

Suppose now that $n_1^B < n_2^B$. The first-order derivatives of social welfare with respect to n_2^A and n_1^B are mirror images of one another. Similarly to the case in which $n_1^B > n_2^B$,

$$\begin{aligned} \frac{\partial W^A\left(n_1^A, n_2^A; n_1^B, n_2^B\right)}{\partial n_2^A} &= n_2^A - 2n_2^A n_2^B + 2\left(1-\alpha\right)n_2^B - \left(1-\alpha\right) \\ &= 2n_2^A n_1^B - n_2^A - 2\left(1-\alpha\right)n_1^B + \left(1-\alpha\right) \\ \frac{\partial W^B\left(n_1^A, n_2^A; n_1^B, n_2^B\right)}{\partial n_2^A} &= -\frac{\partial W^B\left(n_1^A, n_2^A; n_1^B, n_2^B\right)}{\partial n_1^A} \\ &= \left(n_1^B\right)^2 - 2\left(1-\alpha\right)n_1^B - \alpha + \frac{1}{2}. \end{aligned}$$

The derivative with respect to n_2^A thus equals

$$\begin{aligned} \frac{\partial W\left(n_1^A, n_2^A, n_1^B, n_2^B\right)}{\partial n_2^A} &= \frac{\partial}{\partial n_2^A}\left[W^A\left(n_1^A, n_2^A; n_1^B, n_2^B\right) + W^B\left(n_1^B, n_2^B; n_1^A, n_2^A\right)\right] \\ &= 2n_2^A n_1^B - n_2^A + \left(n_1^B\right)^2 - 4\left(1-\alpha\right)n_1^B - 2\alpha + \frac{3}{2}. \end{aligned}$$

Regarding n_1^B, an analogous calculation is presented:

$$\frac{\partial W\left(n_1^A, n_2^A, n_1^B, n_2^B\right)}{\partial n_1^B} = \left(n_2^A\right)^2 - 4\left(1-\alpha\right)n_2^A + 2n_2^A n_1^B - n_1^B - 2\alpha + \frac{3}{2}.$$

Both derivatives correspond to the values of $-\partial W(n_1^A, n_2^A, n_1^B, n_2^B)/\partial n_1^A$ and $-\partial W(n_1^A, n_2^A,$

$n_1^B, n_2^B)/\partial n_1^B$ obtained when $n_1^B > n_2^B$, respectively, but replace n_1^A with n_2^A. The first-order conditions can thus be rewritten as analogous equalities to those in the preceding paragraph. The own-variable second-order derivatives equal those in that paragraph that are multiplied by -1 and replace n_1^A with n_2^A; thus, the own-variable second-order conditions hold (because $0 < n_2^A < 1/2$ and $0 < n_1^B < 1/2$). Moreover, the cross-variable second-order derivatives are also those in that paragraph that are multiplied by -1 and replace n_1^A with n_2^A, which implies that the determinant of the Hessian matrix is analogous. One can therefore establish the absence of an interior welfare-maximizing outcome such that $n_1^B < n_2^B$ for any α in the same way as above. I summarize below how to prove this statement. First, the first-order conditions imply that

$$n_2^A = n_1^B \quad \text{or} \quad n_2^A + n_1^B = 3 - 4\alpha,$$

but the latter equality is incompatible with the condition of n_2^A. Second, if $n_2^A = n_1^B$, the second-order condition with regard to the Hessian matrix does not hold for any α.

Welfare-Maximizing Allocation

The preceding discussion establishes that the welfare-maximization problem has a corner solution only: $n_1^{A**} = 1$ and $n_2^{A**} = 0$. Social welfare can thus be expressed as $\overline{W}(n_1^B) \equiv W(1, 0, n_1^B, 1 - n_1^B)$. If $1/4 < \alpha < 3/4$, n_1^{B**} is n_1^B derived from the first-order condition to maximize $\overline{W}(n_1^B)$:

$$\frac{\mathrm{d}\overline{W}\left(n_1^B\right)}{\mathrm{d}n_1^B} = \frac{1-2\alpha}{2} + \left(1 - \alpha - n_1^B\right) = 0 \iff n_1^B = \frac{3-4\alpha}{2},$$

which satisfies the second-order condition $(\mathrm{d}^2\overline{W}(n_1^B)/\mathrm{d}(n_1^B) = -1 < 0$ for any $n_1^B)$. Otherwise, $n_1^{B**} = 1$ if $0 < \alpha \leq 1/4$, and $n_1^{B**} = 0$ if $3/4 \leq \alpha < 1$.

2.B Discussions on the Major Assumptions

This section reviews a few major assumptions made in the main text. I relax these assumptions and examine their impacts on this chapter's main statements. For simplicity, this section focuses on the case in which conditions (2.1) and (2.7) in the main text hold.

2.B.1 Efficiency of Full Coverage

To examine the relevance of focusing on the full-coverage case in the welfare analysis, consider the welfare impacts of a deviation from the outcome in Proposition 2.2 such that an agent who has the lowest type ($\theta = -\alpha$) exits. First, I remark that the lowest-type agents on side A are assigned with platform 1 for any α in the case of welfare maximization. Suppose that $0 < \alpha \leq 1/4$, where the lowest-type agents on side B are also assigned with platform 1. Exit by an agent of the lowest type on side A reduces social welfare in

$$\underbrace{(v - \alpha)}_{\text{side } A} + \underbrace{\frac{1-2\alpha}{2}}_{\text{side } B},$$

which is positive for any strictly positive v because this decrement equals

$$v + \underbrace{\left(-2\alpha + \frac{1}{2}\right)}_{\in\left[0,\frac{1}{2}\right)} > 0.$$

The decrement of social welfare as an agent of the lowest type on side B exits is analogous. Suppose that $1/4 < \alpha < 1/2$, where the lowest-type agents on side B are assigned with platform 2. Exit by an agent of the lowest type on side A reduces social welfare in

$$\begin{aligned}\underbrace{\left(v - \alpha n_1^{B**}\right)}_{\text{side } A} + \underbrace{\frac{2(1-\alpha) - \left(n_1^{B**}\right)^2}{2}}_{\text{side } B} &= v - \left[\frac{\left(n_1^{B**}\right)^2}{2} + \alpha n_1^{B**}\right] + (1-\alpha) \\ &= v + \frac{4\alpha - 1}{8},\end{aligned}$$

which is positive for any strictly positive v because

$$0 < \frac{4\alpha - 1}{8} < \frac{1}{8}.$$

The decrement of social welfare as an agent of the lowest type on side B exits is just $v > 0$ because platform 2 obtains no market share on side A. Suppose that $1/2 < \alpha < 3/4$.[20] Notice that the lowest-type agents on each side are assigned the same as when $1/4 < \alpha < 1/2$ (i.e., platform 1 for those on side A and platform 2 for those on side B) and that the mathematical form of n_1^{B**} is also the same, which implies that social welfare decreases as an agent of the lowest type on each side exits for any strictly positive v. If $3/4 \leq \alpha < 1$, for any strictly positive v, exit by an agent of the lowest type on each side reduces social welfare in v because all agents incur no negative externality (i.e., $n_1^{A**} = 1$ but $n_1^{B**} = 0$ and $n_2^{A**} = 0$ but $n_2^{B**} = 1$).

I now discuss the welfare implications of the above result. Social welfare decreases for any strictly positive v as an agent of the lowest type exits from the market. Exit by any other agent on each side also reduces social welfare for any strictly positive v because, under the assumption that both platforms have the same intrinsic value, the agent obtains a higher benefit than that of the lowest type. Therefore, the policymaker does not have an incentive to make the market partially covered for any strictly positive v.

2.B.2 Side-Asymmetric Type Distributions

This subsection discusses how the main results are changed if the types of agents on each side are asymmetrically distributed. I maintain the type distribution on side A but modify the type distribution on side B in that $(\alpha + \epsilon)$ replaces α, where $\epsilon \in \mathbb{R}$ is an exogenous parameter such that $0 < (\alpha + \epsilon) < 1$ (i.e., $-\alpha < \epsilon < 1 - \alpha$), which means that the externality is negative for $(\alpha + \epsilon)$ potential users on side B. In sum, the main results are robust unless the type distributions on both sides substantially differ.

[20] The statement below holds if $\alpha = 1/2$ by relaxing condition (2.7) regarding the allocation on side B.

I first investigate the impact of this extension on the equilibrium configuration. Equations (2.8) to (2.11) apply to (p_1^{A*}, p_2^{A*}) and (n_1^{A*}, n_2^{A*}) with α being replaced by $(\alpha+\epsilon)$ and to (p_1^{A*}, p_2^{A*}) and (n_1^{A*}, n_2^{A*}) with no change. The equilibrium allocation on side B is

$$n_1^{B*} = \frac{2-(\alpha+\epsilon)}{3} \qquad n_2^{B*} = \frac{1+(\alpha+\epsilon)}{3},$$

which implies that

$$\begin{aligned}
n_1^{B*} > n_2^{B*} &\iff \frac{1}{2} < n_1^{B*} < 1 \\
&\iff \underbrace{-1-\alpha}_{<-\alpha} < \epsilon < \frac{1}{2} - \alpha \\
n_1^{B*} < n_2^{B*} &\iff 0 < n_1^{B*} < \frac{1}{2} \\
&\iff \frac{1}{2} - \alpha < \epsilon < \underbrace{2-\alpha}_{>1-\alpha}.
\end{aligned}$$

The equilibrium allocation on side A is unchanged; therefore, its property is the same as in the original case. The equilibrium prices on side B are

$$p_1^{B*} = \frac{[2-(\alpha+\epsilon)](1-2\alpha)}{9} \qquad p_2^{B*} = \frac{[1+(\alpha+\epsilon)](1-2\alpha)}{9},$$

which means that $p_1^{B*} > p_2^{B*}$ if $0 < \alpha < 1/2$ and $n_1^{B*} > n_2^{B*}$ and that $p_1^{B*} < p_2^{B*}$ if $1/2 < \alpha < 1$ and $n_1^{B*} < n_2^{B*}$. The properties of p_1^{A*} and p_2^{A*} are qualitatively the same as in the original case because p_1^{A*} and p_2^{A*} do not depend on ϵ if n_1^{B*} and n_2^{B*} are given. Proposition 2.1 is therefore robust to the extent that

$$\begin{cases} -\alpha < \epsilon < \frac{1}{2} - \alpha & \text{if } 0 < \alpha < \frac{1}{2} \\ \frac{1}{2} - \alpha < \epsilon < 1 - \alpha & \text{if } \frac{1}{2} < \alpha < 1. \end{cases}$$

Consider how the side-asymmetric type distributions affect welfare maximization. Appendix 2.A shows that no interior solution exists under the symmetric type distributions because the second-order conditions do not totally hold. The statements of that section are robust if α is not close to 1/4 and $|\epsilon|$ is moderate. The proof of Proposition 2.2 applies with the following modification:

$$\begin{aligned}\frac{\mathrm{d}W\left(1,0,n_1^B,1-n_1^B\right)}{\mathrm{d}n_1^B} &= \frac{1-2\alpha}{2}+\left[1-(\alpha+\epsilon)-n_1^B\right]=0\\ \Longleftrightarrow n_1^{B**} &= \frac{3-4\left(\alpha+\frac{\epsilon}{2}\right)}{2},\end{aligned}$$

where the marginal benefit yielded on side A is unchanged. The efficient allocation configuration has the same properties except that $(\alpha+\epsilon/2)$ replaces α.

2.B.3 Active Beliefs

In this subsection, the concept called "active beliefs" (Gabszewicz and Wauthy 2004) or "responsive expectations" (Hagiu and Hałaburda 2014) applies to the formation of each potential user's demand expectation. This concept allows for agents who form the rational expectations of the opposite-side market demand functions, which depend on the opposite-side prices. If the price of platform 1 increases on side A, for instance, potential users on side B expect the platform to lose some users on side A. Each platform considers this additional price effect when the platform determines its price strategy. To briefly see the impact of this expectation formation, I examine how the platforms deviate from the equilibrium price strategies in the main text and how their deviations change the equilibrium allocation.[21]

Suppose that $0<\alpha<1/2$. The threshold types are positive in the original equilibrium. By reducing the participation fees on a side, each platform can attract additional agents not only on the same side but also on the other side. This implies that the participation fees decrease on both sides. The new threshold types are positive because the price effects shrink as the threshold types approach zero. Platform 1 keeps its fees and market share higher because the platform attracts a larger number of agents. Therefore, the equilibrium configuration is qualitatively unchanged from the equilibrium one.

Suppose next that $1/2<\alpha<1$. The threshold types are negative in the original

[21] Note that how potential users form the rational expectations does not affect welfare maximization.

equilibrium. By raising the participation fees on a side, each platform loses some agents on the same side but attracts additional agents on the other side. Each platform can grow the side with a larger number of users by charging higher participation fees on the side with fewer users. This pricing also enables the platform to raise its price on the former side with the number of users kept larger than that in the original equilibrium. The new threshold types approach zero but are nonpositive because expression (2.2) implies that some agents of positive types on each side would join the platform with fewer opposite-side users under higher fees if the threshold type were strictly positive. In sum, the property of the equilibrium allocation becomes clearer, and the other part of the equilibrium configuration has qualitatively the same properties as the original one in the sense that the threshold types are negative.

2.B.4 Welfare Maximization Given Agents' Expectations

This subsection investigates social-welfare maximization when the policymaker also treats each agent's market-share expectation as given. The main text follows the work of Gabszewicz and Wauthy (2004, 2014) and assumes in the equilibrium analysis that agents form market-share expectations independently of the opposite-side prices and that their expectations are fulfilled with the realizations. The first assumption is adapted to welfare maximization by redefining social welfare as

$$\begin{aligned}&\widetilde{W}\left(n_1^A, n_2^A, n_1^B, n_2^B; n_1^{Ae}, n_2^{Ae}, n_1^{Be}, n_2^{Be}\right)\\ \equiv &W^A\left(n_1^A, n_2^A; n_1^{Be}, n_2^{Be}\right) + W^B\left(n_1^B, n_2^B; n_1^{Ae}, n_2^{Ae}\right),\end{aligned}$$

where the total benefit equals $W^A(n_1^A, n_2^A; n_1^{Be}, n_2^{Be})$ on side A and $W^B(n_1^B, n_2^B; n_1^{Ae}, n_2^{Ae})$ on side B. The second assumption is adapted by reformalizing welfare maximization as the problem to maximize $\widetilde{W}(n_1^A, n_2^A, n_1^B, n_2^B; \cdot)$ with respect to n_1^A, n_2^A, n_1^B, and n_2^B (given n_1^{Ae}, n_2^{Ae}, n_1^{Be}, and n_2^{Be}) subject to $n_1^{Ae} = n_1^{A**}$, $n_2^{Ae} = n_2^{A**}$, $n_1^{Be} = n_1^{B**}$, and $n_2^{Be} = n_2^{B**}$. Notice that the first-order derivatives of the function contain no term of the realized market shares on the respective other sides under this framework, which implies that the second-order condition with regard to the Hessian matrix does not need to be examined.

I solve the problem above. Recall that $\mathrm{d}n_2^A/\mathrm{d}n_1^A = \mathrm{d}n_2^B/\mathrm{d}n_1^B = -1$ because both sides are fully covered. Consider first the case in which $n_1^{Be} > n_2^{Be}$. The first-order derivatives of social welfare are

$$
\begin{aligned}
\frac{\partial \widetilde{W}\left(n_1^A, n_2^A, n_1^B, n_2^B; \cdot\right)}{\partial n_1^A} &= \left(1-\alpha-n_1^A\right) n_1^{Be} - \left(n_2^A - \alpha\right) n_2^{Be} \\
&\quad \left(\text{The calculation process for } \frac{\partial W^A(\cdot)}{\partial n_1^A} \text{ in Appendix 2.A applies.}\right) \\
&= n_1^A - 2n_1^A n_1^{Be} + 2(1-\alpha) n_1^{Be} - (1-\alpha) \\
&= \left(1-2n_1^{Be}\right) n_1^A - (1-\alpha)\left(1-2n_1^{Be}\right) \\
\frac{\partial \widetilde{W}\left(n_1^A, n_2^A, n_1^B, n_2^B; \cdot\right)}{\partial n_1^B} &= \left(1-\alpha-n_1^B\right) n_1^{Ae} - \left(n_2^B - \alpha\right) n_2^{Ae} \\
&\quad \left(\text{A similar calculation process to that of } \frac{\partial \widetilde{W}(\cdot)}{\partial n_1^A} \text{ applies.}\right) \\
&= \left(1-2n_1^{Ae}\right) n_1^B - (1-\alpha)\left(1-2n_1^{Ae}\right),
\end{aligned}
$$

and the second-order derivatives are

$$
\frac{\partial^2 \widetilde{W}\left(n_1^A, n_2^A, n_1^B, n_2^B; \cdot\right)}{\partial \left(n_1^A\right)^2} = 1-2n_1^{Be} \qquad \frac{\partial^2 \widetilde{W}\left(n_1^A, n_2^A, n_1^B, n_2^B; \cdot\right)}{\partial \left(n_1^B\right)^2} = 1-2n_1^{Ae}.
$$

The first-order conditions are that

$$
\begin{aligned}
\frac{\partial \widetilde{W}\left(n_1^A, n_2^A, n_1^B, n_2^B; \cdot\right)}{\partial n_1^A} = 0 &\iff n_1^A = 1-\alpha \\
\frac{\partial \widetilde{W}\left(n_1^A, n_2^A, n_1^B, n_2^B; \cdot\right)}{\partial n_1^B} = 0 &\iff n_1^B = 1-\alpha,
\end{aligned}
$$

and the second-order conditions hold. Notice that the allocations derived above can arise if and only if $0 < \alpha < 1/2$. Welfare maximization is characterized in this case by the following outcome: $n_1^{A**} = n_1^{B**} = 1-\alpha$ and $n_2^{A**} = n_2^{B**} = \alpha$. Suppose next that $n_1^{Be} < n_2^{Be}$. The first-order and second-order conditions with respect to n_1^B are unchanged from those in the preceding case. The conditions with respect to n_2^A (not n_1^A) are parallel to those with respect to n_1^A in the preceding case. Social welfare is thus maximized in the current case by the following outcome: $n_1^{A**} = n_2^{B**} = \alpha(> 1/2)$ and $n_2^{A**} = n_1^{B**} = 1 - \alpha(< 1/2)$.

An interpretation of this result is that all agents of positive types should join the platform with the larger market share on the other side and the other agents should participate in the other platform. This outcome is equivalent to what would arise if both platforms chose identical prices and each potential user joined the platform to maximize his/her payoff. The outcome differs (except when $\alpha \to 0$ or $\alpha \to 1$) from that in Proposition 2.2 because the former one abstracts the cross-side welfare impacts of each agent's participation in a particular platform. Social welfare is therefore enhanced the most only in the case of Proposition 2.2.

Chapter 3

Content Provision, Advertising, and Capacity-Constrained Platforms

3.1 Introduction

The media industry comprises two-sided markets in which a medium behaves as a platform between third-party firms and consumers by two different ways. The first way is to allow third-party firms to provide their contents for consumers through the platform. For instance, TV channels often sign contracts with external firms called "production companies" and broadcast their contents. Third-party contents entertain consumers and thus help the platform obtain a market share on the consumer side, although the platform needs to make payments to the contracting firms. The second way is to allow firms to advertise their products to consumers in spaces designated by the platform as advertising slots. The platform can earn additional profits from advertising firms. However, selling advertising slots does not necessarily help the platform obtain a market share on the consumer side because advertisements themselves do not usually aim to purely entertain consumers and may result in consumer disutility.

The above feature of the media industry raises questions if such a platform is a broadcast channel or live streaming service. This type of platform broadcasts (or streams) specific third-party contents and specific advertisements at specific periods of time, following the timetable that the platform has beforehand prepared. The platform in this sense faces a capacity constraint on the total amount of slots (i.e., time) that it can allocate for either third-party content provision or advertising. The platform is thus concerned not just with the number of content-providing or advertising firms but also with the proportion of third-party contents and advertisements that the platform broadcasts (or streams) to consumers. Which does each platform place higher priority on third-party contents or advertisements, and how does the platform's decision making affects its competitior's managerial policy and the consumer allocation? Moreover, how should each platform's policy be evaluated from a perspective of social welfare?

This chapter studies duopolistic competition between platforms that are concerned with their proportions of third-party contents and advertisements. To model this type of competition, I develop the following one-shot game of platform competition. There are two different groups of potential platform users in the model: firms and consumers. Firms are payoff-maximizing agents who choose, for each platform, whether to provide their contents and whether to show their advertisements in designated slots. Consumers behave as potential end users who determine which platform they use so that the respective payoff (utility) functions can be maximized, assumed not to subscribe to multiple platforms. Third-party contents and advertisements play different roles. First, consumers obtain higher benefits by seeing an amount of third-party contents than the same amount of advertisements. However, platforms face tradeoffs such that they need to pay compensations to content-providing firms on the one hand and can earn revenues by selling their slots to advertising firms on the other hand. The supply side of the market consists of two competing platforms. Each platform faces a capacity constraint such that the sum of the content and advertisement amounts does not exceed the number of slots prepared by the platform's self. Platforms thus choose (i) content-provision, advertising, and subscription prices and (ii) content-advertisement proportions such that the respective profits can be maximized. This chapter aims to describe each platform's managerial policy on its content-advertisement proportion by conducting an equilibrium analysis and to discuss the efficiency of the above competition through a welfare analysis.

The equilibrium analysis, first, obtains two symmetric configurations such that both platforms offer all of their slots to firms of the same type when consumers are sensitive to platforms' intrinsic product characteristics. The first configuration is that each platform signs contracts with as many content-providing firms as possible and broadcasts (or streams) their contents with no advertisement, which tends to occur if the total impact of the indirect network externalities exerted with regard to third-party content provision is large or content-providing firms incur low technological costs. The second configuration is that each platform sells all of its slots to advertising firms and does not broadcast (or stream) any third-party content, which is more likely to occur as the total impact of the indirect network externalities exerted with regard to advertising is larger or firms incur lower technological advertising costs.

Also, I establish that there exists an equilibrium in which one platform fills all of its slots to third-party contents but the other platform sells all of its slots to advertising firms if consumers are not sufficiently sensitive to how platforms differentiate their stand-alone services (e.g., their original contents). This equilibrium configuration describes a special type of vertical differentiation by which each platform attracts content-providing or advertising firms. First, the platform with third-party contents exhibits a higher quality on the consumer side than that with advertisements because consumers strictly prefer third-party contents than advertisements, and the former platform charges higher subscription fees. However, the platform with third-party contents does not necessarily earn a higher profit than its rival. I should point out the relation between this result and the findings from Zennyo (2016), who studies duopolistic competion between vertically differentiated two-sided platforms. Both Zennyo (2016) and this chapter show the possibility that the higher-quality platform may earn a lower profit. The difference is that this chapter endogenizes vertical differentiation: platforms differentiate themselves by which type of firms they attract inside the model.

The welfare analysis finds that the equilibrium and efficient configurations have similar properties but do not equal one another. A symmetric equilibrium and the corresponding efficient outcome coincide in the firm and consumer allocations but differ in the condition to arise; thus, a symmetric equilibrium may maximize social welfare although whether the equilibrium is really efficient depends on the market structure. An asymmetric equilibrium and the corresponding efficient outcome yield the same firm allocation but differ in the consumer allocation, which implies that social welfare is not maximized if an asymmetric equilibrium arises.

Such inefficiency occurs because equilibrium and welfare-maximizing outcomes describe different situations. In equilibrium, each platform competes with its rival and maximizes its own profit, not concerned with any other player's payoff. In welfare maximization, the policymaker takes the impacts of a firm's or consumer's platform adoption into account and maximizes social welfare as a single decision maker.

The rest of this chapter proceeds as follows. The next section reviews the related literature. Section 3.3 develops the model in this chapter, characterizes a competitive outcome as an equilibrium, and formulates welfare maximization. Section 3.4 derives and analyzes an equilibrium outcome. Section 3.5 solves welfare maximization and discusses the welfare implications of the derived equilibrium outcome. This chapter concludes in section 3.6. Appendicies 3.A and 3.B prove the propositions established in this chapter.

3.2 Related Literature

The most important feature of this chapter is that I develop a model to describe a platform's choice between third-party contents and advertisements. I begin by reviewing existing theoretical and empirical papers on advertising-supported media regarded as two-sided platforms to examine how those papers address the industry in which both third-party content provision and advertising matter. This section then proceeds to this chapter's standpoint in the literature that develops general models of two-sided (or network-good) markets.

The majority of existing theoretical and empirical papers on advertising-supported media in the context of platform markets do not explicitly allow for third-party content provision. The research topics in such theoretical papers include the roles of advertising and market two-sidedness in market outcome (e.g., Anderson and Coate 2005; Armstrong 2006), enrichment of subscription behavior (e.g., Reisinger 2012), product differentiation in original content (e.g., Gabszewicz, Laussel, and Sonnac 2001, 2004; Peitz and Valletti 2008; von Ehrlich and Greiner 2013), and targeted advertising (e.g., Kox, Straathof, and Zwart 2017).[1] The research topics in such empirical papers include evaluation of the indirect network externalities and the two-sided pricing strategies in a particular industry (e.g., Rysman 2004; Kaiser and Wright 2006; Sokullu 2016a, 2016b), of mergers (e.g., Chandra and Collard-Wexler 2009; Fan 2013), and of targeted advertising (e.g., Chandra 2009).[2] Although some of them (e.g., Anderson and Coate 2005; Armstrong 2006; Peitz and Valletti 2008; Reisinger 2012) partly adopt similar frameworks to my model, the aforementioned works significantly differ from this chapter in that the former do not incorporate to their models content-provision behavior exhibited by third-party firms.

Several papers in the theoretical literature investigate the role of third-party content provision in the media industry. Weeds (2014) and D'Annunzio (2017) study duopolistic competition between advertising-supported TV channels such that a third-party firm produces a premium content as a upstream monopolist and then the two channels, given whether they can broadcast the premium content, attract potential subscribers. Carroni and Paolini (2017, 2019) model the decision made by a monopolistic freemium platform that offers contents provided by third-party firms with or without advertisements, focusing on the platform's choice between freemium and subscription-only services (2017) or its investment to raise the quality of its premium service (2019). Those papers have similar research motivations to this chapter in that third-party firms do not only exhibit advertising behavior but also endogenously make content-provision choices as independent players (decision makers). However, it is an important difference that the above

[1] Another related paper is Hagiu and Jullien (2011). They consider the situation in which a monopolistic search engine offers both a content and an advertisement. However, their paper assumes that the search engine produces the content by itself.

[2] The most related paper among those works is Fan (2013). She develops a structural (empirical) model of the advertising-supported newspaper industry in which each publisher determines the product characteristics of its newspaper(s) and each potential subscriber cares about those characteristics, which means that she explicitly allows for media contents in her model. However, she does not incorporate content-providing firms as players to her model.

papers abstract a platform's capacity constraint on the total amount of slots for third-party platform participation and thus do not model the platform's strategic choice on its content-advertisement proportion.

Moreover, Weyl (2010) develops a general framework of multi-sided markets and proposes an interpretation of market multi-sidedness such that (i) all sides of a multi-sided market belong to either of two side groups, (ii) agents do not exert an externality on the own side or any other side in the same group, and (iii) agents exhibit discrimination toward each side of the other side group. This chapter applies to Weyl's (2010) concept with simplifications such that (i) the firm side consists of two groups, (ii) the consumer side is a single side, and (iii) the distributions of firm and consumer characteristics are specified, which enable one to obtain equilibrium outcomes explicitly and discuss the interpretations of those outcomes. Existing papers that allow for platform markets with more than two agent groups (e.g., Chen, Zenou, and Zhou 2018; Tan and Zhou 2019) are related in this sense, although those papers could not fully explain each platform's strategic choice between third-party contents and advertisements without any modification because they do not specialize in the media industry but build general models.[3]

I should, lastly, remark that several theoretical and empirical papers incorporate a similar type of two-sided market to this chapter. The firm side is considered by consumers to consist of multiple agent groups in papers that allow for differentiation among third-party firms' products (e.g., Galeotti and Moraga-González 2009; Bresnahan, Orsini, and Yin 2015). The consumer side consists of multiple agent groups from firms' perspectives in papers on targeted advertising (see the second paragraph for examples). Those existing works, to the best of my knowledge, again, do not consider each platform's capacity constraint on the total number of slots for third-party firms, which is the central research focus of this chapter.

[3]Chen, Zenou, and Zhou (2018) describe the situation in which each agent discriminates the others by whether they do not only use the same platform but also belong to the same network in terms of social networks.

3.3 Model

There exists a two-sided market in which two platforms, labeled 1 and 2, offer slots for content provision or advertising to third-party firms (simply called “firms” hereafter). Each platform faces a capacity constraint such that the sum of the content and advertisement amounts on the platform cannot exceed the total number of slots offered by the platform, which is normalized to one thoughout this chapter. I place a few assumptions on the model to simplify the game and the welfare-maximization problem. First, both platforms maximize the respective profits if they fill all of their slots with third-party contents or advertisements (see footnote 17, which appears in Appendix 3.A, for the relevance of this assumption). Second, the policymaker does not intervene in platform entry but allows both platforms to supply their services, and social welfare in this case is maximized if the two platforms offer all of their slots to firms (see footnote 14, which appears in the last subsection, for the relevance of the latter assumption). This section formulates each player's bahavior and an equilibrium in the first two subsections, and social welfare and welfare maximization in the last subsection.

3.3.1 Firms and Consumers

The market consists of firm and consumer sides. The firm side is occupied by a unit mass of economic agents, called “firms,” who are potential content providers or potential advertisers. Firms behave as multihomers: they regard each platform as a monopolist and join any platform that brings nonnegative payoffs to them. For simplicity, third-party content provision and advertising are symmetrically formulated and mutually independent. The consumer side is occupied by a unit mass of economic agents, called “consumers,” each of whom chooses a single platform to maximize his/her payoff (i.e., behaves as a singlehomer). I adapt the formulation of the demand sides introduced in Choi (2007) and applied in Rasch and Wenzel (2013, 2014)[4] to this chapter's context — the coex-

[4]Although several papers use this formulation, there are technical differences. The demand sides in this chapter are closer to those in Choi (2007) in that firms incur arbitrary participation costs and to those in Rasch and Wenzel (2013, 2014) in that consumers explicitly obtain benefits from the intrinsic (stand-alone) services of platforms. Hagiu and Hałaburda (2014) also adopt qualitatively the same approach but do not explicitly define each firm's or consumer's payoff function.

istence of multi-type firms and the existence of capacity constraints — except that all content-providing and advertising firms have respective identical payoff functions.[5]

Begin by formulating third-party content provision. Suppose that an arbitrary firm is planning to participate in platform 1. This firm obtains a payoff of

$$\pi_1^C\left(p_1^C, n_1\right) \equiv \alpha^C n_1 - p_1^C - \gamma^C$$

if the platform attracts $n_1 \in [0,1]$ consumers, which can be interpreted as follows. The firm obtains a (firm-common) benefit of $\alpha^C \in \mathbb{R}_+$ per consumer.[6] The firm pays a lump-sum fee of $p_1^C \in \mathbb{R}$ to the platform. Lastly, the firm incurs a homogeneous participation cost of $\gamma^C \in \mathbb{R}_{++}$, where I presume that $\alpha^C \leq \gamma^C$ to guarantee each platform to announce a (weakly) negative content-provision price to content-providing firms in equilibrium. Notice that the first term simply equals the product of α^C and n_1, which implicitly assumes that the firm provides a single content for all consumers on the platform and engages in no intraplatform competition. The firm therefore makes a content-provision decision such that

$$\begin{cases} \text{the firm joins platform 1} & \text{if } \pi_1^C\left(p_1^C, n_1\right) \geq 0 \\ \text{the firm does not join platform 1} & \text{if } \pi_1^C\left(p_1^C, n_1\right) < 0. \end{cases} \tag{3.1}$$

Expression (3.1) represents not only the firm's demand but also the market demand for third-party content provision on platform 1 because all firms have identical payoff functions and do not compete with each other. The market demand for third-party content provision on platform 2 can be formulated in the same way, where the definitions and interpretations of $\pi_2^C(p_2^C, n_2)$, $p_2^C \in \mathbb{R}$, and $n_2 \in [0,1]$ are analogous to those of $\pi_1^C(\cdot)$,

[5]The formulation of the firm side is also related in this sense to those in Hagiu (2006), Armstrong and Wright (2007), and Reisinger (2012). The differences from the firm-side formulation in Hagiu (2006) are that he allows firms to be charged two-part tariffs and considers firms to make decisions before (all or some) consumers. The differences from the firm-side formulation in Armstrong and Wright (2007) are that firms endogenously choose which to multihome or singlehome and obtain zero (dis)utility from the stand-alone services of platforms. The difference from the firm-side formulation in Reisinger (2012) is that firms incur zero technological cost.

[6]In the broadcast industry, for instance, consumers who watch (or listen to) one of a production company's programs possibly purchase a DVD of that program in the future or become interested in that company's other programs.

p_1^C, and n_1, respectively.

Platforms may also sell advertising slots on the firm side. Let $\alpha^A \in \mathbb{R}_{++}$, $\gamma^A \in \mathbb{R}_+$, $p_1^A \in \mathbb{R}$, $p_2^A \in \mathbb{R}$, and

$$\pi_1^A\left(p_1^A, n_1\right) \equiv \alpha^A n_1 - p_1^A - \gamma^A$$

denote the respective counterparts for α^C, γ^C, p_1^C, p_2^C, and $\pi_1^C(\cdot)$, and define $\pi_2^A(p_2^A, n_2)$ analogously to $\pi_1^A(\cdot)$. Each firm's advertising decision and the market demand for advertising on each platform are formulated similarly to that firm's content-provision decision and the market demand for third-party content provision on that platform, respectively. The parameters α^A and γ^A are specified such that each platform in equilibrium announces a (weakly) positive advertising price. I also assume (i) that $\alpha^C < \alpha^A$ to make the impact of the network externality larger on advertising firms than on content-providing firms and (ii) that $\gamma^C > \gamma^A$ because content provision is in general likely to be more costly than advertising.

Each platform faces a physical constraint on the total number of slots offered to firms. I focus on the situation in which each platform can maximize its profit by selling all of its slots to firms of either type, as mentioned at the beginning. To guarantee this situation to happen, first, platforms are supposed to announce moderate prices such that all firms have incentives to participate. Moreover, the parameters in the model are assumed to take such values that platforms can in equilibrium earn nonnegative profits and social welfare is maximized (as long as both platforms enter the market) if they fill all of their slots. Each platform thus controls its content-advertisement proportion. Platform 1 opens $q_1 \in [0, 1]$ slots for third-party content provision and $1 - q_1$ slots for advertising. Define $q_2 \in [0, 1]$ analogously.

The consumer side is formulated as a unit interval $[0, 1]$ of the Hotelling type. Platforms 1 and 2 are located at the positions of 0 and 1, respectively, and the consumers are uniformly located in that interval. The payoff function for a consumer at the location $x \in [0, 1]$ is characterized such that he/she obtains a payoff of

$$u_1\left(s_1, q_1; x\right) \equiv v_0 + \beta^C q_1 + \left(1 - q_1\right)\beta^A - s_1 - tx$$

from platform 1 and

$$u_2(s_2, q_2; x) \equiv v_0 + \beta^C q_2 + (1 - q_2)\beta^A - s_2 - (1 - x)t$$

from platform 2, which can be interpreted as follows.[7] When using a particular platform, this consumer obtains a platform-common benefit of $v_0 \in \mathbb{R}_{++}$ from a stand-alone service of that platform. The consumer receives a payoff of $\beta^C \in \mathbb{R}_{++}$ per content provided through the platform and $\beta^A \in \mathbb{R}$ per advertisement shown on the platform, where $-\alpha^A < \beta^A < \beta^C$.[8] The consumer pays a subscription fee that the platform charges, denoted by $s_1 \in \mathbb{R}$ for platform 1 and $s_2 \in \mathbb{R}$ for platform 2. The last component of the consumer's payoff is the linear transportation cost of the Hotelling type, where $t \in \mathbb{R}_{++}$ denotes the cost parameter. On the consumer side, the market demand for each platform can be determined as follows. The variable v_0 is assumed throughout this chapter to take a high enough value that (i) all consumers can in equilibrium obtain nonnegative payoffs by subscribing to either platform and (ii) social welfare is maximized if all consumers participate. Each consumer selects the (single) platform that maximizes the payoff for himself/herself, where the consumer chooses platform 1 if he/she is indifferent between both platforms. The resulting consumer allocation is characterized by the location of x such that

$$\begin{aligned} & u_1(\cdot; x) = u_2(\cdot; x) \\ \iff & x = \frac{1}{2} + \frac{(q_1 - q_2)\beta^C + [(1 - q_1) - (1 - q_2)]\beta^A - (s_1 - s_2)}{2t} \equiv \widetilde{x}(s_1, s_2; q_1, q_2). \end{aligned}$$

If $\widetilde{x}(\cdot) \in [0, 1]$, $n_1 = \widetilde{x}(\cdot)$ because all consumers such that $0 \leq x \leq \widetilde{x}(\cdot)$ choose platform 1. Although it is possible to happen that $\widetilde{x}(\cdot) < 0$ or $\widetilde{x}(\cdot) > 1$, this chapter focuses on

[7] If appropriate, I keep the notations $1 - q_1$ and $1 - q_2$ in mathematical expressions to show the impacts of content-providing and advertising firms.

[8] This definition of β^A enables one to avoid a discussion on whether consumers actually obtain positive or negative utility from advertisements and to distinguish the role of advertisements from that of third-party contents. The condition that $-\alpha^A < \beta^A$ is a necessity for the marginal welfare of advertising in a platform to be nonnegative given the number of content providers on that platform. The condition that $\beta^A < \beta^C$ means that consumers receive strictly lower payoffs from advertisements than from third-party contents.

the case of an interior consumer allocation. The assumption of full coverage yields the relation that $n_2 = 1 - n_1$. Let $n_1(s_1, s_2; q_1, q_2)$ and $n_2(s_2, s_1; q_2, q_1)$ denote the market demand functions for platforms 1 and 2, respectively, on the consumer side.

3.3.2 Platform Competition

Platforms 1 and 2 determine the content-provision prices, advertising prices, subscription fees, and content-advertisement proportions. This chapter adapts the formulations of platform competition and equilibrium from the competition stage in Anderson and Coate (2005) and Peitz and Valetti (2008) in the following sense.[9] First, each platform derives its firm-side prices from the inverse firm demand for that platform and chooses a quantity on the firm side. Second, an equilibrium is thus constituted by a bundle of prices and quantities.

I begin with platform 1's profit maximization. The platform earns a profit of

$$\Pi_1\left(p_1^C, p_1^A, s_1, q_1; n_1\right) \equiv p_1^C q_1 + (1 - q_1)\, p_1^A + s_1 n_1$$

which is interpreted as follows. The platform attracts q_1 content-providing firms, $1 - q_1$ advertising firms, and n_1 consumers. It announces the participation prices of p_1^C, p_1^A, and s_1 to the respective agents. Accordingly, platform 1 maximizes $\Pi_1(\cdot)$ with respect to p_1^C, p_1^A, s_1, and q_1 under condition (3.1) for all content-providing firms, an analogous condition for all advertising firms, and the condition that $n_1 = n_1(\cdot)$. Let $p_1^C(s_2, q_2)$, $p_1^A(s_2, q_2)$, $s_1(s_2, q_2)$, and $q_1(q_2, s_2)$ denote a bundle of p_1^C, p_1^A, s_1, and q_1, respectively, that solves the problem, which depends on neither p_2^C nor p_2^A because firms are multihoming agents and platform 2 directly chooses q_2 (as discussed below). I address platform 1's profit maximization by the following multiple steps.[10]

1. Maximize the platform's profit with respect to the firm-side prices of its service given

[9] Those papers allow for pre-competition business practices at the first stage, such as entry (Anderson and Coate 2005) and product differentiation (Peitz and Valetti 2008), which are abstracted from this chapter.

[10] The technical differences from Anderson and Coate (2005) and Peitz and Valetti (2008) are that this chapter regards the firm-side prices of the platform as part of its strategy and solves its profit maximization as a set of multiple problems.

(s_1, q_1) and (s_2, q_2). The content-provision and advertising prices of the platform equal

$$p_1^C = \begin{cases} \alpha^C n_1 - \gamma^C & \text{if } 0 < q_1 \leq 1 \\ 0 & \text{if } q_1 = 0 \end{cases} \qquad p_1^A = \begin{cases} \alpha^A n_1 - \gamma^A & \text{if } 0 \leq q_1 < 1 \\ 0 & \text{if } q_1 = 1, \end{cases}$$

respectively. These prices are determined such that both types of firms obtain zero payoff from the platform.[11] If the platform attracts no content-providing or advertising firm, the content-provision or advertising price, respectively, is fixed to zero. Let $\overline{\Pi}_1(s_1, q_1; s_2, q_2)$ denote the profit function for the platform that incorporates the p_1^C and p_1^A obtained above.

2. Choose s_1 to maximize $\overline{\Pi}_1(\cdot)$ with q_1 being fixed, and define $\overline{\overline{\Pi}}_1(q_1; s_2, q_2)$ as the function obtained by incorporating the s_1 derived here to $\overline{\Pi}_1(\cdot)$.
3. Maximize $\overline{\overline{\Pi}}_1(\cdot)$ with respect to q_1, and let $\overline{\overline{\overline{\Pi}}}_1(s_2, q_2)$ denote the function the function obtained by incorporating the q_1 derived here to $\overline{\overline{\Pi}}_1(\cdot)$.
4. Lastly, obtain $p_1^C(\cdot)$, $p_1^A(\cdot)$, $s_1(\cdot)$, and $q_1(\cdot)$.

All of the detailed conditions to solve the platform's profit maximization appear in Appendix 3.A.

One can analogously formulate and solve platform 2's profit maximization. Define $\Pi_2(p_2^C, p_2^A, s_2, q_2; n_2)$, $\overline{\Pi}_2(s_2, q_2; s_1, q_1)$, $\overline{\overline{\Pi}}_2(s_2; s_1, q_1)$, and $\overline{\overline{\overline{\Pi}}}_2(s_1, q_1)$ similarly. Let $p_2^C(s_1, q_1)$, $p_2^A(s_1, q_1)$, $s_2(s_1, q_1)$, and $q_2(q_1, s_1)$ denote a bundle of p_2^C, p_2^A, s_2, and q_2, respectively, that solves the problem.

This chapter formalizes the result of the platform competition described above as a pure-strategy Nash equilibrium. An equilibrium equals $(p_1^C, p_1^A, s_1, q_1; p_2^C, p_2^A, s_2, q_2; n_1, n_2)$

[11] These prices follow the pricing concept introduced in Armstrong (2006) and further discussed in White and Weyl (2016). In Armstrong's (2006) words, the platform chooses the prices to offer zero payoff to firms regardless of the platform's market share on the consumer side. Under White and Weyl's (2016) frameworks, the platform chooses the prices to cause firms to take dominant strategies against the number of consumers who use the platform. In both cases, the market demand for the platform on the firm side is determined independently of the platform's consumer-side market share.

such that

$$
\begin{aligned}
s_1 &= s_1(s_2, q_2) & s_2 &= s_2(s_1, q_1) \\
p_1^C &= p_1^C(s_2, q_2) & p_2^C &= p_2^C(s_1, q_1) \\
p_1^A &= p_1^A(s_2, q_2) & p_2^A &= p_2^A(s_1, q_1) \\
q_1 &= q_1(q_2, s_2) & q_2 &= q_2(q_1, s_1) \\
n_1 &= n_1(s_1, s_2; q_1, q_2) & n_2 &= n_2(s_1, s_2; q_1, q_2)
\end{aligned}
$$

and is denoted by a pair of $(p_1^{C*}, p_1^{A*}, s_1^*, q_1^*; p_2^{C*}, p_2^{A*}, s_2^*, q_2^*; n_1^*, n_2^*)$, where the values of p_1^{C*}, p_1^{A*}, p_2^{C*}, and p_2^{A*} can be easily derived by using the value of n_1^* or n_2^*. Define Π_1^* and Π_2^* as the equilibrium profits earned by platforms 1 and 2, respectively.

3.3.3 Welfare Maximization

Social welfare in my model is the sum of total surplus on each side, which equals the difference between all agents' payoffs and payments on that side. Recall that the following assumptions apply to the welfare analysis. First, the policymaker does not intervene in platform entry. Second, each platform faces a capacity constraint such that the sum of the content and advertisement amounts on the platform cannot exceed one. Third, social welfare is maximized if both platforms allocate all of their slots to firms (i.e., participation by firms yields high enough benefits or is not too costly) and each consumer uses either platform (i.e., each platform's intrinsic value is large enough). These assumptions imply in the welfare analysis that (q_1, q_2) represents the allocation on the firm side and that $n_2 = 1 - n_1$. Social welfare in this chapter can therefore be defined as

$$W(q_1, q_2, n_1) \equiv S^C(q_1, q_2; n_1) + S^A(q_1, q_2; n_1) + V(n_1; q_1, q_2),$$

which consists of total surplus with regard to content provision, advertising, and subscription, respectively. Total surplus on the firm side equals

$$S^C(\cdot) \equiv \left(\alpha^C n_1 - \gamma^C\right) q_1 + \left[(1-n_1)\,\alpha^C - \gamma^C\right] q_2$$

regarding content provision and

$$S^A(\cdot) \equiv \left(\alpha^A n_1 - \gamma^A\right)(1-q_1) + \left[(1-n_1)\,\alpha^A - \gamma^A\right](1-q_2)$$

regarding advertising. Total surplus on the consumer side is[12]

$$\begin{aligned} V(\cdot) \equiv v_0 - \left[\int_0^{n_1} x \mathrm{d}x + \int_{n_1}^1 (1-x)\,\mathrm{d}x\right] t \\ + \left[q_1 n_1 + (1-n_1)\,q_2\right]\beta^C + \left[(1-q_1)\,n_1 + (1-q_2)(1-n_1)\right]\beta^A. \end{aligned}$$

Welfare maximization is the problem to (i) choose a bundle of q_1, q_2, and n_1 that maximizes $W(\cdot)$ and (ii) choose n_2 such that $n_2 = 1 - n_1$. This problem can simply be solved by two steps.[13] The first step consists of the following substeps, analogous to profit maximization.

1. Maximize $W(\cdot)$ with respect to q_1 and q_2 given n_1, and then define $\overline{W}(n_1)$ as the function that incorporates the q_1 and q_2 obtained here to $W(\cdot)$.
2. Maximize $\overline{W}(\cdot)$ with respect to n_1.
3. Obtain the optimal values of q_1, q_2, and n_1, and derive the optimal value of n_2 from the relation that $n_2 = 1 - n_1$.

As found in Appendix 3.B, however, this step yields multiple corner solution candidates and cannot immediately select the actual solution. This issue happens because, for in-

[12]Total surplus on the consumer side in this chapter maintains that in Rasch and Wenzel (2013, 2014) but allows for the situation in which multi-type firms exist and content-advertisement proportions matter.

[13]Anderson and Coate (2005) also define welfare maximization as multiple steps although the steps *per se* differ from those in this chapter in that Anderson and Coate (2005) (i) derive the efficient allocation of consumers first and the efficient amount of advertisements second and (ii) discuss social welfare with one and two platform(s).

stance, the marginal welfare of q_1 is constant in the own variable:[14]

$$\begin{aligned}\frac{\partial W(\cdot)}{\partial q_1} &= \left(\alpha^C n_1 - \gamma^C\right) - \left(\alpha^A n_1 - \gamma^A\right) + \left(\beta^C - \beta^A\right) n_1 \\ &= \left[\left(\beta^C - \beta^A\right) - \left(\alpha^A - \alpha^C\right)\right] n_1 - \left(\gamma^C - \gamma^A\right).\end{aligned}$$

Thus, as the second step, I calculate the increment of social welfare when the market allocation is changed from a particular solution candidate to another one and select the efficient outcome. Define q_1^{**}, q_2^{**}, n_1^{**}, and n_2^{**} as the values of q_1, q_2, n_1, and n_2, respectively, derived in the above steps.

The rest of this chapter assumes that

$$\begin{aligned}\frac{\partial W(q_1, q_2, 1)}{\partial q_1} &= \left[\left(\beta^C - \beta^A\right) - \left(\alpha^A - \alpha^C\right)\right] - \left(\gamma^C - \gamma^A\right) \\ &= \left[\left(\alpha^C + \beta^C\right) - \gamma^C\right] - \left[\left(\alpha^A + \beta^A\right) - \gamma^A\right] > 0. \qquad (3.2)\end{aligned}$$

This assumption means that social welfare increases as platform 1 attracts all consumers and marginally raises its content-advertisement proportion (i.e., the relative number of content providers). If the assumption does not hold, social welfare cannot be maximized once the platform sells its slots to content-providing firms. Condition (3.2) is therefore a necessity for third-party content provision to enhance social welfare.

3.4 Equilibrium

This section obtains and analyzes the equilibrium outcome. The first subsection considers a symmetric equilibrium such that both platforms offer all of their slots to firms of the same type (i.e., either content-providing or advertising firms depending on the market

[14]Note that

$$\left.\frac{\partial W(\cdot)}{\partial q_1}\right|_{1-q_1 \text{ is fixed}} = \left(\alpha^C n_1 - \gamma^C\right) + \beta^C n_1 = \left(\alpha^C + \beta^C\right) n_1 - \gamma^C$$

and one could analogously obtain $\partial W(\cdot)/\partial(1-q_1)|_{q_1 \text{ is fixed}}$ if platform 1 did not face a capacity constraint with regard to the firm side. The platform should therefore attract as many content-providing and/or advertising firms as possible if the above marginal welfare were positive. This analysis supports the robustness of the presumption that social welfare can be maximized if each platform chooses a content-advertisement proportion.

structure), which exists if and only if consumers incur sufficiently high transportation costs. The second subsection focuses on the case of moderate transportation costs, in which there exists an asymmetric equilibrium such that one platform allocates all of its slots to content-providing firms and the other platform sells all of its slots to advertising firms. For a proof of each proposition in the section, see Appendix 3.A.

3.4.1 Symmetric Equilibrium

First, I establish in the below proposition that there can arise an equilibrium in which both platforms offer all of their slots to the same type of firms and when such an equilibrium exists.

Proposition 3.1. Under the conditions that

$$t > \frac{\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)}{3} \neq \gamma^C - \gamma^A \tag{3.3}$$

$$t > \frac{\left[\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)\right]^2}{6\left|\left[\left(\alpha^C + \beta^C\right) - 3\gamma^C\right] - \left[\left(\alpha^A + \beta^A\right) - 3\gamma^A\right]\right|} \tag{3.4}$$

$$t > 2\gamma^C, \tag{3.5}$$

there exists a unique equilibrium that depends on the sign of

$$\left(\frac{\alpha^C + \beta^C}{3} - \gamma^C\right) - \left(\frac{\alpha^A + \beta^A}{3} - \gamma^A\right). \tag{3.6}$$

If the above expression is strictly positive, the equilibrium outcome and profits are characterized such that

$$\begin{aligned} p_1^{C*} = p_2^{C*} = \frac{\alpha^C}{2} - \gamma^C \qquad & p_1^{A*} = p_2^{A*} = 0 \\ s_1^* = s_2^* = t - \alpha^C \qquad & q_1^* = q_2^* = 1 \\ n_1^* = n_2^* = \frac{1}{2} \qquad & \Pi_1^* = \Pi_2^* = \frac{t}{2} - \gamma^C. \end{aligned}$$

If expression (3.6) is strictly negative, the equilibrium outcome and profits are characterized such that

$$\begin{aligned} p_1^{C*} = p_2^{C*} &= 0 \qquad & p_1^{A*} = p_2^{A*} &= \frac{\alpha^A}{2} - \gamma^A \\ s_1^* = s_2^* &= t - \alpha^A \qquad & q_1^* = q_2^* &= 0 \\ n_1^* = n_2^* &= \frac{1}{2} \qquad & \Pi_1^* = \Pi_2^* &= \frac{t}{2} - \gamma^A. \end{aligned}$$

This proposition identifies two symmetric equilibrium configurations. The first is that both platforms fill all of their slots with third-party contents. Each platform in this case signs contracts with several content-providing firms and pays compensations to those firms, and broadcasts (or streams) the contracting firms' contents with no advertisement and charges consumers subscription fees. In this sense, the configuration describes the situation that might be observed in media markets for premium broadcasting (or streaming) services. The second equilibrium configuration is that both platforms sell all of their slots to advertising firms. This configuration can be interpreted such that each platform mainly broadcasts (or streams) its original content such as news programs, has its profit supported by advertising firms, and determines whether and how much it charges consumers subscription fees by considering each consumer's sensitivity to its original content (t) and each firm's valuation of advertising in the platform ($-\alpha^A$).[15] The configuration might, for instance, suit media markets for community-based and specialized channels, which do not necessarily aim to broadcast (or stream) various types of contents but differentiate their original contents. Below, some major properties of these equilibrium configurations are discussed.

At the beginning, I remark why each platform in equilibrium does not choose an option to attract both content-providing and advertising firms. Consider platform 1's behavior.

[15] The above proposition shows that platforms possibly choose strictly negative subscription prices. Some justifications are that platforms may exempt subscription fees for a few months and that platforms may offer giveaways to their subscribers, both of which can be observed in the media industry. One might also interpret this possibility as a technical limitation that abstracts the situation in which platforms do not generally offer monetary benefits to consumers from the model. For instance, Armstrong and Wright (2007) allow for each platform's constraint that it cannot choose a strictly negative price on each side, which is an approach that could apply to the consumer side in this chapter as well. This chapter, however, does not incorporate such a price constraint but focuses its equilibrium analysis on realized content-advertisement proportions.

First, the platform chooses a subscription price based on the first-order condition. This price consists of (i) the market-power term á la Hotelling and (ii) the discount term derived from the indirect network externality that consumers exert on the firm side, as in Armstrong (2006). The (negative) profit that the platform obtains from the latter term cancels out the (positive) profit that the platform earns derived from the indirect network externality exerted on the firm side, which is called "profit neutrality" by Peitz and Valletti (2008). Second, if this pricing applies on the consumer side, the profit function that the platform faces at the step of choosing q_1 equals

$$\overline{\overline{\Pi}}_1(\cdot) = 2\left[\overline{n}_1(s_2; q_1, q_2)\right]^2 t - \gamma^C q_1 - (1-q_1)\gamma^A,$$

where $\overline{n}_1(\cdot)$ denotes the expected number of the platform's subscribers under the above pricing given platform 2's strategy (see Appendix 3.A). Given $\overline{n}_1(\cdot)$, $\overline{\overline{\Pi}}_1(\cdot)$ can be treated as a linear function of q_1. Appendix 3.A establishes that $\overline{n}_1(\cdot)$ is also a linear function of q_1 with the coefficient equal to $[(\alpha^C + \beta^C) - (\alpha^A + \beta^A)]/4t$. The second-order derivative of the above profit function with respect to q_1 never takes a strictly negative value. Therefore, platform 1 in equilibrium chooses a corner content-advertisement proportion. One can reach an analogous consequence in the case of platform 2's behavior. The above logic again applies in the next subsection, where I discuss an equilibrium in which each platform fills all of its slots to a different type of firms.

Next, if condition (3.4) holds, Proposition 3.1 shows that which symmetric equilibrium arises depends on the sign of expression (3.6). First, under condition (3.3) (which contains technical assumptions to guarantee an interior consumer allocation and avoid zero division), each platform has an incentive to assign all of their slots to third-party content provision only if expression (3.6) is strictly positive. The platform is more likely to choose attracting content-providing firms as the marginal welfare impacts of the indirect network externality on each side of the market regarding third-party content provision are higher (α^C or β^C is higher), which helps the platform by filling all of its slots with third-party contents to attract consumers. The platform tends to possess a similar incentive if content-providing firms incur low technological costs (γ^C is low), where it can pay low compensations paid to those firms. Second, under condition (3.3), each platform has an incentive to assign all of their slots to advertising only if expression (3.6) is strictly negative. The platform is more likely to choose selling its slots to advertising firms as the marginal welfare impacts of the indirect network externality on each side regarding advertising are higher (α^A or β^A is higher) or advertising firms incur lower technological costs (γ^A is lower), which helps the platform raise its advertising fees although discouraging it from attracting consumers. In particular, the platform views selling all of its slots to advertising firms as a non-negligible option because those firms receive a higher indirect network externality ($\alpha^A > \alpha^C$) and incur lower technological costs ($\gamma^A < \gamma^C$) than content-providing firms.

Proposition 3.1 and the proposition to appear in the next subsection establish that the equilibrium configurations in the former proposition do not arise if condition (3.4) is violated in the strict sense. The left-hand side of the condition equals t, which implies that the condition is more likely to hold as consumers incur higher transportation costs. Platform 1's marginal profit of attracting consumers, for instance, increases as t rises; thus, the platform tends to choose $q_1 = 1$ to keep the number of its subscribers if $q_2 = 1$. The difference between platform 1's market shares on the consumer side when $q_1 = 1$ and when $q_1 = 0$ shrinks as t increases; thus, the platform is unlikely to choose $q_1 = 0$ if $q_2 = 0$. Moreover, the right-hand side of condition (3.4) contains a denominator obtained by multiplying the absolute value of expression (3.6) with 3. Condition (3.4) tends to hold if the right-hand side of condition (3.4) has a high denominator (i.e., in the cases of (i) high α^C, high β^C, or low γ^C and (ii) high α^A, high β^A, or low γ^A), which is related to whether a symmetric equilibrium exists with regard to expression (3.6). Conversely, condition (3.4) is less likely to hold as the right-hand side of condition (3.4) has a lower denominator.

3.4.2 Asymmetric Equilibria

Suppose that each platform offers all of its slots with a different type of firms. I find that this type of allocation arises in equilibrium if consumers are not sufficiently sensitive to product differentiation between platforms. The below proposition establishes the value and properties of such an equilibrium.

Proposition 3.2. Suppose that conditions (3.5) and (3.3) holds but condition (3.4) is violated in the strict sense. There only exists a pair of (i) an asymmetric equilibrium such that

$$
\begin{aligned}
&p_1^{C*} = \frac{3t + \left[\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)\right]}{6t}\alpha^C - \gamma^C \qquad p_1^{A*} = 0 \\
&p_2^{C*} = 0 \qquad p_2^{A*} = \frac{3t - \left[\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)\right]}{6t}\alpha^A - \gamma^A \\
&s_1^* = t + \frac{\left(-2\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)}{3} \\
&s_2^* = t - \frac{\left(\alpha^C + \beta^C\right) - \left(-2\alpha^A + \beta^A\right)}{3} \\
&q_1^* = 1 \qquad n_1^* = \frac{1}{2} + \frac{\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)}{6t} \\
&q_2^* = 0 \qquad n_2^* = \frac{1}{2} - \frac{\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)}{6t},
\end{aligned}
$$

in which each platform's profit equals

$$
\begin{aligned}
\Pi_1^* &= \frac{\left\{3t + \left[\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)\right]\right\}^2}{18t} - \gamma^C \\
\Pi_2^* &= \frac{\left\{3t - \left[\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)\right]\right\}^2}{18t} - \gamma^A,
\end{aligned}
$$

and (ii) an analogous equilibrium such that $(q_1^*, q_2^*) = (0, 1)$. The platform with third-party contents in equilibrium announces a higher subscription price than that with advertisements in any case but earns a strictly higher profit if and only if

$$
\left[\frac{2\left(\alpha^C + \beta^C\right)}{3} - \gamma^C\right] - \left[\frac{2\left(\alpha^A + \beta^A\right)}{3} - \gamma^A\right] > 0. \tag{3.7}
$$

This proposition shows a pair of asymmetric equilibria such that one platform fills all of its slots with third-party contents and the other platform sells all of its slots to advertising firms. Each platform's policies on its content-advertisement proportion and subscription price in this case can similarly be interpreted to that in the case of Proposition 3.1. The platform with third-party contents in equilibrium broadcasts (or streams) the contents licensed with charge by firms and earns a profit only from consumers. The other platform in equilibrium runs an advertising-supported broadcasting (or streaming) service that provides its content for consumers with or without charge (see footnote 15 for a discussion on subscription prices). This equilibrium configuration has both the same properties as those discussed in the case of Proposition 3.1 and features that are unique in the case of Proposition 3.2. The discussions in the case of Proposition 3.1 apply to the interpretation of subscription prices, the reason for each platform choosing a corner content-advertisement proportion, and the roles of expressions (3.3) and (3.4); thus, see the preceding subsection. The reminder of the current subsection addresses the meaning of the configuration in the case of Proposition 3.2.

One can find from Proposition 3.2 that the two platforms in equilibrium differentiate their services by which they specialize in third-party contents or advertisements. First, I again examine each platform's strategy from a perspective of product differentiation. The platform with third-party contents offers higher benefits from the indirect network externality (because $\beta^C > \beta^A$) to consumers than that with advertisements. The former platform in this case always announces a higher subscription price than the latter. Choosing third-party contents or advertisements in this sense plays a role for each platform as vertical product differentiation from a consumer-side perspective. The platform with third-party contents can attract a larger number of consumers than that with advertisements, which is consistent with the consequence of standard vertical product differentiation in the sense that the platform bringing higher benefits (except for transportation costs) obtains a larger market share. However, it is a notable difference that the platform with third-party contents earns a strictly *lower* profit if condition (3.7) is violated in the strict sense. Expression (3.7) is more likely to hold as the marginal welfare impacts of the indirect network externality on each side regarding third-party content provision are higher (α^C or β^C is higher) or content-providing firms incur lower technological costs (γ^C is lower), which enhances the value of third-party content provision adjusted by that of advertising. The expression is more likely to be violated as the marginal welfare impacts of the indirect network externality on each side regarding advertising are higher (α^A or β^A is higher) and as advertising firms incur lower technological costs (γ^A is lower), which enhances the value of advertising adjusted by that of third-party content provision.

Zennyo (2016) models a duopolistic two-sided market with vertical product differentiation between platforms in which high quality means high intrinsic values for consumers but high technological costs for firms. Notably, Zennyo (2016) shows the possibility that the lower-quality platform may also earn a *higher* profit in equilibrium, which has a close relation with Proposition 3.2 of this chapter although there are substantial differences from a modeling viewpoint.[16] The platform with third-party contents offers higher benefits to consumers than that with advertisements, which means as mentioned above that the former platform provides a higher-quality service than the latter from a consumer-side perspective. The firms that participate in the former platform incur higher technological costs than those that belong to the latter (i.e., $\gamma^C > \gamma^A$); thus, both platforms are also vertically differentiated on the firm side in Zennyo's (2016) sense. This chapter in this context establishes that the platform with third-party contents cannot necessarily earn a higher profit although it is always labeled the higher-quality platform. However, the reasons for the existence of such an equilibrium differ. Zennyo (2016) assumes that the quality of each platform has been intrinsicly determined outside his model and describes the situation in which both platforms choose prices accordingly to their qualities. This chapter formulates both platforms symmetrically and shows that each platform chooses one between the higher-quality and lower-quality services to provide for consumers. In this sense, the chapter endogenizes each platform's behavior of vertical product differentiation by allowing for the platform's choice of a content-advertisement proportion.

[16] Zennyo (2016) follows an approach applied in the literature on vertical product differentiation such that agents are heterogeneous in sensitivity to quality, analyzes a two-stage game such that platforms choose firm prices at the first stage and consumer prices at the second stage, and importantly does not allow for the coexistence of third-party content provision and advertising or capacity constraints with regard to slots offered to firms.

3.5 Welfare Maximization and Its Implications

This section addresses social-welfare maximization and discusses the efficiency of the competitive outcome. The first subsection analyzes the situation in which both platforms should choose symmetric corner content-advertisement proporations, which occurs in the case of high transportation costs and corresponds to that stated in Proposition 3.1. The second subsection examines an asymmetric efficient outcome such that each platform allocates all of its slots to firms of a different type, which arises in the case of low transportation costs and is related to that established in Proposition 3.2. The last subsection has a brief discussion on the welfare implications of the respective corresponding equilibrium configurations. Appendix 3.B contains the detailed processes to establish the propositions in this section.

3.5.1 Symmetric Efficient Outcome

Below, I state that social welfare is maximized with a symmetric allocation on each side if consumers incur high transportation costs and establish the formal conditions for such a welfare-maximization pattern to arise.

Proposition 3.3. Under expression (3.2) and the conditions that

$$\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)<t\leq 4\min\left\{\left(\alpha^A+\beta^A\right)-2\gamma^A,\left(\alpha^C+\beta^C\right)-2\gamma^C\right\} \tag{3.8}$$

$$t>\frac{\left[\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)\right]^2}{2\left|\left[\left(\alpha^C+\beta^C\right)-2\gamma^C\right]-\left[\left(\alpha^A+\beta^A\right)-2\gamma^A\right]\right|} \tag{3.9}$$

$$\left(\frac{\alpha^C+\beta^C}{2}-\gamma^C\right)-\left(\frac{\alpha^A+\beta^A}{2}-\gamma^A\right)\neq 0, \tag{3.10}$$

there exists a unique efficient outcome such that

$$\begin{cases}(q_1^{**},q_2^{**};n_1^{**},n_2^{**})=\left(1,1;\frac{1}{2},\frac{1}{2}\right) & \text{if } \left(\frac{\alpha^C+\beta^C}{2}-\gamma^C\right)-\left(\frac{\alpha^A+\beta^A}{2}-\gamma^A\right)>0\\ (q_1^{**},q_2^{**};n_1^{**},n_2^{**})=\left(0,0;\frac{1}{2},\frac{1}{2}\right) & \text{if } \left(\frac{\alpha^C+\beta^C}{2}-\gamma^C\right)-\left(\frac{\alpha^A+\beta^A}{2}-\gamma^A\right)<0.\end{cases}$$

This proposition shows two efficient allocation patterns such that both platforms allocate all of their slots to firms of the same type, which arise in the case of high transportation costs incurred on the consumer side. These patterns can be analogously interpreted to those established in Proposition 3.1. The first pattern says that both platforms should attract as many content-providing firms as possible and broadcast (or stream) their contents with no advertisement. The second pattern suggests that each platform focus on its original content only and provide an advertising-supported broadcasting (or streaming) service for consumers. The equilibrium and efficient configurations coincide in terms of resulting allocations. Expressions (3.8) to (3.10) guarantee the interiority of the efficient consumer allocation and the nonnegativity of social welfare, determine which allocation pattern is efficient, and avoid zero division in condition (3.9), respectively, yielding implications analogous to the corresponding conditions stated in the equilibrium analysis but quantitatively different because the welfare analysis considers the policymaker to coordinate the allocations of firms and consumers accordingly (see the last subsection for the roles of these differences).

I remark why social welfare is maximized if each platform chooses a corner content-advertisement proportion. The preceding section explains the reason for the existence of an equilibrium in which both platforms choose corner content-advertisement proportions such that each platform announces a subscription price causing “profit neutrality” (Peitz and Valletti 2008) and then cannot maximize its profit with any interior content-advertising proportion. On the other hand, the reason for the efficiency of the case in which each platform chooses a corner content-advertisement proportion is a consequence of the following technical calculation. Social welfare in the case of a strictly interior content-advertisement proportion for a particular platform is enhanced the most if and only if the other platform chooses a corner content-advertisement proportion. If there exists a platform with a corner content-advertisement proportion, however, social welfare can be at least improved (not necessarily maximized) by changing the content-advertisement proportions for both platforms such that $q_1 = q_2$. Hence, it is a necessity for welfare maximization that both platforms choose corner content-advertisement proportions. This logic applies to Proposition 3.4 as well.

3.5.2 Asymmetric Efficient Outcome

Consider next social-welfare maximization attained if consumers incur relatively low transportation costs. Under the presumption that two platforms enter the market and do not exit, one can in this case obtain a pair of efficient outcomes that correspond to the asymmetric equilibrium pair. The following proposition summarizes the result of welfare maximization in the case.

Proposition 3.4. If expressions (3.2), (3.8), and (3.10) hold but condition (3.9) is violated in the strict sense, there only exists a pair of an efficient outcome such that

$$(q_1^{**}, q_2^{**}) = (1, 0) \qquad n_1^{**} = \frac{1}{2} + \frac{\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)}{2t}$$
$$n_2^{**} = \frac{1}{2} - \frac{\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)}{2t}$$

and such an analogous outcome that $(q_1^{**}, q_2^{**}) = (0, 1)$.

This proposition states a pair of efficient outcomes such that each platform allocates all of its slots for a different type of firm-side participation, which maximizes social welfare if consumers incur relatively low transportation costs. One can interpret these outcomes analogously to those in the case of Proposition 3.2. A platform, not both, should attract as many content-providing firms as possible and broadcast (or stream) their contents without any advertisement. The other platform should sell its slots to as many advertising firms as possible and focus on providing its original content for consumers. The configurations in the cases of Propositions 3.2 and 3.4, however, differ from one another in resulting consumer allocation, which implies that no equilibrium in the former proposition maximizes social welfare in any case (see the next subsection for details). The properties of the efficient outcomes stated in Propositions 3.3 and 3.4 are analogous except for the resulting market allocation; thus, see the preceding subsection.

3.5.3 Welfare Implications of the Equilibrium Outcome(s)

Suppose, first, that an equilibrium in the case of Proposition 3.1 arises as a competitive outcome. One can say that such an equilibrium may be efficient because Proposition 3.3 states an efficient allocation pattern that corresponds to the equilibrium. Nevertheless, it depends on the following two conditions whether social welfare is really maximized in this case. The first condition is that the parameters satisfy both expressions (3.4) and (3.9), which means that consumers are relatively sensitive to each platform's product characteristics. If expression (3.4) holds but expression (3.9) is violated in the strict sense, an equilibrium in the case of Proposition 3.1 arises although the combination of Propositions 3.3 and 3.4 implies that no symmetricoutcome can be efficient. If the former expression is violated in the strict sense but the latter holds, an equilibrium in the case of Proposition 3.2 arises although Proposition 3.3 implies that a symmetric outcome maximizes social welfare. The second condition is, under the assumption that the first condition holds, that both expression (3.6) and the left-hand side of expression (3.10) have the same sign. If the equilibrium in which $(q_1^*, q_2^*) = (1, 1)$ arises, the relation that

$$\begin{aligned}(0 <) \left(\frac{\alpha^C + \beta^C}{3} - \gamma^C\right) - \left(\frac{\alpha^A + \beta^A}{3} - \gamma^A\right)\\ < \left(\frac{\alpha^C + \beta^C}{2} - \gamma^C\right) - \left(\frac{\alpha^A + \beta^A}{2} - \gamma^A\right)\end{aligned}$$

implies that the second condition is satisfied. If the equilibrium in which $(q_1^*, q_2^*) = (1, 1)$ arises, however, social welfare cannot maximized if the bundle of the parameters takes a value such that

$$\begin{aligned}&\left(\frac{\alpha^C + \beta^C}{3} - \gamma^C\right) - \left(\frac{\alpha^A + \beta^A}{3} - \gamma^A\right) < 0\\ \text{and} \quad &\left(\frac{\alpha^C + \beta^C}{2} - \gamma^C\right) - \left(\frac{\alpha^A + \beta^A}{2} - \gamma^A\right) > 0.\end{aligned}$$

Next, consider the situation in which an equilibrium that Proposition 3.2 establishes is realized. Social welfare in this case can never be maximized because, although the firm allocations are equal, the consumer allocations stated in Propositions 3.2 and 3.4

differ. The reason for the occurrence of this difference is that platforms and the policymaker approach consumers in different ways. Platforms do not only make choices between broadcasting (or streaming) third-party contents and advertisements but also decide whether and how much they charge consumers subscription fees. Each platform competes for consumers and thus possesses an incentive to control its subscription price so that the platform can attract them. Consumers in equilibrium are thus not sufficiently sensitive to which each platform broadcasts (or streams) in the sense that the market-share difference on the consumer side is relatively small:

$$2 \cdot \frac{\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)}{6t} = \frac{\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)}{3t}.$$

On the other hand, welfare maximization is formulated as an optimization problem with a single decision maker in which the firm and consumer allocations directly matter. The policymaker obtains an efficient market allocation by coordinating the welfare effects of allocating a firm or consumer to a particular platform so that social welfare can be maximized. As a result, consumers should be allocated such that the market-share difference equals

$$2 \cdot \frac{\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)}{2t} = \frac{\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)}{t}.$$

3.6 Conclusion

This chapter studies duopolistic competition between two-sided platforms that may attract content-providing and advertising firms as well as consumers. Each platform is considered to face a capacity constraint on the number of slots assigned by that platform to firms; thus, the platform determines its proportion of third-party contents and advertisements. Under the above setting, I conduct an equilibrium analysis to investigate each platform's managerial policy especially on the firm side and a welfare analysis to discuss the welfare implications of that managerial policy.

The equilibrium analysis establishes that platforms choose corner content-advertising proportions. If consumers are sensitive to each platform's intrinsic product characteristics, there exists a symmetric equilibrium in which both platforms allocate all of their slots to firms of the same type (i.e., either content-providing or advertising firms) accordingly to the market structure and obtain equal market shares on the consumer side. Otherwise, there exists an equilibrium in which one platform fills all of its slots to third-party contents and the other sells all of its slots to advertising firms. The features of this configuration are that each platform is vertically differentiated by which type of firms the platform attracts and that the platform with third-party contents does not always earn a higher profit than its rival. These features are related to the findings from Zennyo (2016), who discusses duopolistic competion between vertically differentiated platforms and points out the possibility that the lower-quality platform earns a higher profit, but different in that this chapter endogenizes the process of vertical platform differentiation based on the firm allocation.

The welfare analysis shows that the equilibrium and efficient configurations have similar features in terms of the allocation on the firm side but do never coincide. A symmetric equilibrium can be efficient but does not always maximize social welfare because the conditions for such an equilibrium to exist differ from those for the corresponding allocation to solve welfare maximization. An equilibrium of vertical differentiation is inefficient in the sense that the equilibrium market-share difference on the consumer side is smaller than the efficient level. One can interpret these inefficiency patterns as the difference that (i) two competing platforms make their optimal decisions as well as consumers in equilibrium and (ii) the policymaker is a single decision maker directly concerned with the welfare impacts of each firm's or consumer's platform adoption in welfare maximization.

I should remark two major remaining problems that may affect the interpretation of this chapter's result. The first problem is that the model formulates the firm side in a sufficiently simple way from both perspectives of platforms' and firms' decision making, which (i) simplifies the structure of a profit function through the nature called "profit neutrality" in Peitz and Valletti (2008) and (ii) excludes the possibility of a platform facing no capacity constraint. The second problem is that the model focuses on the case of subscription fees charged, which limits the scope of my analysis in the sense that each platform is then likely to attract firms of both types because the platform does not face such a simplified profit function if it chooses zero consumer price (Peitz and Valletti 2008).

Few theoretical papers analyze two-sided markets in which both third-party content provision and advertising matter. This chapter regards as its primary mission a discussion on competition such that platforms have opportunities to attract content-providing and advertising firms under their capacity constraints as a primary mission; thus, it should be considered a way of stylization that the above problems remain. In this sense, this chapter can be understood as an earlier step of research in this field.

Appendicies

3.A Details on the Equilibrium Analysis (Propositions 3.1 and 3.2)

This section consists of three subsections that as a whole prove the propositions established in section 3.4. The first subsection shows each platform's optimal behavior given its rival's strategy, after which the second subsection derives an equilibrium and the corresponding pair of profits. The last subsection investigates the properties of an asymmetric equilibrium stated in Proposition 3.2. Throughout the section, I assume that

$$\left(\frac{\alpha^C + \beta^C}{3} - \gamma^C\right) - \left(\frac{\alpha^A + \beta^A}{3} - \gamma^A\right) \neq 0,$$

which appears as the second half of expression (3.3) in the main text.

3.A.1 Profit Maximization

This subsection investigates platform behavior. The below discussion focuses on solving platform 1's profit maximization. The findings apply to platform 2's problem because both platforms are formulated analogously; thus, I omit the details of platform 2's profit maximization.

Because section 3.3 obtains the pair of p_1^C and p_1^A to maximize platform 1's profit, I begin with s_1. The first-order and second-order conditions apply. The first-order condition is that

$$\begin{aligned} \frac{\partial \overline{\Pi}_1(\cdot)}{\partial s_1} &= -\frac{\alpha^C}{2t} q_1 - \frac{\alpha^A}{2t}(1 - q_1) - \frac{1}{2t} s_1 + \widetilde{x}(\cdot) = 0 \\ \Longleftrightarrow s_1 &= 2t\widetilde{x}(\cdot) - \alpha^C q_1 - (1 - q_1)\alpha^A, \end{aligned}$$

which implicitly yields $s_1(\cdot)$. The second-order condition is constituted by a single inequality and holds for any s_1:

$$\frac{\partial^2 \overline{\Pi}_1(\cdot)}{\partial s_1^2} = -\frac{1}{2t} - \frac{1}{2t} = -\frac{1}{t} < 0.$$

The platform expects, given q_1 and (s_2, q_2), that it obtains a market share of

$$\begin{aligned}
\widetilde{x}(\cdot) &= \frac{1}{2} + \frac{(q_1 - q_2)\beta^C + [(1-q_1) - (1-q_2)]\beta^A}{2t} \\
&\quad - \frac{\left[2t\widetilde{x}(\cdot) - \alpha^C q_1 - (1-q_1)\alpha^A\right] - s_2}{2t} \\
\Longleftrightarrow \widetilde{x}(\cdot) &= \frac{1}{4} + \frac{\left[\left(\alpha^C + \beta^C\right) q_1 + \left(\alpha^A + \beta^A\right)(1-q_1)\right] - \left[\beta^C q_2 + (1-q_2)\beta^A - s_2\right]}{4t} \\
&\equiv \overline{n}_1(q_1; s_2, q_2)
\end{aligned}$$

on the consumer side, whose first-order derivative with respect to q_1 equals

$$\frac{\partial \overline{n}_1(\cdot)}{\partial q_1} = \frac{\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)}{4t}.$$

The platform thus charges a subscription fee of

$$s_1 = 2t\overline{n}_1(\cdot) - \alpha^C q_1 - (1-q_1)\alpha^A.$$

Platform 1 earns a profit of

$$\begin{aligned}
\overline{\overline{\Pi}}_1(\cdot) &= \left[\alpha^C \overline{n}_1(\cdot) q_1 - \gamma^C q_1\right] + \left[\alpha^A \overline{n}_1(\cdot)(1-q_1) - (1-q_1)\gamma^A\right] \\
&\quad + \left\{2\left[\overline{n}_1(\cdot)\right]^2 t - \alpha^C \overline{n}_1(\cdot) q_1 - \alpha^A \overline{n}_1(\cdot)(1-q_1)\right\} \\
&= 2\left[\overline{n}_1(\cdot)\right]^2 t - \gamma^C q_1 - (1-q_1)\gamma^A
\end{aligned}$$

and maximizes it with respect to q_1. I examine the first-order and second-order derivatives. The first-order derivative with respect to q_1 equals

$$\begin{aligned}
\frac{\partial \overline{\overline{\Pi}}_1(\cdot)}{\partial q_1} &= 4t\overline{n}_1(\cdot) \frac{\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)}{4t} - \left(\gamma^C - \gamma^A\right) \\
&= \left[\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)\right] \overline{n}_1(\cdot) - \left(\gamma^C - \gamma^A\right).
\end{aligned}$$

One can obtain the second-order derivative as follows:

$$\frac{\partial^2 \overline{\overline{\Pi}}_1(\cdot)}{\partial q_1^2} = \frac{\left[\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)\right]^2}{4t} > 0.$$

The platform thus chooses either $q_1 = 0$ or $q_1 = 1$ to maximize its profit.[17]

The platform's reaction correspondence and profit can now be characterized. First, I discuss $s_1(\cdot)$, $q_1(\cdot)$, and $\overline{\overline{\overline{\Pi}}}_1(\cdot)$. The platform may choose $q_1(\cdot) = 0$, in which case

$$\begin{aligned} s_1(\cdot) &= 2t\overline{n}_1(0; s_2, q_2) - \alpha^A \\ \overline{\overline{\overline{\Pi}}}_1(\cdot) &= 2\left[\overline{n}_1(0; s_2, q_2)\right]^2 t - \gamma^A. \end{aligned}$$

The platform may also choose $q_1(\cdot) = 1$, in which case

$$\begin{aligned} s_1(\cdot) &= 2t\overline{n}_1(1; s_2, q_2) - \alpha^C \\ \overline{\overline{\overline{\Pi}}}_1(\cdot) &= 2\left[\overline{n}_1(1; s_2, q_2)\right]^2 t - \gamma^C. \end{aligned}$$

Second, $p_1^C(\cdot)$ and $p_1^A(\cdot)$ are derived as in section 3.3.

3.A.2 Equilibrium Derivation

First, recall that platform 2 in equilibrium takes an analogous strategy to platform 1. Platform 2 thus chooses either $q_2 = 0$ or $q_2 = 1$ as its optimal content-advertisement proportion and

$$s_2(\cdot) = 2t\overline{n}_2(q_2; s_1, q_1) - \alpha^C q_2 - (1 - q_2)\alpha^A$$

[17]One can establish the robustness of the presumption that each platform chooses a content-advertisement proportion in this chapter's context: the corresponding derivatives would equal

$$\begin{aligned} \left.\frac{\partial \overline{\overline{\Pi}}_1(\cdot)}{\partial q_1}\right|_{(1-q_1)\text{ is fixed}} &= 4t\overline{n}_1(\cdot)\frac{\alpha^C + \beta^C}{4t} - \gamma^C = \left(\alpha^C + \beta^C\right)\overline{n}_1(\cdot) - \gamma^C \\ \left.\frac{\partial^2 \overline{\overline{\Pi}}_1(\cdot)}{\partial q_1^2}\right|_{(1-q_1)\text{ is fixed}} &= \frac{\left(\alpha^C + \beta^C\right)^2}{4t} > 0 \end{aligned}$$

with regard to q_1 and be analogously calculated with regard to $1 - q_1$ if platform 1 did not face a capacity constraint regarding the firm side.

$$= \begin{cases} 2t\overline{n}_2(1;\cdot) - \alpha^C & \text{if } q_2 = 1 \\ 2t\overline{n}_2(0;\cdot) - \alpha^A & \text{if } q_2 = 0 \end{cases}$$

as its optimal subscription price, where $\overline{n}_2(\cdot)$ is the platform's counterpart for $\overline{n}_1(\cdot)$, resulting in a profit of

$$\begin{aligned} \overline{\overline{\Pi}}_2(\cdot) &= 2\left[\overline{n}_2(\cdot)\right]^2 t - \gamma^C q_2 - (1-q_2)\gamma^A \\ &= \begin{cases} 2\left[\overline{n}_2(1;\cdot)\right]^2 t - \gamma^C & \text{if } q_2 = 1 \\ 2\left[\overline{n}_2(0;\cdot)\right]^2 t - \gamma^A & \text{if } q_2 = 0 \end{cases} \end{aligned}$$

for the platform. The number of consumers who choose each platform given (q_1, q_2) equals

$$\begin{aligned} \overline{n}_1\left[q_1; s_2(\cdot), q_2\right] &= \frac{1}{4} + \frac{\left(\alpha^C + \beta^C\right) q_1 + \left(\alpha^A + \beta^A\right)(1-q_1)}{4t} \\ &\quad - \frac{\beta^C q_2 + (1-q_2)\beta^A - \left\{2t\overline{n}_2\left[q_2; s_1(\cdot), q_1\right] - \alpha^C q_2 - (1-q_2)\alpha^A\right\}}{4t} \\ \Longleftrightarrow n_1 &= \frac{1}{2} + \frac{\left(\alpha^C + \beta^C\right)(q_1 - q_2) + \left(\alpha^A + \beta^A\right)\left[(1-q_1) - (1-q_2)\right]}{6t} \equiv \overline{\overline{n}}_1(q_1, q_2) \\ n_2 &= 1 - \overline{\overline{n}}_1(q_1, q_2) \equiv \overline{\overline{n}}_2(q_2, q_1), \end{aligned}$$

which can be rewritten as

$$\begin{aligned} \overline{\overline{n}}_1(\cdot) &= \frac{1}{2} + \frac{\left(\alpha^C + \beta^C\right)(q_1 - q_2) + \left(\alpha^A + \beta^A\right)(-q_1 + q_2)}{6t} \\ &= \frac{1}{2} + \frac{\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)}{6t}(q_1 - q_2) \\ \overline{\overline{n}}_2(\cdot) &= \frac{1}{2} + \frac{\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)}{6t}(q_2 - q_1), \end{aligned}$$

where the assumption of full coverage applies.

Suppose at the beginning of the process to derive an equilibrium that platform 2 chooses $q_2 = 0$. I first consider the case in which $q_1 = 0$. The number of consumers who choose each platform is symmetric:

$$\overline{\overline{n}}_1(0,0)=\overline{\overline{n}}_2(0,0)=\frac{1}{2}.$$

Both platforms take symmetric strategies such that

$$\begin{aligned} p_1^C = p_2^C &= 0 \qquad & p_1^A = p_2^A &= \frac{\alpha^A}{2}-\gamma^A \\ s_1 = s_2 = t-\alpha^A & & q_1 = q_2 &= 0 \end{aligned}$$

in this case. They then earn symmetric profits of

$$\overline{\overline{\overline{\Pi}}}_1(\cdot)=\overline{\overline{\overline{\Pi}}}_2(\cdot)=\frac{t}{2}-\gamma^A.$$

Next, consider the case in which $q_1=1$. The two platforms obtain market shares of

$$\begin{aligned} \overline{\overline{n}}_1(1,0) &= \frac{1}{2}+\frac{\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)}{6t} \\ \overline{\overline{n}}_2(0,1) &= \frac{1}{2}-\frac{\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)}{6t} \end{aligned}$$

on the consumer side, where the allocation of consumers is strictly interior under condition (3.3). Each platform takes a strategy such that

$$\begin{aligned} p_1^C &= \frac{3t+\left[\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)\right]}{6t}\alpha^C-\gamma^C \\ p_1^A &= 0 \\ s_1 &= \underbrace{t+\frac{\left(-2\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)}{3}}_{=2t\cdot\frac{3t+\left[\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)\right]}{6t}-\alpha^C} \\ q_1 &= 1 \\ p_2^C &= 0 \\ p_2^A &= \frac{3t-\left[\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)\right]}{6t}\alpha^A-\gamma^A \end{aligned}$$

$$
\begin{aligned}
s_2 &= \underbrace{t - \frac{\left(\alpha^C + \beta^C\right) - \left(-2\alpha^A + \beta^A\right)}{3}}_{=2t \cdot \frac{3t - \left[\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)\right]}{6t} - \alpha^A} \\
q_2 &= 0
\end{aligned}
$$

and earns a profit of

$$
\begin{aligned}
\overline{\overline{\overline{\Pi}}}_1 (\cdot) &= \frac{\left\{3t + \left[\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)\right]\right\}^2}{18t} - \gamma^C \\
\overline{\overline{\overline{\Pi}}}_2 (\cdot) &= \frac{\left\{3t - \left[\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)\right]\right\}^2}{18t} - \gamma^A.
\end{aligned}
$$

Platform 1 earns a strictly higher profit by choosing $q_1 = 0$ if and only if

$$
\begin{aligned}
& \left(\frac{t}{2} - \gamma^A\right) - \left\{\frac{\left\{3t + \left[\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)\right]\right\}^2}{18t} - \gamma^C\right\} > 0 \\
\iff & \frac{6\left\{\left[3\gamma^C - \left(\alpha^C + \beta^C\right)\right] - \left[3\gamma^A - \left(\alpha^A + \beta^A\right)\right]\right\} t}{18t} > \frac{\left[\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)\right]^2}{18t} \\
\iff & 6 \underbrace{\left\{\left[3\gamma^C - \left(\alpha^C + \beta^C\right)\right] - \left[3\gamma^A - \left(\alpha^A + \beta^A\right)\right]\right\}}_{\neq 0 \text{ by assumption (3.3)}} t > \left[\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)\right]^2.
\end{aligned}
$$

If the left-hand side is negative, this inequality never holds; thus, platform 1 has an incentive to choose $q_1 = 1$ (and no incentive to choose $q_1 = 0$). If the left-hand side is strictly positive, the above inequality holds if and only if

$$
t > \frac{\left[\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)\right]^2}{6\left\{\left[3\gamma^C - \left(\alpha^C + \beta^C\right)\right] - \left[3\gamma^A - \left(\alpha^A + \beta^A\right)\right]\right\}}, \tag{3.11}
$$

in which case the platform has an incentive to choose $q_1 = 0$ (and no incentive to choose $q_1 = 1$). Either case may happen under condition (3.3) because it holds for some, not all, parameter bundles that[18]

$$
\begin{aligned}
& \frac{\left[\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)\right]^2}{6\left\{\left[3\gamma^C - \left(\alpha^C + \beta^C\right)\right] - \left[3\gamma^A - \left(\alpha^A + \beta^A\right)\right]\right\}} > \frac{\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)}{3} \\
\iff & \left[\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)\right] > 2\left\{\left[3\gamma^C - \left(\alpha^C + \beta^C\right)\right] - \left[3\gamma^A - \left(\alpha^A + \beta^A\right)\right]\right\}
\end{aligned}
$$

[18] As for the direction of the below inequality and the analogous inequality stated in the next paragraph, note that $(\alpha^C + \beta^C) - (\alpha^A + \beta^A) > \gamma^C - \gamma^A > 0$ under expression (3.2).

$$\Longleftrightarrow 3\left\{\left[\left(\alpha^C+\beta^C\right)-2\gamma^C\right]-\left[\left(\alpha^A+\beta^A\right)-2\gamma^A\right]\right\}>0$$
$$\Longleftrightarrow \left(\frac{\alpha^C+\beta^C}{2}-\gamma^C\right)-\left(\frac{\alpha^A+\beta^A}{2}-\gamma^A\right)>0,$$

which is compatible with the condition that

$$\left[3\gamma^C-\left(\alpha^C+\beta^C\right)\right]-\left[3\gamma^A-\left(\alpha^A+\beta^A\right)\right]>0$$
$$\Longleftrightarrow \left(\frac{\alpha^C+\beta^C}{3}-\gamma^C\right)-\left(\frac{\alpha^A+\beta^A}{3}-\gamma^A\right)<0.$$

Suppose next that platform 2 chooses $q_2=1$. Consider the case in which platform 1 chooses $q_1=0$. The outcome in this case is analogous to that when $(q_1,q_2)=(1,0)$. Thus,

$$\begin{aligned}\overline{\overline{n}}_1(0,1)&=\frac{1}{2}-\frac{\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)}{6t}\\ \overline{\overline{n}}_2(1,0)&=\frac{1}{2}+\frac{\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)}{6t}\end{aligned}$$

with regard to the consumer allocation,

$$\begin{aligned}
p_1^C&=0\\
p_1^A&=\frac{3t-\left[\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)\right]}{6t}\alpha^A-\gamma^A\\
s_1&=\underbrace{t-\frac{\left(\alpha^C+\beta^C\right)-\left(-2\alpha^A+\beta^A\right)}{3}}_{=2t\cdot\frac{3t-\left[\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)\right]}{6t}-\alpha^A}\\
q_1&=0\\
p_2^C&=\frac{3t+\left[\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)\right]}{6t}\alpha^C-\gamma^C\\
p_2^A&=0\\
s_2&=\underbrace{t+\frac{\left(-2\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)}{3}}_{=2t\cdot\frac{3t+\left[\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)\right]}{6t}-\alpha^C}\\
q_2&=1
\end{aligned}$$

with regard to the strategy that each platform takes, and

$$\overline{\overline{\overline{\Pi}}}_1(\cdot) = \frac{\left\{3t - \left[\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)\right]\right\}^2}{18t} - \gamma^A$$
$$\overline{\overline{\overline{\Pi}}}_2(\cdot) = \frac{\left\{3t + \left[\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)\right]\right\}^2}{18t} - \gamma^C$$

with regard to the profit that each platform earns. Next, I consider the case in which platform 1 choose $q_1 = 1$ under the assumption that $q_2 = 1$. The number of consumers who choose each platform is symmetric:

$$\overline{\overline{n}}_1(1,1) = \overline{\overline{n}}_2(1,1) = \frac{1}{2}$$

Both platforms take symmetric strategies such that

$$s_1 = s_2 = t - \alpha^C \qquad p_1^C = p_2^C = \frac{\alpha^C}{2} - \gamma^C$$
$$p_1^A = p_2^A = 0 \qquad q_1 = q_2 = 0.$$

Their resulting profits equal

$$\overline{\overline{\overline{\Pi}}}_1(\cdot) = \overline{\overline{\overline{\Pi}}}_2(\cdot) = \frac{t}{2} - \gamma^C.$$

Platform 1 earns a strictly higher profit by choosing $q_1 = 1$ if

$$\left(\frac{t}{2} - \gamma^C\right) - \left\{\frac{\left\{3t - \left[\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)\right]\right\}^2}{18t} - \gamma^A\right\} > 0$$
$$\Longleftrightarrow 6\underbrace{\left\{\left[\left(\alpha^C + \beta^C\right) - 3\gamma^C\right] - \left[\left(\alpha^A + \beta^A\right) - 3\gamma^A\right]\right\}}_{\neq 0 \text{ by assumption (3.3)}} t > \left[\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)\right]^2$$
$$\Longleftrightarrow t > \frac{\left[\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)\right]^2}{6\left\{\left[\left(\alpha^C + \beta^C\right) - 3\gamma^C\right] - \left[\left(\alpha^A + \beta^A\right) - 3\gamma^A\right]\right\}},$$

whose interpretation is analogous to that in the preceding paragraph. Platform 1 has an incentive to choose $q_1 = 1$ (and no incentive to choose $q_1 = 0$) only if the left-hand side of this inequality is strictly positive and to choose $q_1 = 0$ otherwise. When the left-hand side is strictly positive, the above inequality holds if and only if

$$t > \frac{\left[\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)\right]^2}{6\left\{\left[\left(\alpha^C + \beta^C\right) - 3\gamma^C\right] - \left[\left(\alpha^A + \beta^A\right) - 3\gamma^A\right]\right\}}. \tag{3.12}$$

Either of the cases in which $q_1 = 1$ and in which $q_1 = 0$ may happen under condition (3.3) because, again, it holds for some parameter bundles that

$$\begin{aligned}
&\frac{\left[\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)\right]^2}{6\left\{\left[\left(\alpha^C + \beta^C\right) - 3\gamma^C\right] - \left[\left(\alpha^A + \beta^A\right) - 3\gamma^A\right]\right\}} > \frac{\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)}{3} \\
\iff &\left[\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)\right] > 2\left\{\left[\left(\alpha^C + \beta^C\right) - 3\gamma^C\right] - \left[\left(\alpha^A + \beta^A\right) - 3\gamma^A\right]\right\} \\
\iff &\left[\left(\alpha^C + \beta^C\right) - 6\gamma^C\right] - \left[\left(\alpha^A + \beta^A\right) - 6\gamma^A\right] > 0 \\
\iff &\left(\frac{\alpha^C + \beta^C}{6} - \gamma^C\right) - \left(\frac{\alpha^A + \beta^A}{6} - \gamma^A\right) > 0,
\end{aligned}$$

which is compatible with the condition that

$$\begin{aligned}
&\left[\left(\alpha^C + \beta^C\right) - 3\gamma^C\right] - \left[\left(\alpha^A + \beta^A\right) - 3\gamma^A\right] > 0 \\
\iff &\left(\frac{\alpha^C + \beta^C}{3} - \gamma^C\right) - \left(\frac{\alpha^A + \beta^A}{3} - \gamma^A\right) > 0.
\end{aligned}$$

I now make some preparations to derive an equilibrium. Define

$$t > \frac{\left[\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)\right]^2}{6\left|\left[\left(\alpha^C + \beta^C\right) - 3\gamma^C\right] - \left[\left(\alpha^A + \beta^A\right) - 3\gamma^A\right]\right|}$$

as a summary of expressions (3.11) and (3.12), which appears as condition (3.4). Recall that platform 2's decision making with respect to its content-advertisement proportion is analogous to platform 1's because both platforms are symmetrically formulated. Moreover, I below establish that each platform strictly prefers choosing the different corner content-advertisement proportion from its rival's (i.e., $q_1 \neq q_2$) if condition (3.4) is violated in the strict sense. For instance, suppose that $(q_1, q_2) = (1, 0)$ and that platform 2 chooses so because the condition is violated in the strict sense. The relation that

$$\left(\frac{\alpha^C+\beta^C}{3}-\gamma^C\right)-\left(\frac{\alpha^A+\beta^A}{3}-\gamma^A\right)>0$$

needs to be satisfied for condition (3.4) to matter in the platform's decision making. Platform 2 in this case never chooses $q_2=0$ because the condition that

$$\left(\frac{\alpha^C+\beta^C}{3}-\gamma^C\right)-\left(\frac{\alpha^A+\beta^A}{3}-\gamma^A\right)<0,$$

which is a necessity for the platform to choose $q_2=0$, does not hold. The cases (i) in which $(q_1,q_2)=(1,0)$ and platform 1 cares about expression (3.4) affects platform 1's decision making and (ii) in which $(q_1,q_2)=(0,1)$ are analogous.

The equilibrium-derivation process concludes as follows. Consider first the equilibrium pair of content-advertisement proportions. Under condition (3.4), it only arises in equilibrium that $(q_1,q_2)=(1,1)$ if

$$\left(\frac{\alpha^C+\beta^C}{3}-\gamma^C\right)-\left(\frac{\alpha^A+\beta^A}{3}-\gamma^A\right)>0$$

and that $(q_1,q_2)=(0,0)$ if the above inequality is violated in the strict sense. If expression (3.4) is violated in the strict sense, it only arises in equilibrium that $(q_1,q_2)=(0,1)$ or $(q_1,q_2)=(1,0)$. The values of the other equilibrium components and the corresponding profits for both platforms are derived in the second and third paragraphs of this subsection.

3.A.3 Properties of the Asymmetric Equilibrium Configuration

Because the two asymmetric equilibria are analogous to one another, this subsection focuses on the case in which $q_1^*=1$. Platform 1 in equilibrium chooses a higher subscription price because it is always satisfied that

$$s_1^*-s_2^*=\left[t+\frac{\left(-2\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)}{3}\right]-\left[t-\frac{\left(\alpha^C+\beta^C\right)-\left(-2\alpha^A+\beta^A\right)}{3}\right]$$

$$
\begin{aligned}
&= \frac{\left(-\alpha^C + 2\beta^C\right) + \left(\alpha^A - 2\beta^A\right)}{3} \\
&= \frac{2\left(\alpha^A - \alpha^C\right) + 2\left(\beta^C - \beta^A\right)}{3} > 0,
\end{aligned}
$$

where $\alpha^A > \alpha^C$ and $\beta^C > \beta^A$ by assumption. The platform in equilibrium earns a higher profit if and only if

$$
\begin{aligned}
\Pi_1^* - \Pi_2^* = &\left[\frac{\left\{3t + \left[\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)\right]\right\}^2}{18t} - \gamma^C\right] \\
&- \left[\frac{\left\{3t - \left[\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)\right]\right\}^2}{18t} - \gamma^A\right] \\
= &\frac{2\left[\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)\right]}{3} - \left(\gamma^C - \gamma^A\right) \\
= &\frac{\left[2\left(\alpha^C + \beta^C\right) - 3\gamma^C\right] - \left[2\left(\alpha^A + \beta^A\right) - 3\gamma^A\right]}{3} \\
= &\left[\frac{2\left(\alpha^C + \beta^C\right)}{3} - \gamma^C\right] - \left[\frac{2\left(\alpha^A + \beta^A\right)}{3} - \gamma^A\right] > 0.
\end{aligned}
$$

3.B Details on the Welfare Analysis (Propositions 3.3 and 3.4)

This section proves the propositions stated in section 3.5. The first subsection derives the candidates of a solution to social-welfare maximization, which exclude several allocation patterns that cannot be efficient. The second subsection obtains the efficient outcome(s).

3.B.1 Solution Candidates

This subsection begins by deriving all solution candidates. First, the optimal values of q_1 and q_2 are obtained given n_1. Section 3.3 establishes that the first-order derivatives of social welfare with respect to q_1 and q_2 are constant in the respective own variables. Each of the efficient content-advertisement proportions is thus 1, 0, and any value in $[0, 1]$ if the respective derivatives are strictly positive, strictly negative, and zero, respectively. Second, I maximize $\overline{W}(\cdot)$ with respect to n_1. The first-order and second-order derivatives of this function with respect to n_1 equal

$$\begin{aligned}
\frac{\partial \overline{W}(\cdot)}{\partial n_1} &= \frac{\partial W(\cdot)}{\partial n_1} \\
&= (q_1 - q_2)\left(\alpha^C + \beta^C\right) + \underbrace{\left[(1-q_1) - (1-q_2)\right]}_{=q_1 - q_2}\left(\alpha^A + \beta^A\right) - (2n_1 - 1)\, t \\
&= \left[\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)\right](q_1 - q_2) - (2n_1 - 1)\, t \\
\frac{\partial^2 \overline{W}(\cdot)}{\partial n_1^2} &= -2t,
\end{aligned}$$

respectively, because q_1 and q_2 are constant in n_1. The first-order condition is thus that

$$\begin{aligned}
\frac{\partial \overline{W}(\cdot)}{\partial n_1} = 0 \iff 2tn_1 &= t + \left[\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)\right](q_1 - q_2) \\
\iff n_1 &= \frac{1}{2} + \frac{\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)}{2t}(q_1 - q_2),
\end{aligned}$$

where $n_1 > 1/2$ if and only if $q_1 > q_2$. The second-order condition with respect to n_1 is satisfied for any t. The first-order condition characterizes the efficient allocation of consumers if

$$0 < n_1 = \frac{t + \left[\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)\right](q_1 - q_2)}{2t} < 1,$$

which implies that the strict interiority of the efficient consumer allocation is guaranteed if and only if

$$\begin{aligned}
0 < \max_{n_1 \in (0,1)} n_1 \left(= \max_{n_2 \in (0,1)} n_2\right) < 1 &\iff t + \left[\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)\right] < 2t \\
&\iff \underbrace{t > \left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)}_{\text{the first half of condition (3.8)}}.
\end{aligned}$$

One can select some of the solution candidates by excluding the allocations that never maximize social welfare. First, consider the case in which the consumers are allocated such that $n_1 = n_2 = 1/2$. The first-order derivatives with respect to q_1 and q_2 are calculated as

$$\begin{aligned}\frac{\partial W(\cdot)}{\partial q_1} &= \frac{\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)}{2}-\left(\gamma^C-\gamma^A\right)\\ &= \left(\frac{\alpha^C+\beta^C}{2}-\gamma^C\right)-\left(\frac{\alpha^A+\beta^A}{2}-\gamma^A\right)=\frac{\partial W(\cdot)}{\partial q_2}.\end{aligned}$$

It is a unique solution candidate in this case that $q_1 = q_2 = 1$ if

$$\left(\frac{\alpha^C+\beta^C}{2}-\gamma^C\right)-\left(\frac{\alpha^A+\beta^A}{2}-\gamma^A\right)>0$$

and that $q_1 = q_2 = 0$ if

$$\left(\frac{\alpha^C+\beta^C}{2}-\gamma^C\right)-\left(\frac{\alpha^A+\beta^A}{2}-\gamma^A\right)<0.$$

The other pairs of q_1 and q_2 are never efficient in the case. Next, I consider the case in which the consumers are allocated such that $n_1 \neq n_2$. Suppose also that $q_1 \in (0,1)$ and $q_2 \in (0,1)$. Under the assumption that q_1 is interior,

$$\begin{aligned}\frac{\partial W(\cdot)}{\partial q_1}=0 &\iff \left[\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)\right]n_1=\left(\gamma^C-\gamma^A\right)\\ &\iff n_1=\frac{\gamma^C-\gamma^A}{\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)}\in(0,1).\end{aligned}$$

The first-order derivative with respect to q_2 in this case equals

$$\begin{aligned}\frac{\partial W(\cdot)}{\partial q_2} &= \left\{\left[\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)\right]-\left[\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)\right]n_1\right\}-\left(\gamma^C-\gamma^A\right)\\ &= \left\{\left[\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)\right]-\left(\gamma^C-\gamma^A\right)\right\}-\left[\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)\right]n_1\\ &= \left[\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)\right]-2\left(\gamma^C-\gamma^A\right)\\ &= 2\left[\left(\frac{\alpha^C+\beta^C}{2}-\gamma^C\right)-\left(\frac{\alpha^A+\beta^A}{2}-\gamma^A\right)\right],\end{aligned}$$

which implies that $q_2 = 0$ or $q_2 = 1$. Social welfare thus cannot be maximized. Consider the case in which q_2 takes the efficient corner value. If n_1 continues to take the above value and be treated as given, social welfare is unchanged as q_1 increases or decreases to q_2 because the partial derivative with respect to q_1 is constant in the variable. Some consumers can reduce their transportation costs by switching accordingly, which does not change the total benefit from the indirect network externalities exerted on both sides if $q_1 = q_2$. These imply that social welfare can be improved as q_1 is changed from any interior value to q_2, although this change does not necessarily cause welfare maximization. Thus, there does not exist an efficient outcome such that $n_1 \neq n_2$, $q_1 \in (0,1)$, and $q_2 \in \{0,1\}$. The same result can be obtained by assuming that $q_1 \in \{0,1\}$ and $q_2 \in (0,1)$. Consequently, the selected solution candidates can be listed as follows:

$$
\begin{aligned}
&(q_1, q_2) = (1,1) \qquad n_1 = \frac{1}{2} \ \text{ if } \left(\frac{\alpha^C + \beta^C}{2} - \gamma^C\right) - \left(\frac{\alpha^A + \beta^A}{2} - \gamma^A\right) > 0 \\
&(q_1, q_2) = (0,0) \qquad n_1 = \frac{1}{2} \ \text{ if } \left(\frac{\alpha^C + \beta^C}{2} - \gamma^C\right) - \left(\frac{\alpha^A + \beta^A}{2} - \gamma^A\right) < 0 \\
&(q_1, q_2) = (0,1) \qquad n_1 = \frac{1}{2} - \frac{\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)}{2t} \\
&(q_1, q_2) = (1,0) \qquad n_1 = \frac{1}{2} + \frac{\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)}{2t},
\end{aligned}
$$

where $n_2 = 1 - n_1$.

Before proceeding by deriving the efficient outcome(s), I should examine the possibility that any of the above solution candidates is actually inefficient because social welfare takes a strictly negative value with the candidate. One can prevent the occurrence of this possibility by placing additional assumptions on the parameters based on the cases of the symmetric solution candidates:

$$
\begin{aligned}
W\left(1,1,\frac{1}{2}\right) &= \left[\left(\frac{\alpha^C}{2} - \gamma^C\right) + \left(\frac{\alpha^C}{2} - \gamma^C\right)\right] \\
&\quad + \left\{-\left[\int_0^{\frac{1}{2}} x \mathrm{d}x + \int_{\frac{1}{2}}^1 (1-x)\,\mathrm{d}x\right] t + \left(\frac{1}{2} + \frac{1}{2}\right)\beta^C\right\} \\
&= \left[\left(\alpha^C + \beta^C\right) - 2\gamma^C\right] - \frac{1}{4}t \geq 0 \\
W\left(0,0,\frac{1}{2}\right) &= \left[\left(\frac{\alpha^A}{2} - \gamma^A\right) + \left(\frac{\alpha^A}{2} - \gamma^A\right)\right] \\
&\quad + \left\{-\left[\int_0^{\frac{1}{2}} x \mathrm{d}x + \int_{\frac{1}{2}}^1 (1-x)\,\mathrm{d}x\right] t + \left(\frac{1}{2} + \frac{1}{2}\right)\beta^A\right\}
\end{aligned}
$$

$$= \left[\left(\alpha^A + \beta^A\right) - 2\gamma^A\right] - \frac{1}{4}t \geq 0.$$

These assumptions yield the condition that

$$\begin{aligned}
& W\left(1, 1, \frac{1}{2}\right) \geq 0 \text{ or } W\left(0, 0, \frac{1}{2}\right) \geq 0 \\
\Longleftrightarrow\ & t \leq 4\left[\left(\alpha^C + \beta^C\right) - 2\gamma^C\right] \text{ or } t \leq 4\left[\left(\alpha^A + \beta^A\right) - 2\gamma^A\right] \\
\Longleftrightarrow\ & t \leq 4 \min\left\{\left(\alpha^A + \beta^A\right) - 2\gamma^A, \left(\alpha^C + \beta^C\right) - 2\gamma^C\right\},
\end{aligned}$$

which appears as the second half of expression (3.8).

3.B.2 Efficient Outcome(s)

Suppose that

$$\left(\frac{\alpha^C + \beta^C}{2} - \gamma^C\right) - \left(\frac{\alpha^A + \beta^A}{2} - \gamma^A\right) > 0,$$

in which case it might be efficient that $q_1 = q_2 = 1$ and it cannot be efficient that $q_1 = q_2 = 0$. Consider then the welfare change as q_2 decreases to 0 (and q_1 is unchanged). Notice that the first-order and second-order conditions with respect to n_1 obtained in the first paragraph can apply in this case because those conditions are derived given q_1 and q_2; thus,

$$n_1 = \frac{t + \left[\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)\right]}{2t}.$$

The change in social welfare can be decomposed into two parts. The first is the welfare change as q_1 decreases to zero given that $n_1 = 1/2$:

$$\begin{aligned}
-\int_0^1 \left.\frac{\partial W(\cdot)}{\partial q_2}\right|_{n_1 = \frac{1}{2}} \mathrm{d}q_2 &= -\left[\frac{\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)}{2} - \left(\gamma^C - \gamma^A\right)\right] \\
&= -\left[\left(\frac{\alpha^C + \beta^C}{2} - \gamma^C\right) - \left(\frac{\alpha^A + \beta^A}{2} - \gamma^A\right)\right] \\
&= -\frac{\left[\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)\right] - 2\left(\gamma^C - \gamma^A\right)}{2},
\end{aligned}$$

where the last expression enhances the readability of the following calculation. The second part is the welfare change as n_1 increases to the value above given that $(q_1, q_2) = (1, 0)$:

$$\begin{aligned}
&\int_{\frac{1}{2}}^{\frac{t+\left[\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)\right]}{2t}} \left.\frac{\partial W(\cdot)}{\partial n_1}\right|_{q_1=1 \text{ and } q_2=0} \mathrm{d}n_1 \\
=&\int_{\frac{1}{2}}^{\frac{t+\left[\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)\right]}{2t}} \left\{\left[\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)\right]-(2n_1-1)\,t\right\} \mathrm{d}n_1 \\
=&\frac{\left[\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)\right]^2}{4t}.
\end{aligned}$$

The total welfare change is thus strictly negative if and only if

$$\begin{aligned}
&-\frac{\left[\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)\right]-2\left(\gamma^C-\gamma^A\right)}{2}+\frac{\left[\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)\right]^2}{4t}<0 \\
\iff &t>\frac{\left[\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)\right]^2}{2\left\{\left[\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)\right]-2\left(\gamma^C-\gamma^A\right)\right\}}, \qquad (3.13)
\end{aligned}$$

which does not necessarily hold because it is not always satisfied that

$$\begin{aligned}
&\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)>\frac{\left[\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)\right]^2}{2\left\{\left[\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)\right]-2\left(\gamma^C-\gamma^A\right)\right\}} \\
\iff &2\left[\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)\right]\left\{\left[\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)\right]-2\left(\gamma^C-\gamma^A\right)\right\} \\
&\qquad >\left[\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)\right]^2 \\
\iff &\left[\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)\right]^2-4\left[\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)\right]\left(\gamma^C-\gamma^A\right)>0 \\
\iff &\left[\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)\right]-4\left(\gamma^C-\gamma^A\right)>0 \\
\iff &2\left[\left(\frac{\alpha^C+\beta^C}{2}-\gamma^C\right)-\left(\frac{\alpha^A+\beta^A}{2}-\gamma^A\right)\right]>2\left(\gamma^C-\gamma^A\right).
\end{aligned}$$

It is thus efficient that only $q_1 = q_2 = 1$ if condition (3.13) holds and that only $(q_1, q_2) = (0, 1)$ if the condition is strictly violated. The analysis of this case concludes by rewriting the condition as

$$t>\frac{\left[\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)\right]^2}{2\left\{\left[\left(\alpha^C+\beta^C\right)-2\gamma^C\right]-\left[\left(\alpha^A+\beta^A\right)-2\gamma^A\right]\right\}}. \qquad (3.14)$$

One can discuss the case in which $q_1 = 1$ and $q_2 = 0$ analogously, and the same result as above applies.

Next, suppose that

$$\left(\frac{\alpha^C + \beta^C}{2} - \gamma^C\right) - \left(\frac{\alpha^A + \beta^A}{2} - \gamma^A\right) < 0,$$

in which case it might be efficient that $q_1 = q_2 = 0$ and it cannot be efficient that $q_1 = q_2 = 1$. Consider the welfare change as $q_1 = 1$, similar to the preceding paragraph. I decompose the welfare change below. The welfare change as q_1 increases given that $n_1 = 1/2$ equals

$$\begin{aligned}
\int_0^1 \left.\frac{\partial W(\cdot)}{\partial q_1}\right|_{n_1=\frac{1}{2}} \mathrm{d}q_2 &= \left[\frac{\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)}{2} - \left(\gamma^C - \gamma^A\right)\right] \\
&= \left[\left(\frac{\alpha^C + \beta^C}{2} - \gamma^C\right) - \left(\frac{\alpha^A + \beta^A}{2} - \gamma^A\right)\right] \\
&= \frac{\left[\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)\right] - 2\left(\gamma^C - \gamma^A\right)}{2},
\end{aligned}$$

which is strictly negative. The welfare change as n_1 increases given that $(q_1, q_2) = (1, 0)$ takes the same value as above:

$$\frac{\left[\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)\right]^2}{4t}.$$

The total welfare change is thus strictly negative if and only if

$$\begin{aligned}
&\frac{\left[\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)\right] - 2\left(\gamma^C - \gamma^A\right)}{2} + \frac{\left[\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)\right]^2}{4t} < 0 \\
\Longleftrightarrow\ & -2\underbrace{\left\{2\left(\gamma^C - \gamma^A\right) - \left[\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)\right]\right\}}_{>0} t < -\left[\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)\right]^2 \\
\Longleftrightarrow\ & t > \frac{\left[\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)\right]^2}{2\left\{2\left(\gamma^C - \gamma^A\right) - \left[\left(\alpha^C + \beta^C\right) - \left(\alpha^A + \beta^A\right)\right]\right\}},
\end{aligned} \tag{3.15}$$

which does not necessarily hold because it is not always satisfied that

$$\begin{aligned}
&\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right) > \frac{\left[\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)\right]^2}{2\left\{2\left(\gamma^C-\gamma^A\right)-\left[\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)\right]\right\}} \\
\iff & 2\left[\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)\right]\left\{2\left(\gamma^C-\gamma^A\right)-\left[\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)\right]\right\} \\
&\qquad > \left[\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)\right]^2 \\
\iff & 4\left[\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)\right]\left(\gamma^C-\gamma^A\right)-3\left[\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)\right]^2>0 \\
\iff & 4\left(\gamma^C-\gamma^A\right)-3\left[\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)\right]>0 \\
\iff & 2\left\{2\left(\gamma^C-\gamma^A\right)-\left[\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)\right]\right\}-\left[\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)\right]>0 \\
\iff & -4\underbrace{\left[\left(\frac{\alpha^C+\beta^C}{2}-\gamma^C\right)-\left(\frac{\alpha^A+\beta^A}{2}-\gamma^A\right)\right]}_{<0} > \left[\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)\right].
\end{aligned}$$

It is thus efficient that only $q_1=q_2=0$ if condition (3.15) holds and that only $(q_1,q_2)=(0,1)$ if the condition is strictly violated. I rewrite condition (3.15) as follows:

$$t > \frac{\left[\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)\right]^2}{2\left\{\left[2\gamma^C-\left(\alpha^C+\beta^C\right)\right]-\left[2\gamma^A-\left(\alpha^A+\beta^A\right)\right]\right\}}.$$

The case in which $q_1=1$ and $q_2=0$ can, again, be analyzed analogously.

This subsection concludes by stating the efficient outcome. Suppose that

$$t > \frac{\left[\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)\right]^2}{2\left|\left[\left(\alpha^C+\beta^C\right)-2\gamma^C\right]-\left[\left(\alpha^A+\beta^A\right)-2\gamma^A\right]\right|}, \tag{3.16}$$

which summarizes conditions (3.14) and (3.16). First, $q_1^{**}=q_2^{**}=1$ and $n_1^{**}=n_2^{**}=1/2$ if

$$\left(\frac{\alpha^C+\beta^C}{2}-\gamma^C\right)-\left(\frac{\alpha^A+\beta^A}{2}-\gamma^A\right)>0.$$

Second, $q_1^{**}=q_2^{**}=0$ and $n_1^{**}=n_2^{**}=1/2$ under the condition that

$$\left(\frac{\alpha^C+\beta^C}{2}-\gamma^C\right)-\left(\frac{\alpha^A+\beta^A}{2}-\gamma^A\right)<0.$$

Consider next the case in which

$$\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)<t<\frac{\left[\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)\right]^2}{2\left|\left[\left(\alpha^C+\beta^C\right)-2\gamma^C\right]-\left[\left(\alpha^A+\beta^A\right)-2\gamma^A\right]\right|}.$$

It holds in this case that

$$\begin{aligned}
(q_1^{**},q_2^{**})&=(0,1) \qquad n_1^{**}=\frac{1}{2}-\frac{\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)}{2t}\\
(q_1^{**},q_2^{**})&=(1,0) \qquad n_1^{**}=\frac{1}{2}+\frac{\left(\alpha^C+\beta^C\right)-\left(\alpha^A+\beta^A\right)}{2t}.
\end{aligned}$$

The efficient number of consumers on platform 2 in the case can be derived from the relation that $n_2^{**}=1-n_1^{**}$.

Chapter 4

Two-Sided Platform Competition with Biased Expectations

4.1 Introduction

Two-sided platforms, or platforms in a two-sided market, are intermediaries between two distinguished groups of economic agents who make communications, transactions, or otherwise interactions with one another. Such platforms include C-to-C intermediaries for sharing economies (e.g., Uber and Airbnb), operating systems and the online marketplaces for those systems (e.g., iOS and App Store), a type of technologies (e.g., Blu-ray Disc), and payment services (e.g., PayPal). A notable feature of a two-sided market is the existence of (positive) indirect network externalities: each platform usually needs to attract a certain number of potential users on one side to obtain a sufficient market share on the other side because most part of the utility from that platform increases as the platform attracts a larger number of potential users on the opposite side. However, in most cases, the number of platform users is realized as a consequence of each potential user's platform adoption. The platform can thus attract potential users only if those on the opposite side *expect* the platform to in advance. The formulation of potential users' expectations in this sense matters in predicting the market outcome and evaluating its welfare consequence.

Most papers in the literature adopt a type of the rational-expectation concept. Each potential user is assumed under that concept to make a platform choice observing all prices announced on both sides, using the knowledge of all profit and utility functions, and correctly expecting the allocation of potential users on the opposite side. This assumption guarantees that all platforms choose their profit-maximizing prices and all potential users make their utility-maximizing platform choices expecting the other players to take their optimal strategies, in which sense the market outcome can be theoretically expressed as a Nash equilibrium.

However, I argue that the concept of rational expectation might not fully explain potential users' expectations formed in a two-sided market. Suppose, for instance, that there emerges a new market with two incompatible platforms. Potential users on each side can observe little market information about the opposite side at that point because the market is at its earliest stage. It is thus a natural conjecture that some potential users may fail in correctly expecting each platform's market share and hold idiosyncratic expectations while making their platform choices. Particularly, it is a possible scenario that those potential users form biased expectations toward a specific platform. An important property of biased expectations is that they affect the expectations formed by potential users with rational expectations and the price strategies taken by platforms because those players can incorporate the impacts of biased expectations to their decisions. In this sense, biased expectations may play a crucial role in the consequence of platform competition. This possibility cannot be captured under the concept of rational expectation.

This chapter studies duopolistic competition between two-sided platforms when some potential users form biased expectations of the opposite-side market shares toward a particular platform. To highlight the features of biased expectations, I formulate platform competition as the following simple one-shot game based on the Armstrong (2006) framework. The market consists of two sides characterized as Hotelling lines: on each side, there exist two platforms located at the respective corners and a unit mass of economic agents uniformly located in the line. Both platforms at the beginning simultaneously choose the prices on both sides that maximize their own profits, after which each potential user on each side simultaneously chooses such a platform that his/her utility can be maximized.[1] Notice that each agent's expectation of the opposite-side allocation matters in the market outcome under this formulation because all platform choices are made simultaneously. Most existing papers, including Armstrong (2006), assume that all potential users form rational expectations. On the other hand, this chapter incorporates the possibility that some agents hold biased expectations to the above competition game as below.

Biased expectations are formulated by the following steps. First, suppose that potential users form heterogeneous allocation expectations in the following way, as in Hagiu and Hałaburda (2014). All agents on one side and some potential users on the other side can form rational expectations of each platform's opposite-side market share as discussed above. On the other hand, the other agents (on the latter side) cannot form such rational expectations but are simply considered to hold certain expectations. The distributions of expectation patterns and agent characteristics are independent. Hagiu and Hałaburda (2014) require the latter type of expectations to be fulfilled as an additional assumption to keep the expectations consistent with the actual opposite-side allocation. On the contrary, I allow those expectations to be biased in the following sense. Potential users with such expectations believe that a particular platform attracts all potential users on the other side, which is adapted from the situation that Caillaud and Jullien (2001, 2003), Hagiu (2006), and their subsequent papers originally consider under the assumption of rational expectations. This chapter departs from those existing works in that biased expectations may be inconsistent with the resulting allocation on the other side.

Notably, the competitive outcome under the above formulation cannot necessarily be obtained as an equilibrium in the standard sense because the concept of Nash equilibrium requires all players to form rational expectations at the time of their decision making.[2] This chapter thus adopts a solution concept that relaxes Nash equilibrium by the following

[1]The words "agents" and "potential users" are interchangeably used thoughout this chapter as words for economic agents who are facing platform-choice problems.

[2]This chapter interchangeably uses the words "competitive outcome" and "market outcome" except when a certain platform dominates a side or the market group of agents with biased expectations (and thus does not actually compete for potential users on that side or in that group).

three steps. First, the two platforms and all potential users on both sides maximize the respective profit and utility functions, respectively. Second, all players but those with biased expectations form consistent expectations. Third, biased expectations are allowed to remain inconsistent with the arising market shares. Here, I should also remark that the market outcome constitutes a Nash equilibrium if biased expectations are consistent. This case can happen if and only if the platform believed by agents with biased expectations to dominate the opposite side really attracts all potential users on that side. When the market outcome can be obtained as an equilibrium is therefore one of the major research questions in this chapter.

The analysis of the competition game shows two different market-outcome patterns. The first pattern is that the platform with an advantage from biased expectations obtains larger market shares on both sides but does not attract all agents on each side, which occurs in the case of moderate indirect network externalities. The advantageous platform in this case announces relatively high prices and selects the groups of agents who exhibit relatively high willingness to pay by exploiting its advantage from biased expectations, which enables the disadvantageous platform to attract some potential users. This type of market outcome can arise even if the indirect network externality exerted on each side is so intense that a platform would dominate at least one side under the Armstrong (2006) framework,[3] in which sense biased expectations reduce the possibility of a particular platform dominating one or both side(s). The second pattern of the market outcome is that the advantageous platform attracts all agents on both sides of the market, which happens if and only if an indirect network externality is intensely exerted on each side and, surprisingly, sufficiently *few* agents form biased expectations (see the next paragraph for a discussion on this condition). This pattern describes the situation in which the dominant platform possesses an advantage from biased expectations such that the dominated platform has no room to attract any potential user on each side. The market-outcome pattern is particularly notable that all of the agents with biased expectations form cor-

[3]The technical meaning of this condition is that the second-order condition with respect to the Hessian matrix of each platform's profit function would be violated under the original Armstrong (2006) framework yet is satisfied in my model.

rect expectations, which means that a particular platform's expectation-driven behavior of market dominance can be obtained as a Nash equilibrium by allowing for the concept of biased expectation.

I also examine the transition between the first and second market-outcome patterns when the market structure is changed. Two conditions matter in whether the latter pattern arises. First, it is a necessary condition that the indirect network externality exerted on each side is sufficiently strong. Second, the latter pattern becomes more likely to occur as the number of potential users with biased expectations *decreases*. These conditions can be interpreted as below. If few agents hold biased expectations, the advantageous platform tends to choose a low price on the biased side because the platform cannot utilize a strong advantage from biased expectations. The disadvantageous platform responds to this pricing and also chooses a relatively low price on the side. Platforms engage in more severe price reduction as a stronger indirect network externality is exerted on each side (where the cross-side price effects are more intense) or a smaller number of agents form biased expectations (where the advantageous platform's market power is weaker). Such price reduction helps the advantageous platform stimulate the demand from potential users with biased expectations and possibly enables the platform to dominate the market group of those potential users. The competition-game analysis finds that the advantageous platform obtains market shares of 100 percent on both sides once it attracts all of the agents with biased expectations.

The welfare analysis establishes that biased expectations tend to play a negative role in the welfare consequence of price competition between two-sided platforms. At the beginning, there are mainly two efficient allocation configurations, which *per se* is equivalent to those in Armstrong (2006) models although the welfare implications differ. Social welfare is maximized if both platforms obtain equal market shares on both sides in the case of moderate indirect network externalities because agents incur transportation costs while choosing the more distant platform. Social welfare is maximized if a particular platform gathers all agents on both sides in the case of sufficiently intensive indirect network externalities. The market outcome discussed above maximizes social welfare if and only if the indirect network externality exerted on each side and the number of agents who hold biased expectations are strong and small, respectively, enough for the advantageous platform to choose market dominance. In other words, social welfare is not maximized if platforms coexist as a competitive outcome.

From the welfare analysis, I find that the platform-coexistence market outcome does not maximize social welfare for different reasons. The competitive outcome is inefficient if an indirect network externality is intensely exerted on each side but no market dominance occurs as a consequence of price competition because the advantageous platform misses some potential users. One can obtain a twofold welfare implication if the indirect network externality exerted on each side is so moderate that market dominance does not happen under price competition but should occur from a welfare perspective. The competitive outcome is inefficient because the advantageous platform misses some agents. The competitive outcome can, however, be seen to improve social welfare compared with that in original Armstrong (2006) models in the sense that the advantageous platform attracts a larger number of agents on each side. The competitive outcome in this chapter is inefficient if the indirect network externality exerted on each side is weak, in which case the advantageous platform obtains larger market shares than the efficient levels for that platform.

The rest of this chapter is organized as follows. The next section reviews related papers. Section 4.3 develops the model in this chapter. After the model is constructed, I discuss the properties of the market outcome in section 4.4 and conduct a welfare analysis in section 4.5. The main text concludes in section 4.6. Appendix contains the proofs of all propositions.

4.2 Related Literature

This chapter adopts a static framework to describe the situation in which a two-sided platform competes for potential users and attracts a larger number of them because that platform is expected so.[4] Caillaud and Jullien (2001, 2003) construct a duopoly model in which identical potential users on a side initially expect a particular one of the two identical platforms to dominate the other side and establish that (if each agent chooses at most one platform) the other platform may not attract any agent because there is a price strategy for the former platform to attract all agents in the entire market. Several papers, including Jullien (2001, 2011), Hagiu (2006), Hagiu and Spulber (2013), and Hałaburda and Yehezkel (2013), adopt Caillaud and Jullien's (2001, 2003) framework to discuss several contexts of duopolistic price competition. Hałaburda and Yehezkel (2016) study duopolistic competition between platforms that connect sellers with buyers and adapt Caillaud and Jullien's (2001, 2003) expectation concept to the situation in which buyers expect a certain platform to dominate the market with a higher probability (not 100 percent) if there exist multiple equilibria of market dominance. Gabszewicz and Wauthy (2014) also construct a variant of Caillaud and Jullien's (2001, 2003) model that differs in that each potential user has a different valuation of a network benefit per user and obtain an equilibrium in which one platform attracts a larger number (but not all) of agents on each side.[5] Ambrus and Argenziano (2009) extend Gabszewicz and Wauthy's (2014) model in the distribution of potential users' valuations and establish that one platform may dominate the entire market if the variation of the valuations is small.[6] Ko and Shen (2016) develop a model in which potential users on one side (formulated á la Hotelling) exhibit an asymmetric allocation, including a corner allocation, because the

[4]There are also a few theoretical papers that develop infinite-period models of duopolistic two-sided market, which might be related to this chapter in that some researchers do not follow the concept of rational expectation in order to capture the path between the beginning and steady state of platform competition (e.g., Sun and Tse 2007). However, to the best of my knowledge, none of those papers allows for the possibility of market-share expectations being biased toward a specific platform as in this chapter. This literature review therefore focuses on existing papers that construct single-period games with one or multiple stage(s).

[5]de Palma and Leruth (1996) adopt a similar approach and obtain an equilibrium with asymmetric market shares in the case of network goods, where potential users constitute a single side and users of a network good exert a *direct* network externality on themselves.

[6]Ambrus and Argenziano (2009) are the most remarkable for the following two contributions although those are less related to the methodology and findings in this chapter. First, Ambrus and Argenziano (2009) allow for sophisticated expectation formation by adopting the concept of "coalitional rationalizability" (Ambrus 2006), under which a group of potential users rule out the strategies that they expect to improve the payoff for no one in the same group. Second, Ambrus and Argenziano (2009) find that the majority of agents may choose the platform with the smaller market share on the opposite side if the variation of potential users' valuations is small.

other side consists of identical agents who make platform choices before those on the former side make decisions. Moreover, a large number of duopoly models with horizontal differentiation, which include those in Armstrong (2006) and Armstrong and Wright (2007), potentially yield a market outcome in which a particular platform dominates the market. This type of market dominance occurs if the second-order condition with regard to the Hessian matrix of a platform's profit function is violated, intuitively where the strength of indirect network externalities is large enough that the platform can enhance its profit the most by coordinating its prices on both sides and attracting as many agents as possible. Potential users' market-share expectations play a role in determining which platform eventually dominates the entire market. The aforementioned papers presume all potential users to form identical expectations consistent with the resulting number of platform users and regard which platform dominates competition solely as a matter of which equilibrium they select. This chapter, however, analyzes the situation in which potential users form different expectations that can be biased and inconsistent for some agents, which means incorporating each agent's idiosyncratic guess.[7] From a technical viewpoint, this chapter needs to adopt a solution concept that relaxes Nash equilibrium to derive the market outcome because some potential users do not necessarily form consistent (i.e., rational) market-share expectations.

This chapter is thus sufficiently close to works that describe static competition between two-sided platforms when potential users form different expectations. Hagiu and Hałaburda (2014) construct a model that allows for the following two types of expectations. The agents on one side and some agents on the other side can form correct opposite-side allocation expectations by utilizing the information of the opposite-side market demand functions and all announced prices, responding to opposite-side price changes and adjust their expectations. The other agents (who exist on the latter side) cannot form such sophisticated expectations due to lack of price information but do hold some fixed expectations that are assumed to be fulfilled; thus, these agents form rational

[7] This chapter limits my focus to the case in which the second-order conditions are satisfied and describes a particular platform's larger market share on (or dominance of) each side induced by that platform's advantage from biased expectations although those conditions can also be violated in this chapter's model.

expectations as in models with imperfect information. Hagiu and Hałaburda (2014) and this chapter are particularly close in the distribution of expectation types and the formulation of the latter demand side but crucially differ in that this chapter allows the fixed expectations to be biased and inconsistent. Jullien and Pavan (2019) develop a model of global games to describe two-sided markets in which potential users have heterogeneous valuations of each platform's stand-alone service and hold opposite-side market-share expectations based on their own valuations because no potential user is informed of the opposite-side valuation distribution. Jullien and Pavan (2019) are close in that they allow for somewhat idiosyncratic expectations and define utility functions as a generalization of Armstrong's (2006) but different in that Jullien and Pavan (2019) assume all agents to hold rational expectations given their valuations and adopt an equilibrium concept with incomplete information. Moreover, Jullien and Pavan (2019) do not obtain a market outcome in an explicit form because the distribution of valuations on each side is general, which differs in that this chapter develops a stylized linear model and calculates the exact value of the market outcome to show the impacts of biased expectations. Hossain and Morgan (2013) describe herding behavior that enables a certain platform to dominate the market by using a model in which potential users have heterogeneous cognitive levels such that (i) agents at a strictly positive level only know of those at lower levels while forming expectations and (ii) those at level 0 randomly choose a platform. Hossain and Morgan (2013) assume that each potential user forms a rational expectation given his/her congnitive level and thus use equilibrium as a solution concept, relatively close to the papers mentioned in this paragraph in this sense. On the other hand, again, this chapter adopts a solution concept other than equilibrium because biased expectations in this chapter do not need to be consistent with the realized number of platform users. In sum, the aforementioned works are different from this chapter in that the latter rely on rational-expectation and equilibrium concepts of particular types.

This section concludes by examining the relations to several works that focus on the biases exhibited by agents toward a particular platform. Among those works, Hałaburda and Yehezkel (2016) are notable in that they allow for the possibility of some buyers (not sellers, though) being loyal to a certain platform as well. Hałaburda and Yehezkel (2016) apply Narasimhan's (1988) model and regard loyalty as responses to horizontal product differentiation such that buyers loyal to a particular platform choose either it or nothing to join with rational expectations of how many sellers participate in the platform. This chapter considers biases not to depend on agents' tastes for platforms' stand-alone services but to arise in the expectations formed by certain potential users, different from loyalty in Hałaburda and Yehezkel's (2016) context. Second, several papers describe biases that constitute a certain platform's advantage in more direct ways. Vasconcelos (2015) constructs a one-period model of duopolistic competition with predatory pricing in which a specific platform behaves the incumbent one that possesses an installed base treated as given. Gold and Hogendorn (2016) and White and Weyl (2016) alter Armstrong's (2006) duopoly model such that each platform incurs a different marginal cost. Those papers by assumption make payoff (profit or utility) functions directly biased toward a specific platform, which should be distinguished from my approach in that I incorporate biases not to objective functions but to expectations. Zennyo (2016) analyzes the situation in which platforms for developers and consumers of software are vertically differentiated such that developers incur higher costs and consumers obtain higher benefits as platform quality increases.[8] All papers mentioned in this paragraph presume that biases intrinsicly exist in platforms' or potential users' payoff functions, which means that those biases constitute part of social welfare as an objective function and affect the value of the first-best solution. On the other hand, biases in this chapter have no impact on social welfare or welfare maximization because the chapter formulates those biases as expectations oriented to a specific platform. This difference plays a non-negligible role in the welfare analysis of the market outcome and in its implications from a policy perspective.

[8]Hałaburda and Yehezkel (2016) also allow platforms to be vertically differentiated on the buyer side; however, this feature plays a less decisive role than the two features explained above in the market outcome in Hałaburda and Yehezkel (2016).

4.3 Model

This section constructs a model of a duopolistic two-sided market that allows some potential platform users to form biased expectations of the opposite-side allocation. The first subsection explains the structure of this market and formalizes biased expectations. The next two subsections formulate platform adoption and profit maximization, respectively, in this situation. The fourth subsection introduces a solution concept to describe the competitive outcome with biased expectations. The last subsection discusses social-welfare maximization.

4.3.1 Market Structure and Biased Expectations

There exists a duopolistic platform market of the Armstrong (2006) type (see the next few subsections for detailed formulations and notations). The market consists of two sides named A and B, each of which is occupied by a unit mass of economic agents who desire to interact (e.g., transact or communicate) with those on the other side. Two platforms, labeled 1 and 2, run intermediation services that enable interactions between their users on one and the other sides. Each potential user's utility from a platform thus increases as that platform attracts a larger number of agents on the opposite side. In other words, a positive indirect network externality is exerted on each side of the market. Platforms also provide their users with stand-alone services, from which the benefits do not depend on the number of opposite-side platform users. These services are assumed to bring such high benefits that all agents obtain nonnegative payoffs by using either platform (i.e., the market is fully covered).[9] Platforms horizontally differentiate their stand-alone services as in (pricing-only) Hotelling models: both platforms are positioned at the respective corners of a unit interval $[0, 1]$ in which the potential users are uniformly located, and the distance between a platform and an agent represents the difference between the product characteristics possessed by that platform and favored by that agent. Under this setting, the game proceeds as follows. Each platform simultaneously chooses and announces a

[9] Armstrong (2006) actually does not explicitly express the benefit from a stand-alone service in the definition of an agent's utility although assuming that the market is fully covered in equilibrium.

pair of side-A and side-B lump-sum prices such that its total profit is maximized. Each agent on each side simultaneously selects the (single) platform to yield the highest payoff for that agent after both platforms announce their prices. Here, one should particularly note that all potential users need to expect the allocation on the opposite side at the time of their decision making because they simultaneously make platform adoption with those on the opposite side.

The main difference between Armstrong (2006) or his subsequent papers and this chapter is that I formulate the situation in which some agents form biased allocation expectations. At the beginning, suppose that potential users exhibit the expectation patterns introduced in Hagiu and Hałaburda (2014). There coexist agents who exhibit two expectation patterns on side A. First, $\rho \in (0, 1)$ potential users form rational expectations in the sense that they can accurately calculate the number of opposite-side users based on the information of the opposite-side market demand functions and all announced prices. The other $(1 - \rho)$ agents, however, cannot form such a rational expectation but do simply hold agent-common fixed expectations. On side B, all potential users form rational expectations in the above sense. Assume that the distribution of expectation patterns is known by all players, and that the expectation patterns and the agent locations are mutually independent.[10] Hagiu and Hałaburda (2014) originally place an additional assumption that requires all fixed expectations to be fulfilled (i.e., to equal the realized market shares).[11] This chapter, however, deviates from such a fulfillment assumption by allowing these agents to form biased expectations in the following sense. First, these agents expect platform 1 to attract all potential users on side B, which is an adaptation of the treatment for agents' expectations adopted in Caillaud and Jullien (2001, 2003), Hagiu (2006), and their subsequent works. Second, these expectations can be inconsistent

[10]Hagiu and Hałaburda (2014) also place these assumptions, formulating the side with two expectation patterns in a similar way to Armstrong (2006) and this chapter.

[11]As Hagiu and Hałaburda (2014) argue, both expectation patterns cause rational expectations in the sense of consistency with the realized outcome but different in whether potential users of interest can respond to price changes on the other side while forming their expectations. This categorization cannot necessarily apply in this chapter, where fixed expectations are allowed to be inconsistent as stated below. This chapter thus uses the word "rational" only for expectations of the first pattern in most cases and also for fixed expectations only when the rationality of those expectations is of interest.

with the realized side-B allocation, which crucially differs from those existing papers.[12]

It should be remarked again that potential users do not necessarily form rational expectations in this chapter. The concept of Nash equilibrium cannot always apply to describe the market outcome because it requires all expectations to be consistent with the actual number of platform users. This chapter therefore adopts a solution concept that relaxes Nash equilibrium by allowing some agents to hold inconsistent expectations at the time of their decision making, which is discussed later.

4.3.2 Demand Sides

This chapter formulates the demand sides as in a side-symmetric Armstrong (2006) model (Gold and Hogendorn 2016; White and Weyl 2016) but incorporates the concept of biased expectation discussed above to the formulation. The addition of this expectation concept makes no substantial difference in platform-adoption behavior and the allocation on each side given each potential user's allocation expectation, which are formalized in the next two paragraphs. The definition of market demand functions, however, significantly differs in that the existence of agents with biased expectations affects the value of the functions through the effects on the total allocation of side-A potential users, which is discussed in the last paragraph.

Begin by specifying side A. Suppose that a potential user located at $x \in [0,1]$ is choosing a platform. The potential user obtains a payoff of

$$u_1^A\left(p_1^A, n_1^B; x\right) = v + bn_1^B - p_1^A - tx$$

from platform 1 if the platform charges a participation fee of $p_1^A \in \mathbb{R}$ on the side and attracts $n_1^B \in [0,1]$ agents on side B, where $v \in \mathbb{R}_{++}$ is the value of the platform's stand-alone service, $b \in \mathbb{R}_{++}$ denotes a utility per cross-side interaction, and $t \in \mathbb{R}_{++}$ is the

[12]The combination of this and the preceding assumptions means that potential users with biased expectations do not consider actual market shares on the other side while choosing platforms, in which sense the assumption that such potential users can also know the distribution of expectation types does not matter although it might sound weird.

parameter for transportation costs.[13] The payoff from platform 2 equals

$$u_2^A\left(p_2^A, n_2^B; x\right) = v + bn_2^B - p_2^A - (1-x)\,t,$$

where $p_2^A \in \mathbb{R}$ and $n_2^B \in [0,1]$ are analogously defined. Here, v, b, and t are exogenous parameters common between both platforms and among all agents. The potential user selects the platform that maximizes his/her payoff expecting platforms 1 and 2 to attract $\widehat{n}_1^B \in [0,1]$ and $\widehat{n}_2^B \in [0,1]$ side-B agents, respectively, and observing the prices announced by both platforms. Let $d_1^A(p_1^A, p_2^A; \widehat{n}_1^B)$ and $d_2^A(p_2^A, p_1^A; \widehat{n}_2^B)$ denote the proportions of potential users who choose platforms 1 and 2, respectively, given the expectation pair $(\widehat{n}_1^B, \widehat{n}_2^B)$.[14] The threshold location on the side given $(\widehat{n}_1^B, \widehat{n}_2^B)$ is such x that

$$\begin{aligned} & u_1^A\left(\widehat{n}_1^B, \widehat{n}_2^B; x\right) = u_2^A\left(\widehat{n}_2^B, \widehat{n}_1^B; x\right) \\ \iff & x = \frac{1}{2} + \frac{\left(\widehat{n}_1^B - \widehat{n}_2^B\right) b + \left(p_2^A - p_1^A\right)}{2t} \equiv \widetilde{x}^A\left(p_1^A, p_2^A; \widehat{n}_1^B, \widehat{n}_2^B\right). \end{aligned}$$

The proportion of potential users with $(\widehat{n}_1^B, \widehat{n}_2^B)$ who join platform 1 equals

$$d_1^A\left(p_1^A, p_2^A; \widehat{n}_1^B, \widehat{n}_2^B\right) = \begin{cases} 0 & \widetilde{x}^A(\cdot) < 0 \\ \widetilde{x}^A\left(p_1^A, p_2^A; \widehat{n}_1^B, \widehat{n}_2^B\right) & 0 \le \widetilde{x}^A(\cdot) \le 1 \\ 1 & \widetilde{x}^A(\cdot) > 1. \end{cases}$$

The assumption of full coverage implies that

$$d_2^A\left(p_2^A, p_1^A; \widehat{n}_2^B, \widehat{n}_1^B\right) = 1 - d_1^A\left(p_1^A, p_2^A; \widehat{n}_1^B, \widehat{n}_2^B\right).$$

The specification of side B is similar to that of side A. Let the three exogenous

[13]Notice that $u_1^A(\cdot)$ is expressed as a function of the platform's actual market share on the other side, whichever expectation the potential user forms, to define the function as the payoff eventually enjoyed by the potential user from the platform. This distinction is important in the welfare analysis because the payoffs for agents constitute part of social welfare. This definition of payoff functions also helps one evaluate the impacts of biased expectations on the coordination that occurs as a consequence of the platform competition modeled in this section.

[14]The number of platform users thus equals the weighted mean of proportions given expectations.

parameters defined above, v, b, and t, be common between both sides as well. The definitions of $p_1^B \in \mathbb{R}$, $p_2^B \in \mathbb{R}$, $\widehat{n}_1^A \in [0,1]$, $\widehat{n}_2^A \in [0,1]$, $d_1^B(p_1^B, p_2^B; \widehat{n}_1^A, \widehat{n}_2^A)$, and $d_2^B(p_2^B, p_1^B; \widehat{n}_2^A, \widehat{n}_1^A)$ are analogous.

The market demand for each platform is obtained now. Notice that the mean of expectations formed on each side matters to characterize the side-level allocation on the same side. The mean of expectations formed by side-A agents with regard to platform 1's market share on side B equals

$$\begin{aligned} \mathrm{E}\left[\widehat{n}_1^B\right] &= \rho n_1^B + (1-\rho)\cdot 1 \\ &= 1 - \left(1 - n_1^B\right)\rho, \end{aligned} \tag{4.1}$$

where $n_1^B \in [0,1]$ depends on the side-B prices (because the variable denotes platform 1's actual side-B market share). The mean of expectations held by side-A potential users regarding platform 2's market share on side B is

$$\mathrm{E}\left[\widehat{n}_2^B\right] = 1 - \mathrm{E}\left[\widehat{n}_1^B\right] \tag{4.2}$$

because rational agents know the full coverage of side B and the others expect platform 1 to attract all agents. The mean of expectations formed by side-B potential users with regard to each platform's market share on side A is consistent with its actual value:

$$\mathrm{E}\left[\widehat{n}_1^A\right] = n_1^A \qquad \mathrm{E}\left[\widehat{n}_2^A\right] = 1 - n_1^A.$$

The market demand functions are therefore $D_1^A(p_1^A, p_2^A; p_1^B, p_2^B)$, $D_2^A(p_2^A, p_1^A; p_2^B, p_1^B)$, $D_1^B(p_1^B, p_2^B; p_1^A, p_2^A)$, and $D_2^B(p_2^B, p_1^B; p_2^A, p_1^A)$, which are defined as n_1^A, n_2^A, n_1^B, and n_2^B, respectively, such that

$$n_1^A = \mathrm{E}\left[d_1^A\left(\cdot; \widehat{n}_1^B, \widehat{n}_2^B\right)\right] \qquad n_2^A = \mathrm{E}\left[d_2^A\left(\cdot; \widehat{n}_2^B, \widehat{n}_1^B\right)\right] \tag{4.3}$$

$$n_1^B = d_1^B\left(\cdot; n_1^A, n_2^A\right) \qquad n_2^B = d_2^B\left(\cdot; n_2^A, n_1^A\right) \tag{4.4}$$

under equations (4.1) and (4.2).

4.3.3 Platforms

Each platform maximizes its profit with respect to its prices. First, I formulate platform 1's profit maximization. The platform earns a total profit of

$$\pi_1\left(p_1^A, p_1^B; p_2^A, p_2^B\right) \equiv p_1^A D_1^A\left(p_1^A, p_2^A; p_1^B, p_2^B\right) + p_1^B D_1^B\left(p_1^B, p_2^B; p_1^A, p_2^A\right),$$

which can be interpreted as follows. On side A, the platform charges each agent a lump-sum fee of p_1^A and attracts $D_1^A(\cdot)$ agents. The platform attracts and charges side-B potential users analogously. The marginal and fixed costs are normalized to zero for the platform. Platform 1 thus faces the following profit-maximization problem:

$$\max_{\left(p_1^A, p_1^B\right)} \pi_1\left(p_1^A, p_1^B; p_2^A, p_2^B\right) \quad \text{given} \ \left(p_2^A, p_2^B\right),$$

which is solved by $(p_1^A(p_2^A, p_2^B), p_1^B(p_2^B, p_2^A))$. Second, platform 2's profit maximization can be formalized analogously. Let $\pi_2(p_2^A, p_2^B; p_1^A, p_1^B)$ denote the platform's profit, and define it as symmetric to platform 1's. Platform 2 chooses $(p_2^A(p_1^A, p_1^B), p_2^B(p_1^B, p_1^A))$ to maximize its profit. Notice that neither platform earns a strictly negative profit under this setting because each platform can obtain zero profit by attracting no potential user.

Suppose that each platform's profit maximization yields an interior solution. I begin by stating the first-order and second-order conditions for platform 1's problem. The first-order conditions are that

$$\frac{\partial \pi_1(\cdot)}{\partial p_1^A} = 0 \qquad \frac{\partial \pi_1(\cdot)}{\partial p_1^B} = 0.$$

The second-order conditions consist of three inequalities:

$$\frac{\partial^2 \pi_1(\cdot)}{\partial\left(p_1^A\right)^2} < 0 \qquad \frac{\partial^2 \pi_1(\cdot)}{\partial\left(p_1^B\right)^2} < 0$$

$$H_1\left(p_1^A, p_1^B; p_2^A, p_2^B\right) \equiv \frac{\partial^2 \pi_1(\cdot)}{\partial \left(p_1^A\right)^2} \cdot \frac{\partial^2 \pi_1(\cdot)}{\partial \left(p_1^B\right)^2} - \frac{\partial^2 \pi_1(\cdot)}{\partial p_1^A \partial p_1^B} \cdot \frac{\partial^2 \pi_1(\cdot)}{\partial p_1^B \partial p_1^A} > 0,$$

where $H_1(\cdot)$ denotes the determinant of the Hessian matrix derived from the platform's profit. The first-order and second-order conditions for platform 2's profit maximization are analogous, where the determinant of the Hessian matrix is denoted by $H_2(p_2^A, p_2^B; p_1^A, p_1^B)$.

There exist three possible patterns of a corner solution to profit maximization. The first two are that the prices derived from the first-order conditions result in a corner allocation of agents although the second-order conditions hold, which occurs because biased expectations make the allocation asymmetric and the number of potential users is finite. Specifically, the first pattern is that a particular platform attracts all of the agents with biased expectations although the entire allocation remains interior on each side, in which case the analysis proceeds by adding the assumption that those agents prefer the platform because both platforms may compete for potential users with rational expectations. The second pattern is that a platform attracts all potential users on either side or dominates the entire market, in which case (i) the dominant platform chooses a price on the dominated side such that agents who obtain the lowest payoffs from the platform are indifferent and (ii) the other platform adopts the price strategy on that side based on the first-order and second-order conditions.[15] The third pattern is that the second-order condition with regard to a Hessian matrix no longer holds because the cross-side price effects are too large. The reason for the occurrence of this pattern does not differ from those in standard Armstrong (2006) and his subsequent papers, which especially include Gold and Hogendorn (2016) and White and Weyl (2016), except that the platform with a disadvantage from biased expectations is unlikely to dominate the entire market in this chapter because it cannot easily attract agents with such expectations. This chapter therefore omits a discussion on this pattern to focus on the first two.

[15]Gold and Hogendorn (2016) and White and Weyl (2016) address a corner allocation of this pattern under similar frameworks of the demand sides except that they assume all agents to form rational expectations, consider an asymmetric allocation to arise the cost difference between platforms, and do not explicitly investigate the market outcome of the pattern but focus on the condition for the market allocation to remain interior.

4.3.4 Market Outcome

Consider the $(p_1^{A*}, p_1^{B*}; p_2^{A*}, p_2^{B*})$ and $(n_1^{A*}, n_2^{A*}; n_1^{B*}, n_2^{B*})$ defined by the twofold process below. First, $(p_1^{A*}, p_1^{B*}; p_2^{A*}, p_2^{B*})$ denotes the $(p_1^A, p_1^B; p_2^A, p_2^B)$ that solves the following system:

$$p_1^A = p_1^A\left(p_2^A, p_2^B\right) \qquad p_1^B = p_1^B\left(p_2^B, p_2^A\right) \tag{4.5}$$

$$p_2^A = p_2^A\left(p_1^A, p_1^B\right) \qquad p_2^B = p_2^B\left(p_1^A, p_1^B\right), \tag{4.6}$$

where each platform maximizes its profit with the rational expectation of the other platform's price strategy and the knowledge about the distribution of expectation patterns on each side. Second, $(n_1^{A*}, n_2^{A*}; n_1^{B*}, n_2^{B*})$ is defined as

$$n_1^{A*} \equiv D_1^A\left(p_1^{A*}, p_2^{A*}; p_1^{B*}, p_2^{B*}\right) \qquad n_2^{A*} \equiv D_2^A\left(p_2^{A*}, p_1^{A*}; p_2^{B*}, p_1^{B*}\right) \tag{4.7}$$

$$n_1^{B*} \equiv D_1^B\left(p_1^{B*}, p_2^{B*}; p_1^{A*}, p_2^{A*}\right) \qquad n_2^{B*} \equiv D_2^B\left(p_2^{B*}, p_1^{B*}; p_2^{A*}, p_1^{A*}\right), \tag{4.8}$$

where all potential users choose platforms such that their payoffs are maximized given their expectations of market shares. This pair of prices and market shares constitutes a competitive outcome, which solves the competition game.

The market outcome defined above does not necessarily constitute a Nash equilibrium because some potential users may hold inconsistent allocation expectations with the realizations. This chapter thus adopts a solution concept relaxed such that potential users are not required to form rational expectations, retaining the principle that all players maximize the respective objective functions. Nevertheless, I should also remark that this solution concept can yield an equilibrium if all potential users form consistent expectations, which case occurs if and only if platform 1 attracts all agents on side B.[16]

[16]The competitive outcome can almost be considered a Nash equilibrium as $\rho \to 1$, corresponding to Armstrong (2006) and other papers that construct Armstrong (2006) models; however, this chapter does not precisely treat that outcome as an equilibrium because the set of ρ is an open interval whose upper bound equals one.

4.3.5 Social-Welfare Maximization

Welfare maximization should be formulated as the problem to obtain a first-best outcome from a perspective of social welfare. Potential users with biased expectations are assumed in the competition game to make platform choices that depend on their own market-share expectations but supposed (like those with rational expectations) to receive payoffs that are determined based on the actual opposite-side allocation. The welfare impact of the total production cost can be ignored in this chapter because the marginal and fixed costs are normalized to zero for each platform and thus independent of which I calculate them based on each agent's expectations or the actual allocations. This chapter therefore determines social welfare as the sum of each player's final payoff and defines a first-best outcome as a pair of allocations that maximizes social welfare in this sense, following the formulation that the literature on two-sided markets usually adopts.

Thus, social welfare and welfare maximization are formulated as those in White and Weyl's (2016) simplified version of the Armstrong (2006) model.[17] Social welfare in this chapter is expressed as the sum of total surplus on each side of the market. Total surplus on side A equals

$$w^A\left(n_1^A, n_1^B\right) \equiv v + \left[n_1^A n_1^B + \left(1 - n_1^A\right)\left(1 - n_1^B\right)\right] b - \left[\int_0^{n_1^A} x \mathrm{d}x + \int_{n_1^A}^1 (1 - x) \, \mathrm{d}x\right] t$$

because all agents on each side should be allocated such that those located in $[0, \widetilde{x}^A]$ (on side A) or $[0, \widetilde{x}^B]$ (on side B) choose platform 1 and all of the others should join platform 2 under the assumption that v guarantees full coverage to be efficient on both sides, where $n_2^A = 1 - n_1^A$ and $n_2^B = 1 - n_1^B$, and $\widetilde{x}^A$ and $\widetilde{x}^B$ denote the threshold locations on the respective sides.[18] Define analogously $w^B(n_1^B, n_1^A)$ as total surplus on side B. Therefore,

[17] White and Weyl (2016) cover the situation considered in my model except that they assume all potential users to form rational expectations and introduce the equilibrium concept for duopoly models in which each platform announces two-part tariffs of a special type. The differences in equilibrium and pricing concepts do not affect the welfare analysis in this chapter in the sense that (i) an equilibrium concept applies not to welfare maximization but only to platform competition and (ii) the total payment incurred by agents cancels out the total revenue obtained by platforms. The formulations of social welfare and welfare maximization in this chapter are thus equivalent to those in White and Weyl (2016) although the welfare implications are different.

[18] Although the utility function for each agent is defined the same, Gold and Hogendorn (2016) consider a different surplus structure in that they presume platforms to incur asymmetric marginal costs.

social welfare equals

$$
\begin{aligned}
W\left(n_1^A, n_1^B\right) &\equiv w^A\left(n_1^A, n_1^B\right) + w^B\left(n_1^B, n_1^A\right) \\
&= 2v + 2\left[n_1^A n_1^B + \left(1 - n_1^A\right)\left(1 - n_1^B\right)\right] b \\
&\quad - \left[\int_0^{n_1^A} x \mathrm{d}x + \int_0^{n_1^B} x \mathrm{d}x + \int_{n_1^A}^1 (1-x)\,\mathrm{d}x + \int_{n_1^B}^1 (1-x)\,\mathrm{d}x\right] t.
\end{aligned}
$$

Welfare maximization is formalized as the following optimization problem:

$$
\max_{\left(n_1^A, n_1^B\right)} W\left(n_1^A, n_1^B\right).
$$

Let (n_1^{A**}, n_1^{B**}) denote a solution to this problem. The efficient market shares on sides A and B for platform 2 are defined such that $n_2^{A**} \equiv 1 - n_1^{A**}$ and $n_2^{B**} \equiv 1 - n_1^{B**}$, respectively.

The above problem can be solved by multiple steps.[19] First of all, examine the first-order and second-order conditions for that problem. The first-order conditions are that

$$
\frac{\partial W(\cdot)}{\partial n_1^A} = 0 \qquad \frac{\partial W(\cdot)}{\partial n_1^B} = 0.
$$

The second-order conditions consist of the following three inequalities:

$$
\begin{aligned}
&\frac{\partial^2 W(\cdot)}{\partial \left(n_1^A\right)^2} < 0 \qquad \frac{\partial^2 W(\cdot)}{\partial \left(n_1^B\right)^2} < 0 \\
&H_0\left(n_1^A, n_1^B\right) \equiv \frac{\partial^2 W(\cdot)}{\partial \left(n_1^A\right)^2}\frac{\partial^2 W(\cdot)}{\partial \left(n_1^B\right)^2} - \frac{\partial^2 W(\cdot)}{\partial n_1^A \partial n_1^B} \cdot \frac{\partial^2 W(\cdot)}{\partial n_1^B \partial n_1^A} > 0,
\end{aligned}
$$

where $H_0(\cdot)$ denotes the determinant of the Hessian matrix derived from social welfare. However, the second-order condition with regard to the Hessian matrix of social welfare can be violated by some sets of the parameters that satisfy all second-order conditions for profit maximization. The welfare-maximization problem in this case needs to be solved

[19] White and Weyl (2016) adopt a different way to solve welfare maximization in that they do not explicitly examine a first-order or second-order condition but use the fact that social welfare is maximized if a platform dominates the entire market or attracts 1/2 agents on each side and investigate which case is efficient.

on the case-by-case basis. To keep this section concise, the process to solve the problem in that case appears in the proof of Proposition 4.3 (Appendix 4.A.3).

4.4 Market Outcome with Biased Expectations

This section discusses the market outcome when some potential users form biased expectations. The first subsection analyzes the case in which the allocations on side A, on side B, and of agents with biased expectations are weakly interior, and especially examines the relation between the proportion of potential users who form biased expectations and the existence of such an interior outcome. The second subsection focuses on the case of a market outcome such that all of the agents with biased expectations prefer a certain platform, in which case I show that all of the agent groups exhibit corner allocations. The processes to derive and investigate the market outcome correspond to those in a standard equilibrium analysis except that some agents hold biased expectations.

4.4.1 Interior Competitive Outcome

This subsection begins by deriving a competitive outcome in the case of an interior solution. The interiority of a market outcome is characterized by two properties. The first is that the resulting price strategies satisfy all of the first-order and second-order conditions for profit maximization. The second property is that the allocations of side-A agents, side-B agents, and agents with biased expectations are interior. The proposition below states the value and major properties of the outcome.

Proposition 4.1. There arises an interior competitive outcome such that

$$p_1^{A*} = (t-b) + 2\left(t\epsilon^{A*} - b\epsilon^{B*}\right) \qquad p_1^{B*} = (t-b\rho) + 2\left(t\epsilon^{B*} - b\rho\epsilon^{A*}\right)$$
$$p_2^{A*} = (t-b) - 2\left(t\epsilon^{A*} - b\epsilon^{B*}\right) \qquad p_2^{B*} = (t-b\rho) - 2\left(t\epsilon^{B*} - b\rho\epsilon^{A*}\right)$$

regarding the prices and

$$n_1^{A*} = \frac{1}{2} + \epsilon^{A*} \qquad n_2^{A*} = \frac{1}{2} - \epsilon^{A*}$$
$$n_1^{B*} = \frac{1}{2} + \epsilon^{B*} \qquad n_2^{B*} = \frac{1}{2} - \epsilon^{B*}$$

regarding the market shares, where

$$\epsilon^{A*} \equiv \frac{3\left(t^2 - b^2\rho\right)\left(1-\rho\right)bt}{2\left\{9t^4 - 2\left[\left(\rho+7\right)\rho+1\right]b^2t^2 + \left(2\rho+1\right)\left(\rho+2\right)b^4\rho\right\}} \in \left(0, \frac{1}{2}\right)$$
$$\epsilon^{B*} \equiv \frac{\left(t^2 - b^2\rho\right)\left(2\rho+1\right)\left(1-\rho\right)b^2}{2\left\{9t^4 - 2\left[\left(\rho+7\right)\rho+1\right]b^2t^2 + \left(2\rho+1\right)\left(\rho+2\right)b^4\rho\right\}} \in \left(0, \epsilon^{A*}\right),$$

under the condition that

$$4\left(t\epsilon^{A*} - b\epsilon^{B*}\right) + \left(t-b\right) \geq 0. \tag{4.9}$$

This outcome arises if $t \geq b$ and is a unique solution to the competition game if condition (4.9) holds as a strict inequality. If $t < b$, there exists $\underline{\rho} \in (0,1)$ such that (i) condition (4.9) is violated for any $\rho \geq \underline{\rho}$ and (ii) $\partial\underline{\rho}/\partial b < 0$.

Proof. See Appendix 4.A.1. □

This proposition shows a market-outcome configuration in which platform 1 takes an advantage derived from biased expectations. First, platform 1 attracts a larger number of potential users on side A than on side B and obtains a larger market share on each side than platform 2: $n_1^{A*} > n_1^{B*}$, $n_1^{A*} > n_2^{A*}$, and $n_1^{B*} > n_2^{B*}$. Platform 1 can attract a larger number of agents on side A because the side includes potential users with biased expectations, who expect to obtain higher network benefits from that platform. The platform can also obtain a larger market share on side B because a stronger indirect network externality is exerted through that platform on that side. The relation that $n_1^{A*} > n_1^{B*}$ shows that platform 1's advantage from biased expectations exceeds its advantage from the stronger externality on side B. Second, look through the arising prices. The realized price of each platform on each side comprises two components of the Armstrong (2006) type with zero marginal cost (e.g., White and Weyl 2016) although each component is adjusted by the existence of biased expectations. The first is a component that consists of the first and third terms, which expresses each platform's market power of the Hotelling type derived from product differentiation and platform 1's advantage from biased expectations. The second component is the sum of the second and fourth terms. This component describes a cross-side subsidy effect derived from the indirect network externality on the other side but adjusted by biased expectations. The component on side B is discounted by the parameter ρ because potential users with biased expectations, who exist on the opposite side (i.e., side A), do not consider the market shares that are eventually realized on side B at the time of their decision making. Moreover, one can see platform 1 exploiting its advantage from biased expectations in pricing. This argument with regard to side A is proven by the following calculation:[20]

$$p_1^{A*} - p_2^{A*} = 4\left(t\epsilon^{A*} - b\epsilon^{B*}\right) \begin{cases} = 4\left[\left(\epsilon^{A*} - \epsilon^{B*}\right)t + (t-b)\,\epsilon^{B*}\right] > 0 & \text{if } t \geq b \\ \geq -(t-b) > 0 & \text{if } t < b. \end{cases}$$

As for side B,

$$\begin{aligned} p_1^{B*} - p_2^{B*} &= 4\left(t\epsilon^{B*} - b\rho\epsilon^{A*}\right) \\ &= \frac{2\left(t^2 - b^2\rho\right)(1-\rho)^2 b^2 t}{9t^4 - 2\left[(\rho+7)\rho+1\right]b^2t^2 + (2\rho+1)(\rho+2)\,b^4\rho} > 0, \end{aligned}$$

where the condition that $t^2 - b^2\rho > 0$ is satisfied under expression (4.9) (see the proof) and guarantees finite positive price effects. Recall that the market outcome discussed here is interior: platform 1 attracts a larger number of potential users in the entire market but misses some agents on both sides and even in the group of agents with biased expectations. The platform therefore selects specific groups of potential users among those both with biased expectations and with rational expectations; and then platform 2 can attract the agents missed by platform 1. The result that platform 2 attracts agents with biased expectations notably distinguishes this chapter from Hałaburda and Yehezkel (2016), who analyze the situation in which some potential users are loyal to a specific

[20] The result in the latter case can be derived from condition (4.9).

platform, in the sense that the latter abstract the possibility of those potential users choosing the other platform.

I now proceed with a discussion on inequality (4.9) as a sufficient condition to induce an interior market outcome. First, the inequality expresses the condition under which the proportion of platform-1 users among agents with biased expectations does not exceed one. Second, each side of the market needs to exhibit an interior allocation for the market outcome to be interior. The side-A part of this condition is satisfied using the relation that $(1/2 <)n_1^{A*} < d_1^A(p_1^{A*}, p_2^{A*}; 1, 0) \leq 1$ because $\widehat{n}_1^B > n_1^{B*}$ for agents who form biased expectations, which also guarantees that the proportion of platform-1 users among agents with biased expectations cannot be negative. The side-B part is met because $1/2 < n_1^{B*} < n_1^{A*} < 1$. Third, the second-order conditions for each platform's profit maximization also need to hold. All own-variable conditions are always satisfied. The condition with regard to the Hessian matrix of each platform's profit function is that

$$t > \frac{b}{2}\left(\rho + 1\right).$$

This condition has a similar property to that in side-symmetric Armstrong (2006) models (Gold and Hogendorn 2016; White and Weyl 2016) in the relation between the degree of product differentiation (t) and the impacts of the indirect network externality exerted on each side (b); however, the condition in this chapter differs in that the chapter allows for biased expectations discounting the price effects related to the externality. One can establish through technical calculation (see the proof) that the condition holds if expression (4.9) is met on the one hand and that the latter is not necessarily satisfied even if the former holds on the other hand. All of the conditions for the existence of an interior competitive outcome can therefore be represented by a single inequality expressed as condition (4.9). In addition, the uniqueness of such an outcome is also guaranteed unless the condition holds as an equality, where the group of agents with biased expectations exhibits an interior allocation that may also be treated as a corner allocation.

Moreover, Proposition 4.1 establishes two statements on whether the interior competitive outcome can arise. The first statement is a twofold condition: platform 1 attracts all of the potential users with biased expectations only if the impacts of the indirect network externality exerted on each side are large enough ($t < b$), and such dominance is more likely to occur as the number of them is *smaller* (ρ equals a particular value or *higher*). Platform 1 takes a weaker advantage from biased expectations as the number of agents with such expectations decreases, which encourages the platform to announce a lower price on side A in order to attract a sufficient number of potential users.[21] Platform 2 then obtains an incentive to choose a lower price on that side as a response, which has an additional effect to reduce the price of platform 1 on the side. This price reduction is more severe as the indirect network externalities exerted on both sides, which determine the cross-side effects of changing prices, are more intense (b is higher) or platform 1 takes a weaker advantage from biased expectations (ρ is *higher*). If the indirect network externality exerted on each side is sufficiently strong and the number of agents with biased expectations is sufficiently large, therefore, platform 1 chooses a low enough price for all of the potential users with biased expectations to join the platform. The second statement is that the threshold number of agents with biased expectations (the infimum of ρ) to induce such dominance increases as the indirect network externality exerted on each side becomes stronger (b increases); thus, platform 1 chooses this type of dominance for various numbers of those agents in the case of sufficiently strong indirect network externalities.

4.4.2 Corner Market Outcome

Consider the situation in which the second-order conditions for profit maximization would hold if Proposition 4.1 were the case but expression (4.9) no longer holds. All of the potential users with biased expectations are supposed in this situation to prefer platform 1, which plays a role as a strong advantage for that platform.[22] The following proposition

[21] Note here that lowering the side-A price of platform 1 stimulates the market demand for the platform not only on the same side but also on the other side.

[22] Some existing papers also analyze the case in which some agents are supposed to use a specific platform. Hossain and Morgan (2013) allow for random platform adoption in the context of bounded rationality (low-level cognition), and Vasconcelos (2015) presume the existence of an installed base in the context of predatory pricing. However, it is a significant difference from those papers that the agents with biased expectations in this chapter *choose* platform 1 to maximize their own payoffs.

shows the existence, uniqueness, and value of such a market outcome.

Proposition 4.2. If $t > (\rho+1)b/2$ and expression (4.9) is violated, there arises a unique market outcome such that

$$\begin{aligned} p_1^{A*} = p_1^{B*} = b - t \qquad & p_2^{A*} = p_2^{B*} = 0 \\ n_1^{A*} = n_1^{B*} = 1 \qquad & n_2^{A*} = n_2^{B*} = 0, \end{aligned}$$

which constitutes a Nash equilibrium.

Proof. See Appendix 4.A.2. □

This proposition establishes that platform 1 dominates the entire market if the impacts of the indirect network externality exerted on each side are not too large and the platform attracts all of the agents with biased expectations, which consists of three statements. First, platforms do not choose prices to induce an outcome such that $0 < n_1^A < 1$, $0 < n_1^B < 1$, $0 < n_2^A < 1$, and $0 < n_2^B < 1$. The second-order conditions for profit maximization in this case can be summarized as the condition that $\rho/(t^2 - b^2\rho) > 0$, which is derived from the condition with regard to the determinant of each Hessian matrix, and found to hold if those in the case of Proposition 4.1 are satisfied. Nevertheless, platform 1 has an incentive to utilize its advantage from biased expectations and choose prices that enable it to dominate at least one side of the market, which occurs because the mass of agents is finite on each side. Second, there is no possibility of platform 2 attracting an agent on any side. Platform 1 desires to attract all potential users on side B if it dominates side A, although the second-order conditions for its profit-maximization problem hold, due to the same reason as above. The platform gathers all agents on side A if it dominates side B because potential users with rational expectations on side A expect the same as those with biased expectations, all of whom choose the platform in the case of Proposition 4.2. Lastly, each platform has no incentive to deviate from such a price strategy that platform 1 can attract all potential users on both sides. Platform 1 keeps its price (i) on side A because it is expected to arise that $n_1^B = 1$ and (ii) on side B because the platform's marginal profit of reducing p_1^B in the case of Proposition 4.2

evaluated at the market outcome in question equals that in the case of Proposition 4.1. Platform 2, which chooses prices based on the first-order and second-order conditions, retains its prices on both sides because those prices satisfy those conditions in the case of platform 1's market dominance. In sum, both platforms choose prices such that platform 1 dominates both sides of the market not because a second-order condition is violated but due to the combination of platform 1's advantage from biased expectations and the market-size finiteness on each side.

Proposition 4.2 states that platform 1 can attract all potential users on both sides just because some agents have formed market-share expectations oriented to the platform. Several papers describe the situation in which two platforms face price competition under the assumption of rational expectations formed by agents and the market is entirely dominated by the platform that potential users on each side expect to attract all agents on the other side (see section 4.2 for details). However, those papers usually pay little attention to the question of which platform potential users expect to dominate the market and regard it as a matter of selection between multiple equilibria, whichever the agents on one (Ko and Shen 2016), both (e.g., Caillaud and Jullien 2003), or neither (White and Weyl 2016) side(s) have identical utility functions.[23] The above papers thus differ from this chapter in that the chapter treats biased expectations as an advantage for platform 1 regardless of whether those expectations are consistent. Some papers develop duopoly models in which a certain platform takes an advantage; an installed base (Vasconcelos 2015), a cost difference (Gold and Hogendorn 2016; White and Weyl 2016), and loyalty (Hałaburda and Yehezkel 2016) are examples for such an advantage. This type of advantage affects the objective function or the strategy space for a player directly and thus differs from an advantage based on biased expectations, which has no meaning in an objective function or a strategy space. This difference is especially notable in that the latter

[23] White and Weyl (2016) analyze a simplified Armstrong (2006) model as a benchmark and in Lemma 9 of their Online Appendix establish the existence of a corner equilibrium that looks similar to the market outcome stated in Proposition 4.2 of this chapter. However, the outcome derived by White and Weyl (2016) describes the situation in which the dominant platform chooses zero price on both sides and the dominated platform announces strictly negative participation fees. This difference might be interpreted such that biased expectations in this chapter help the advantageous platform (i.e., platform 1) dominate the entire market without severe proce reduction by assuring potential users its market dominance.

advantage does not affect the definition of social welfare unlike the former. Therefore, this chapter analyzes the case in which a particular platform takes an advantage based on expectations that are formulated from a more primitive perspective.

The proposition also establishes that the market outcome in the current case is a Nash equilibrium. As discussed in the preceding section, there can exist an equilibrium in my model only if platform 1 attracts all agents on side B. The competitive outcome in the case of Proposition 4.1 indeed never constitutes an equilibrium because the allocation on side B is always interior. On the other hand, the market outcome in the case of Proposition 4.2 expresses platform 1's dominance of side B. Notice that Hagiu and Hałaburda (2014) and Proposition 4.2 of this chapter analyze the case in which some potential users form fixed (price-independent) market-share expectations and those expectations are fulfilled. Hagiu and Hałaburda (2014) impose an addition assumption that requires fixed expectations to equal the respective realized market shares to make those expectations always rational. In this chapter, however, biased expectations are fulfilled not by assumption but as a consequence of platform. Hagiu and Hałaburda's (2014) expectation concept is therefore adapted in this chapter in the assumption placed on fixed expectations.

4.5 Welfare Implications

This section discusses the efficiency of the market outcome with biased expectations. The first subsection addresses welfare maximization and analyzes the efficient outcome(s). After that, I compare the market and efficient outcomes to investigate the welfare impacts of biased expectations.

4.5.1 Welfare Maximization

Consider the allocations that maximize social welfare. Recall that the formulations of social welfare and welfare maximization are the same as those in standard Armstrong (2006) models because each agent obtains a payoff based on not his/her allocation expectation but the actual opposite-side allocation. In particular, the following proposition states the efficient configurations of allocations established in White and Weyl (2016) except for that when $t = 2b$.

Proposition 4.3. The efficient allocations on both sides are characterized as follows.

1. If $(0 <)t < 2b$, $n_1^{A**} = n_1^{B**} = 1$ and $n_2^{A**} = n_2^{B**} = 0$ or $n_1^{A**} = n_1^{B**} = 0$ and $n_2^{A**} = n_2^{B**} = 1$.
2. If $t = 2b$, there exist a continuum of efficient allocation pairs such that $0 \leq n_1^{A**} = n_1^{B**} \leq 1$, $n_2^{A**} = 1 - n_1^{A**}$, and $n_2^{B**} = 1 - n_1^{B**}$.
3. If $t > 2b$, $n_1^{A**} = n_2^{A**} = n_1^{B**} = n_2^{B**} = 1/2$.

Proof. See Appendix 4.A.3. □

Begin the interpretation of this proposition with the case in which $t \neq 2b$. If $t < 2b$, a single platform should gather all agents on both sides because the second-order condition with regard to the Hessian matrix of social welfare is not satisfied. If $t > 2b$, both platforms should attract 1/2 agents on both sides, which is the pair of allocations that satisfies all first-order and second-order conditions for welfare maximization. These allocation configurations appear also in White and Weyl (2016) but suggest different welfare implications due to the existence of biased expectations in this chapter (see the next subsection).

Proposition 4.3 establishes that all side-symmetric allocation pairs are efficient if $t = 2b$. This result can be derived from the following logic. At the beginning, the first-order conditions for welfare maximization imply that $n_1^A = n_1^B$ and $n_2^A = n_2^B$. Second, I reformulate welfare maximization as the problem to maximize social welfare subject to $n_1^A = n_1^B = n_1$, $n_2^A = 1 - n_1$, and $n_2^B = 1 - n_1$, which any $n_1 \in [0, 1]$ solves because the first-order derivative of social welfare in this problem equals zero if $t = 2b$. The result is theoretically meaningful in that the continuum of solutions includes both corner and symmetric pairs of allocations; thus, the efficient outcome is continuous in the pair of b and t.

4.5.2 Efficiency of Platform Competition

I now examine the efficiency of the market outcome in the case of the second-order conditions for both platforms' profit-maximization problems (and the conditions for finite positive price effects on both sides) being satisfied. To focus on such a market outcome, the following assumptions are placed:

$$\begin{cases} \text{the parameters satisfy the condition that } t > \dfrac{b}{2}\left(\rho + 1\right), \text{ and} \\ \text{social welfare is maximized subject to } n_1^A \geq n_2^A. \end{cases} \tag{4.10}$$

The first assumption guarantees the aforementioned conditions to hold. The second assumption enables one to abstract an allocation such that platform 2 obtains a strictly larger market share, which can maximize social welfare but does never occur by platform competition under this chapter's setting. The following proposition summarizes the result of the comparison between the market and efficient outcomes.

Proposition 4.4. Under condition (4.10), the market outcome is efficient in the case of Proposition 4.2 and inefficient in the following sense in the case of Proposition 4.1.

1. If $t < 2b$ (except in the case of Proposition 4.2), $1/2 < n_1^{B*} < n_1^{A*} < n_1^{A**} = n_1^{B**}$ and $n_2^{A**} = n_2^{B**} < n_2^{A*} < n_2^{B*} < 1/2$.
2. If $t = 2b$, (i) $n_1^{A*} > n_1^{B*} = n_1^{A**} = n_1^{B**}$ and $n_2^{A*} < n_2^{B*} = n_2^{A**} = n_2^{B**}$ or (ii) $n_1^{B*} < n_1^{A*} = n_1^{A**} = n_1^{B**}$ and $n_2^{B*} > n_2^{A*} = n_2^{A**} = n_2^{B**}$.
3. If $t > 2b$, $n_1^{B*} < n_1^{A*} < 1/2 = n_1^{A**} = n_1^{B**}$ and $n_2^{A*} < n_2^{B*} < 1/2 = n_2^{A**} = n_2^{B**}$.

Proof. See Appendix 4.A.4. □

This proposition states that the market outcome is efficient only in the case of Proposition 4.2. The latter proposition shows a market outcome such that platform 1 attracts all potential users on both sides of the market. Proposition 4.1 implies that such an outcome can arise only if $t < b$, in which case Proposition 4.3 establishes that social welfare is maximized if one platform gathers all agents on both sides. The market outcome in the case of Proposition 4.2 is thus efficient from a welfare viewpoint.

Proposition 4.4 also establishes that the competitive outcome in the case of Proposition 4.1 is inefficient and that the reason for inefficiency differs according to the value of (b, t). Consider the case in which $t < 2b$ under condition (4.10). Proposition 4.1 states that platform 1 attracts a larger number of potential users on each side than platform 2 but dominates neither side. However, again, Proposition 4.3 shows that a single platform should dominate both sides. The competitive outcome in this case is inefficient in the sense that the advantageous platform obtains a smaller market share than it should on each side. Next, suppose that $t = 2b$. Proposition 4.1 establishes that platform 1 attracts a larger number of potential users on side A than on side B. On the other hand, Proposition 4.3 states that each platform should equalize its side-A and side-B market shares. The market outcome in this case is inefficient because the side-A and side-B market shares differ for both platforms. Suppose, lastly, that $t > 2b$. Proposition 4.1, as discussed above, shows that platform 1 attracts a larger number of agents on each side than platform 2. However, Proposition 4.3 establishes that social welfare is maximized if the allocations are symmetric on both sides. The competitive outcome in this case is inefficient in the sense that platform 1 attracts a larger number of agents than it should on each side.

This section concludes by obtaining the welfare implications of Proposition 4.4. Suppose at the beginning that almost no potential user holds a biased expectation, which approaches the situation in White and Weyl (2016). There arises a symmetric competitive outcome if $t > b$ and a corner market outcome on each side if $t < b$ (where the second-order condition with regard to a Hessian matrix is violated).[24] The market outcome is therefore efficient if $t < b$ or $t > 2b$ and inefficient if $b < t < 2b$. The above discussion enables one to investigate the welfare impacts of biased expectations. If $((\rho+1)b/2 <)t < b$, biased expactations cause welfare maximization in the case of Proposition 4.2 and make the market outcome less efficient in the case of Proposition 4.1; thus, the discussion in the preceding section implies that the number of agents with those expectations should be

[24]See Lemma 9 in White and Weyl's (2016) Online Appendix for a proof that establishes the existence of such a corner market outcome in the latter case, which exhibits a different property from what may arise in this chapter in general (as mentioned in footnote 23) but can almost apply to the current discussion because potential users with biased expectations have almost no impact on any market demand function.

sufficiently low to apply the former case. If $b < t < 2b$, such expectations might be considered to improve social welfare in that the allocation on each side is oriented to platform 1 although they do not maximize social welfare. If $t > 2b$, those expectations always make the market outcome less efficient due to an asymmetric allocation on each side. Biased expectations therefore induce an inefficient market outcome unless $b < t < 2b$.

4.6 Conclusion

This chapter investigates the market outcome and its efficiency in the case of duopolistic price competition between two-sided platforms that allows for biased expectations by some potential users toward a particular platform. In particular, the chapter adopts a solution concept for the competition game that relaxes Nash equilibrium to incorporate biased expectations because the latter concept does not allow each player to form an inconsistent expectation with the market outcome. The analysis of the competition game finds two different market-outcome patterns. The advantageous platform (i) attracts not all but a larger number of agents on each side in the case of moderate indirect network externalities and (ii) dominates the entire market if and only if indirect network externalities are strongly exerted on both sides and few potential users form biased expectations. Remarkably, the market outcome in the latter case describes the advantageous platform's expectation-driven dominance of both sides as a Nash equilibrium. The welfare analysis establishes that biased expectations cause the market outcome to maximize social welfare only in the case of market dominance; thus, this type of expectation may play a role in welfare reduction. These analyses can therefore be summarized such that biased expectations do or do not induce a certain platform's dominance of the entire market or social-welfare maximization depending on the market structure.

As the final remarks, there are two major limitations contained in this chapter. The first is that the model adopts a simplified formulation of the demand sides. The two sides are symmetrically characterized except that some agents form biased expectations on one side. This simplification abstracts each platform's cross-side price coordination that allows for the cross-side difference in agent characteristics, which is well known as one of the notable properties in two-sided markets. The simplification is, however, worth adopting in order to emphasize the impacts of biased expectations. On the biased side, I focus on the situation in which potential users with biased expectations are extremely oriented to a specific platform. This formulation of biased expectations could be questioned although the first section shows an example that might justify the formulation. Nevertheless, the assumption on the distribution of expectation patterns implies that the process to derive the market outcome would be unchanged and that its value would only quantitatively differ if the value of each agent's biased expectation were changed unless the number of expectation patterns increases or the biasless side also contains potential users with biased expectations. The second limitation is that all agents are restricted to use multiple platforms, or to multihome. This limitation might play a substantial role in the market outcome and its welfare implications because platforms would not actually engage in competition for multihomers but face more severe price competition for singlehomers (each of whom can only use a single platform) if multihoming were allowed. Relaxing the limitation could also make it difficult to justify why potential users with biased expectations form such extreme expectations if agents on the biasless side were able to join both platforms. By regarding this chapter's analysis as a first step, however, I consider the results under the singlehoming assumption meaningful. The remaining problems discussed above should be addressed in future research.

Appendix

4.A Proofs

4.A.1 Proof of Proposition 4.1

This proof is organized by four parts. First, I derive the market demand functions for both platforms on both sides and the corresponding price effects. Second, the first-order and second-order conditions for each platform's profit maximization are identified. Third, I calculate the differences between platform 1's and platform 2's resulting market shares on both sides, and find on which side the difference is larger. Lastly, the proof concludes by characterizing a competitive outcome and establishing several properties of the outcome. The proof assumes that the resulting allocations on side A, on side B, and of agents who form biased expectations are interior, which includes the case of demand that exactly equals 1 when all prices are derived from the first-order and second-order conditions for profit maximization.

Market Demand Functions and Price Effects

This part begins with the demand equations. The demand equation for platform 1 on side A comprises the demand by agents with rational expectations and that by those with biased expectations. The former demand equals

$$\begin{aligned} d_1^A\left(\cdot; n_1^B, n_2^B\right) &= \frac{1}{2} + \left[\left(n_1^B - n_2^B\right) b + \left(p_2^A - p_1^A\right)\right] \tau \\ &= \left[2bn_1^B + \left(p_2^A - p_1^A\right) - b\right] \tau + \frac{1}{2}, \end{aligned}$$

where $\tau \equiv 1/2t$ (which means that $t = 1/2\tau$) and the assumption of full coverage applies to the expression $n_1^B - n_2^B$. The latter demand is

$$d_1^A\left(\cdot; 1, 0\right) = \left[b + \left(p_2^A - p_1^A\right)\right] \tau + \frac{1}{2},$$

which only differs from $d_1^A(\cdot; n_1^B, n_2^B)$ in that the term with a coefficient of $b\tau$. The demand equation is the weighted mean of both expressions above:

$$\begin{aligned} n_1^A &= d_1^A\left(\cdot; n_1^B, n_2^B\right)\rho + d_1^A\left(\cdot; 1, 0\right)(1-\rho) \\ &= \left[2b\rho n_1^B + \left(p_2^A - p_1^A\right) - 2b\rho + b\right]\tau + \frac{1}{2}. \end{aligned} \tag{4.11}$$

The market demand for platform 1 on side B is characterized analogously to the demand by side-A agents with rational expectations as

$$n_1^B = \left[2bn_1^A + \left(p_2^B - p_1^B\right) - b\right]\tau + \frac{1}{2}. \tag{4.12}$$

One can immediately obtain the demand equations for platform 2 on sides A and B from the assumption of full coverage.

The market demand functions can be derived as a solution to equations (4.11) and (4.12) under the assumption that

$$t^2 - b^2\rho \neq 0. \tag{4.13}$$

The market demand function for platform 1 on side A is obtained as follows:

$$\begin{aligned} & n_1^A = 4b^2\tau^2\rho n_1^A + \left(p_2^A - p_1^A\right)\tau + 2\left(p_2^B - p_1^B\right)b\tau^2\rho - 2b^2\tau^2\rho - b\tau\rho + b\tau + \frac{1}{2} \\ \Longleftrightarrow\ & n_1^A = 4b^2\tau^2\rho n_1^A + \left(p_2^A - p_1^A\right)\tau + 2\left(p_2^B - p_1^B\right)b\tau^2\rho + \left(\frac{1}{2} - 2b^2\tau^2\rho\right) \\ & \qquad - b\tau\rho + b\tau \\ \Longleftrightarrow\ & D_1^A(\cdot) = \frac{1}{2} + \frac{\left(p_2^A - p_1^A\right)\tau + 2\left(p_2^B - p_1^B\right)b\tau^2\rho + (1-\rho)b\tau}{1 - 4b^2\tau^2\rho} \\ \Longleftrightarrow\ & D_1^A(\cdot) = \frac{1}{2} + \frac{\left(p_2^A - p_1^A\right)t + \left(p_2^B - p_1^B\right)b\rho + (1-\rho)bt}{2\left(t^2 - b^2\rho\right)}. \end{aligned}$$

That on side B equals

$$D_1^B(\cdot) = \left[b\tau + \frac{2\left(p_2^A - p_1^A\right)b\tau^2 + 4\left(p_2^B - p_1^B\right)b^2\tau^3\rho + 2(1-\rho)b^2\tau^2}{1 - 4b^2\tau^2\rho}\right]$$

$$
\begin{aligned}
&+\left(p_2^B - p_1^B\right)\tau - b\tau + \frac{1}{2} \\
=&\ \frac{1}{2} + \frac{2\left(p_2^A - p_1^A\right)b\tau^2 + \left(p_2^B - p_1^B\right)\tau + 2\left(1-\rho\right)b^2\tau^2}{1 - 4b^2\tau^2\rho} \\
=&\ \frac{1}{2} + \frac{\left(p_2^A - p_1^A\right)b + \left(p_2^B - p_1^B\right)t + \left(1-\rho\right)b^2}{2\left(t^2 - b^2\rho\right)}.
\end{aligned}
$$

The market demand functions for platform 2 are

$$
D_2^A\left(\cdot\right) = 1 - D_1^A\left(\cdot\right) \qquad D_2^B\left(\cdot\right) = 1 - D_1^B\left(\cdot\right)
$$

due to the assumption of full coverage.

To easily calculate the first-order and second-order conditions, I beforehand obtain the price effects on the market demand under condition (4.13). Suppose that condition (4.13) holds. The first-order derivatives of each market demand function are derived in the following. Regarding platform 1,

$$
\begin{aligned}
\frac{\partial D_1^A\left(\cdot\right)}{\partial p_1^A} &= -\frac{\partial D_1^A\left(\cdot\right)}{\partial p_2^A} = -\frac{t}{2\left(t^2 - b^2\rho\right)} \\
\frac{\partial D_1^A\left(\cdot\right)}{\partial p_1^B} &= -\frac{\partial D_1^A\left(\cdot\right)}{\partial p_2^B} = -\frac{b\rho}{2\left(t^2 - b^2\rho\right)}
\end{aligned}
$$

on the side-A market demand and

$$
\begin{aligned}
\frac{\partial D_1^B\left(\cdot\right)}{\partial p_1^B} &= -\frac{\partial D_1^B\left(\cdot\right)}{\partial p_2^B} = -\frac{t}{2\left(t^2 - b^2\rho\right)} \\
\frac{\partial D_1^B\left(\cdot\right)}{\partial p_1^A} &= -\frac{\partial D_1^B\left(\cdot\right)}{\partial p_2^A} = -\frac{b}{2\left(t^2 - b^2\rho\right)}
\end{aligned}
$$

on the side-B market demand. The effects on the market demand for platform 2 are analogous. These price effects have to be strictly negative due to the following reason. Each potential user on each side has a payoff function such that the indirect utility from a platform (directly) decreases as the own-side price of the platform increases. The user mass of a side exerts a strictly positive indirect network externality on each agent who joins the same platform on the other side, which means that each user of a platform loses a fraction of his/her payoff if the opposite-side price of the platform increases. This implies that own-side price effects are magnified (in the same direction) by the existence of indirect network externalities and eventually remain strictly negative. Formally,

$$t^2 - b^2\rho > 0. \tag{4.14}$$

This condition also guarantees negative cross-side price effects. I focus on the case in which condition (4.14) holds throughout this proof.[25] Notice that the second-order partial derivatives equal zero with respect to all own-side and cross-side prices of both platforms.

First-Order and Second-Order Conditions

I obtain the first-order and second-order conditions for platform 1's profit maximization that are evaluated at the arising market outcome.[26] The first-order conditions are that

$$\begin{aligned}
&\frac{\partial \pi_1(\cdot)}{\partial p_1^A} = -\frac{t}{2\left(t^2 - b^2\rho\right)} p_1^A + n_1^A - \frac{b}{2\left(t^2 - b^2\rho\right)} p_1^B = 0 \\
\iff &p_1^A = \frac{2\left(t^2 - b^2\rho\right)}{t} n_1^A - \frac{b}{t} p_1^B \\
&\frac{\partial \pi_1(\cdot)}{\partial p_1^B} = -\frac{b\rho}{2\left(t^2 - b^2\rho\right)} p_1^A - \frac{t}{2\left(t^2 - b^2\rho\right)} p_1^B + n_1^B = 0 \\
\iff &p_1^B = \frac{2\left(t^2 - b^2\rho\right)}{t} n_1^B - \frac{b\rho}{t} p_1^A,
\end{aligned} \tag{4.15}$$

from which the prices of the platform can be derived as

$$\begin{aligned}
&p_1^A = \frac{2\left(t^2 - b^2\rho\right)}{t} n_1^A - \frac{b}{t}\left[\frac{2\left(t^2 - b^2\rho\right)}{t} n_1^B - \frac{b\rho}{t} p_1^A\right] \\
\iff &\frac{t^2 - b^2\rho}{t^2} p_1^A = \frac{2\left(t^2 - b^2\rho\right)}{t} n_1^A - \frac{2\left(t^2 - b^2\rho\right) b}{t^2} n_1^B \\
\iff &p_1^A = 2t n_1^A - 2b n_1^B
\end{aligned}$$

[25]The proof rules out the possibility of the case in which $t^2 - b^2\rho = 0$ to avoid zero division, and this restriction is just a mathematical issue because the result of this proposition may apply as $t^2 - b^2\rho \to 0$ and does not matter in this proof because it is to be shown that the second-order conditions are sufficient.

[26]The p_1^A and p_1^B that satisfy the first-order conditions are written as expressions of n_1^A and n_1^B because they are evaluated somewhere (n_1^A, n_1^B) arises. Notice that these expressions are conceptually different from the pricing introduced as "insulating tariffs" by Weyl (2010) and White and Weyl (2016) because platforms in my model charge standard participation fees as in Armstrong (2006). This statement applies to the p_2^A and p_2^B obtained in the next paragraph as well.

regarding side A and

$$
\begin{aligned}
p_1^B &= \frac{2\left(t^2-b^2\rho\right)}{t}n_1^B - \frac{b\rho}{t}\left(2tn_1^A - 2bn_1^B\right)\\
&= 2tn_1^B - 2b\rho n_1^A
\end{aligned}
$$

regarding side B. The second-order condition is that

$$
\frac{\partial^2 \pi_1(\cdot)}{\partial\left(p_1^A\right)^2} = -\frac{t}{\left(t^2-b^2\rho\right)} < 0 \qquad \frac{\partial^2 \pi_1(\cdot)}{\partial\left(p_1^B\right)^2} = -\frac{t}{\left(t^2-b^2\rho\right)} < 0
$$

with respect to each of the own prices and

$$
\begin{aligned}
H_1(\cdot) &= \frac{t^2}{\left(t^2-b^2\rho\right)^2} - \frac{b^2}{4\left(t^2-b^2\rho\right)^2}(\rho+1)^2 > 0\\
\Longleftrightarrow t^2 - \frac{(\rho+1)^2}{4}b^2 > 0 \;&\Longleftrightarrow\; t > \frac{b}{2}(\rho+1)
\end{aligned}
\tag{4.16}
$$

with regard to the Hessian matrix, where

$$
\frac{\partial^2 \pi_1(\cdot)}{\partial p_1^A \partial p_1^B} = \frac{\partial^2 \pi_1(\cdot)}{\partial p_1^B \partial p_1^A} = -\frac{b}{2\left(t^2-b^2\rho\right)}(\rho+1).
$$

The own-price conditions hold if and only if expression (4.14) is met (because $t > 0$). On the other hand, expression (4.14) is a necessity for condition (4.16) because it holds for any $\rho \in (0,1)$ that

$$
t^2 - \frac{(\rho+1)^2}{4}b^2 < t^2 - b^2\rho \;\Longleftrightarrow\; \underbrace{(\rho+1)^2 - 4\rho}_{=(1-\rho)^2} > 0.
$$

All second-order conditions are therefore satisfied if and only if expression (4.16) holds.

The first-order and second-order conditions for platform 2's profit maximization are symmetric. One can obtain the prices of the platform derived from the first-order conditions as

$$p_2^A = 2\underbrace{\left(1 - n_1^A\right)}_{=n_2^A} t - 2\underbrace{\left(1 - n_1^B\right)}_{=n_2^B} b = \underbrace{-2tn_1^A + 2bn_1^B}_{=-p_1^A} + (2t - 2b)$$

$$p_2^B = 2\underbrace{\left(1 - n_1^B\right)}_{=n_2^B} t - 2\underbrace{\left(1 - n_1^A\right)}_{=n_2^A} b\rho = \underbrace{-2tn_1^B + 2b\rho n_1^A}_{=-p_1^B} + (2t - 2b\rho),$$

which implies that

$$p_2^A - p_1^A = -2p_1^A + (2t - 2b) = 4\left(bn_1^B - tn_1^A\right) + 2(t - b)$$

$$p_2^B - p_1^B = -2p_1^B + (2t - 2b\rho) = 4\left(b\rho n_1^A - tn_1^B\right) + 2(t - b\rho).$$

The second-order conditions hold if and only if condition (4.16) is met because they are analogous to those for platform 1's problem.

Resulting Market Shares

This part begins with the realized allocation on each side given the number of platform-1 users on the opposite side. Consider the allocation on each side given that on the other side. Platform 1 obtains a market share of n_1^A on side A such that

$$\begin{aligned} n_1^A = {} & \frac{1}{2} + \frac{\left[\left(4btn_1^B - 4t^2 n_1^A\right) + \left(2t^2 - 2bt\right)\right] + \left[\left(4b^2\rho^2 n_1^A - 4bt\rho n_1^B\right) + \left(2bt\rho - 2b^2\rho^2\right)\right]}{2\left(t^2 - b^2\rho\right)} \\ & + \frac{(1-\rho)\, bt}{2\left(t^2 - b^2\rho\right)} \\ \iff 2\left[3t^2 - (2\rho + 1)\, b^2\rho\right] n_1^A = {} & 4(1-\rho)\, btn_1^B \\ & + \left(2t^2 - 2bt\right) + \left(2bt\rho - 2b^2\rho^2\right) + \left(t^2 - b^2\rho\right) + (1-\rho)\, bt \\ \iff 2\left[3t^2 - (2\rho + 1)\, b^2\rho\right] n_1^A = {} & 4(1-\rho)\, btn_1^B \\ & \left[3t^2 - (2\rho+1)\, b^2\rho\right] - (1-\rho)\, bt, \end{aligned} \tag{4.17}$$

whereas platform 2 attracts $1 - n_1^A$ $(= n_2^A)$ potential users on that side. Platform 1's market share equals n_1^B such that

$$\begin{aligned}
n_1^B &= \frac{1}{2} + \frac{\left[\left(4b^2 n_1^B - 4btn_1^A\right) + \left(2bt - 2b^2\right)\right] + \left[\left(4bt\rho n_1^A - 4t^2 n_1^B\right) + \left(2t^2 - 2bt\rho\right)\right]}{2\left(t^2 - b^2\rho\right)} \\
&\quad + \frac{(1-\rho)\, b^2}{2\left(t^2 - b^2\rho\right)} \\
\Longleftrightarrow\ & 2\left[3t^2 - (\rho + 2)\, b^2\right] n_1^B = 4(\rho - 1)\, btn_1^A \\
&\quad + \left(2bt - 2b^2\right) + \left(2t^2 - 2bt\rho\right) + \left(t^2 - b^2\rho\right) + (1-\rho)\, b^2 \\
\Longleftrightarrow\ & 2\left[3t^2 - (\rho + 2)\, b^2\right] n_1^B = 4(\rho - 1)\, btn_1^A \\
&\quad + \left[3t^2 - (\rho + 2)\, b^2\right] + (b + 2t)(1-\rho)\, b, \qquad (4.18)
\end{aligned}$$

whereas platform 2 attracts $1 - n_1^B$ $(= n_2^B)$ agents on the side. For any n_1^A and n_1^B, define $\epsilon^A \in \mathbb{R}$ and $\epsilon^B \in \mathbb{R}$ such that $n_1^A = 1/2 + \epsilon^A$ and $n_1^B = 1/2 + \epsilon^B$. Expression (4.17) is thus rewritten as

$$\begin{aligned}
\Longleftrightarrow\ & 2\left[3t^2 - (2\rho + 1)\, b^2\rho\right]\left(\frac{1}{2} + \epsilon^A\right) = 4(1-\rho)\left(\frac{1}{2} + \epsilon^B\right) bt \\
& \left[3t^2 - (2\rho + 1)\, b^2\rho\right] - (1-\rho)\, bt \\
\Longleftrightarrow\ & 2\left[3t^2 - (2\rho + 1)\, b^2\rho\right]\epsilon^A = 4(1-\rho)\, bt\epsilon^B + (1-\rho)\, bt. \qquad (4.19)
\end{aligned}$$

As for expression (4.18),

$$\begin{aligned}
& 2\left[3t^2 - (\rho + 2)\, b^2\right]\left(\frac{1}{2} + \epsilon^B\right) = 4(\rho - 1)\left(\frac{1}{2} + \epsilon^A\right) bt \\
&\quad + \left[3t^2 - (\rho + 2)\, b^2\right] + (b + 2t)(1-\rho)\, b \\
\Longleftrightarrow\ & 2\left[3t^2 - (\rho + 2)\, b^2\right]\epsilon^B = 4(\rho - 1)\, bt\epsilon^A + (1-\rho)\, b^2. \qquad (4.20)
\end{aligned}$$

One can find the existence of a unique solution to the equation system above and obtain the value of the solution. The resulting value of ϵ^A equals

$$4\left[3t^2 - (2\rho + 1)\, b^2\rho\right]\left[3t^2 - (\rho + 2)\, b^2\right]\epsilon^A = 2\left[3t^2 - (\rho + 2)\, b^2\right]\epsilon^B \cdot 4(1-\rho)\, bt$$

$$
\begin{aligned}
&\quad + 2\left[3t^2 - (\rho+2)\, b^2\right](1-\rho)\, bt \\
\Longleftrightarrow\ & 4\left[3t^2 - (2\rho+1)\, b^2\rho\right]\left[3t^2 - (\rho+2)\, b^2\right]\epsilon^A \\
&\quad = 4\,(1-\rho)\, bt \cdot \left[4\,(\rho-1)\, bt\epsilon^A + (1-\rho)\, b^2\right] + 2\left[3t^2 - (\rho+2)\, b^2\right](1-\rho)\, bt \\
\Longleftrightarrow\ & 2\left\{9t^4 - 2\left[(\rho+7)\,\rho+1\right] b^2t^2 + (2\rho+1)\,(\rho+2)\, b^4\rho\right\}\epsilon^A \\
&\quad = 3\left(t^2 - b^2\rho\right)(1-\rho)\, bt \\
\Longleftrightarrow\ & \epsilon^A = \frac{3\,(t^2 - b^2\rho)\,(1-\rho)\, bt}{2\left\{9t^4 - 2\left[(\rho+7)\,\rho+1\right] b^2t^2 + (2\rho+1)\,(\rho+2)\, b^4\rho\right\}},
\end{aligned}
$$

where the properties that the ϵ^A above takes a strictly positive value and that the denominator of it is nonzero are guaranteed by the following process. First, as established in the next paragraph, the denominator of the ϵ^A derived above is always strictly positive. Second, the numerator of the ϵ^A derived above is strictly positive because $(1-\rho)bt > 0$ by definition, and it holds from inequality (4.14) (which holds if condition (4.16) is met) that $3(t^2 - b^2\rho) > 0$. The resulting value of ϵ^A is therefore strictly positive. I obtain the resulting value of ϵ^B from equations (4.20) and (4.19):

$$
\begin{aligned}
& 4\left[3t^2 - (2\rho+1)\, b^2\rho\right]\left[3t^2 - (\rho+2)\, b^2\right]\epsilon^B = 2\left[3t^2 - (2\rho+1)\, b^2\rho\right]\epsilon^A \cdot 4\,(\rho-1)\, bt \\
&\quad + 2\left[3t^2 - (2\rho+1)\, b^2\rho\right](1-\rho)\, b^2 \\
\Longleftrightarrow\ & 4\left[3t^2 - (2\rho+1)\, b^2\rho\right]\left[3t^2 - (\rho+2)\, b^2\right]\epsilon^B \\
&\quad = 4\,(\rho-1)\, bt \cdot \left[4\,(1-\rho)\, bt\epsilon^B + (1-\rho)\, bt\right] + 2\left[3t^2 - (2\rho+1)\, b^2\rho\right](1-\rho)\, b^2 \\
\Longleftrightarrow\ & 2\left\{9t^4 - 2\left[(\rho+7)\,\rho+1\right] b^2t^2 + (2\rho+1)\,(\rho+2)\, b^4\rho\right\}\epsilon^B \\
&\quad = \left(t^2 - b^2\rho\right)(2\rho+1)\,(1-\rho)\, b^2 \\
\Longleftrightarrow\ & \epsilon^B = \frac{(t^2 - b^2\rho)\,(2\rho+1)\,(1-\rho)\, b^2}{2\left\{9t^4 - 2\left[(\rho+7)\,\rho+1\right] b^2t^2 + (2\rho+1)\,(\rho+2)\, b^4\rho\right\}},
\end{aligned}
$$

whose properties of strict positivity and no zero division can be shown in the following way. First, the denominator of the ϵ^B derived above equals that of the ϵ^A obtained in this proof and thus takes a strictly positive value. Second, the numerator of the ϵ^B above is also strictly positive because $t^2 - b^2\rho > 0$ (which condition (4.14) just says), $2\rho + 1 > 0$, and $(1-\rho)b^2 > 0$. The ϵ^B derived above therefore takes a strictly positive value. The existence and uniqueness of (ϵ^A, ϵ^B) to solve the equation system in the preceding paragraph are thus proved as well.

Establish now that the denominator of the ϵ^A derived above is strictly positive. Condition (4.16) implies that the denominator divided by 2 can be rewritten as follows:

$$\begin{aligned}
&9b^4k^4 - 2\left[(\rho+7)\rho+1\right]b^4k^2 + (2\rho+1)(\rho+2)b^4\rho \\
=&\left[9k^4 + \left(-2\rho^2 - 14\rho - 2\right)k^2 + \left(2\rho^3 + 5\rho^2 + 2\rho\right)\right]b^4,
\end{aligned}$$

where $t = bk$ ($b \neq 0$ because $t \neq 0$) and $k > (\rho+1)/2$. The partial derivative of the divided denominator with respect to k is

$$\left[36k^3 + 2\left(-2\rho^2 - 14\rho - 2\right)k\right]b^4 = \left[36k^2 - 4\left(\rho^2 + 7\rho + 1\right)\right]b^4k,$$

which always takes a strictly positive value because it has an infimum that arises as $k \to \inf k = (\rho+1)/2$ and equals

$$\begin{aligned}
&\frac{(\rho+1)\,b^4}{2}\left[9(\rho+1)^2 - 4\left(\rho^2 + 7\rho + 1\right)\right] \\
=&\frac{5(\rho+1)\,b^4}{2}(1-\rho)^2 > 0.
\end{aligned}$$

The divided denominator approaches

$$\begin{aligned}
&\left[\frac{9}{16}(\rho+1)^4 + \frac{1}{4}\left(-2\rho^2 - 14\rho - 2\right)(\rho+1)^2 + \left(2\rho^3 + 5\rho^2 + 2\rho\right)\right]b^4 \\
=&\frac{b^4}{16}\left[(1-\rho)^4 + 48\rho^2\right] > 0
\end{aligned}$$

as $k \to \inf k = (\rho+1)/2$. The divided denominator is thus strictly positive. This means that the denominator itself takes a strictly positive value.

It is helpful to find the relation between the realized values of ϵ^A and ϵ^B. The difference between the numerators of these values equals

$$\begin{aligned}
&\underbrace{3\left(t^2-b^2\rho\right)(1-\rho)\,bt}_{\text{from }\epsilon^A}-\underbrace{\left(t^2-b^2\rho\right)(2\rho+1)(1-\rho)\,b^2}_{\text{from }\epsilon^B}\\
&=\left[3t-(2\rho+1)\,b\right]\underbrace{\left(t^2-b^2\rho\right)(1-\rho)\,b}_{>0}.
\end{aligned}$$

The following calculation can be obtained, where $t=kb$ and $k>(\rho+1)/2$:

$$\begin{aligned}
3t-(2\rho+1)\,b &= 3kb-(2\rho+1)\,b\\
&=\left[3k-(2\rho+1)\right]b\\
&>\left[\frac{3}{2}(\rho+1)-(2\rho+1)\right]b\\
&=\frac{b}{2}(1-\rho)>0
\end{aligned}$$

for any $\rho\in(0,1)$. Therefore, it always hold that $\epsilon^A>\epsilon^B$.

Value and Major Properties of the Interior Solution

Each component of a competitive outcome can be described as an expression of ϵ^A and/or ϵ^B. Platform 1's market shares are derived, by definition, from the ϵ^A and ϵ^B that I show above. Platform 2's market shares equal

$$\begin{aligned}
n_2^A &= 1-\left(\frac{1}{2}+\epsilon^A\right)=\frac{1}{2}-\epsilon^A\\
n_2^B &= 1-\left(\frac{1}{2}+\epsilon^B\right)=\frac{1}{2}-\epsilon^B.
\end{aligned}$$

Platform 1 chooses a pair of prices such that

$$\begin{aligned}
p_1^A &= 2\left(\frac{1}{2}+\epsilon^A\right)t-2\left(\frac{1}{2}+\epsilon^B\right)b=(t-b)+2\left(t\epsilon^A-b\epsilon^B\right)\\
p_1^B &= 2\left(\frac{1}{2}+\epsilon^B\right)t-2\left(\frac{1}{2}+\epsilon^A\right)b\rho=(t-b\rho)+2\left(t\epsilon^B-b\rho\epsilon^A\right).
\end{aligned}$$

As for platform 2,

$$
\begin{aligned}
p_2^A &= -\left[(t-b)+2\left(t\epsilon^A - b\epsilon^B\right)\right] + (2t-2b) = (t-b) - 2\left(t\epsilon^A - b\epsilon^B\right) \\
p_2^B &= -\left[(t-b\rho)+2\left(t\epsilon^B - b\rho\epsilon^A\right)\right] + (2t-2b\rho) = (t-b\rho) - 2\left(t\epsilon^B - b\rho\epsilon^A\right).
\end{aligned}
$$

The variables obtained here constitute a market outcome if $0 \leq n_1^A = 1/2 + \epsilon^A \leq 1$, $0 \leq n_1^B = 1/2 + \epsilon^B \leq 1$,

$$
\begin{aligned}
d_1^A(\cdot; 1, 0) &= \frac{(b+t) + \left(p_2^A - p_1^A\right)}{2t} \\
&= \frac{(b+t) + 4\left(bn_1^B - tn_1^A\right) + 2(t-b)}{2t} \\
&= \frac{4\left(bn_1^B - tn_1^A\right) + (3t-b)}{2t} \\
&= \frac{4\left(b\epsilon^B - t\epsilon^A\right) + (t+b)}{2t} \in [0,1],
\end{aligned}
$$

and inequality (4.16) holds, all of which conditions are found to be represented by a single condition in the following two paragraphs.

The first three conditions for the existence of an interior market outcome can be digested into a single inequality. The preceding part establishes that $\epsilon^A > \epsilon^B > 0$, where $n_1^A > n_1^B > 1/2$. Next, it holds that

$$
\begin{aligned}
\left(\frac{1}{2} <\right) n_1^A &= d_1^A\left(\cdot; n_1^B, n_2^B\right)\rho + d_1^A(\cdot; 1, 0)(1-\rho) \\
&= d_1^A(\cdot; 1, 0) - \underbrace{\left[d_1^A(\cdot; 1, 0) - d_1^A\left(\cdot; n_1^B, n_2^B\right)\right]}_{\geq 0}\rho \\
&\leq d_1^A(\cdot; 1, 0)
\end{aligned}
$$

because potential users with biased expectations on side A evaluate platform 1 more highly than other agents on that side. This result guarantees that $d_1^A(\cdot; 1, 0)(> 1/2) > 0$. The only condition to state with regard to $d_1^A(\cdot; 1, 0)$ is thus that

$$
d_1^A(\cdot; 1, 0) \leq 1 \iff 4\left(b\epsilon^B - t\epsilon^A\right) + (t+b) - 2t \leq 0
$$

$$\iff 4\left(t\epsilon^A - b\epsilon^B\right) + (t-b) \geq 0,$$

which appears as expression (4.9). This condition also restricts the upper bound of platform 1's resulting market share on side A such that $n_1^A < d_1^A(\cdot;1,0) \leq 1$. These calculations imply that

$$\frac{1}{2} < n_1^B < n_1^A < d_1^A\left(\cdot;1,0\right) \leq 1,$$

or that the three existence conditions are satisfied under condition (4.9).

One can summarize conditions (4.9) and (4.16) as a single condition by three steps, which also yield a sufficient condition that satisfies all existence conditions. The first step is to examine whether the latter condition holds when the former condition is satisfied. Condition (4.9) always holds if $t \geq b$ because

$$4\left(t\epsilon^A - b\epsilon^B\right) \geq 4\underbrace{\left(\epsilon^A - \epsilon^B\right)}_{>0} b > 0$$

and $t - b \geq 0$. Suppose next that $(\rho+1)b/2 < t < b$, where expression (4.9) can be rewritten as

$$\begin{aligned} & t\epsilon^A - b\epsilon^B \geq \frac{b-t}{4} \\ \iff & \left[3t^2 - (2\rho+1)\,b^2\right]\underbrace{\left(t^2 - b^2\rho\right)}_{>0}\underbrace{(1-\rho)\,b}_{>0} \\ & \geq \underbrace{\frac{b-t}{2}}_{>0}\underbrace{\left\{9t^4 - 2\left[(\rho+7)\,\rho+1\right]b^2t^2 + (2\rho+1)\,(\rho+2)\,b^4\rho\right\}}_{>0}. \end{aligned} \tag{4.21}$$

This expression holds only if

$$\begin{aligned} 3t^2 - (2\rho+1)\,b^2 > 0 &\iff 3k^2b^2 - (2\rho+1)\,b^2 > 0 \\ &\iff \left[3k^2 - (2\rho+1)\right]b^2 > 0 \\ &\iff k > \sqrt{\frac{2\rho+1}{3}} \end{aligned} \tag{4.22}$$

from the fact that the right-hand side of inequality (4.21) takes a strictly positive value, where $(\rho+1)/2 < k < 1$. One can then find a value of $\rho \in (0,1)$ such that the left-hand side of condition (4.22) (which equals k) is weakly lower than the right-hand side because

$$\sup k = 1 \qquad \sup \sqrt{\frac{2\rho+1}{3}} = 1$$
$$\frac{\mathrm{d}}{\mathrm{d}\rho}\frac{2\rho+1}{3} = \frac{2}{3} > 0,$$

which suggests the existence of ρ that violates the condition for any $k \in ((\rho+1)/2, 1)$. Expression (4.9) is therefore a stronger condition than expression (4.16) if $(\rho+1)b/2 < t < b$. Lastly, consider the case in which $(0 <)t \leq (\rho+1)b/2$. Expressions (4.21) and (4.22) suggest that condition (4.9) is never met, in addition to the violation of condition (4.16). This result means that condition (4.9) does not hold if condition (4.16) does not, which implies that condition (4.16) is satisfied if condition (4.9) is. One can therefore obtain the following findings from the above analysis. First, expression (4.9) is a (not necessary but) sufficient condition for expression (4.16). Second, there arises a market outcome characterized in the first paragraph of this part if $t \geq b$.

Moreover, I derive the condition for the above market outcome to arise as a unique solution to the competition game. Suppose that condition (4.9) holds as a strict inequality. All of the allocations on side A, on side B, and in the group of agents with biased expectations are strictly interior:

$$\frac{1}{2} < n_1^B < n_1^A < d_1^A(\cdot; 1, 0) < 1.$$

Condition (4.16) also holds, as stated above. The competitive outcome in question is therefore a unique solution. On the contrary, it is impossible to establish the uniqueness of the outcome in the case of condition (4.9) being met as an equality because the group of agents with biased expectations exhibits the mixture of interior and corner allocations (see also the proof of the next proposition).

The current proof concludes with a further discussion on the relation between the value of ρ and the violation of condition (4.9). The preceding analysis establishes that condition (4.9) is never violated if $t \geq b$. Below, suppose that $t < b$. Partially differentiating inequality (4.21) with respect to ρ yields

$$\frac{\partial}{\partial\rho}\left[3t^2-(2\rho+1)\,b^2\right]\left(t^2-b^2\rho\right)(1-\rho)\,b$$

regarding the left-hand side and

$$\begin{aligned}
&\frac{\partial}{\partial\rho}\frac{b-t}{2}\left\{9t^4-2\left[(\rho+7)\,\rho+1\right]b^2t^2+(2\rho+1)\,(\rho+2)\,b^4\rho\right\}\\
=&\frac{\partial}{\partial\rho}\frac{b-t}{2}\left[\left(-2\rho^2-14\rho-2\right)b^2t^2+\left(2\rho^3+5\rho^2+2\rho\right)b^4\right]\\
=&\left[\left(3\rho^2+5\rho+1\right)b^2-(2\rho+7)\,t^2\right](b-t)\,b^2
\end{aligned}$$

regarding the right-hand side. The first-order derivative of the left-hand side is strictly negative for any ρ and decreases as b increases. The first-order derivative of the right-hand side has an ambiguous sign. However, notice that both the derivative and the difference $(3\rho^2+5\rho+1)b^2-(2\rho+7)t^2$ increase in b, and that the derivative is strictly positive for some ρ and increases in ρ because

$$\begin{aligned}
&\frac{\partial}{\partial\rho}\left[\left(3\rho^2+5\rho+1\right)b^2-(2\rho+7)\,t^2\right](b-t)\,b^2\\
=&\left[(6\rho+5)\,b^2-2t^2\rho\right](b-t)\,b^2\\
=&\left[2\left(b^2-t^2\right)\rho+(4\rho+5)\,b^2\right](b-t)\,b^2>0\\
&\left(3\rho^2+5\rho+1\right)b^2-(2\rho+7)\,t^2>0\iff\underbrace{\frac{t^2}{b^2}}_{<1}<\underbrace{\frac{3\rho^2+5\rho+1}{2\rho+7}}_{\to 1\text{ as }\rho\to 1}.
\end{aligned}$$

The right-hand side of inequality (4.21) therefore has a first-order derivative with respect to ρ whose value always increases in ρ or b. To investigate the impacts of increase in b on expression (4.21) as $\rho\to 1$, I rewrite the left-hand and right-hand sides of expression (4.21) as

$$\begin{aligned}
&\left[3t^2-(2\rho+1)\,b^2\right]\left(t^2-b^2\rho\right)(1-\rho)\,b\\
=&\left[3k^2b^2-(2\rho+1)\,b^2\right]\left(k^2b^2-b^2\rho\right)(1-\rho)\,b
\end{aligned}$$

$$
\begin{aligned}
&= \left\{ \left[2\rho^2 + \left(1 - 5k^2\right)\rho + \left(3k^4 - k^2\right) \right] + \left[-2\rho^3 + \left(5k^2 - 1\right)\rho^2 + \left(-3k^4 + k^2\right)\rho \right] \right\} b^5 \\
&= \left[-2\rho^3 + \left(5k^2 + 1\right)\rho^2 + \left(-3k^4 - 4k^2 + 1\right)\rho + \left(3k^4 - k^2\right) \right] b^5 \\
&\frac{b - t}{2} \left\{ 9t^4 - 2\left[(\rho + 7)\rho + 1\right] b^2 t^2 + (2\rho + 1)(\rho + 2) b^4 \rho \right\} \\
&= \frac{1 - k}{2} \left[2\rho^3 + \left(-2k^2 + 5\right)\rho^2 + \left(-14k^2 + 2\right)\rho + \left(9k^4 - 2k^2\right) \right] b^5 \\
&= \frac{b^5}{2} \left[2\rho^3 + \left(-2k^2 + 5\right)\rho^2 + \left(-14k^2 + 2\right)\rho + \left(9k^4 - 2k^2\right) \right] \\
&\quad - \frac{b^5}{2} \left[2k\rho^3 + \left(-2k^3 + 5k\right)\rho^2 + \left(-14k^3 + 2k\right)\rho + \left(9k^5 - 2k^3\right) \right] \\
&= \frac{b^5}{2} \left[(2 - 2k)\rho^3 + \left(2k^3 - 2k^2 - 5k + 5\right)\rho^2 + \left(14k^3 - 14k^2 - 2k + 2\right)\rho \right] \\
&\qquad + \frac{b^5}{2} \left(-9k^5 + 9k^4 + 2k^3 - 2k^2\right).
\end{aligned}
$$

Expression (4.21) can then be transformed as

$$
\begin{aligned}
&\left[-2\rho^3 + \left(5k^2 + 1\right)\rho^2 + \left(-3k^4 - 4k^2 + 1\right)\rho + \left(3k^4 - k^2\right) \right] b^5 \\
&\geq \frac{b^5}{2} \left[(2 - 2k)\rho^3 + \left(2k^3 - 2k^2 - 5k + 5\right)\rho^2 + \left(14k^3 - 14k^2 - 2k + 2\right)\rho \right] \\
&\qquad + \frac{b^5}{2} \left(-9k^5 + 9k^4 + 2k^3 - 2k^2\right) \\
\iff &\left[(2k - 6)\rho^3 + \left(-2k^3 + 12k^2 + 5k - 3\right)\rho^2 + \left(-6k^4 - 14k^3 + 6k^2 + 2k\right)\rho \right] b^5 \\
\iff &\qquad + \underbrace{\left(9k^5 - 3k^4 - 2k^3\right) b^5}_{=9t^5 - 3bt^4 - 2b^2t^3} \geq 0.
\end{aligned}
$$

The left-hand side of this inequality approaches

$$
9t^5 - 3bt^4 - 2b^2t^3 = \left(9t^2 - 3bt - 2b^2\right) t^3
$$

as $\rho \to 0$, and this value decreases in b. The analysis in this paragraph establishes (i) that the infimum of ρ to violate expression (4.21), or condition (4.9), decreases as b increases (and if $t < b$) and (ii) that the expression never holds for any higher ρ than the infimum because the set described by inequality (4.21) shrinks and the shrinking impacts of ρ on the set grows as b increases.

4.A.2 Proof of Proposition 4.2

This proof consists of three parts. The first part derives the market demand functions and the conditions for each platform's profit maximization in the case of an outcome such that all of the agents with biased expectations *strictly* prefer platform 1 but neither platform dominates side A entirely or side B, which is here called a quasi-interior competitive outcome. The second part maintains the condition of agents who form biased expectations and establishes the absence of a quasi-interior outcome and also a side-dominance market outcome, which would occur if one side were dominated but the allocation on the other side were strictly interior. The third part relaxes the condition of agents with biased expectations such that all of them at least weakly prefer platform 1 and discusses a market outcome such that a particular platform (namely, platform 1) attracts all potential users on both sides, which is here called a market-dominance outcome and constitutes a Nash equilibrium.

Market Demand and Profit Maximization

A quasi-interior competitive outcome technically has two properties. First, platform 2 can attract no agent by marginal price reduction because any potential user with a biased expectation strictly prefers platform 1. Second, each platform chooses the price pair(s) derived from the first-order and second-order conditions if the entire allocation on side A and the allocation on side B that eventually arise are weakly interior. This part thus proceeds under the condition that $d_1^A(\cdot; 1, 0) = 1$.

I begin with the market demand for each platform in this situation. The market demand functions for platform 1 are derived under assumption (4.13) as a solution to equation (4.12) and the following equation:

$$\begin{aligned} n_1^A &= d_1^A\left(\cdot; n_1^B, n_2^B\right)\rho + \underbrace{d_1^A\left(\cdot; 1, 0\right)(1-\rho)}_{=1-\rho} \\ &= \left[2bn_1^B + \left(p_2^A - p_1^A\right) - b\right]\tau\rho + \frac{1-\rho}{2} + \frac{1}{2}. \end{aligned}$$

The function on side A is

$$\begin{aligned}
n_1^A &= \underbrace{\left[4b^2\tau^2\rho n_1^A + 2\left(p_2^B - p_1^B\right)b\tau^2\rho - 2b^2\tau^2\rho + b\tau\rho\right]}_{=2b\tau\rho n_1^B} + \left[\left(p_2^A - p_1^A\right)\tau\rho - b\tau\rho\right] \\
&\quad + \left(\frac{1-\rho}{2} + \frac{1}{2}\right) \\
\iff \left(1 - 4b^2\tau^2\rho\right) n_1^A &= \left(\frac{1}{2} - 2b^2\tau^2\rho\right) + \left(p_2^A - p_1^A\right)\tau\rho + 2\left(p_2^B - p_1^B\right)b\tau^2\rho + \frac{1-\rho}{2} \\
\iff D_1^A(\cdot) &= \frac{1}{2} + \frac{\left(p_2^A - p_1^A\right)\tau\rho + 2\left(p_2^B - p_1^B\right)b\tau^2\rho + (1-\rho)/2}{1 - 4b^2\tau^2\rho} \\
\iff D_1^A(\cdot) &= \frac{1}{2} + \frac{\left(p_2^A - p_1^A\right)t\rho + \left(p_2^B - p_1^B\right)b\rho + (1-\rho)t^2}{2\left(t^2 - b^2\rho\right)}.
\end{aligned}$$

On side B,

$$\begin{aligned}
D_1^B(\cdot) &= \left[b\tau + \frac{2\left(p_2^A - p_1^A\right)b\tau^2\rho + 4\left(p_2^B - p_1^B\right)b^2\tau^3\rho + (1-\rho)b\tau}{1 - 4b^2\tau^2\rho}\right] \\
&\quad + \left[\left(p_2^B - p_1^B\right)\tau - b\tau + \frac{1}{2}\right] \\
&= \frac{1}{2} + \frac{\left(p_2^B - p_1^B\right)\tau + 2\left(p_2^A - p_1^A\right)b\tau^2\rho + (1-\rho)b\tau}{1 - 4b^2\tau^2\rho} \\
&= \frac{1}{2} + \frac{\left(p_2^B - p_1^B\right)t + \left(p_2^A - p_1^A\right)b\rho + (1-\rho)bt}{2\left(t^2 - b^2\rho\right)}.
\end{aligned}$$

The assumption of full coverage immediately yields the market demand functions for platform 2.

One can derive the price effects on the market demand by a similar process to that adopted in the proof of Proposition 4.1 (Appendix 4.A.1). The differences between that and the current proofs are as below:

$$\begin{aligned}
\frac{\partial D_1^A(\cdot)}{\partial p_1^A} &= -\frac{\partial D_1^A(\cdot)}{\partial p_2^A} = -\frac{t\rho}{2\left(t^2 - b^2\rho\right)} \\
\frac{\partial D_1^B(\cdot)}{\partial p_1^A} &= -\frac{\partial D_1^B(\cdot)}{\partial p_2^A} = -\frac{b\rho}{2\left(t^2 - b^2\rho\right)}.
\end{aligned}$$

The price effects with regard to the side-B prices, the result that all (own-side or cross-side) second-order partial derivatives equal zero, and the fact that condition (4.14) needs to hold are unchanged.

I now address the first-order and second-order conditions for platform 1's profit maximization.[27] The first-order condition is that

$$\begin{aligned} \frac{\partial \pi_1(\cdot)}{\partial p_1^A} &= -\frac{t\rho}{2\left(t^2-b^2\rho\right)}p_1^A + n_1^A - \frac{b\rho}{2\left(t^2-b^2\rho\right)}p_1^B = 0 \\ \iff p_1^A &= \frac{2\left(t^2-b^2\rho\right)}{t\rho}n_1^A - \frac{b}{t}p_1^B \end{aligned}$$

on side A and expression (4.15) on side B; thus, the optimal price of the platform equals

$$\begin{aligned} p_1^A &= \frac{2\left(t^2-b^2\rho\right)}{t\rho}n_1^A - \frac{b}{t}\left[\frac{2\left(t^2-b^2\rho\right)}{t}n_1^B - \frac{b\rho}{t}p_1^A\right] \\ \iff t^2\rho p_1^A &= 2\left(t^2-b^2\rho\right)tn_1^A + \left[b^2\rho^2 p_1^A - 2\left(t^2-b^2\rho\right)b\rho n_1^B\right] \\ \iff \left(t^2-b^2\rho\right)\rho p_1^A &= 2\left(t^2-b^2\rho\right)tn_1^A - 2\left(t^2-b^2\rho\right)b\rho n_1^B \\ \iff p_1^A &= \frac{2t}{\rho}n_1^A - 2bn_1^B \end{aligned}$$

on side A and

$$\begin{aligned} p_1^B &= \frac{2\left(t^2-b^2\rho\right)}{t}n_1^B - \frac{b\rho}{t}\left(\frac{2t}{\rho}n_1^A - 2bn_1^B\right) \\ &= 2tn_1^B - 2bn_1^A \end{aligned}$$

on side B if both sides of the market exhibit interior allocations under this pricing. One can establish that all second-order conditions for the platform's problem are satisfied under condition (4.16) (which also suffices for condition (4.14) to hold). The conditions with respect to the own prices are that

$$\frac{\partial^2 \pi_1(\cdot)}{\partial\left(p_1^A\right)^2} = -\frac{t\rho}{t^2-b^2\rho} < 0 \qquad \frac{\partial^2 \pi_1(\cdot)}{\partial\left(p_1^B\right)^2} = -\frac{t}{t^2-b^2\rho} < 0.$$

[27] See footnote 26 for why the first-order and second-order derivatives of a platform's profit in this proof contain n_1^A and n_1^B.

The condition with regard to the Hessian matrix is that

$$\begin{aligned}H_1(\cdot) &= \frac{t^2\rho}{\left(t^2-b^2\rho\right)^2} - \frac{b^2\rho^2}{\left(t^2-b^2\rho\right)^2}\\ &= \frac{\rho}{\left(t^2-b^2\rho\right)^2}\left(t^2-b^2\rho\right)\\ &= \frac{\rho}{t^2-b^2\rho} > 0,\end{aligned}$$

where

$$\frac{\partial^2\pi_1(\cdot)}{\partial p_1^A\partial p_1^B} = \frac{\partial^2\pi_1(\cdot)}{\partial p_1^B\partial p_1^A} = -\frac{b\rho}{t^2-b^2\rho}.$$

The first-order and second-order conditions for platform 2's profit maximization are analogous. The prices of the platform derived from the first-order conditions equal

$$\begin{aligned}p_2^A &= \frac{2t}{\rho}\underbrace{\left(1-n_1^A\right)}_{=n_2^A} - 2\underbrace{\left(1-n_1^B\right)}_{=n_2^B} b = \underbrace{\left(-\frac{2t}{\rho}n_1^A + 2bn_1^B\right)}_{=-p_1^A} + \frac{2}{\rho}(t-b\rho)\\ p_2^B &= 2\underbrace{\left(1-n_1^B\right)}_{=n_2^B} t - 2\underbrace{\left(1-n_1^A\right)}_{=n_2^A} b = \underbrace{\left(-2tn_1^B + 2bn_1^A\right)}_{=-p_1^B} - 2(b-t),\end{aligned}$$

which implies that

$$\begin{aligned}p_2^A - p_1^A &= -2p_1^A + \frac{2}{\rho}(t-b\rho) = \left(-\frac{4t}{\rho}n_1^A + 4bn_1^B\right) + \frac{2}{\rho}(t-b\rho)\\ p_2^B - p_1^B &= -2p_1^B - 2(b-t) = \left(-4tn_1^B + 4bn_1^A\right) - 2(b-t).\end{aligned}$$

The second-order conditions are analogous to those for platform 1's problem and thus hold under expression (4.16).

Absence of a Quasi-Interior or Side-Dominance Outcome

This part considers the implications of the resulting market shares if both platforms choose the prices that satisfy the first-order and second-order conditions for the respective platforms' problems. Platform 1 attracts the following number of agents on side A:

$$\begin{aligned}
n_1^A &= \frac{1}{2} + \frac{\left(-4t^2 n_1^A + 4bt\rho n_1^B\right) + 2\left(t - b\rho\right)t}{2\left(t^2 - b^2\rho\right)} \\
&\quad + \frac{\left[\left(-4bt\rho n_1^B + 4b^2\rho n_1^A\right) - 2\left(b - t\right)b\rho\right] + \left(1 - \rho\right)t^2}{2\left(t^2 - b^2\rho\right)} \\
\Longleftrightarrow n_1^A &= \frac{1}{2} + \frac{-2 \cdot 2\left(t^2 - b^2\rho\right)n_1^A + 2\left(t^2 - b^2\rho\right) + \left(1 - \rho\right)t^2}{2\left(t^2 - b^2\rho\right)} \\
\Longleftrightarrow n_1^A &= \frac{1}{2} - 2n_1^A + 1 + \frac{\left(1 - \rho\right)t^2}{2\left(t^2 - b^2\rho\right)} \\
\Longleftrightarrow n_1^A &= \frac{1}{2} + \frac{\left(1 - \rho\right)t^2}{6\left(t^2 - b^2\rho\right)} \left(> \frac{1}{2}\right).
\end{aligned}$$

This market share takes a weakly interior value if and only if

$$\begin{aligned}
n_1^A \leq 1 \iff & 3\left(t^2 - b^2\rho\right) \geq \left(1 - \rho\right)t^2 \\
\iff & \left(3t^2 - 3b^2\rho\right) + \left(3t^2\rho - 3t^2\right) \geq \left(t^2 - t^2\rho\right) + \left(3t^2\rho - 3t^2\right) \\
\iff & -3\left(b^2 - t^2\right)\rho \geq -2\left(1 - \rho\right)t^2 \\
\iff & 3\left(b - t\right)\rho \leq \frac{2\left(1 - \rho\right)t^2}{b + t}.
\end{aligned} \tag{4.23}$$

Recall that all of the potential users with biased expectations strictly prefer the platform:

$$\begin{aligned}
d_1^A\left(\cdot; 1, 0\right) \not\leq 1 \iff & 3b\rho - t\rho > 2t \\
\iff & \left(3b\rho - t\rho\right) - 2t\rho > 2t - 2t\rho \\
\iff & 3\left(b - t\right)\rho > \left(1 - \rho\right)t,
\end{aligned}$$

where

$$\begin{aligned}
p_2^A - p_1^A &= \left(-\frac{4t}{\rho}n_1^A + 4bn_1^B\right) + \frac{2}{\rho}\left(t - b\rho\right) \\
&= -\frac{2\left(1 - \rho\right)t}{3\rho} \\
d_1^A\left(\cdot; 1, 0\right) &= \frac{\left(b + t\right) + \left(p_2^A - p_1^A\right)}{2t} \qquad \text{(see the last part of Appendix 4.A.1)} \\
&= \frac{\left(3b + 5t\right)\rho - 2t}{6t\rho}.
\end{aligned}$$

There does not exist (b, t, ρ) that satisfies both the above condition and expression (4.23) because

$$(1-\rho)\, t - \frac{2\,(1-\rho)\, t^2}{b+t} = \frac{(1-\rho)\,(b-t)\, t}{b+t} > 0.$$

The above calculation implies that platform 2 cannot attract any potential user on side A. Although the prices under the first-order and second-order conditions are found to be no longer relevant, I continue examining platform 1's market share on side B in this case in order to determine the next step of this proof. The platform would obtain a market share of n_1^B on the side such that

$$\begin{aligned}
n_1^B &= \frac{1}{2} + \frac{\left[\left(-4t^2 n_1^B + 4btn_1^A\right) - 2\,(b-t)\, t\right] + \left[\left(-4btn_1^A + 4b^2\rho n_1^B\right) + 2\,(t-b\rho)\, b\right]}{2\,(t^2 - b^2\rho)} \\
&\quad + \frac{(1-\rho)\, bt}{2\,(t^2 - b^2\rho)} \\
\Longleftrightarrow n_1^B &= \frac{1}{2} - 2n_1^B + 1 + \frac{(1-\rho)\, bt}{2\,(t^2 - b^2\rho)} \\
\Longleftrightarrow n_1^B &= \frac{1}{2} + \frac{(1-\rho)\, bt}{6\,(t^2 - b^2\rho)} > n_1^A > 1
\end{aligned}$$

because $t < b \iff t^2 < bt$. Platform 2, again, cannot attract any agent on side B.

One therefore needs to investigate the consequences that occur (i) if platform 1 dominates side A without any restriction being imposed on side B and (ii) if platform 1 dominates side B without any restriction being imposed on side A. Suppose first that platform 1 attracts all potential users on side A, where $n_1^A = 1$. The platform's market share on side B equals

$$n_1^B = \frac{(b+t) - 4tn_1^B + 2\,(b+t)}{2t} \iff n_1^B = \frac{b+t}{2t},$$

where

$$\begin{aligned}
p_2^B - p_1^B &= \left(-4tn_1^B + 4b\right) - 2\,(b-t) \\
&= -4tn_1^B + 2\,(b+t) \\
n_1^B &= d_1^B\,(\cdot; 1, 0)
\end{aligned}$$

$$= \frac{(b+t) + \left(p_2^B - p_1^B\right)}{2t} \quad \left(\text{the preceding calculation of } d_1^A(\cdot;1,0) \text{ applies}\right).$$

This market share exceeds one:

$$\begin{aligned} n_1^B &= \frac{1}{2t}\left[(b+t) + (t-t)\right] \\ &= \frac{2t + (b-t)}{2t} = 1 + \frac{b-t}{2t} > 1. \end{aligned}$$

Suppose next that platform 1 attracts all potential users on side B, where $n_1^B = 1$. Recall that the agents with biased expectations are assumed to strictly prefer platform 1, and notice that they and those with rational expectations on side A have identical payoff functions given x. There is thus no possibility of platform 2 attracting any potential user on side A. Hence, neither a quasi-interior competitive outcome nor a side-dominance market outcome arises.

Market-Dominance Outcome

The existence of a market-dominance outcome needs to be formally examined because there exists an agent with a biased expectation who is indifferent between both platforms, which violates the condition imposed above in this subsection, if the outcome arises. This part first derives the (single) candidate of such an outcome. After that, the absence of any platform's strict incentive to deviate from its candidate price strategy is stated, which establishes that the candidate constitutes a market outcome. The last subsection establishes some properties of a market-dominance outcome to conclude this proof.

Suppose that platform 1 attracts all potential users on both sides, and assume that each platform maximizes its profit in this situation. There arises an allocation on each side such that $n_1^A = n_1^B = 1$ and $n_2^A = n_2^B = 0$. Platform 1 in this case chooses a price pair such that the platform can exactly obtain the market shares of 100 percent on both sides:

$$n_1^A = \rho d_1^A\left(p_1^A, p_2^A; 1, 0\right) + (1-\rho) \cdot 1 = 1$$

$$
\begin{aligned}
&\Longleftrightarrow \frac{\left[(b+t)+\left(p_2^A-p_1^A\right)\right]\rho}{2t}+(1-\rho)=1\\
&\Longleftrightarrow \underbrace{\frac{(b+t)+\left(p_2^A-p_1^A\right)}{2t}}_{=d_1^A\left(p_1^A,p_2^A;1,0\right)}=1\\
&\Longleftrightarrow p_1^A=p_2^A+(b-t)
\end{aligned}
$$

regarding side A, and

$$
n_1^B=\underbrace{\frac{(b+t)+\left(p_2^B-p_1^B\right)}{2t}}_{=d_1^B\left(p_1^B,p_2^B;1,0\right)}=1 \iff p_1^B=p_2^B+(b-t)
$$

regarding side B. Because the second-order conditions are not violated, platform 2 in the case chooses a price strategy based on the first-order conditions:

$$
\begin{aligned}
p_2^A &= \frac{2t}{\rho}\cdot\underbrace{0}_{=n_2^A}-2b\cdot\underbrace{0}_{=n_2^B}=0\\
p_2^B &= 2t\cdot\underbrace{0}_{=n_2^B}-2b\cdot\underbrace{0}_{=n_2^A}=0.
\end{aligned}
$$

Platform 1's resulting price on each side is thus

$$
p_1^A=p_1^B=b-t>0.
$$

The above bundle of prices and market shares therefore constitutes a market outcome if both platforms really maximize the respective profits by taking the respective price strategies.

Next, examine whether platforms may deviate in the strict sense from the respective price pairs that constitute the market-outcome candidate to the prices based on the respective best-response strategies in the case of Proposition 4.1. Consider first platform 1's incentive given that $p_2^A=p_2^B=0$. The platform chooses $p_1^A=b-t$ by the assumption that expression (4.9) holds, which excludes the possibility that potential users with biased expectations prefer platform 2 and thus guarantees that $d_1^A(p_1^A,0;1,0)=1$ (where $p_2^A=$

0). There is also no reason for the platform to change p_1^B because the first-order derivatives with respect to that variable in the current and preceding propositions are equal (see Appendix 4.A.1 for the latter). Thus, the platform does not need to deviate from its price strategy found above. Consider next platform 2's incentive given that $p_1^A = p_1^B = b - t$. The first-order and second-order conditions for the platform's problem in the case of the preceding proposition, not only in that of the current proposition, are satisfied if $p_2^A = p_2^B = 0$ and $n_2^A = n_2^B = 0$. The platform thus does not need to deviate from its price strategy derived above. Accordingly, neither platform possesses at least a strict incentive of deviation. The market-outcome candidate obtained in the preceding paragraph is therefore realized as a market outcome.

This proof concludes by proving the properties of the above market outcome mentioned in Proposition 4.1. First, recall that expression (4.9) is violated, that condition (4.16) holds, and that the market-outcome pattern obtained in this part is unique. The first statement suggests that a corner market outcome induced by the violation of a second-order condition never occurs. The second statement means that no interior competitive outcome arises, as established in Proposition 4.1. These two statements also enable one to establish that any solution to the competition game is an outcome of platform 1's entire-market dominance because the preceding paragraphs show that none of quasi-interior and side-dominance outcomes arises. The combination of the above implications and the third statement implies that the market outcome derived in this part is a unique solution to the competition game. Moreover, the arising market outcome can immediately be found to constitutes a Nash equilibrium because all players, who include all of the agents with biased expectations (which are that $\widehat{n}_1^B = 1$ and $\widehat{n}_2^B = 0$), form consistent expectations with the realized components of that outcome.

4.A.3 Proof of Proposition 4.3

This proof begins with the efficient outcome to which the first-order and second-order conditions apply. The first-order conditions for welfare maximization are that

$$\begin{aligned}
&\frac{\partial W(\cdot)}{\partial n_1^A} = 2\left(2n_1^B - 1\right)b - \left(2n_1^A - 1\right)t = 0 \\
\iff &n_1^A = \frac{2b}{t}n_1^B + \frac{t-2b}{2t} \\
&\frac{\partial W(\cdot)}{\partial n_1^B} = 2\left(2n_1^A - 1\right)b - \left(2n_1^B - 1\right)t = 0 \\
\iff &n_1^B = \frac{2b}{t}n_1^A + \frac{t-2b}{2t}.
\end{aligned}$$

The second-order conditions are that

$$\begin{aligned}
&\frac{\partial^2 W(\cdot)}{\partial \left(n_1^A\right)^2} = \frac{\partial^2 W(\cdot)}{\partial \left(n_1^B\right)^2} = -2t < 0 \\
&H_0(\cdot) = \underbrace{(-2t)\cdot(-2t) - (4b\cdot 4b)}_{=4t^2-16b^2=4(t-2b)(t+2b)} > 0 \iff t > 2b.
\end{aligned}$$

Under these conditions, the combination of the first-order conditions yields a candidate pair of n_1^{A**} and n_1^{B**} such that

$$\begin{aligned}
&n_1^A = \frac{4b^2}{t^2}n_1^A + \frac{2bt-4b^2}{2t^2} + \frac{t-2b}{2t} \\
\iff &\underbrace{\frac{t^2-4b^2}{t^2}}_{\neq 0}n_1^A = \frac{t^2-16b^2}{t^2} \\
\iff &n_1^A = \frac{1}{2} \\
&n_1^B = \frac{2b}{t}\cdot\frac{1}{2} + \frac{t-2b}{2t} = \frac{1}{2}.
\end{aligned}$$

Social welfare is maximized if and only if $n_1^A = n_1^B = 1/2$ under the assumption that $t > 2b$.

Next, suppose that $t = 2b$. The first-order condition with respect to each market share implies that $n_1^A = n_1^B = n_1 \in [0, 1]$. Welfare maximization thus becomes the optimization problem to maximize

$$\begin{aligned}
\overline{W}\left(n_1\right) &\equiv W\left(n_1, n_1\right) \\
&= 2v + 2\left[n_1^2 + \left(1-n_1\right)^2\right]b - \left[2\int_0^{n_1} x\mathrm{d}x + 2\int_{n_1}^{1}\left(1-x\right)\mathrm{d}x\right]t \\
&= 2\left[v + \left(2n_1^2 - 2n_1 + 1\right)b - \left(\int_0^{n_1} x\mathrm{d}x + \int_0^{1-n_1} x\mathrm{d}x\right)t\right]
\end{aligned}$$

with respect to n_1. The first-order derivative equals

$$\frac{\mathrm{d}\overline{W}\left(\cdot\right)}{\mathrm{d}n_1} = 2\left[2\left(2b-t\right)n_1 + \left(t-2b\right)\right] = 0$$

for any n_1 because $t - 2b = 2b - t = 0$, which implies that any n_1 yields the same value of social welfare. Welfare maximization therefore occurs if and only if $n_1^A = n_1^B \in [0, 1]$ in this case.

Consider the case in which $(0 <)t < 2b$. All agents on at least one side should participate in a single platform. Suppose, for instance, that platform 1 attracts all agents on side A. The first-order derivative of social welfare with respect to n_1^B is a linear decreasing function of n_1^B such that

$$\frac{\partial W\left(\cdot\right)}{\partial n_1^B} = 2b - \left(2n_1^B - 1\right)t \geq 2b - t > 0.$$

Social welfare is thus maximized if and only if $n_1^B = 1$. An analogous result can be found if one starts the discussion from any of the situations in which platform 1 attracts all agents on side B, in which platform 2 attracts all agents on side A, and in which platform 2 attracts all agents on side B. Therefore, social welfare is maximized if and only if $n_1^A = n_1^B \in \{0, 1\}$ in this case.

One can derive the efficient outcome for a bundle of b and t. The efficient pair of platform 1's market shares, (n_1^{A**}, n_1^{B**}), is obtained as in the preceding paragraphs. The efficient pair of platform 2's market shares, (n_2^{A**}, n_2^{B**}), is derived such that $n_2^{A**} = 1 - n_1^{A**}$ and $n_2^{B**} = 1 - n_1^{B**}$.

4.A.4 Proof of Proposition 4.4

Suppose that $t < 2b$. Propositions 4.1 and 4.2 imply that one of two different configurations might occur. In the case of Proposition 4.1, there arises an outcome such that $1/2 < n_1^{B*} < n_1^{A*} < 1$ and $0 < n_2^{A*} < n_2^{B*} < 1/2$. In the case of Proposition 4.2, there arises an outcome such that $n_1^{A*} = 1$, $n_1^{B*} = 1$, $n_2^{A*} = 0$, and $n_2^{B*} = 0$. Proposition 4.3 establishes under condition (4.10) that $n_1^{A**} = n_1^{B**} = 1$ and $n_2^{A**} = n_2^{B**} = 0$. Therefore, $1/2 < n_1^{B*} < n_1^{A*} < n_1^{A**} = n_1^{B**}$ and $n_2^{A**} = n_2^{B**} < n_2^{A*} < n_2^{B*} < 1/2$ in the case of Proposition 4.1, and $n_1^{A*} = n_1^{A**}$, $n_1^{B*} = n_1^{B**}$, $n_2^{A*} = n_2^{A**}$, and $n_2^{B*} = n_2^{B**}$.

Consider the case in which $t \geq 2b$. Proposition 4.1 states, again, that $1/2 < n_1^{B*} < n_1^{A*} < 1$ and $0 < n_2^{A*} < n_2^{B*} < 1/2$. Suppose that $t = 2b$. Proposition 4.3 shows the existence of multiple efficient outcomes such that $n_1^{A**} = n_1^{B**} = 1 - n_2^{A**} = 1 - n_2^{B**}$. These imply that (i) $n_1^{A*} > n_1^{B*} = n_1^{A**} = n_1^{B**}$ and $n_2^{A*} < n_2^{B*} = n_2^{A**} = n_2^{B**}$ or (ii) $n_1^{B*} < n_1^{A*} = n_1^{A**} = n_1^{B**}$ and $n_2^{B*} > n_2^{A*} = n_2^{A**} = n_2^{B**}$. Suppose that $t > 2b$. Proposition 4.3 establishes that $n_1^{A**} = n_2^{A**} = n_1^{B**} = n_2^{B**} = 1/2$. Thus, $n_1^{B*} < n_1^{A*} < 1/2 = n_1^{A**} = n_1^{B**}$ and $n_2^{A*} < n_2^{B*} < 1/2 = n_2^{A**} = n_2^{B**}$.

Chapter 5

Concluding Remarks

5.1 Summary of this Dissertation

This dissertation studies two-sided platform competition when potential users are heterogeneous in terms of indirect network externality and expectation. Heterogeneity in indirect network externality may occur from two different possibilities: potential users have heterogeneous tastes for cross-side interactions (e.g., subscribers to advertising-supported media) and play heterogeneous roles (e.g., third-party firms as content providers or advertisers). As for heterogeneity in expectation, I focus on the possibility that some potential users may fail in correctly expecting the number of platform users and form idiosyncratic expectations biased toward a particular platform. The reminder of this section summarizes the preceding three chapters.

Chapter 2 studies duopolistic price competition in a two-sided market with positive and negative indirect network externalities on both sides. I develop a model in which the indirect network externality is positive for some agents and negative for the others on each side. The chapter shows that (i) a platform in equilibrium attracts a larger number of agents on both sides if the proportion of agents who incur an indirect network negative externality is small and (ii) each platform in equilibrium obtains a larger market share on one side and a lower market share on the other side if the proportion is large. Social welfare is not maximized in these equilibria because the platform with the lower market share on each side attracts too many agents in the former case while each platform attracts too many agents on the side with a lower market share in the latter case.

Chapter 3 studies duopolistic platform competition such that the proportion of third-party contents and advertisements matters in the consumer valuations of each platform. I model a two-sided market in which (i) third-party firms behave as content providers and advertisers, (ii) consumers strictly prefer third-party contents to advertisements, (iii) platforms pay compensations to content-providing firms but receive fees from advertising firms, and (iv) each platform faces a constraint on the number of its slots that can be offered to content-providing and advertising firms. The equilibrium analysis shows symmetric and asymmetric corner market-outcome configurations, depending on the market structure. In the symmetric case, both platforms allocate all of their slots to firms of the same type (i.e., either content-providing or advertising firms). In the asymmetric case, one and the other platforms fill all of their slots with third-party contents and advertisements, respectively, which describes a special type of vertical differentiation such that the platform with third-party contents does not always earn a higher profit despite its larger consumer-side market share. The welfare analysis establishes that social welfare is or is not maximized in equilibrium although the equilibrium and efficient outcomes coincide in the resulting firm allocation; particularly, an asymmetric equilibrium is inefficient because the equilibrium and efficient consumer allocations differ.

Chapter 4 investigates the consequence of platform competition in a duopolistic two-sided market with potential users forming biased expectations toward a specific platform. The chapter constructs a simple one-shot competition game and incorporates to it the situation in which some agents on a certain side believe that a particular platform dominates the opposite side. In particular, the chapter adopts a solution concept relaxing Nash equilibrium such that potential users may form inconsistent expectations with the opposite-side allocation to describe the consequence of this competition game. The analysis of the competition game finds that the platform with an advantage from biased expectations (i) obtains larger market shares on both sides but (ii) dominates the entire market if and only if the indirect network externality exerted on each side is sufficiently intense and few agents hold biased expectations. The welfare analysis establishes that biased expectations in most cases, not always though, reduce the value of social welfare realized in platform competition. Moreover, the chapter describes a certain platform's expectation-driven dominance of the entire market as a Nash equilibrium under a framework that allows for biased expectations.

5.2 Discussion: Remaining Problems

The above three studies have two major remaining problems, one of which is that each study adopts a simple framework to explicitly obtain the market outcome and its welfare implications. Chapter 2 uses a simplified concept of rational expectation to formulate platform-choice behavior. Chapter 3 assumes that content-providing and advertising firms respectively have identical payoff functions, and that both platforms always coexist in the equilibrium and welfare analyses. Chapter 4 adopts a side-symmetric formulation of demand sides and thus partially simplifies cross-side price coordination by platforms. Moreover, chapters 2 and 4 abstract the possibility that some potential users might be multihoming agents. Although chapter 2 contains a brief analysis of the case in which potential users can form more sophisticated allocation expectations, the robustness of the other aforementioned simplifications should be examined in future research.

The second problem is that each of the three studies only obtains the consequence of duopolistic competition, solves welfare maximization, and discusses the interpretation and welfare implications of the competitive outcome. For instance, this dissertation cannot capture the impacts of entry, mergers, and other business practices than competition on profits, allocations, and social welfare. The dissertation also abstracts other types of competition policies such as consumer and small-firm protection. These topics deserve to be covered in future research.

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