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主　論　文　の　要　旨

論 文 題 目　Geometric Relations among the Bott-Virasoro Group, Equicentroaffine Curves and the KdV Equation (Bott-Virasoro 群と Equicentroaffine 曲線および KdV 方程 式の間の幾何的な関係)

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論　文　内　容　の　要　旨

As the title says, in this thesis, we would like to build a bridge between two different mathematical objects: the Bott-Virasoro group and the space of equicentroaffine curves. The key will be the Korteweg-de Vries equation:

$$\frac{\partial u}{\partial t} = -a\frac{\partial^3 u}{\partial t^3} - 3u\frac{\partial u}{\partial x}, \tag{1}$$

where $u : S^1 \times \mathbb{R} \to \mathbb{R}$ is a function of $(x, t) \in S^1 \times \mathbb{R}$ and a a positive constant.

In Chapter 1, we reviwed the Euler equation and applied it to the Bott-Virasoro group. We saw that it amounts to the KdV equation (1). On the other hand, by making use of a smooth Euler cocycle, we generalized the Bott-Virasoro group, and, as one of our main results, obtained a generalized version of the KdV eqaution:

$$\dot{u} = -3uu' - \beta au''' + \alpha au'. \tag{2}$$

By setting $u = v + \frac{1}{3}a\alpha$, and $\beta = 1$, it leads to (1), which is nothing but the KdV equation.

In Chapter 2, we turned our attention to $\mathcal{M}$, the connected component of the space of equicentroaffine curves that contains the unit circle c. An element in $\mathcal{M}$ is a plain curve $\gamma : S^1 \to \mathbb{R}^2$ satisfying

$$\det(\gamma, \gamma') = 1.$$

We reviewed two presymplectic forms $\hat{\omega}_0$ and $\hat{\omega}_1$ on $\mathcal{M}$, as well as a Hamiltonian function H on $\mathcal{M}$, whose presymplectic gradient X_H with respect to $\hat{\omega}_1$ satisfies

$$X_H(\gamma) = \frac{1}{2}\kappa'\gamma - \kappa\gamma',$$

where $\kappa : S^1 \to \mathbb{R}$ is the equicentroaffine curvature determined by γ. Given an integral curve $\tilde{\gamma}$ of X_H, let $\tilde{\kappa}$ be its equicentroaffine curvature flow. Then, we verified that $\tilde{\kappa}$ satisfies the KdV equation (1).

It is very interesting that the KdV equation appears in such two completely different objects, the Bott-Virasoro group and the space of equicentroaffine curves $\mathcal{M}$. We thought that there may exist some mathematical connection behind them, and this was one of our motivations for the main results.

In Chapter 3, we reviewed two well-known forms, the canonical symplectic form $d\Theta$ on $G \ltimes \mathfrak{g}^*$ and the Kirillov-Kostant-Souriau form ω_{KKS} on $\mathfrak{g}^*$. We introduced a right action of G on $G \ltimes \mathfrak{g}^*$. Denoting by $\underline{U}$ its fundamental vector field and $\mathrm{pr}_2 : G \ltimes \mathfrak{g}^* \to \mathfrak{g}^*$ the projection map, we proved:

Theorem 1. *For the Kirillov-Kostant-Souriau form ω_{KKS} on $\mathfrak{g}^*$ and the canonical symplectic form $d\Theta$ on $G \ltimes \mathfrak{g}^*$, we have*

$$\mathrm{pr}_2^* \omega_{KKS}(\underline{U}_{(g,\sigma)}, \underline{V}_{(g,\sigma)}) = d\Theta(\underline{U}_{(g,\sigma)}, \underline{V}_{(g,\sigma)}),$$

where $U, V \in \mathfrak{g}$ and $(g, \sigma) \in G \ltimes \mathfrak{g}^*$. ■

On, the other hand, we defined a function E on $G \ltimes \mathfrak{g}^*$. Let X_E be the symplectic gradient of E with respect to $d\Theta$ and (φ, ξ) its integral curve. We showed that ξ satisfies the Euler equation. Setting G to be the Bott-Virasoro group, we verified that ξ satisfies:

$$\dot{\xi} = \frac{1}{2}\xi''' + 3\xi'\xi, \tag{3}$$

which is only different from the KdV equation (1) by a minus sign. Observing this, we deduced that we may find a way to explain why the KdV equation appears in both cases by revealing the relationship between $d\Theta$ and $\hat{\omega}_1$, as well as X_E and X_H. As a result, we proved that:

Theorem 2. *Given $\gamma \in \mathcal{M}$, we take $\psi \in \mathrm{Diff}(S^1)$ such that $\gamma = c \cdot \psi$. Let $\sigma_1 : \mathcal{M} \to G \ltimes \mathfrak{g}^*$ be a map on $\mathcal{M}$ given by*

$$\sigma_1(\gamma) := ((\psi, 0), (-\kappa dx \otimes dx, -\frac{1}{2})),$$

where κ is the equicentroaffine curvature of γ. Then, we have

$$\sigma_1^* d\Theta = \hat{\omega}_1$$

where $d\Theta$ is the canonical symplectic form on $G \ltimes \mathfrak{g}^*$ and $\hat{\omega}_1$ the Fujioka-Kurose 2-form on $\mathcal{M}$. ■

By making use of the map σ_1, we obtained a relation between X_H and X_E:

$$\sigma_1(X_H(\gamma)) = X_E(\sigma_1(\gamma)) + X \tag{4}$$

where $X \in T_{\sigma_1(\gamma)}((\mathrm{Diff}(S^1) \times_B \mathbb{R}) \ltimes (\mathfrak{X}(S^1) \times_\omega \mathbb{R})^*)$ is a tangent vector such that

$$d\Theta(X_{\sigma_1(\gamma)}, \sigma_{1*}Z) = 0 \tag{5}$$

for all $Z \in T_\gamma \mathcal{M}$. The existence of X in (4) may keep us from getting a strong relation between X_H and X_E. But what we proved in the last theorem cleared the obstruction. This helped us finally answer the question why the KdV equation appears for the space of equicentroaffine curves.