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COMPUTATIONAL MORPHOGENESIS OF DISCRETE STRUCTURES VIA GENETIC ALGORITHMS

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Abstract

Present paper describes the use of a stochastic search procedure based on genetic algorithms (GAs), in developing near-optimal topologies of load-bearing discrete structures. Much work has already been published on the topology optimization of discrete stuctures using genetic algorithms. In most of those papers, the topology of structures are expressed as a simple combination of members, and the existence of each member is directly connected to the genetic code. These methods, however, have a weak point. Namely when these methods are simply applied to express the topology of frame structures, there might be included needless members or those which lie on the other members. In addition to these problems, generated structures are not guaranteed to be structurally stable. These problems become more remarkable when the freedom of the problem becomes large. Present paper proposes new methods for expression of the stable discrete structures. A detail of the proposed methodology is presented as well as the results of numerical examples that clearly show the effectiveness and efficiency of the present methods.

Keywords: genetic algorithm, topology optimization, structural optimization, truss structure, non-linear programming, computational morphogenesis

1 Introduction

Truss topology optimization is one of the most interesting and at the same time difficult problems in structural optimization. There has been much research over three decades all over the world. In the literature, this problem is described as one where a ground structure containing many joints and members defines the discrete version of structural universe, and from which an optimal structure is derived. The ground structure approach was firstly proposed by Dorn et al. [1], where duality was used to formulate the optimal topology problem (minimal weight subject to stress constraints) as a linear programming problem. This approach was highly utilized [2,3]; however,

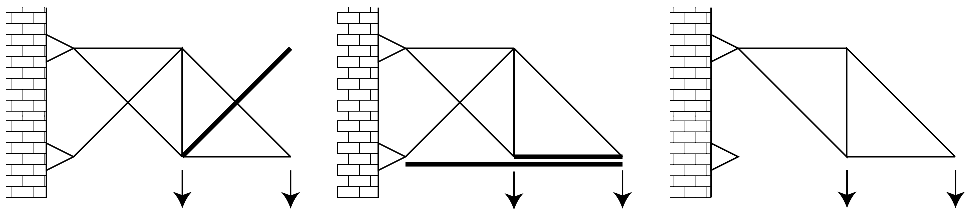


Fig. 1: Undesirable Structures

very technical and complicated calculations had to be used. The main difficulty in the present problem lies in the discreteness of the variables that can not be treated as continuous ones, as in the other ordinary problems.

A genetic algorithm proposed by Holland [4] is very useful and strong for treating such discrete variables and has already been used for the present problem of truss topology optimization [5,6,7,8,9,10,11]. A genetic algorithm is a stochastic search procedure that has its philosophical basis in Darwin's postulate of the "survival of the fittest". In formal ways for pursuing the optimal structures by using genetic algorithms, undesirable structures such as those shown in Fig. 1 can frequently come into being. This is because of the simple ways of the arrangement of the genetic strings which have no information on the condition of existence of the truss as a real structure. Here, if needless members are produced, the problem can be overcome by simply deleting these members. But when undesirable overlap of members or unstable structures are produced, it is very difficult to decide which members should be modified toward reasonable topologies. These problems become more remarkable when the freedom of the problem becomes large as shown in Fig. 2. In these cases, the solutions of topology have been simply killed off or their finesses have been set at relatively small fitness values by making use of penalty parameters so that they are suppressed to be born in successive processes. As a result of such treatment, the number of structurally reasonable individuals becomes small and hence, the probability for obtaining desirable optimal solutions decreases as the generation progresses.

Kwan [12] has already proposed a topological optimization method where the number of joints and bars, as well as their locations, are allowed to change. Using that method, the distribution of nodal points will determine only the topology of the structure. But, the present method deals with not only distributions of nodal points but also the exsistence of each structural member as the design parameters. According to this approach, the same distribution of nodal points creates a large number of structural topologies.

In this work, a new methodology is proposed for producing truss topology, which is based on the idea of triangular elements connected to each other. While the structural members are generated all at once through the ordinary way, the triangular elements are produced little by little in the proposed scheme, in such a manner, that, newly produced triangles are formed so as to be connected to the nodes having been generated up to the step directly before. These procedures are repeated until both supporting and loading points are included as one of the selected nodes. The topologies produced by the present method are guaranteed to have neither needless member nor undesirable overlaps between structural members, and also confirmed to be always stable structures.

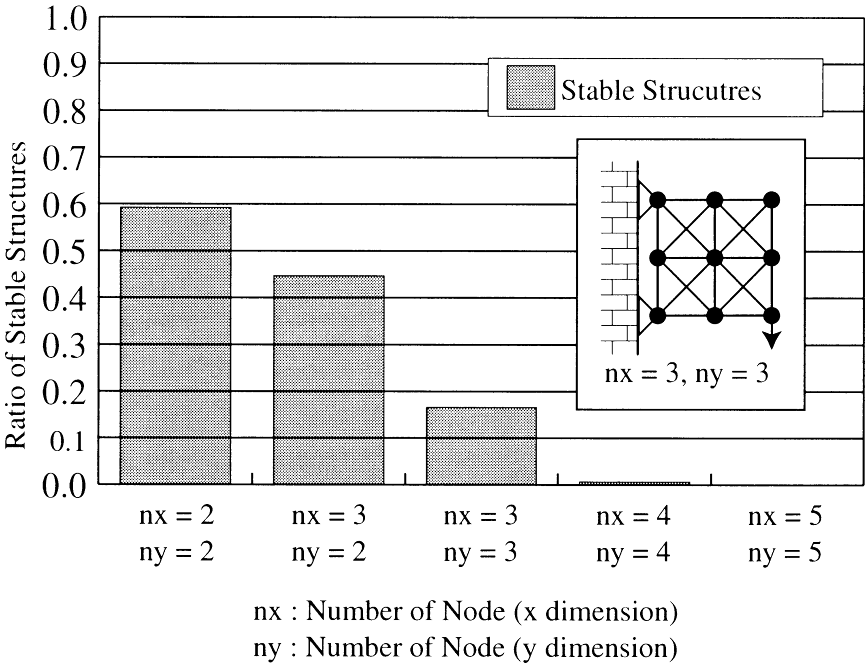


Fig. 2: Probability of Producing Stable Structures

2 Problem Formulation

The present structural optimization problem is stated as follows;

$$\text{minimize} \quad W(\boldsymbol{x}, \boldsymbol{A}) \tag{1}$$

$$\text{subject to} \quad g_i \leq 0 \tag{2}$$

Here W express the total weight, $\boldsymbol{x}$ is for dispositon of the members and nodal dislocation, $\boldsymbol{A}$ is corresponding to cross-sectional area and g_i expresses the i-th constraint condition of the structure. When the aim of the optimization is to find a topology whose weight is minimum and satisfies a certain set of constrains, we can adopt a simple fitness function shown as follows;

$$f = \frac{1}{W(\boldsymbol{x}, \boldsymbol{A})} \prod_i \gamma_i \tag{3}$$

Here, γ_i is a penalty term for the i-th constraint. For example, the stress penalty function forms as follows;

$$\gamma_\sigma = \begin{cases} \dfrac{\sigma_{\lim}}{|\sigma|} & |\sigma| > \sigma_{\lim} \\ 1 & \text{otherwise} \end{cases} \tag{4}$$

Here, γ_σ, σ_{lim} and σ are respresenting the penalty term of stress, the prescribed stress limit and the actual stress, respectively. The stress limit is formed as follows;

$$\sigma^t_{\text{lim}} = \frac{F}{1.5} \tag{5}$$

$$\sigma^c_{\text{lim}} = \begin{cases} \dfrac{1.0 - 0.4\left(\frac{\lambda}{\Lambda}\right)^2}{\frac{3}{2} + \frac{2}{3}\left(\frac{\lambda}{\Lambda}\right)^2} F & \text{if} \quad \lambda \leq \Lambda \\ \dfrac{0.277}{\left(\frac{\lambda}{\Lambda}\right)^2} F & \text{otherwise} \end{cases} \tag{6}$$

Here, σ^t_{lim}, σ^c_{lim}, λ, Λ, and F express the allowable tensile stress, the allowable compressive stress, the slenderness ratio, the critical slenderness ratio, and the referenced strength, respectively.

3 GAs in Topology Optimization

3.1 Generation of Structural Topology

In the followings, the genetic algorithms used in the present research toward the optimal structural topology of plane trusses is explained in detail. As can be seen in Fig. 3, at first, three points are randomly selected in the available area to make the first triangle in such a way that those points are not on the same line (Phase 1). Next, two points among the nodes in the foregoing chain of triangles are selected and another node among the nodes which have not yet been selected in the available area is connected to those points (Phase 2 and Phase 3). In the final stage, the necessary members are created so that each point is connected (Phase 4). These newly produced members will be stable. Until all required points, namely supporting points and loading points, are included

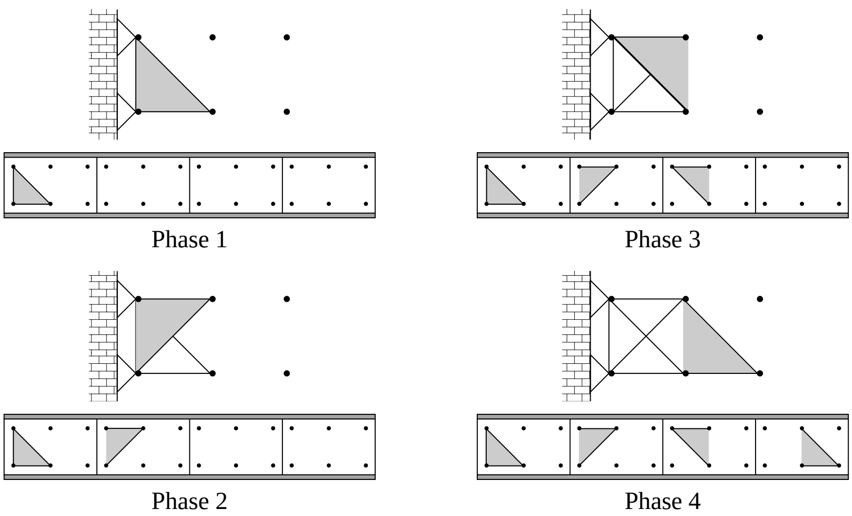


Fig. 3: Truss Topology Consisting of Triangles

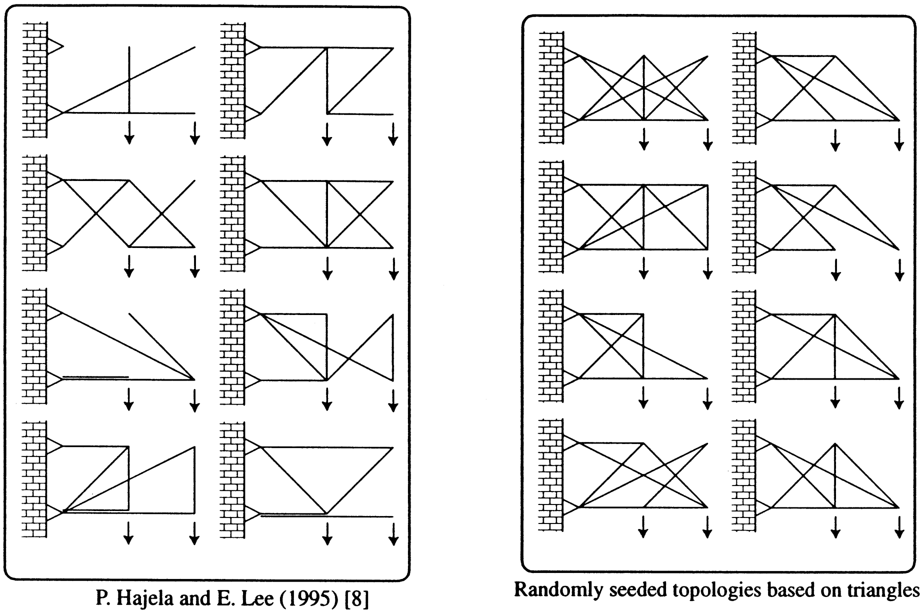


Fig. 4: Comparison between Different Methods

in these selected nodes, the same operations are continued. The required points, e.g. supporting points or loading points, are given by designer. It is not necessary to include all required points. For example, when there are 5 supporting points, at least 3 points among them is required to be selected as the suporting nodes, it is not necessary to include the 5 supporting points. Fig. 4 shows the results obtained by using simple GA coding [8] where those codes directly express the existence of members as well as the results by the present method. As can be seen from the figure, we can make sure that GA coding based on the triangular element is effective for seeding structurally nontrivial topologies.

By changing the triangular element to segment one, we can deal with the frame structural optimization problem just in the same manner as shown in Fig. 5.

Additionally, changing the trianglar element to a tetrahedron one, enable us to deal with the structural optimization problem of the 3-dimensional truss structure in just the same manner as has been shown in the plane problems. The present method has a strong advantage especially in a 3-dimensional problem because it is more difficult to produce candidates which have a stable truss topology when we use ordinary production methods. Fig. 6 shows the mechanism for producing the initial topology of a 3-dimensional truss.

3.2 Coding

In the proposed scheme, chromosomes are as shown in Fig. 7, where information about the triangular element is included in some groups of chromosomes composed of three bits as shown in the figure. The triangular elements, as shown in Fig. 7. b, consist of three nodal numbers as shown in Fig. 7. a. This method makes it possible to always produce stable truss topology from the viewpoint of structural mechanics. However, if as usual, we convert those produced topologies into bit codes based on the existence of each member and throw the bit codes into the GA process,

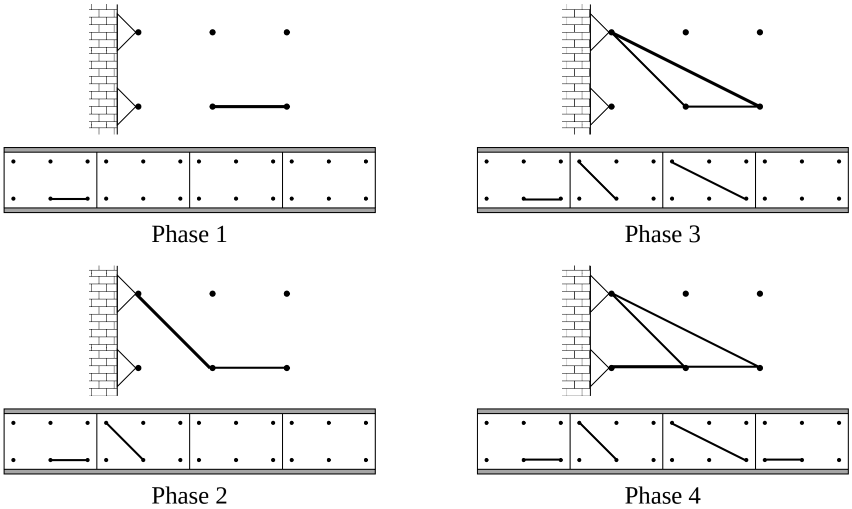


Fig. 5: Frame Topology Consisting of Segments

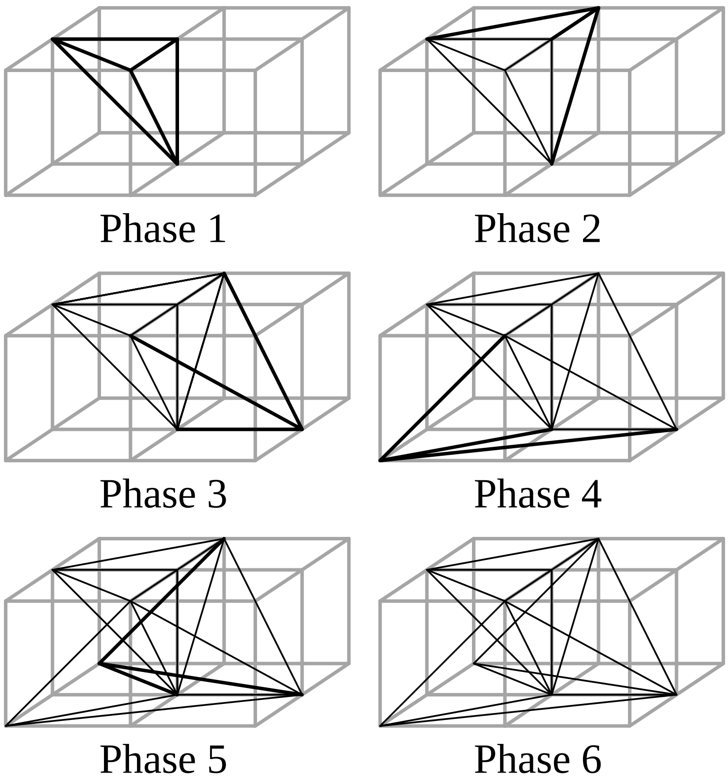


Fig. 6: Producing of The Initial 3-Dimensional Truss Topology

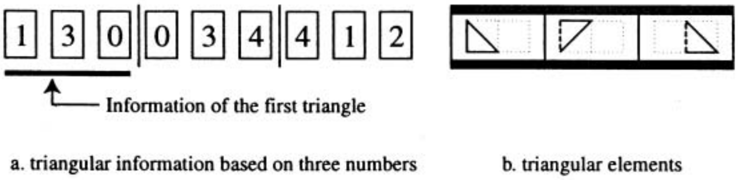


Fig. 7: Expression of Chromosome

such as the crossover or the mutation, each topology is a potentially unstable structure or can cause other problems. Consequently, we have to convert the truss topology in another way which is specially contrived for the present GA procedure. Briefly, the aim of GA is to look for a better element combination. Namely, while the aim in the usual method is to search for a desirable combination of members, in the present method, the desirable combination of triangles is to be pursued. In the present method, not the bit codes but the triangles which consist of three bits are treated as the genes which are the units of the chromosome.

3.3 Transformation Process to Structural Members

Using the methodology shown above gives us the way of expression of the topology of the stable structures by a combination of the codes which are expressed by the arrangement of combination of the fundamental stable elements. However, the conversion process from the expression given by those codes into the actual spatial arrangement of the structural members is needed. In this section, the detail of this conversion process is discussed.

Let us adopt the triangular stable fundamental element for the plane truss structures to explain the way of conversion. Let us suppose that the plane truss structure as shown in the left-hand side picture in Fig. 8 is expressed by the arrangement of the group of the triangular elements as shown in the right-hand side one in the same figure. The way of conversion from each other is as follows; firstly, a simple mapping of the triangular elements on to the region of the problem gives us the shape shown as the left-hand side picture in Fig. 9, where the sides of the elements having the nodes of the ground-structure on them are divided by the nodes themselves and the sides overlapped with several members are modified into a single side. Secondly, the straight members having nodes on them are changed into those which have no node on them, that is, replaced with one straight member. The conversion process given here promises to always bring about the stable structures and the same procedure can be done for the case where the straight linear elements are used for frame structures.

3.4 GA Expression for the Hybrid Structures

According to the procedure proposed up to the previous section, we can deal with so-called the hybrid structures which are the structures composed of trusses and frames. Through simultaneous using of the plural kinds of the fundamental stable element such as the linear element for the frame structures, the triangular element for the plane trusses and the tetrahedron element for the space trusses, we can express the structures composed of a various kind of fundamental elements.

In Fig. 10, an example of the structure composed of two kinds of the fundamental element is illustrated. The way of generation of the plane frame structure using the two kinds of the fundamental elements is as follows; firstly, the kind of the fundamental element generated at the first one is randomly selected. After choosing the arbitrary two or three nodes randomly picked up from the given design space of the problem, structural members having those nodes are generated

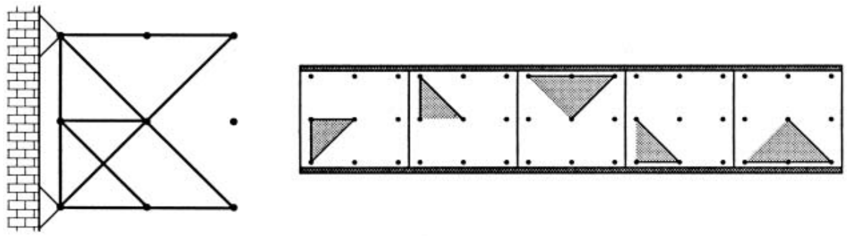


Fig. 8: Expression of Chromosome by Fundamental Stable Elements

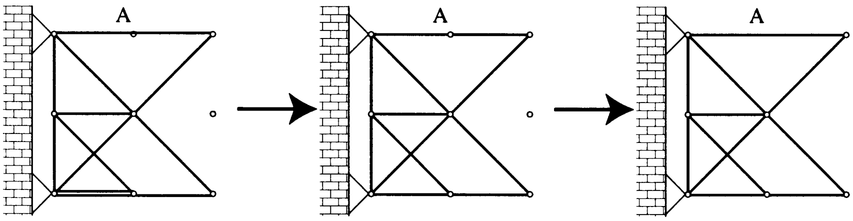


Fig. 9: Transformoing to the Structure from the Combination of Stable Elements

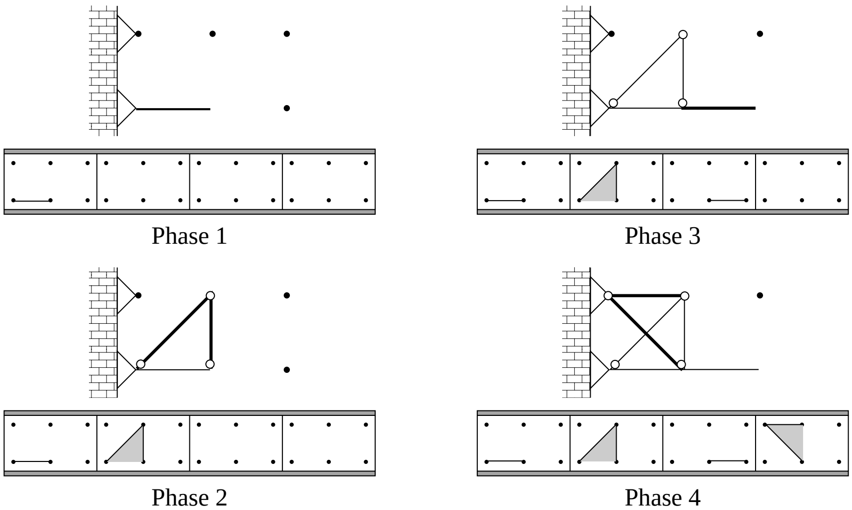


Fig. 10: Hybrid Topology Consisting of Combination of Stable Elements

(Phase 1), where the nodes of the element are supposed to be connected as rigid nodes or pinned joints according to the kind of the generated element, that is, the linear element or the triangular element, respectively. As the next step, a new fundamental element is generated among those nodes, one of which is selected from those which have not yet been selected as the nodes of the structure up to this stage and the other ones are selected from those which are already on the

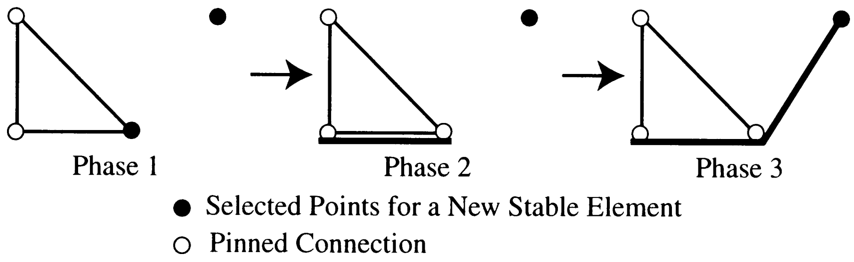


Fig. 11: The Rule for Producing Planer Discrete Structures

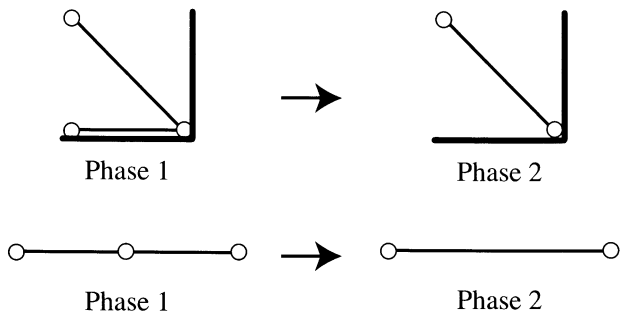


Fig. 12: The rule for Transforming to Structural Members

generated structure (Phase 2). At this point, whether the newly generated element is that of truss or of frame is randomly decided. On the successive steps, generation of the elements is continued until all of the necessary points such as supporting points and loading ones are selected as one of the node of the generated structure (Phase 3 & 4). Finally, every side of the generated structure is converted to that of the actual structure.

In the present scheme, the triangular element for the plane trusses, the linear element for the frames and the tetrahedron element for the space trusses are used so that the generated structures are guaranteed to be structurally stable. In the hybrid structures which are generated as a combination of a various kind of those fundamental elements, there are some possibilities that the obtained structures are not stable according to the way of the combination of the fundamental elements. In order to avoid such situation, the conditions for the generation process as shown in Fig. 11 are newly introduced. Namely, when the node selected for generation of the linear element is the pinned one, that is, the node without rotational rigidity, the linear element is automatically generated so as to overlap with the side of the triangular element including the node. Additionally, the following rules are settled in the process of the conversion from the generated configuration into the final objective structure; 1) the frame structure takes priority to be selected when there is an overlapping between truss and frame members as shown in the upper figure of Fig. 12, 2) when there is a pinned node on a line connected by two truss elements, the node is replaceed with a rigid connection as shown in the lower figure of Fig. 12.

For the three dimensional structures, we can extend the present scheme by replacing the triangular element with the tetrahedron one. However, there should be some additional rules preparing for the case which has not been encountered in the previous plane problems.

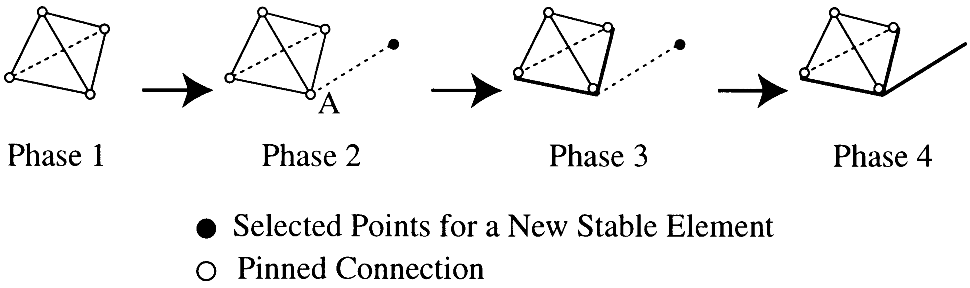


Fig. 13: The rule for producing the 3D structures

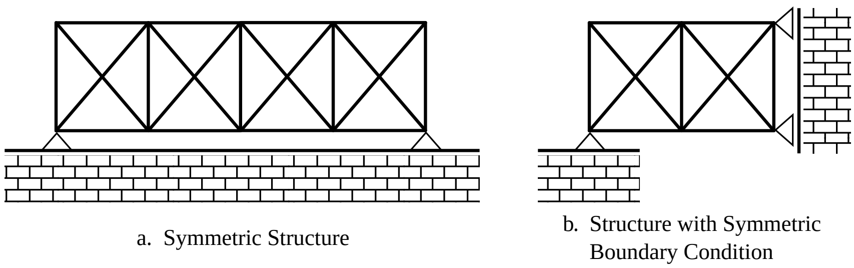


Fig. 14: Structural Analysis with Symmetric Boundry Conditioner

For the plane problem, the rule which controls the generation of unstable structures is to simply replace the side of the triangular truss element with the linear element when the selected node which is one of the linear element has no rotational rigidity. However, such replacement does not always guarantee to generate the stable structure for the three dimensional structures. Consequently, for the three dimensional problem, the new rule by which two sides of tetrahedron element are simultaneously replaced by the linear element as illustrated in Fig. 13. As shown in this figure, let us consider the case where we have the tetrahedron fundamental element as shown in Phase 1 at first and a new linear element is generated as the next step as shown in Phase 2. As the next step, randomly selecting the two sides from those of the tetrahedron element which is connected with the node ‘A’, one node of the former linear element, the two sides are enforced to change into the linear fundamental element as shown in Phase 3. Finally, the new linear element is generated as shown in Phase 4. Through this replacement process, the newly generated hybrid structures can be those which are always structurally stable.

3.5 Consideration of Topological Symmetricity

There are many structures that are symmetric in their topology. In the usual structural analysis, symmetric conditions are often utilized for those structures which have certain symmetricities in both their configuration and loading conditions, as shown in Figs. 14.a and 14.b. On the contrary, in the topology analysis, such symmetricities can not always be utilized. As can be observed in Fig. 15, there can be such cases that the final objective has the structural member which extends itself crossing over the axis of symmetricity. For such cases, it is obvious that the usual technique for structural analysis can not be applied and it is necessary to introduce a certain

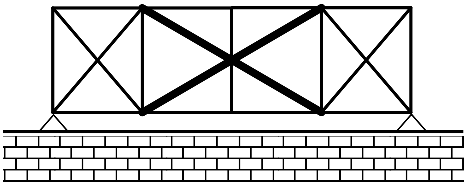


Fig. 15: Topological Symmetricity

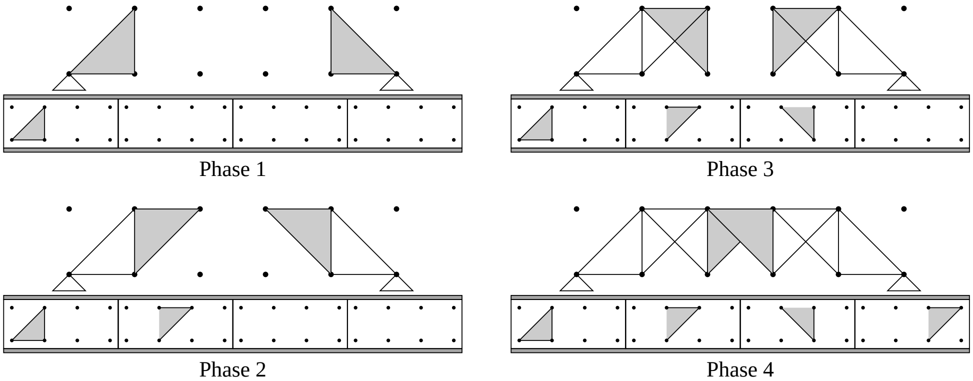


Fig. 16: Generation of Truss Topology with Topological Symmetricity

device by which we can deal with those structures.

Present paper proposes a new method by which we can consider topological symmetricity without addition of boundary condition. Fig. 16 typically shows how the generation of the topology is done in the present method. At first, the arbitrary fundamental stable element is produced and the mirror image of the triangle is produced on the other side (Phase 1). Next, fundamental stable elements are created one after another, and mirror images corresponding to the fundamental stable elements are also produced (Phase 2 and Phase 3). The same operations are continued until all required points are included in the selected nodes and the chain of fundamental stable elements is overlapped with the chain of mirror images at no less than 2 points (Phase 4).

3.6 GA Operator

Crossover The crossover point on each selected parent is arbitrarily decided and the chromosomes are separated into the former part and the latter one. After that, the crossover is performed. If the triangles in the latter part could not be connected to the former part of the chromosome of the new parent, triangles which are connected to the former part are derived from the latter part and jointed with the former part while the other triangles are deleted. And if the produced topology does not include the required points, new triangles will be produced until all points come to be included. Fig. 17 shows how the crossover is accomplished in the present scheme.

Mutation In order to perform the mutation process, we decide two cut points on the chromosome and take the bits away between the cut points. When the chain of triangles does not include the required points, new triangles continue to be reproduced until all required points are included.

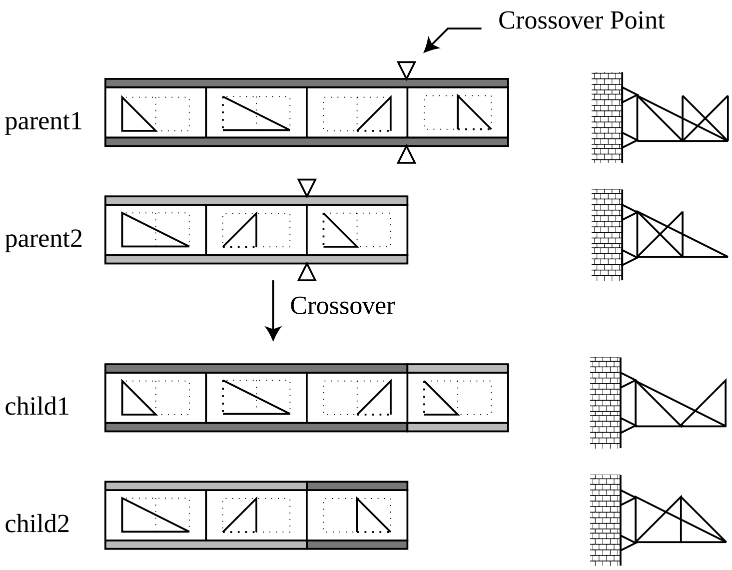


Fig. 17: Crossover Based on Triangular Element

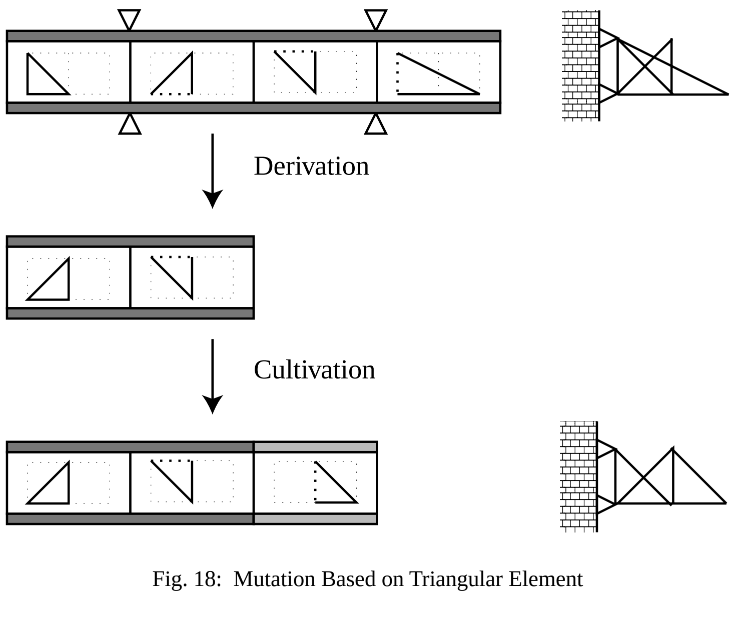


Fig. 18: Mutation Based on Triangular Element

Fig. 18 shows how the present scheme works. This treatment is one of the peculiar treatments to the proposed GA scheme, which is different from the mutation process in the ordinary GA and can be expected to have the same effect onto the GA process.

4 Layered GA

4.1 Simultaneous Optimization for Plural Variables

In structural optimization, we often encounter the problem where the topology is required to be optimized as well as the size or the shape of the structure, simultaneously. In fact, real structural optimizations should be completed through such a process and unexpected solutions might be obtained when we deal with each problem, that is, on topology, on size and on shape, separately. In the literatures, two kinds of methods have been used for such a problem by using GA. One is a double stage method; at first, the shape of the structure is optimized, and next, the optimization of the size of the members is done [8]. The other is the singular chromosome method; all information is coded onto the same chromosome [9]. It is simple to convert all information onto the chromosome directly, that is, information of topology, of sectional area and of shape itself. According to such treatment, we readily encounter a barrier due to the large number of unknown variables.

GA is an algorithm, which imitates a creature's evolution process. Now the environment in which creatures grow up must be considered. At this point, the condition of the creature's evolution depends upon two factors, one is the genetic information in the chromosome and the other is the environment. Naturally, we can imagine the fact that a better environment brings about a better creature. For our purpose of producing good creatures, all we have to do is optimize the environment. We estimate the fitness of the environment by calculating the average of the creature's fitness. This is based on the thought that any creature must be influenced by the environment, and in a good environment, the creature should grow up successfully.

$$F = \sum_{g}^{n} f_g \tag{7}$$

Here, F and f_g are the environment's fitness and the creatures' in the g-th generation. And n is the generation where environments are reproduced.

For the structural optimization problem, topology, size and shape can be dealt with by regarding one of them as a creature and the others as the environments, which has an effect on the growth of the creature, that is, the topology of the structure of interest in the present problem.

We can optimize the creature and the environment using respective GAs, and by regarding the dispositions as the creature and the sectional area as the environment. Optimization of truss topology regarding the change in the sectional area of the members can be done. The way of expressing the truss topology in the present treatment is as shown in Fig. 19.

"Layered GA" is an optimization method composed of plural variables. The main feature of this method is the treatment of variables in a GA operation. The fitness value, which is composed of each variable, is estimated under a constant constraint. The flow chart of this method is shown in Fig. 20.

4.2 Topology Optimization with Subset of Members Selection

Genetic algorithm has the advantage that can optimize of discrete variable easily. In most cases of the real structures, members are selected from standardized building components. The proposed GA method has the ability to treat such an actual scheme. Taking three design variables, that is, layout, sizing and the way of selection of member specification as the unknown variables as shown in Fig. 21, we can realize to optimize the structure in the very close meaning to the actual design problem.

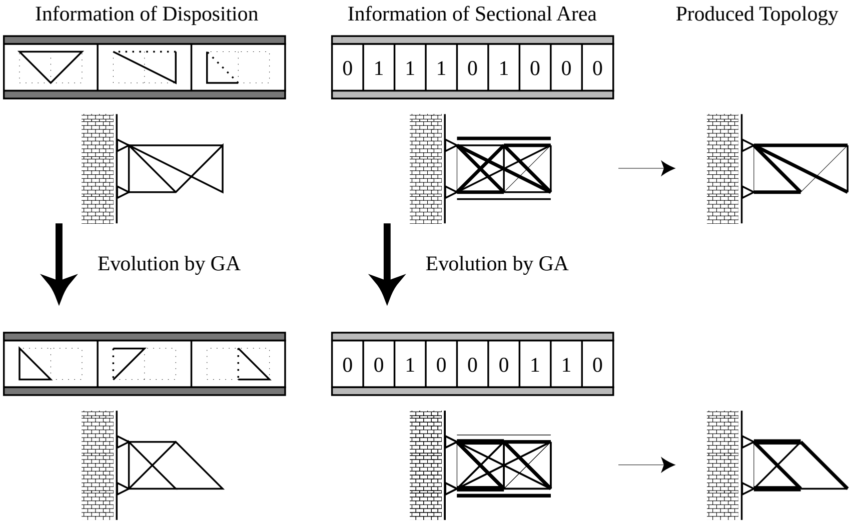


Fig. 19: Expression of Truss Topology by Plural Variables

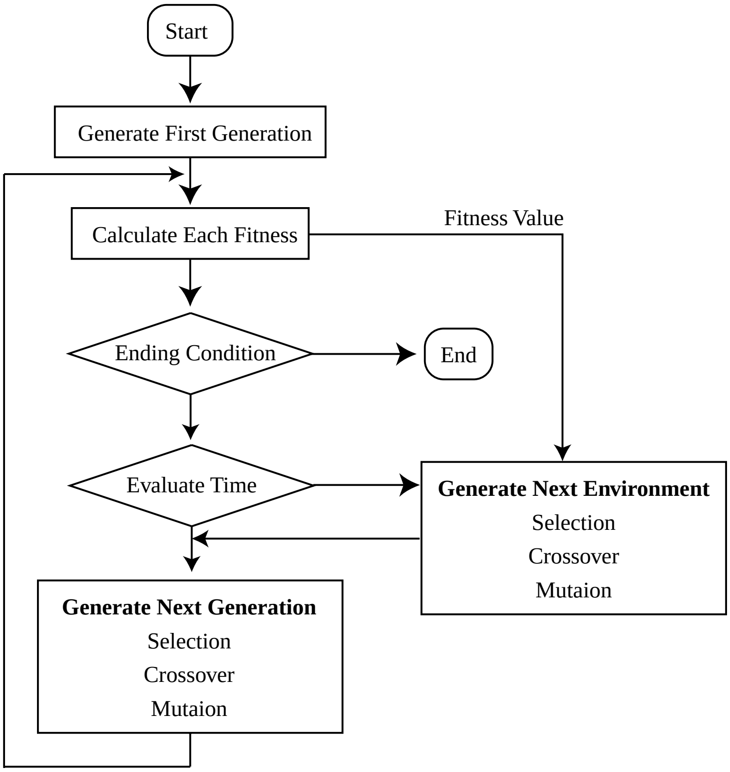


Fig. 20: Flow Chart of Layered GA

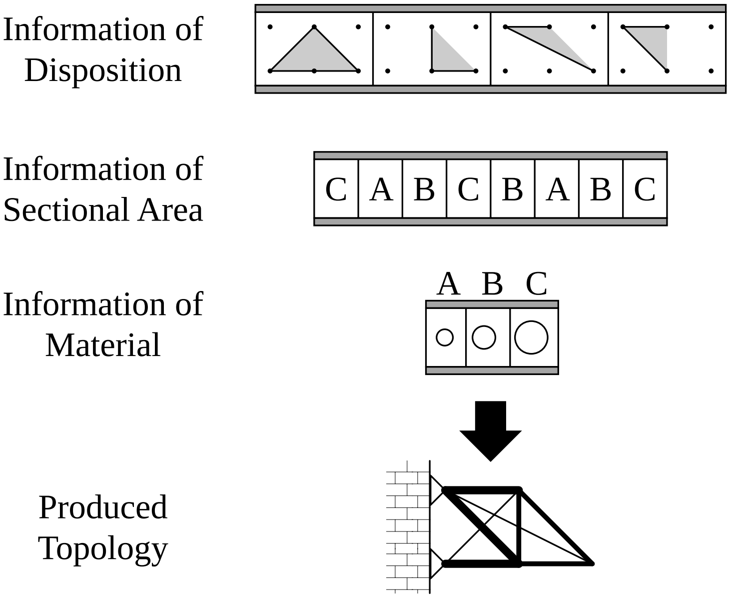
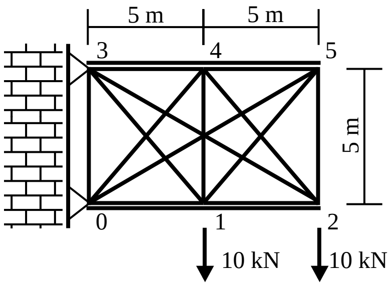


Fig. 21: Topology with Subset of Members Selection



Population	10 (Elite 2)
Selection	Ranking
Generation	50
Probability of Crossover	0.60
Probability of Mutation	0.05
Sectional Area	20 cm^2
Young's Modulus	21 GPa
Displacement Limit	2 mm
Stress Limit	15 MPa

Fig. 22: 15-Bar Cantilever Truss

5 Numerical Examples

5.1 Truss Topology Optimization

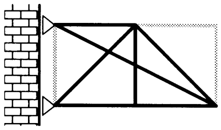
5.1.1 Topology Optimization of Truss

Fig. 22 and 23 show the results of a topology optimization analysis of a 15-bar cantilever truss.

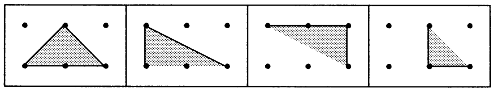
5.1.2 Topology and Size Optimization of Truss

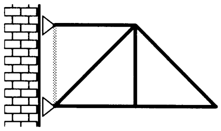
In Fig. 24, the results for the 15-bar cantilever truss which was adopted before in Fig. 22, where only the disposition of the members is considered, are depicted. In Fig. 24, not only the

Member with No Axial Stress

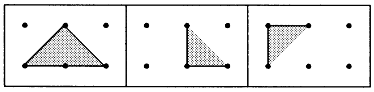


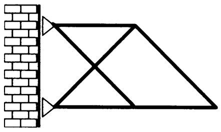
Generation	0
fitness	1.405
mass	712 kg
displacement	1.46 mm
stress	10.2 MPa





Generation	1
fitness	1.426
mass	536 kg
displacement	1.96 mm
stress	15.0 MPa





Generation	3
fitness	1.751
mass	569 kg
displacement	1.51 mm
stress	10.0 MPa

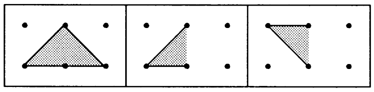
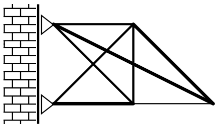
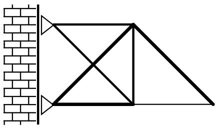


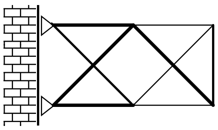
Fig. 23: Cantilever 15-Bar Truss



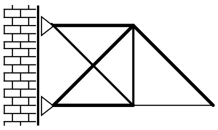
Generation	0
fitness	1.582
mass	500 kg
displacement	1.90 mm
stress	12.7 MPa



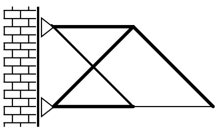
Generation	7
fitness	1.590
mass	515 kg
displacement	1.83 mm
stress	12.7 MPa



Generation	10
fitness	1.610
mass	621 kg
displacement	1.49 mm
stress	11.5 MPa

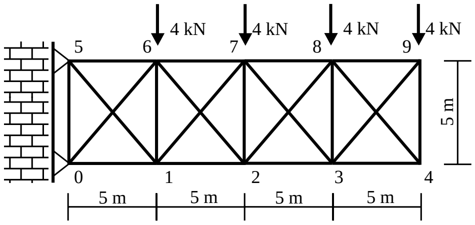


Generation	16
fitness	1.622
mass	528 kg
displacement	1.75 mm
stress	11.1 MPa

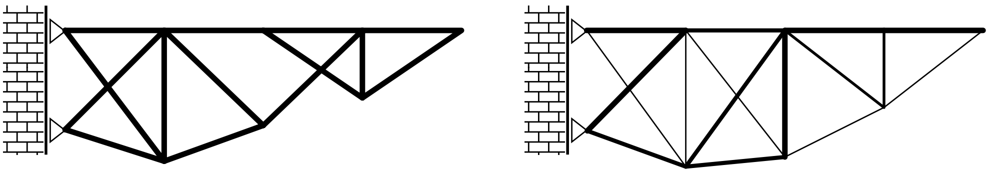


Generation	34
fitness	1.847
mass	479 kg
displacement	1.69 mm
stress	10.5 MPa

Fig. 24: Cantilever 15-Bar Truss



Sectional Area	20 cm^2
Young’s Modulus	21 GPa
Displacement Limit	0.7 mm
Stress Limit	15 MPa
Movable Point	1, 2, 3 and 4 (y dimension)



Shape and Topology Optimal Solution   Size, Shape and Topology Optimal Solution

Fig. 25: 21-Bar Cantilever Truss

disposition of the menbers but also the change of the sectional area of the rods are considered by using “Layrered GA”.

For the problems above, design variables of the sectional area of the members were allowed to vary from 5.0 cm^2 to 20.0 cm^2 with an increment of 1.0 cm^2. The thicknesses of the members in the figures expresses the sectional area proportionally.

5.1.3 Size, Shape and Topology Optimization of Truss

In just the same manner, we can take into consideration all factors which basically should be considered simultaneously for the structural optimization of the actual trusses, that is, size, shape and topology. Fig. 25 shows the results of the 21-bar truss problem where the sectional area of the members, the nodal dislocations and also the dispositions of the members are taken into account simultaneously. In the figure, we can see a clear difference between these results. The shape and topology optimal solution was obtained through optimization without considering a change in the member sectional areas and the other shows the results through optimization considering the effect of a change in the cross sectional area of the truss members.

5.1.4 3-Dimensional Cantilever Truss

Fig. 26 shows the results of the size and topology optimization for the 3-dimensional cantilever truss. The obtained topology has two planer trusses which are fairly similar to the obtained solution at the section 5.1.1 and 5.1.2.

5.1.5 Sphere Truss Dome

Sphere truss model is as shown in Fig. 27. In this example, truss topologies are requested to be generated arbitrarily between two spheres having different diameters. The aim of the problem in the present example is to obtain both the topology and the configuration of a space truss with a minimum weight under the restraint conditions. The discrete variables of sectional area are as shown in Table 1. Fig. 28 shows the convergence history where thickness is 0.5 m and, the transition of the topology is shown in Fig. 29, where only the quarter part of the truss dome is

Sectional Area	5 to 20 cm^2 (Δ = 1.0 cm^2)
Young's Modulus	21 GPa
Displacement Limit	2 mm
Stress Limit	15 MPa

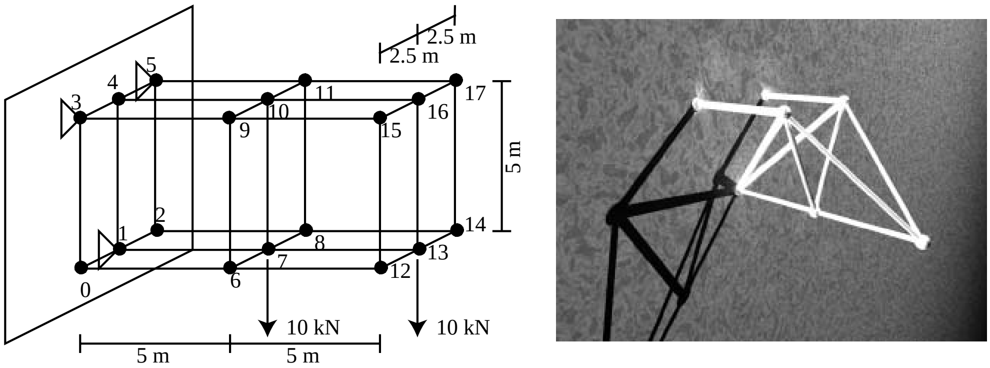
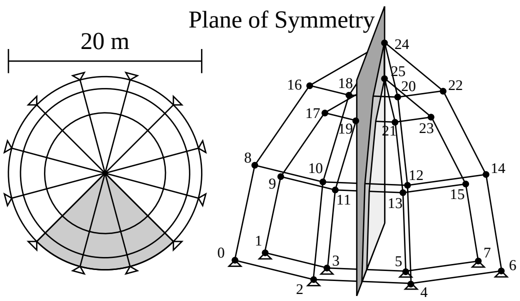


Fig. 26: 3-Dimensional Cantilever Truss



Load	500 Pa
Population	20 (Elite 2)
Selection	Ranking
Generation for Disposition	500
Generation for Sectional Area	500
Generation for Material	50
Displacement Limit	20 mm
Stress Limit (Tension)	160 MPa
(Compression)	Buckling Stress

Fig. 27: Sphere Truss Dome

Table 1: Material List (JIS 3444)

Material No.	0	1	2	3	4	5	6	7	8
Sectional Area (cm^2)	21.7	34.0	42.7	48.6	60.5	76.3	89.1	101.6	114.3
	9	10	11	12	13	14	15	16	17
	139.8	165.2	190.7	216.3	267.4	318.5	355.6	406.4	457.2
	18	19	20	21	22	23	24		
	500.0	508.0	558.8	600.0	609.6	700.0	711.2		

depicted for the sake of simplicity. The density sample of axial stress level is as shown in Fig. 30. In early generations, as can be seen in Fig. 29, there are a large number of members existing and they are so thick. As the generation progresses, it can be seen that the number of members

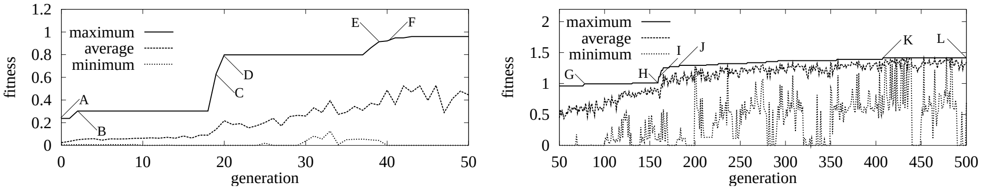
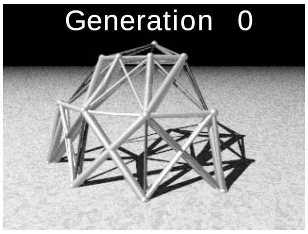
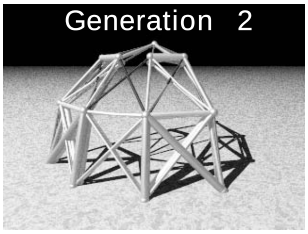


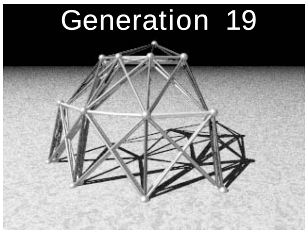
Fig. 28: Convergence History



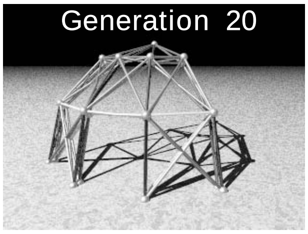
Topology A



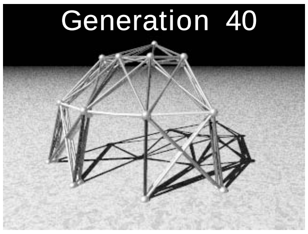
Topology B



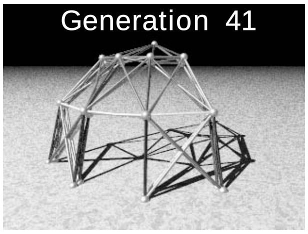
Topology C



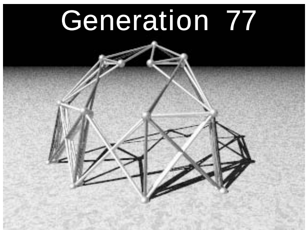
Topology D



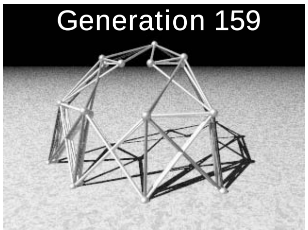
Topology E



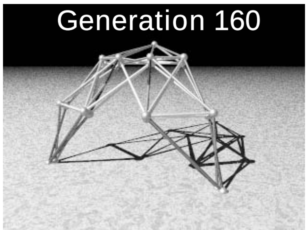
Topology F



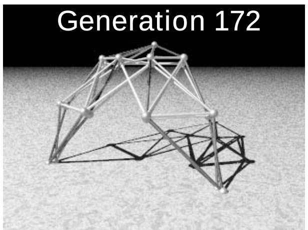
Topology G



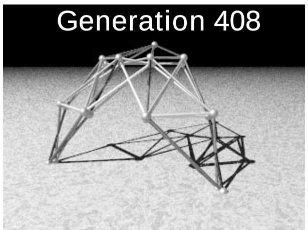
Topology H



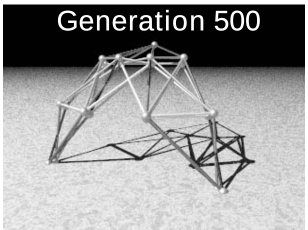
Topology I



Topology J



Topology K



Topology L

Fig. 29: Transitio of Topology (1/4)

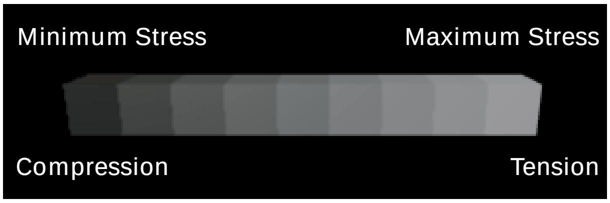
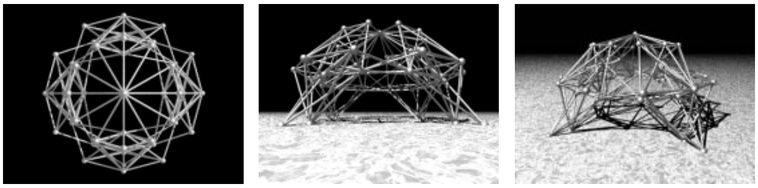


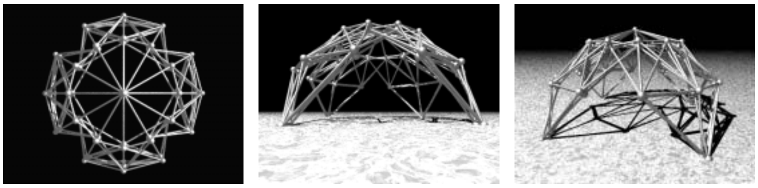
Fig. 30: Density Sample



Fitness	1.57701
Mass	629 kg
Max. Disp.	1.47 mm
Max. Stress	10.55 MPa
Min. Stress	-17.40 MPa

Vertical View Horizontal View Bird's Eye View

Thickness 2.0 m



Fitness	1.41500
Mass	707 kg
Max. Disp.	2.44 mm
Max. Stress	129.4 MPa
Min. Stress	-157.5 MPa

Vertical View Horizontal View Bird's Eye View

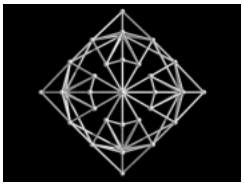
Thickness 0.5 m

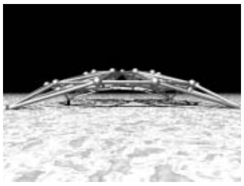
Fig. 31: Solutions with Difference of Thickness

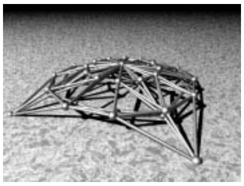
decreases, and the sectional areas become smaller. In the late stage of generation, the geometry itself scarcely changes into the lighter one while the sectional area of members changes more frequently. As depicted in Fig. 29, geometry of the dome changes drastically at the 160th generation while almost no change has been made up to the step from the 77th generation. Corresponding to this geometrical transformation, the value of fitness can be seen to be largely improved. After this point, consecutive improvement has been continued until the final stage. The obtained solutions are as shown in Fig. 31 where the thickness of the shell is changed. Fig. 31 shows the same results as in Fig. 32 corresponding the different values of the rise-to-span ratio.

5.1.6 Flat Roof Space Truss

The total view of the flat roof space truss with the parameters used in GA calculation is as shown in Figs. 33 and 34. There are three different treatments for control parameters for GA process, in which the different conditions concerning with the symmetricity are adopted, that is, symmetry with respect to plane, to the central point and without any of such condition as shown in Fig. 34. Fig. 35 shows the optimal solutions obtained by the present GA scheme. Fig. 36 shows the comparison of the convergence history where symmetric conditions are changed.

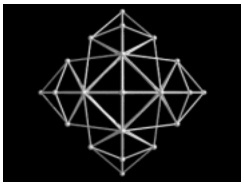

Vertical View

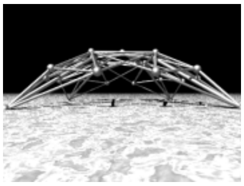

Horizontal View

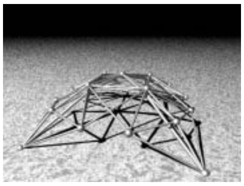

Bird's Eye View

Fitness	1.39993
Mass	714 kg
Max. Disp.	14.86 mm
Max. Stress	70.4 MPa
Min. Stress	-84.8 MPa

Rise 3 m


Vertical View


Horizontal View


Bird's Eye View

Fitness	2.12475
Mass	471 kg
Max. Disp.	1.95 mm
Max. Stress	9.6 MPa
Min. Stress	-32.5 MPa

Rise 5 m

Fig. 32: Solutions with Difference of Rise

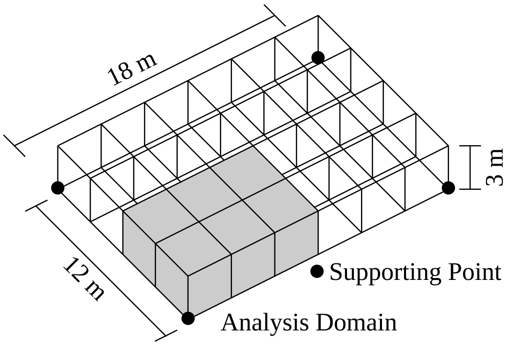
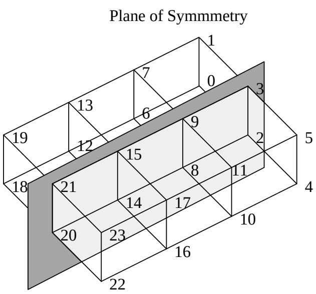
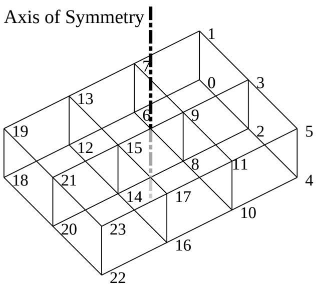


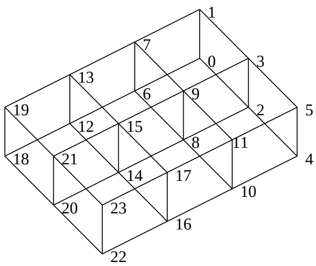
Fig. 33: Total View of Flat Roof Space Truss



Test A ; Symmetry with Respect to Plane

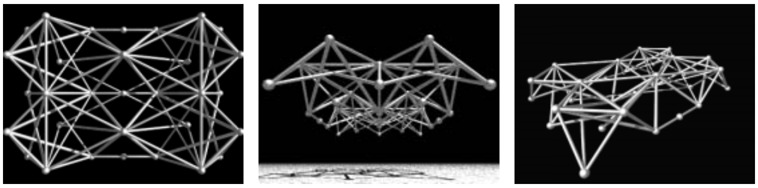


Test B ; xisymmetry



Test C ; Asymmetry

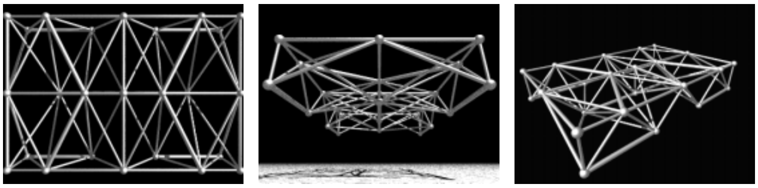
Fig. 34: 1/4 Analysis Domain



Fitness 1.52129
Mass 657 kg
Max. Disp. 4.17 mm
Max. Stress 18.1 MPa
Min. Stress -21.6 MPa

Vertical View   Horizontal View   Bird's Eye View

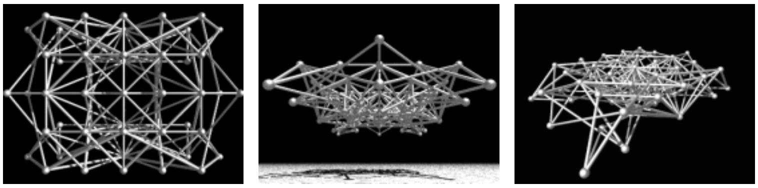
Plane



Fitness 1.36627
Mass 732 kg
Max. Disp. 4.76 mm
Max. Stress 22.3 MPa
Min. Stress -43.5 MPa

Vertical View   Horizontal View   Bird's Eye View

Axisymmetry



Fitness 1.04752
Mass 910 kg
Max. Disp. 9.33 mm
Max. Stress 44.3 MPa
Min. Stress -44.8 MPa

Vertical View   Horizontal View   Bird's Eye View

Asymmetry

Fig. 35: Solutoin with Defference of Symmetricity

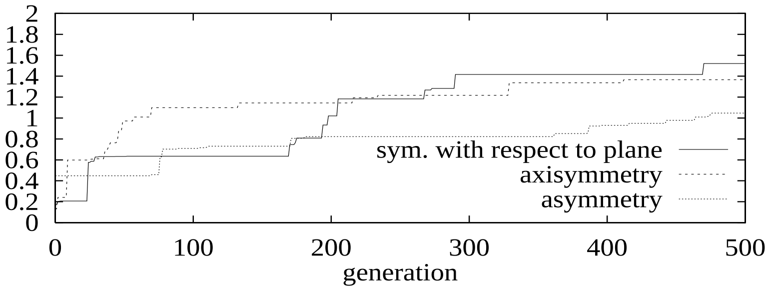
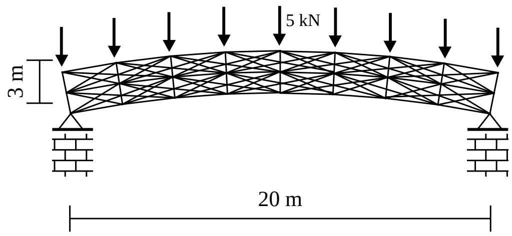


Fig. 36: Convergence History

5.2 Topology Optimization of Frameworks

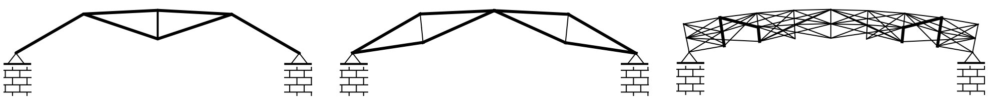
5.2.1 Plane Arch

Fig. 37 shows the ground structure of the 27 points-planer arch problem as well as the parameters adopted for numerical calculation. Fig. 38 shows obtained solutions when the kind of



Young's Modulus	210 Mpa
Load	5 kN × 9
Population	20 (Elite 2)
Generation for Disposition	500
Generation for Sectional Area	500
Generation for Material	250
Displacement Limit	10 mm
Stress Limit (Tension)	160 MPa
(Compression)	Buckling Stress

Fig. 37: 27-points Plane Arch



Segmental Element Triangular Element Bit Code

	Bar	Triangle	Bit
Fitness	5.46565	7.76388	2.26897
Mass	149 kg	129 kg	441 kg
Maximum Displacement	11.17 mm	7.90 mm	0.001 mm
Maximum Stress	26.32 MPa	61.95 MPa	30.25 MPa
Minimum Stress	–65.94 MPa	–58.70 MPa	–52.05 MPa
Maximum Moment	92.5 N·m	42.3 N·m	92800 N·m
CPU Time	61 sec.	68 sec.	119 sec.

[Notes] Bar : segmental stable element, Triangle : triangular stable element, Bit : bit codes

Fig. 38: Optimized Structures

elements for frame structures are changed in 3 types, that is, the linear stable element, the triangular stable element, and the bit code proposed by Hajela [8]. In case of the bit code, the length of chromosome is 154, and it cannot be realized to search the solution as fine as the other cases where the fundamental stable elements are used.

5.2.2 Sphere Frame Dome

Space frame dome structures where each member can resist against bending moments as well as shear forces, and each connection can transmit those stress resultants are dealt with in this section. Single layered dome structures that have 10m in diameter, supported at the ground level with roller-supports in the radial direction and subjected to the dead load are investigated. The data used for the present problem is as shown in Table 2. Design variables are cross-sectional area, selection of the materials, and configurations of the structure. The constraint conditions are subjected on the axial stresses, the bending moments and the nodal displacements. In this problem, four kinds of boundary conditions are set as shown in Fig. 39. In the present problem, the linear element is used for the expression of the structure.

Table 2: Parameters for Sphere Frame Dome

	Division		
	1/4 and 1/6	1/2	1
Load	1000 Pa	1000 Pa	1000 Pa
Maximum Length of Members	9000 mm	9000 mm	8000 mm
Population	20	20	20
Elite	2	2	2
Generation for Disposition	500	1000	1000
Generation for Sectional Area	500	1000	1000
Generation for Material	250	500	500
Displacement Limit	20 mm	20 mm	20 mm
Stress Limit (Tension)	160 MPa	160 N	160 MPa
Stress Limit (Compression)	Buckling Stress	Buckling Stress	Buckling Stress
Moment Limit	$160 \times Z$ N·m	$160 \times Z$ N·m	$160 \times Z$ N·m
Probability of Crossover	0.60	0.60	0.60
Probability of Mutation	0.05	0.05	0.05

[note] Z: Section Modulus

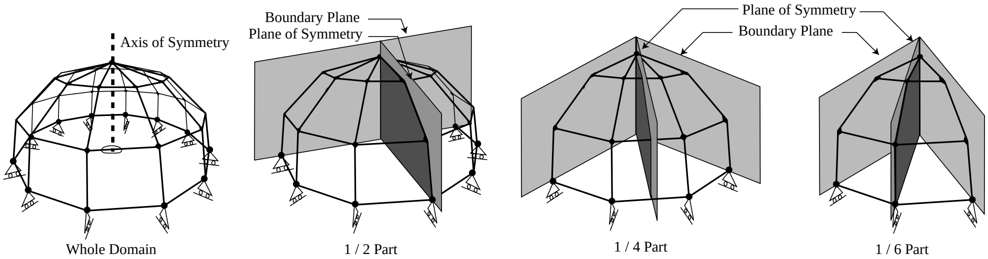


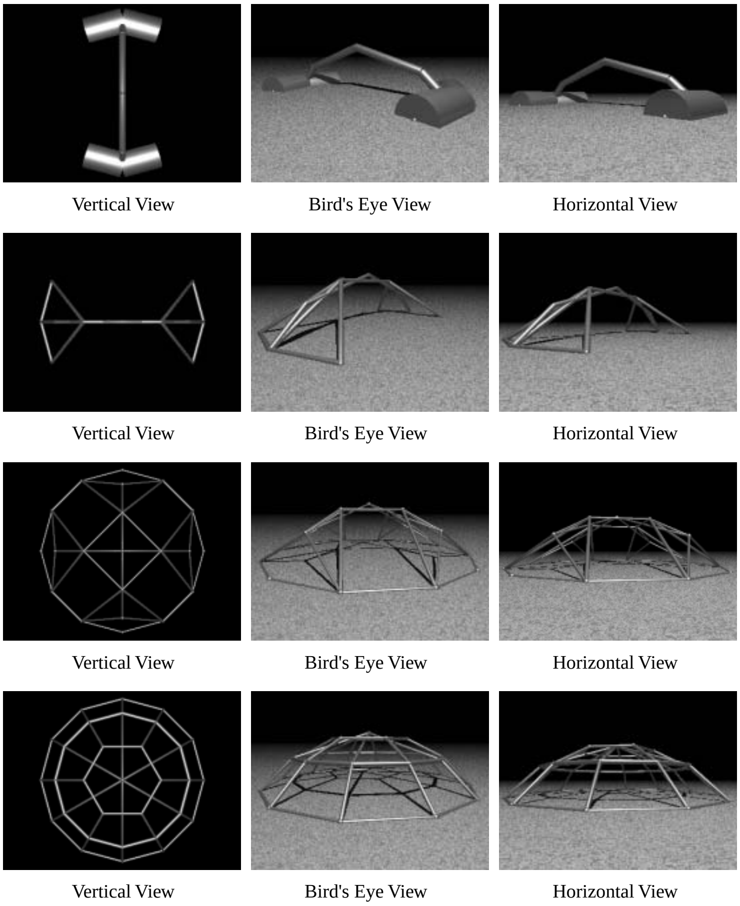
Fig. 39: Sphere Frame Dome

The obtained solutions are shown in Fig. 40. It is unable to express these solutions by triangular fundamental stable elements. In the case of whole domain calculation, a simple configuration, which consists of the four-bars arch resisting to the vertical loads and supporting members, is obtained. From this result, we can see that the supporting members prevent the arch from the displacement in the out-of-plane direction as well as the rigid motion. The difference of boundary condition is turned out to have a big effect to the final result of the structural topology.

5.3 Topology Optimization of Hybrid Structures

5.3.1 Plane Arch

Fig. 41 shows the ground structure of the 27 points-planer arch problem as well as the parameters adopted for numerical calculation. Fig. 42 shows obtained solutions through the genetic progress. The finally obtained solution is similar to the obtained arch in the case of frame structures, but it can be observed that the compressive pin-jointed bar is added at the bottom of the arch.



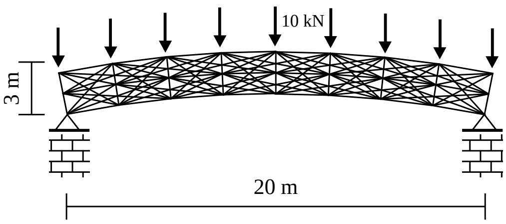
Whole Domain	
Mass	2300 kg
Max. Displacement	19.94 mm
Max. Stress	4.77 MPa
Min. Stress	-29.51 MPa
Max. Moment	74402.32 N·m

2 Parts Division	
Mass	1272 kg
Max. Displacement	19.95 mm
Max. Stress	96.73 MPa
Min. Stress	-34.77 MPa
Max. Moment	565.17 N·m

4 Parts Division	
Mass	1960 kg
Max. Displacement	19.35 mm
Max. Stress	33.69 MPa
Min. Stress	-22.15 MPa
Max. Moment	4541.31 N·m

6 Parts Division	
Mass	2700 kg
Max. Displacement	14.05 mm
Max. Stress	117.52 MPa
Min. Stress	-25.77 MPa
Max. Moment	77.52 N·m

Fig. 40: Obtained Solutions of Sphere Frame Dome



Young's Modulus	210 Gpa
Population	50
Elite	2
Generation for Layout	5000
Generation for Sectional Area	5000
Generation for Material	2500
Generation for Location	2500
Displacement Limit	70 mm
Members' Rotation Limit	0.1 rad
Stress Limit (Tension)	160 MPa
(Compression)	Buckling Stress

Fig. 41: 27-points Planer Hybrid Arch

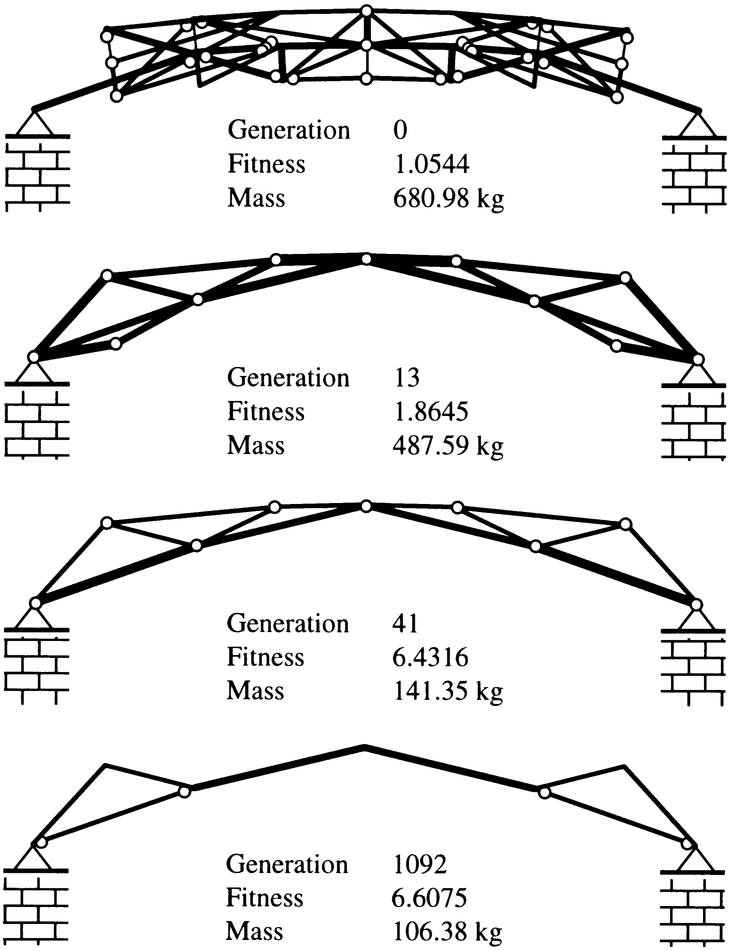
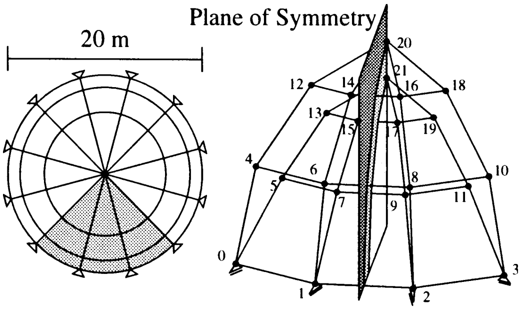


Fig. 42: Obtained Solutions of Planer Hybrid Arch

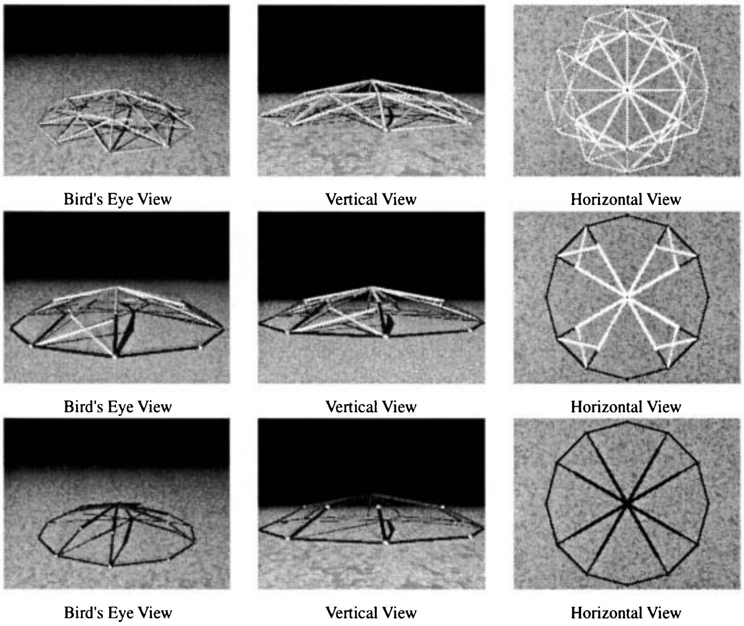
5.3.2 Sphere Hybrid Dome

Fig. 43 shows the problem domain for the sphere hybrid dome and used parameters. Fig. 44 shows the obtained solutions when the value of displacement limit is changed to 100 mm, 160 mm and 200 mm. In these figures, truss members are depicted in white and the frame members in black. As can be seen in this figure, when the displacement criterion is set tight, the solution consists of a lot of thin truss members. However when the displacement criterion is set a little looser, the obtained solution becomes a more frame-like dome which consists of a few members with large sections. When the displacement limit is set at 160 mm, the hybrid structure is finally obtained.



Young’s Modulus	210 Gpa
Population	30
Elite	2
Generation for Layout	2000
Generation for Sectional Area	2000
Generation for Material	1000
Displacement Limit	3 Types
Members’ Rotation Limit	0.1 rad
Stress Limit (Tension)	160 MPa
(Compression)	Buckling Stress

Fig. 43: Sphere Hybrid Dome



Generation	1912
Fitness	2.7000
Mass	337 kg
Max. Disp.	18.57 mm
Max. Stress	102.8 MPa

Generation	1886
Fitness	2.1878
Mass	345 kg
Max. Disp.	28.31 mm
Max. Stress	103.7 MPa

Generation	70
Fitness	2.6955
Mass	198 kg
Max. Disp.	65.12 mm
Max. Stress	151.8 MPa

Fig. 44: Obtained Solutions of Sphere Hybrid Dome

6 Conclusion

A new scheme for the computational morphology of the space structures is proposed where the genetic algorithm (GA) is utilized as an optimization driver through some original modifications by which high efficiency for calculations toward the final optimized space structures can be realized. In this paper, two main ideas are introduced for the GA scheme, that is, the introduction of the structurally stable fundamental element and the technique for treating plural variables to be optimized named “Layered GA”. As the structurally stable fundamental elements, the triangular element for the plane trusses, the linear element for the space frames and the tetrahedron element for the three dimensional space truss structures are newly introduced.

Through a various kind of numerical examples, it has been shown that the proposed GA scheme can be effectively applied to all kinds of space structures, that is, space trusses, space frames as well as hybrid structures which are composed of both of trusses and frames.

Through the proposed scheme, we can obtain the final optimized space structures which rigorously satisfy the given subsidiary conditions and, at the same time, have the extremal value of the objective function such as the cost or the total weight of the structure. However, there might be another aspects for utilization of the proposed scheme, that is, those as a software tool for the brain-storming investigation of rough configurations of the structures which are to be created especially in the beginning stage of the structural design process. Recent astounding advances in hardware technology for high speed and high performance computing can be a favorable wind for that way of usage of computers for the designers of the structures. Using computers as just a stationery which can almost instantaneously propose the structures according to the imposed subsidiary conditions which can be either mechanical or not, we will be able to utilize the time not for the routine works but for more creative and essential ones. The advent of computers with higher speed and performance which will be realized in near future seems to be able to change the way of the structural design process of the space structures. The proposed scheme through GA will be surely utilized for one of effective tools for those processes.

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