

# Hong and Li meet Weyl and Fabinger: Modeling vertical structure by the conduct parameter approach\*

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## Abstract

By using Weyl and Fabinger's (2013) conduct parameter approach, this note extends Hong and Li's (2017) model of vertical structure to include downstream and upstream competition, and thereby generalizes the formula for cost pass-through elasticity. Three channels are identified through which downstream and upstream competition affect the cost pass-through elasticity, and it is argued that competition generally has an ambiguous effect.

Keywords: Vertical relationships; Conduct parameter; Cost pass-through.

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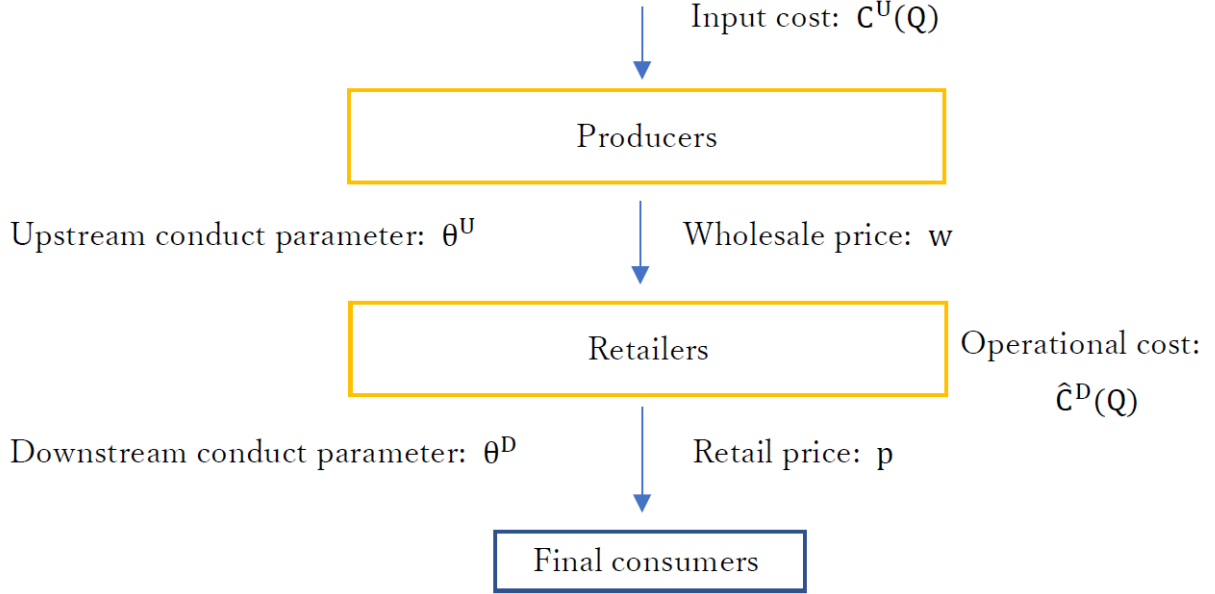
# 1 Introduction

It is well recognized that cost pass-through—how the final price responds to a change in marginal cost— is important to understand the significance of a policy change such as the introduction of a “soda tax” or some other effects such as a change in exchange rate (see Ritz 2018 for an excellent survey). When a vertical relationship is considered, one also needs to distinguish between cost pass-through perceived by downstream and by upstream firms. Hong and Li (2017) empirically study how cost pass-through is affected by vertical and horizontal dimensions based on the formula for cost pass-through elasticity. Whereas the degree of vertical closeness—measured by product-level branding a product (i.e., whether retailer sells a national-brand product under no vertical integration or its own private-brand product under vertical integration)—is considered in their study, horizontal competition is not fully taken into account because Hong and Li’s (2017) model assumes one downstream firm and one upstream firm.

In this note, I extend Hong and Li’s (2017) formula of the cost pass-through to include both downstream and upstream competition by using Weyl and Fabinger’s (2013) conduct parameter approach. One of the useful features of the conduct parameter approach is that one can circumvent unnecessary complications that may arise from modeling strategic interaction directly and yet still manage to focus on the consequences of imperfect competition. After the equilibrium retail and wholesale prices using the upstream and downstream conduct parameters are derived in Section 2, Section 3 presents a generalized version of Hong and Li’s (2017) formula for cost pass-through elasticity.

In Section 4, I identify three channels through which downstream and upstream competition affect cost pass-through elasticity. One is related to downstream competition, where a change in the intensity of downstream competition has an ambiguous effect on cost pass-through elasticity. The other two channels are related to upstream competition. As discussed by Hong and Li (2017), these two channels work in opposite directions when the cost pass-through elasticity is compared across the two regimes (i.e., arm’s-length pricing and vertical integration). It is pointed out that a lesser degree of upstream competition may strengthen or weaken these forces. Lastly, Section 5 concludes the paper.

Figure 1: Vertical structure modeled with conduct parameters.



## 2 Using conduct parameters to model downstream and upstream competition in vertical relationships

The following model is a simplified illustration of Weyl and Fabinger's (2013, pp. 562-64) setup of vertical structure.<sup>1</sup> Upstream firms (producers) sell their products to downstream firms (retailers) with a unit price  $w \geq 0$  (see Figure 1). Then, retailers successively sell these products to final consumers with a unit price  $p \geq 0$ . Now, suppose that each sector is represented by a single (representative) firm. The downstream firm's payment to the upstream firm is  $w \cdot Q$ , where  $Q$  is its sales volume as well as the order quantity. Thus, its total cost is  $C^D(Q) = wQ + \hat{C}^D(Q)$ , and the marginal cost is  $MC^D(Q) = w + \widehat{MC}^D(Q)$ . On the other hand, the upstream firm's cost of producing  $Q$  is given by  $C^U(Q)$  and the marginal cost by  $MC^U(Q)$ .

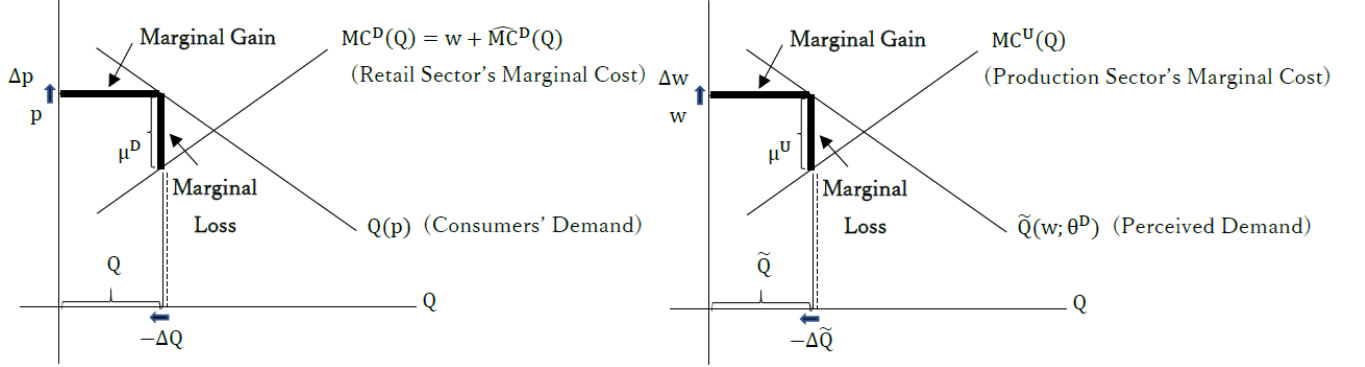
### 2.1 Downstream pricing

Let a small increase in price be denoted by  $\Delta p > 0$ , and the associated reduction in output  $\Delta Q < 0$ . Then, the downstream firm equates the marginal gain in profit with the marginal loss from raising the retail price  $p$  (see the left panel of Figure 2):

$$\underbrace{\theta^D(\Delta p)Q}_{\text{Maginal Gain}} = \underbrace{-\mu^D(\Delta Q)}_{\text{Marginal Loss}},$$

<sup>1</sup>Adachi and Ebina's (2014b) model is a further specialization of Weyl and Fabinger's (2013) setup with Cournot competition and the numbers of upstream and downstream firms being explicitly given.

Figure 2: Downstream/retail (left) and upstream/manufacturing (right) layers.



where  $\theta^D \in [0, 1]$  is the *conduct parameter* for the downstream sector, measuring the intensity of imperfect competition in this layer, and  $\mu^D \equiv p - w - \widehat{MC}^D(Q)$  is the *downstream markup*. Here,  $\theta^D = 1$  means that there is only one monopolistic retailer, whereas perfect competition prevails when  $\theta^D = 0$ . Then, given  $w$ , the equilibrium retail price  $p$  solves

$$\theta^D Q(p) = \{p - w - \widehat{MC}^D[Q(p)]\} \left( -\frac{\Delta Q}{\Delta p}(p) \right), \quad (1)$$

and the solution is denoted by  $p = p(w; \theta^D)$ . The upstream firm perceives the demand as  $Q[p(w; \theta^D)] \equiv \tilde{Q}(w; \theta^D)$ . Rewriting Equation (1), I obtain the following lemma.

**Lemma 1.** *The downstream markup rate is given by*

$$\frac{p - w - \widehat{MC}^D[Q(p)]}{p} = \frac{\theta^D}{\epsilon^D}, \quad (2)$$

where  $\epsilon^D$  is the downstream elasticity of demand:  $\epsilon^D(p) \equiv -Q'(p)p/Q(p)$ .

## 2.2 Upstream pricing

Similarly, the upstream firm equates the marginal gain in profit with the marginal loss from raising the wholesale price  $w$  by  $\Delta w > 0$  (see the right panel of Figure 2), given its perceived demand  $\tilde{Q}(w; \theta^D)$ :

$$\underbrace{\theta^U(\Delta w)\tilde{Q}}_{\text{Maginal Gain}} = \underbrace{-\mu^U(\Delta\tilde{Q})}_{\text{Marginal Loss}},$$

where  $\theta^U \in [0, 1]$  is the *conduct parameter* for the upstream sector,  $\mu^U \equiv w - MC^U(\tilde{Q})$  is the *downstream markup*, and  $\Delta\tilde{Q} < 0$  is the associated reduction in output. Thus, the equilibrium

wholesale price  $w$  solves

$$\theta^U \tilde{Q}(w; \theta^D) = \{w - MC^U[\tilde{Q}(w; \theta^D)]\} \left( -\frac{\Delta \tilde{Q}}{\Delta w}(w) \right), \quad (3)$$

and the solution is denoted by  $w^* = w^*(\theta^U, \theta^D)$ , where

$$\frac{\Delta \tilde{Q}}{\Delta w} = \frac{\Delta Q}{\Delta p} \cdot \frac{\Delta p}{\Delta w}.$$

Let the equilibrium retail price denoted by  $p^* = p^*(\theta^U, \theta^D) \equiv p[w^*(\theta^U, \theta^D); \theta^D]$ , and the equilibrium output by  $Q^* = Q^*(\theta^U, \theta^D) \equiv Q[p^*(\theta^U, \theta^D)]$ . Then, the following lemma is obtained from Equation (3).

**Lemma 2.** *The upstream markup rate is given by*

$$\frac{w^* - MC^U(Q^*)}{w^*} = \frac{\theta^U}{\rho_w \epsilon^D}, \quad (4)$$

where the wholesale price pass-through elasticity is defined by  $\rho_w \equiv [dp(w; \theta^D)/dw](w/p)$ .<sup>2</sup>

### 3 Extending Hong and Li's (2017) arguments by the conduct parameter approach

Now, suppose that the part of additional cost for downstream distribution has a constant marginal cost for an additional unit,  $\kappa^D \geq 0$  (this corresponds to  $\theta_i^r$  in Hong and Li 2017, p.152), that is,  $\widehat{MC}^D(Q) = \kappa^D$ . Furthermore, I assume that the marginal cost of upstream production is also constant:  $MC^U(Q) = c + \kappa^U$ , where  $c > 0$  is the marginal cost of “commodity inputs” (Hong and Li 2017, p.152) and  $\kappa^U \geq 0$  is an additional part (this corresponds to  $\theta_i^m$  in Hong and Li 2017, p.152). Then, Equations (2) and (4) in Lemmas 1 and 2 are simplified to

$$p = \frac{\epsilon^D}{\epsilon^D - \theta^D} (w + \kappa^D), \quad (5)$$

which corresponds to Hong and Li's (2017, p.152)  $p_i = \frac{\epsilon_i}{\epsilon_i - 1} (w_i + \theta_i^r)$ , and

$$w = \frac{\rho_w \epsilon^D}{\rho_w \epsilon^D - \theta^U} (c + \kappa^U), \quad (6)$$

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<sup>2</sup>See Adachi and Ebina (2014a) for an analysis of the role of wholesale pass-through in a model of successive monopoly.

which corresponds to Hong and Li's (2017, p. 152)  $w_i = \frac{\mu_i}{\mu_i - 1} (c + \theta_i^m)$ , respectively. Thus, the equilibrium retail price is expressed by

$$p = \frac{\epsilon^D}{\epsilon^D - \theta^D} \left[ \kappa^D + \frac{\rho_w \epsilon^D}{\rho_w \epsilon^D - \theta^U} (c + \kappa^U) \right].$$

This corresponds to Hong and Li's (2017, p. 153) Equation (1), where downstream and upstream competition is not considered. Now, I generalize Hong and Li's (2017, p. 153) Equation (5), which expresses the *cost pass-through elasticity* defined by  $\frac{dp}{dc} \frac{c}{p}$  for the case of arm's-length pricing between multiple manufacturers and multiple retailers.

**Proposition 1.** *The cost pass-through elasticity under downstream and upstream competition, where the competitiveness of the downstream and upstream layers is measured by  $\theta^D \in [0, 1]$  and  $\theta^U \in [0, 1]$ , respectively, is given by*

$$\frac{dp}{dc} \frac{c}{p} = \frac{1}{1 + \frac{d\epsilon^D}{dp} \frac{p}{\epsilon^D} \frac{\theta^D}{\epsilon^D - \theta^D}} \cdot \frac{1}{1 + \frac{d\mu}{dw} \frac{w}{\mu} \frac{\theta^U}{\mu - \theta^U}} \cdot \frac{c}{c + \kappa^U + \frac{\mu - \theta^U}{\mu} \kappa^D}, \quad (7)$$

where  $\mu(w) \equiv \rho_w(w) \epsilon^D [p(w)]$  is the wholesale price pass-through elasticity perceived by upstream firms.

*Proof.* With the use of Equations (5) and (6) above, it follows that

$$\begin{aligned} \frac{dp}{dc} \frac{c}{p} &= \left( \frac{dp}{dw} \frac{w}{p} \right) \cdot \left( \frac{dw}{dc} \frac{c}{w} \right) \\ &= \frac{\epsilon^D}{\epsilon^D - \theta^D + (p - w - \kappa^D) \cdot (\epsilon^D)'} \cdot \frac{(\epsilon^D - \theta^D)w}{\epsilon^D(w + \kappa^D)} \\ &\quad \times \frac{\mu}{\mu - \theta^U + (w - c - \kappa^U) \cdot (\mu)'} \cdot \frac{(\mu - \theta^D)w}{\mu(c + \kappa^U)} \\ &= \frac{1}{1 + \frac{d\epsilon^D}{dp} \frac{\theta^D p}{\epsilon^D} \frac{1}{\epsilon^D - \theta^D}} \cdot \frac{1}{1 + \frac{d\mu}{dw} \frac{\theta^U w}{\mu} \frac{1}{\mu - \theta^U}} \cdot \left( \frac{w}{w + \kappa^D} \cdot \frac{c}{c + \kappa^U} \right), \end{aligned}$$

where  $p - w - \kappa^D = \frac{\theta^D}{\epsilon^D - \theta^D} (w + \kappa^D) = \frac{\theta^D}{\epsilon^D - \theta^D} \frac{\epsilon^D - \theta^D}{\epsilon^D} p = \frac{\theta^D p}{\epsilon^D}$  and  $w - c - \kappa^U = \frac{\theta^U w}{\mu}$  are used. Lastly, it is shown that

$$\frac{1}{1 + \frac{\kappa^D}{w}} \cdot \frac{c}{c + \kappa^U} = \frac{c}{c + \kappa^U + \frac{\kappa^D}{w} (c + \kappa^U)} = \frac{c}{c + \kappa^U + \frac{\kappa^D}{w} \frac{\mu - \theta^U}{\mu} w} = \frac{c}{c + \kappa^U + \frac{\mu - \theta^U}{\mu} \kappa^D},$$

which provides the desired result.  $\square$

Note that if  $\theta^D = 1$  and  $\theta^U = 1$ , then Equation (7) above coincides with Hong and Li's (2017, p. 153) Equation (5).

## 4 Discussion

Here I discuss how Hong and Li's (2017) arguments are affected by the introduction of  $\theta^D$  and  $\theta^U$ . First, similar to Hong and Li's (2017, p.153) Equation (4), the cost pass-through under vertical integration is given by

$$\frac{dp}{dc} \frac{c}{p} = \frac{1}{1 + \frac{d\epsilon^D}{dp} \frac{p}{\epsilon^D} \frac{\theta^D}{\epsilon^D - \theta^D}} \cdot \frac{c}{c + \kappa^U + \kappa^D}, \quad (8)$$

which is derived from the pricing equation

$$p = \frac{\epsilon^D}{\epsilon^D - \theta^D} (c + \kappa^U + \kappa^D)$$

under vertical integration. This is an extension of Hong and Li's (2017, p.153) Equation (1) to include downstream competition.

For an easier comparison of cost pass-through elasticity under arm's-length pricing and under vertical integration, Equations (7) and (8) are juxtaposed:

$$\frac{dp}{dc} \frac{c}{p} = \underbrace{\frac{1}{1 + \left( \frac{d\epsilon^D}{dp} \frac{p}{\epsilon^D} \right) \frac{\theta^D}{\epsilon^D - \theta^D}}}_{(*)} \cdot \underbrace{\frac{1}{1 + \left( \frac{d\mu}{dw} \frac{w}{\mu} \right) \frac{\theta^U}{\mu - \theta^U}}}_{(i)} \cdot \underbrace{\frac{c}{c + \kappa^U + \frac{\mu - \theta^U}{\mu} \kappa^D}}_{(ii)} \quad (7')$$

$$\frac{dp}{dc} \frac{c}{p} = \underbrace{\frac{1}{1 + \left( \frac{d\epsilon^D}{dp} \frac{p}{\epsilon^D} \right) \frac{\theta^D}{\epsilon^D - \theta^D}}}_{(*)} \times 1 \times \frac{c}{c + \kappa^U + \kappa^D} \quad (8')$$

In comparison of Equations (7') and (8'), Hong and Li (2017, p.153) point out three channels that can lower the cost pass-through under arm's-length pricing more than vertical integration: (i) adjustment through upstream markup, (ii) the presence of the downstream cost, and (iii) the market power channel. The last channel is merely related to the fact that under vertical integration, the retail price is generally lower, and hence the output is larger. This simply means that the cost pass-through elasticity is different across the two regimes.

Instead I point out another channel that is related to downstream competition. This becomes evident once Hong and Li's (2017) formula is generalized. This is expressed by the term (\*) above, which is the *superelasticity* of demand (Kimball 1995; Ritz 2019). This amounts to the "elasticity of the elasticity," and it is known that the superelasticity is greater than or equal to unity if and only if the demand is log-concave. Now, consider a small increase in  $\theta^D$ ; then the retail sector becomes less competitive. If the market demand is log-concave, and hence  $\frac{d\epsilon^D}{dp} > 0$  holds, then this implies that  $\epsilon^D$  also rises through an increase in  $p$ . As a result,  $\frac{\theta^D}{\epsilon^D - \theta^D}$  may

increase or decrease, making the effect of downstream competition on the cost pass-through elasticity ambiguous.

The same reasoning also applies to the other two channels related to upstream competition. As explained by Hong and Li (2017, p.153), the presence of superelasticity of the perceived demand under arm's-length pricing ( $\frac{d\mu}{dw} \frac{w}{\mu} > 0$ ) lowers the cost pass-through elasticity compared with the case of vertical integration. This force is strengthened or weakened if the upstream sector becomes less competitive: this ambiguity comes from, as above, the fact that  $\frac{\theta^U}{\mu - \theta^U}$  may be larger or smaller as a result of an increase in  $\theta^U$ . Lastly, the cost channel, which arises only if  $\kappa^D$  is positive, is captured by  $\frac{\mu - \theta^U}{\mu} \leq 1$ . The sign for the effect of an increase in  $\theta^U$  on this term is generally undetermined as well.

## 5 Concluding remarks

In this note, I analyze a model of vertical structure in which Weyl and Fabinger's (2013) conduct parameter approach is utilized. Specifically, I generalize Hong and Li's (2017) formula for cost pass-through elasticity by introducing the upstream and downstream conduct parameters. The main result is obtained under the assumption of constant marginal cost. However, Ritz (2019) recently points out that if marginal costs are increasing, caution must be paid: a greater intensity of competition may, as opposed to standard intuition, *reduce* cost pass-through. It would be interesting to consider this issue of non-constant marginal costs in the context of vertical structure. It would also be interesting to consider asymmetric firms, although the main thrust would not change significantly.

## References

- Adachi, T., Ebina, T., 2014a. Double marginalization and cost pass-through: Weyl-Fabinger and Cowan meet Spengler and Bresnahan-Reiss. *Econom. Lett.* 122(2), 170–175.
- , —, 2014b. Cost pass-through and inverse demand curvature in vertical relationships with upstream and downstream competition. *Econom. Lett.* 124(3), 465–468.
- Hong, G. H., Li, N., 2017. Market structure and cost pass-through in retail. *Rev. Econ. Stat.* 99(1), 151-166.
- Kimball, M. S., 1995. The quantitative analytics of the basic neomonetarist model. *J. Money, Credit and Banking.* 27(4), Part 2, 1241–1277.
- Ritz, R. A., 2018. Oligopolistic competition and welfare. In: Corchón, L. Marini, M. (Eds.), *Handbook of Game Theory & Industrial Organization*, vol. 1, Edward Elgar, pp. 181-200.



—, 2019. Does competition increase pass-through? Unpublished.

Weyl, E. G., Fabinger, M., 2013. Pass-through as an economic tool: Principles of incidence under imperfect competition. *J. Polit. Econ.* 121(3), 528-583.