

2020 Doctor's Thesis

Essays on Economic Analysis of Standards,  
Patents, and Compatibility

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# Chapter 1

## Introduction

### 1.1 Motivation

Innovation drives economic growth and as the society and economy progress, technology becomes more complex and deeply related. In the first chapter of *The Wealth of Nations*, Book I, Adam Smith points out that division of labor is the key to improve productivity. Economic agents will become more and more specialized in the advanced economy by only producing some parts of goods and services. Meanwhile, products will become more sophisticated and be produced by multiple companies. For example, artificial intelligence will play an important role for autonomous driving, involving the collaboration of automobile, computer hardware and the AI software technology each developed by different companies. In order to combine the components developed by different companies, standards are required and the compatibility among those components needs to be guaranteed. The patents of complex technologies also give difficulties to companies because developing a product using the technologies may cause the infringement of the patents.

The development of technology is inherently cumulative, standing on the shoulders of giants. For instance, patent application documents, which partially reveal inventive activities, show that patents are often cited by others and again those

other patents are cited by others and so forth. These citation relationships imply that the new inventions are related with previous ones and that they have novelty and add some improvement. If the line of innovation development is linear, it is straightforward to understand the relationship and ownership of the patents, making it easy to offer licensing negotiations. It is, however, hard to invent and adopt a new technology if the holders of the patents are diversified and the relationship of the patents are complicated, resulting in a patent thicket. This would become severe if the technology to be developed requires a number of sub-technologies. In some cases, companies mitigate with patent pools the cost of the patent thickets which would be brought by complex technologies. WIPO (2014) defines a patent pool as follows: “A patent pool is an agreement between two or more patent owners to license one or more of their patents to one another or to third parties.” It is necessary to disentangle the relationships of technologies and to know the value of patents in order to design an effective patent pool.

Actually, products manufactured in modern days are comprised of many components such as electronic devices. For example, “[...] the iPhone is made up of hundreds of parts and components manufactured by many different producers [...]” (p.103, Coe et al. (2019)). In addition to the hardware, software such as an operating system and applications are necessary to use them. All of these parts produced by different manufacturers are required to coordinate to function and thus standards are established in some industries. Compatibility standards are sometimes disclosed and the entry of so called third parties is observed while it is not done in some markets. Then, what makes standards disclosed? When do third parties enter?

Some standards can be interpreted as the platforms in a two-sided market. For example, operating systems can be interpreted as standards and as the platforms in a two-sided market. Users cannot do anything with only operating systems while applications provide the functions that users need. Operating systems provide the “places” where applications and users meet. Users choose which operation system to

buy and application developers choose which ones to provide their software for. Although the two-sided structure is not new, it has become increasingly common with the progress of information technology e.g., electronic billing, electronic commerce platforms and online streaming services. This market structure has a propensity to create a monopoly platform since the more agents of side 1 participate in a platform, the more agents of side 2 will participate in the same platform and in turn increase the side 1 agents. Recently, regulatory authorities have expressed antitrust concerns about the behavior of some platforms in two-sided markets. In order to protect consumer and social welfare, it is required to analyze the strategy of two-sided platforms and its economic impact.

In order to address the issues raised above brought by the complex economy, this dissertation attempts to analyze the relationship among technologies and provide implications for standards, patents and compatibility.

## 1.2 Standard and Compatibility

David and Greenstein (1990) define a standard as “a set of technical specifications adhered to by a producer, either tacitly or as a result of a formal agreement”. Standards have several different functions and can be classified into i) reference, ii) minimum quality, and iii) interface or “compatibility” (David (1987)). This dissertation focuses on the compatibility standard.

There are products which can be used only when they are assembled with other components, e.g., audio speakers and music players, video game consoles and game software. They have no value in isolation, that is, they are perfect complementary goods. These components have interfaces to connect with each other and firms decide whether or not to make the interface compatible with the components produced by other firms. Matutes and Regibeau (1988) analyze the compatibility of perfect complementary goods in a duopoly market and find that price competition is alle-



viated by making the components compatible. While both firms in their analysis produce both components, there are markets where firms produce only one kind of compatible component such as developers of video game software, so called third parties. In the video game industry, there are many third-party software developers which provide software for the game consoles produced by different manufacturers while third-party provision of refills in the ballpoint pen industry is rarely observed. The pricing of the components and the entry decision of a third party are analyzed in chapter 2.

While this is an example of the compatibility among multiple different products, single products also encompass standards and compatibility. Usually, these standards entail positive network externalities, which arise if the valuation of a good increases as the number of the users who use the same or compatible good increase. For example, the value of having a phone increases if there are more phone users. Phones are useless if you are the only one to have a phone. Katz and Shapiro (1985) and Farrell and Saloner (1985) study network externalities and compatibility in single product markets. Farrell and Saloner (1985) show that firms may trap in an inferior standard when the information about the firms' willingness to switch to a better alternative is incomplete. Katz and Shapiro (1985) focus on consumers' expectation of the network size and find that large firms are less likely to make their products compatible to others.

As mentioned above, telecommunication services have network externalities, more precisely, direct network externalities. Assuming operating systems are standards, however, network externality should be captured from a different aspect since the utility of adopting an operation system is determined by how many applications are available for users and how many users there are for application developers. The network externality which depends on the the number of the users on the other side, as the example above, is called indirect network externality. Two-sided markets, which involve indirect network externality, have become common recently and the

study of two-sided markets is pioneered by Caillaud and Jullien (2003), Rochet and Tirole (2003), and Armstrong (2006). It is well known that if the indirect network externalities are large, the price on the other side may be less than the marginal cost. Antitrust authorities have been concerned about the market power that giant platforms have nowadays. Baker (2019) points out that two-sided platforms may use predatory pricing and price discrimination in order to gain market power. The progress of information technology has facilitated the collection of purchasing history and online platforms have rich data about the behavior of the participants, which gives the platforms the opportunity to conduct behavior-based price discrimination. However, when price discrimination and predatory pricing are observed, does it always imply that the platforms aim to exclude their rivals and gain market power as Baker (2019) points out? Fudenberg and Tirole (2000) analyze price discrimination based on consumers' purchasing history in one-sided markets but behavior-based price discrimination in two-sided markets has not been studied. Chapter 3 studies the pricing strategy of platforms in two-sided markets when they discriminate their participation fee based on purchasing history and provide policy implications when they set the fee below their marginal costs.

### **1.3 Patent Network and Citation**

Patent documents include the information about citation. If a patent cites other patents, it is assumed that the patent has technological relations with the others and that patents with many citations are important inventions. Trajtenberg (1990) and Albert et al. (1991) point out that citation counts are positively correlated with the quality of the patent. It is known that patent value is extremely skewed and thus citation information can be used to control the quality of patents. Hall et al. (2005) estimate the value of patents using citation count and find that an extra citation increases the market value of the patentee by 3%. However, patents have streams

of citations; a patent is cited by new patents and those new patents are cited by yet new patents. Considering these relations, patent citations should be interpreted as a network defined in a social network analysis and the importance of patents should be evaluated by the positions in the network. There are a number of measures which assess the importance of the nodes in a network. Bonacich (1987) proposed a measure which assigns scores to nodes based on the number of walks, which can be applied to a patent citation network. This measure can take the stream of patent citations into account and in chapter 4, patent value is estimated by constructing a patent citation network in order to include the relationship between patents in a broader level.

## 1.4 Structure of Dissertation

The rest of the dissertation is structured as follows.

**Chapter 2** This chapter considers two types of components, main and peripheral, both indispensable for consumption. There are two first-party incumbent firms, each producing the two components, and one potential third-party entrant that would supply only the peripheral one. By using a two-dimensional Hotelling model, I show that entry takes place if the peripheral component is sufficiently differentiated. Furthermore, entry of the potential entrant does not harm the incumbents. It can even be beneficial when the entrant captures consumers that were not participating in the market, as it generates additional demand for the incumbents. Finally, entry is efficient for society due to a significant increase in the consumer surplus, while the expected profit of the third party does not cover its entry cost for some parameter values. In this case, government intervention to encourage the third party to enter the market may be desirable.

**Chapter 3** This chapter analyzes customer poaching in two-sided markets. It is shown that the platforms may use behavior-based price discrimination and below-cost pricing. The platforms do not have the intention to exclude their rivals, however. While social surplus is deteriorated due to increased transportation costs, consumers are better-off under customer poaching. Additionally, it is known that poaching alleviates price competition at the first period as the discount factor on the second-period profit increases in one-sided markets. I show that poaching intensifies first-period competition as the discount factor increases in two-sided markets with strong indirect network externalities. This result is specific to two-sided markets because when the indirect network externalities are strong, the prices for switchers at the second period decrease below the marginal costs. Forward-looking platforms try to set the prices more aggressively and expand their turfs at the first period in order to reduce the number of switchers at the second period.

**Chapter 4** The patents which are cited by many other patents are thought to be important. Using patent citation data, many studies estimate the value of patents. It is, however, needed to include how many citations the cited patents have. In this chapter, using the methodology of social network analysis, a patent citation network is constructed and the Bonacich centrality is introduced in order to take the higher degree citations.

**Chapter 5** This chapter concludes the dissertation by summarizing the results and proposing future subjects.

# Chapter 2

## Third party complements and market entry<sup>1</sup>

### 2.1 Introduction

In some markets, producers allow third parties to supply compatible components to their products. For instance, several companies supply lenses for the single-lens reflex (SLR) camera<sup>2</sup> market, in which first parties (Canon, Nikon and so on) produce both the bodies of SLR cameras and lenses. The lenses of the third parties are compatible with the cameras of the first parties. An interesting feature of this market is that the third-party lenses are compatible with several first-party cameras, while the lenses of each first party are compatible exclusively with its own bodies.<sup>3</sup> Third-party developers of video games supply the same software for both Sony's PlayStation and Microsoft's Xbox with Sony and Microsoft producing software for their own consoles. In other markets, third-party provision of components is possible yet rarely observed. For instance, there are no third parties in the markets of razors or cheap ballpoint pens, which supply blades for the razor handles of first parties

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<sup>1</sup>This chapter is based on Okumura (2020).

<sup>2</sup>A camera in which the lens is interchangeable.

<sup>3</sup>According to the standards of each first party, third parties provide functionally identical lenses except for their interfaces. This is because the interfaces are different for each first party.

or refills for the bodies of pens, respectively, that the first parties produced. What makes third parties present in one market but absent in another?

This chapter investigates how the entry of a third party affects the pricing and profits of first parties, and the welfare consequences when the firms compete in a horizontally differentiated market. The modeling strategy is as follows. There are two incumbents that produce both a main and a peripheral component, and one potential entrant that would produce only a peripheral component. Both incumbents are first parties and the potential entrant is a third party. Consumers mix and match a main component and a peripheral component. Since the main components of the first parties are compatible with the peripheral component of the third party, consumers can assemble these components. However, it is impossible to assemble the main component of one first party and the peripheral component of the other first party. The consumers have heterogeneous preferences for main and peripheral components and buy them when the utility is above a given reservation level. Accordingly, I adopt a two-dimensional Hotelling model in which the peripheral components of the first parties are differentiated from the peripheral component of the third party, and the main components are differentiated between the first parties. In the first stage, the third party decides whether to enter the market at the expense of an entry cost. In the second stage, all of the firms determine the prices of the main components and the peripheral components simultaneously.

The main results of this chapter are threefold. First, the more differentiated the peripheral components are, the more likely entry by the third party becomes. Less differentiation leads to harsher competition, lower prices and eventually smaller profits. The third party thus does not enter the market. Second, the entry does not damage the first parties. In fact, their profits actually increase if the peripheral components are highly differentiated. This finding and the logic behind it are related to the analysis of Whinston (1990) and Carlton and Waldman (2002) regarding the price setting of the first parties. The demand for the main component

increases because the entry gives the consumers additional options and those who would not buy the system without the entry end up buying it. Obviously, the third party deprives the first parties of the consumers who would buy the peripheral component produced by one of the first parties without the entry. However, this loss is substantially small because the first parties optimally charge a significantly reduced price for the peripheral component and raise the price of the main component to compensate for the loss caused by the reduced price on the peripheral component. Third, when the entry cost is high, the third party stays out of the market even though its entry would increase the profits of the first parties and, eventually, social welfare. The government should intervene in the market to encourage entry in this case.

The main result is consistent with real-world observations. Consumers' tastes for SLR cameras, video games, and audio systems are diversified, and therefore third parties find profitable opportunities to provide differentiated components. Meanwhile, the markets of razors and cheap ballpoint pens are less differentiated, and thus the entry of third parties is unlikely to be profitable. These products are considered daily needs, and consumers who have distinct preferences are few. The results of this chapter also suggest that third parties do not enter the market if the cost of entry is sufficiently high. In the 1990s, Sony launched PlayStation, which used CD-ROMs for the game software storage media, while Nintendo released Nintendo 64, whose storage media were ROM cartridges. CD-ROMs have more data capacity than do ROM cartridges, which made it easier for third-party software developers to make video games with CD-ROMs. Actually, 1335 titles were released for PlayStation in North America<sup>4</sup> and 297 titles for Nintendo 64 in North and South America.<sup>5</sup> This may imply that the high setup cost for selling video games with ROM cartridges prevented some developers from making games for Nintendo 64.

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<sup>4</sup>[http://www.sie.com/en/corporate/data/software\\_title.html](http://www.sie.com/en/corporate/data/software_title.html)

<sup>5</sup>[https://www.nintendo.co.jp/ir/library/historical\\_data/pdf/number\\_of\\_titles\\_e1606.pdf](https://www.nintendo.co.jp/ir/library/historical_data/pdf/number_of_titles_e1606.pdf)

This chapter is organized as follows. Section 2.2 reviews the related literature. Section 4.5 presents the model. Section 2.4 analyzes the pricing and profits of the firms and the entry decision of the third party. Section 2.5 provides the welfare analysis. Section 2.6 concludes. Appendices 2.A and 2.B provide supporting calculations.

## 2.2 Related Literature

This chapter is closely related to the study of compatibility. Matutes and Regibeau (1988) analyze market competition in which firms produce complementary components and point out that compatibility alleviates the competition. Under compatibility, a price cut in one component increases demand only for that component while under incompatibility, it increases demand for both of them. Because the authors consider the symmetry of compatibility in the sense that all of the firms produce every component and all of the components are compatible with each other, the prices of the components are symmetric in their model. In my model, however, the difference in prices between the components arises by introducing the distinction of a first party and a third party. This difference is similar to that observed in the studies of Whinston (1990) and Carlton and Waldman (2002), where a first-party monopolist decides to sell its products in the form of a bundle or sell them independently, which allows third-party competitors to enter the market. They find that allowing the third parties to enter is not less lucrative for the first party, which also happens in my model. This result is obtained by a general specification. However, the Hotelling specification in this chapter, although less general, provides equilibrium characterization and a detailed welfare analysis, which are not present in their papers.

Peitz (2008) analyzes the entry of a single-product firm into a market in which a multi-product incumbent can bundle two products, one of which competes with the



entrant's product. He shows that bundling is the dominant strategy, which works as an entry barrier resulting in a decrease in social welfare. The main difference between his study and mine is that the goods are neutral in his model while they are perfect complements in mine. This results in different consequences: bundling is profitable in his study, whereas it is not in my model. The benefit of bundling comes from making consumers buy a less-preferred product even though their favorite one is made by the bundling firm's rival. However, this benefit does not exist in the case of perfect complements. The reason is as follows. First, suppose (i) a first party chooses its optimal bundle price under a bundling scheme when a third party sells only a peripheral component, (ii) the first party is going to choose the prices of the two components when sold independently, and (iii) the first party has chosen the prices of the main component at the optimal level for the bundle price and of the peripheral component at its marginal cost. Then, if the marginal cost of the peripheral component is zero, the profit is the same as in the case of bundling. If the marginal cost is positive, independent selling produces more profits. This is because, as part of the bundle, it can save the cost of producing the peripheral component for consumers who would buy the bundle even though they prefer the peripheral component of the rival firm. Thus, bundling is not profitable in the case of perfect complements.

Although this chapter focuses on goods that are perfect complements, the first parties face a case similar to that of one-way essential complements (Chen and Nalebuff (2007); Adachi et al. (2017); Broos and Gautier (2017)). The goods analyzed in these studies have one-way essential complementarity in the sense that one good can be used alone (stand-alone) while the other good is available only when you have the stand-alone one (e.g., add-ons for software). Firms would produce the stand-alone and a combination of the stand-alone and add-on. In my model, the first parties sell their main components to some consumers and sell both main and peripheral components to others. The way they sell is similar to the case in one-way essential

complements.

Chen and Nalebuff (2007) and Adachi et al. (2017) analyze the market of one-way essential complements without competition and show that a monopolist sets its add-on price at the marginal cost within some range of parameters. In my analysis, although the first parties set their peripheral component prices at the marginal cost, competition is introduced between the first parties and the third party, and the logic behind it is different. The third party's entry deprives the first parties of demand for their peripheral components. However, the first parties extract surplus from the buyers of the third party's component by setting the prices of their peripheral components at a low level and charging high prices for their main components. The first parties decrease the prices of their peripheral components because the third party expands demand for the main components. Meanwhile the monopolist in the authors' models chooses its add-on price using price discrimination. The monopolist regards the combination of the stand-alone and add-on as a premium version of the stand-alone. When consumers evaluate the add-on's value to be low, however, it is not profitable to price discriminate. This is because some consumers stop buying the stand-alone in addition to the add-on due to the increase in the price of the add-on, since consumers have independent preferences for them. Furthermore, the gain in doing so is small because only a few consumers buy the add-on due to their low valuation of it.

In the context of one-way essential complements, Broos and Gautier (2017) analyze the market of internet service providers (ISPs) that also sell telephone service. Consumers can talk with someone remote also by using a VoIP application provided by a different company via the internet. The internet service is an essential component, and the VoIP application is an optional component that competes with the telephone service. The ISPs decide whether to exclude the VoIP application, which is differentiated from the telephone service. In equilibrium, one of the ISPs allows the VoIP application to operate in order to differentiate itself from its rival

because both the internet and telephone services provided by the ISPs are ex-ante homogeneous. In my model, the main components are ex-ante differentiated, which contrasts with their model. This heterogeneity guarantees the profits of the first parties even if the third party's peripheral component is supplied to both of them.

## 2.3 Model

Consider three firms, A, B, and C. Firms A and B are first-party incumbents, which produce both a main component and a peripheral component. A potential entrant, firm C, is a third party that produces only a peripheral component. I will describe later how the products are differentiated. I assume the following patterns of compatibility. The main component and the peripheral component of firm A are not compatible with those of firm B and vice versa. The peripheral component of firm C is compatible with the main component of either firm A or B. The marginal costs of firms A, B, and C are normalized to zero.<sup>6</sup> The consumers who buy an AA system demand firm A's main unit and firm A's peripheral unit. The consumers who buy an AC system demand firm A's main unit and firm C's peripheral unit. The consumers who buy a BB system and those who buy a BC one demand analogously. Each firm determines its prices in order to maximize its profit.

Each consumer receives utility from a system, i.e., a main component and a peripheral component together. To consider consumers' taste for heterogeneous products, I assume that the consumers are uniformly distributed on a unit square (see Figure 2.1). Firm A is located at  $(g_p, g_m) = (1, 0)$ , firm B is located at  $(g_p, g_m) = (1, 1)$ , and firm C is located at  $g_p = 0$ . The locations of the firms are fixed. The utility function of the consumers when they buy the main and peripheral

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<sup>6</sup>This might seem to be odd in that the third party provides its components for both first parties without additional costs, although the two main components are not compatible. However, the following argument persists qualitatively even if it is assumed that there are two third parties and one provides its peripheral component to one first party while the other third party supplies the other first party.

components from firm  $i = A, B$  and  $j = A, B, C$  respectively is as follows,

$$V - d_m - td_p - P_m^i - P_p^j,$$

where  $V$  is the intrinsic value of consuming the system,  $d_m$  is the distance between a consumer and the firm that supplies a main component, and  $d_p$  is the distance between a consumer and the firm that supplies a peripheral component. For instance, if a consumer of  $(g_p^0, g_m^0)$  buys a main component and a peripheral component from firm A, then  $d_m = g_m^0$ ,  $d_p = 1 - g_p^0$ . If the same consumer buys a main component from firm A and a peripheral component from firm C, then  $d_m = g_m^0$ ,  $d_p = g_p^0$ . Let  $t$  be the transportation cost of a peripheral component. I normalize the transportation cost of a main component to one. Let  $P_m^i$  and  $P_p^j$  denote the prices of firm  $i$ 's main component and firm  $j$ 's peripheral component, respectively, where  $i = A, B$  and  $j = A, B, C$ . The consumers buy the system that gives the highest utility as long as the utility exceeds the reservation level, normalized to zero.

The timing of the game is as follows. In the first stage, firm C determines whether to enter the market, assuming an entry cost  $F > 0$ . In the second stage, all of the firms in the market determine the prices of their products simultaneously.

## 2.4 Equilibrium

Now I solve the game to derive the subgame perfect Nash equilibrium. The values of  $V$  and  $t$  determine the configuration of the competition (see Figure 2.2). Depending on the values of  $V$  and  $t$ , there are two types of competition, full coverage and partial coverage. In order to perform comparative statics with respect to  $t$  after the optimal behavior is derived,  $V$  is fixed in the following analysis. In a fully covered market  $t$  is sufficiently low so that all the consumers in the market buy the system and the firms compete with each other to attract the marginal consumers. In a partially covered market  $t$  is not so low that the consumers who are distant from

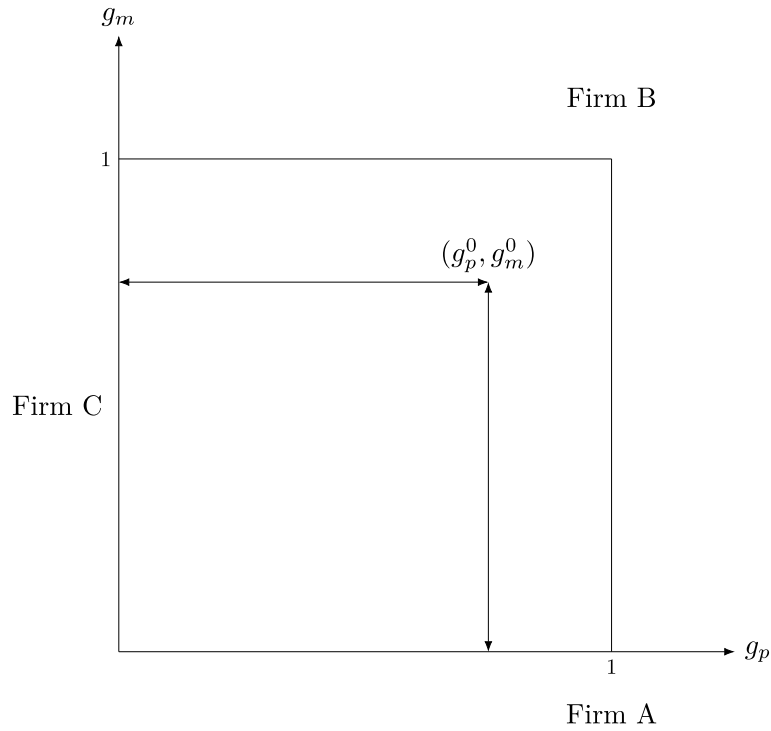


Figure 2.1: Location of firms and consumers.

the firms do not buy the system. This is because it does not yield positive payoffs, yet the firms compete with each other to get the marginal consumers. In addition to these types of competition, if  $t$  is sufficiently high, only the consumers close to the firms buy the system so that the firms do not face with competition and behave as local monopolists. To see the first parties' tradeoff between the competition and the demand expansion effect however, only the cases where competition occurs are analyzed.<sup>7</sup> I first look into the subgame in which the third party entered the market.

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<sup>7</sup>The value of  $V$  is not allowed to be infinitely large. Although the model also works fine in this case, the market is always fully covered so that the transition between full and partial coverage, which I will be shown later, is not occurring.

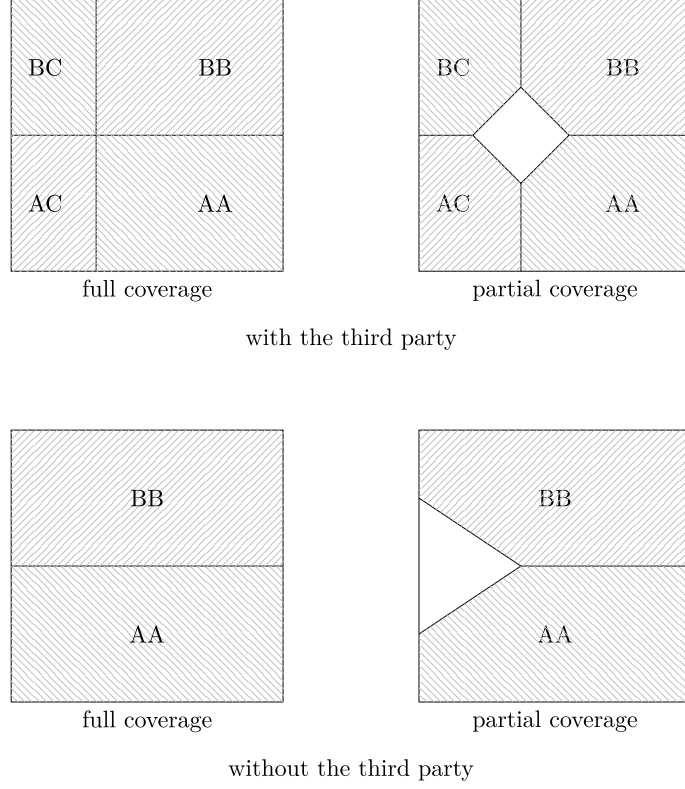


Figure 2.2: Market configurations.

### 2.4.1 Second Stage

**Third Party Entry** The profits of the firms when the market is fully covered are expressed by

$$\begin{aligned}\pi_i &= P_m^i D^{iC} + (P_m^i + P_p^i) D^{ii}, \\ \pi_C &= P_p^C D^{AC} + P_p^C D^{BC} - F,\end{aligned}$$

where  $i = A, B$  and  $D^{kl}$  denotes how many consumers buy the system composed of a main component of firm  $k$  and a peripheral component of firm  $l$  (see Table 2.1). There are two cases which depend on whether  $P_p^A \leq P_p^B$  or  $P_p^A > P_p^B$ . I focus on the case  $P_p^A \leq P_p^B$  to derive a symmetric equilibrium.<sup>8</sup>

<sup>8</sup>The price candidates derived under the case when  $P_p^A > P_p^B$  must satisfy this condition and therefore the prices cannot be symmetric. In fact, in deriving a symmetric equilibrium, the two cases coincide when  $P_p^A$  and  $P_p^B$  approach the same value.

	$p_p^A \leq p_p^B$
$D^{AA}$	$\left(\frac{P_p^B - P_p^A + P_m^B - P_m^A}{2} + \frac{1}{2}\right) \left(\frac{P_p^C - P_p^A}{2t} + \frac{1}{2}\right)$ $-\frac{1}{2} \left[ \left(\frac{P_p^B - P_p^C}{2t} + \frac{1}{2}\right) - \left(\frac{P_p^A - P_p^C}{2t} + \frac{1}{2}\right) \right] \left[ \left(\frac{P_p^B - P_p^A + P_m^B - P_m^A}{2} + \frac{1}{2}\right) - \left(\frac{P_m^B - P_m^A}{2} + \frac{1}{2}\right) \right]$
$D^{BB}$	$\left(\frac{P_p^A - P_p^B + P_m^A - P_m^B}{2} + \frac{1}{2}\right) \left(\frac{P_p^C - P_p^B}{2t} + \frac{1}{2}\right)$
$D^{AC}$	$\left(\frac{P_m^B - P_m^A}{2} + \frac{1}{2}\right) \left(\frac{P_p^A - P_p^C}{2t} + \frac{1}{2}\right)$
$D^{BC}$	$\left(\frac{P_m^A - P_m^B}{2} + \frac{1}{2}\right) \left(\frac{P_p^B - P_p^C}{2t} + \frac{1}{2}\right)$ $-\frac{1}{2} \left[ \left(\frac{P_p^B - P_p^C}{2t} + \frac{1}{2}\right) - \left(\frac{P_p^A - P_p^C}{2t} + \frac{1}{2}\right) \right] \left[ \left(\frac{P_p^B - P_p^A + P_m^B - P_m^A}{2} + \frac{1}{2}\right) - \left(\frac{P_m^B - P_m^A}{2} + \frac{1}{2}\right) \right]$

	$p_p^A > p_p^B$
$D^{AA}$	$\left(\frac{P_p^B - P_p^A + P_m^B - P_m^A}{2} + \frac{1}{2}\right) \left(\frac{P_p^C - P_p^A}{2t} + \frac{1}{2}\right)$
$D^{BB}$	$\left(\frac{P_p^A - P_p^B + P_m^A - P_m^B}{2} + \frac{1}{2}\right) \left(\frac{P_p^C - P_p^B}{2t} + \frac{1}{2}\right)$ $-\frac{1}{2} \left[ \left(\frac{P_p^A - P_p^C}{2t} + \frac{1}{2}\right) - \left(\frac{P_p^B - P_p^C}{2t} + \frac{1}{2}\right) \right] \left[ \left(\frac{P_m^B - P_m^A}{2} + \frac{1}{2}\right) - \left(\frac{P_p^B - P_p^A + P_m^B - P_m^A}{2} + \frac{1}{2}\right) \right]$
$D^{AC}$	$\left(\frac{P_m^B - P_m^A}{2} + \frac{1}{2}\right) \left(\frac{P_p^A - P_p^C}{2t} + \frac{1}{2}\right)$ $-\frac{1}{2} \left[ \left(\frac{P_p^A - P_p^C}{2t} + \frac{1}{2}\right) - \left(\frac{P_p^B - P_p^C}{2t} + \frac{1}{2}\right) \right] \left[ \left(\frac{P_m^B - P_m^A}{2} + \frac{1}{2}\right) - \left(\frac{P_p^B - P_p^A + P_m^B - P_m^A}{2} + \frac{1}{2}\right) \right]$
$D^{BC}$	$\left(\frac{P_m^A - P_m^B}{2} + \frac{1}{2}\right) \left(\frac{P_p^B - P_p^C}{2t} + \frac{1}{2}\right)$

Table 2.1: Demand for each system.

A Nash equilibrium in this subgame is a price profile  $(P_m^*, P_p^*, P_p^{C*})$  satisfying

$$\begin{aligned}\frac{\partial \pi_i}{\partial P_m^i} &= D^{iC*} + P_m^* \frac{\partial D^{iC*}}{\partial P_m^i} + D^{ii*} + (P_m^* + P_p^*) \frac{\partial D^{ii*}}{\partial P_m^i} = 0, \\ \frac{\partial \pi_i}{\partial P_p^i} &= P_m^* \frac{\partial D^{iC*}}{\partial P_p^i} + D^{ii*} + (P_m^* + P_p^*) \frac{\partial D^{ii*}}{\partial P_p^i} = 0, \\ \frac{\partial \pi_C}{\partial P_p^C} &= D^{AC*} + P_p^{C*} \frac{\partial D^{AC*}}{\partial P_p^C} + D^{BC*} + P_p^{C*} \frac{\partial D^{BC*}}{\partial P_p^C} = 0.\end{aligned}$$

Solving these equations I obtain

$$(P_m^*, P_p^*, P_p^{C*}) = (1, 0, \frac{t}{2}),$$

inducing the profit

$$\pi_i^* = \frac{1}{2}, \quad \pi_C^* = \frac{t}{8} - F.$$

This solution is valid when the consumer who is the most distant from the firms,  $(1/4, 1/2)$ , receives a non-negative payoff, that is  $V - (1/4)t - 1/2 - P_p^{C*} - P_m^{A*} \geq 0$ . This boils down to  $V \geq 3/2 + (3/4)t$ .

The first parties charge a significantly low price for their peripheral components while keeping a high price for their main components. The pricing is interpreted as follows. Suppose the market is under full coverage, and firm A sets the total price at  $P_p^A + P_m^A = 1$ , which is determined based on the competition between the first parties. Now I examine the marginal effect on firm A when it changes the price of the main component to  $P_m^A + \epsilon$  and that of the peripheral component to  $P_p^A - \epsilon$ , where  $\epsilon > 0$ . In other words, firm A alters the weights of each price, keeping the total price fixed. This manipulation causes three effects. The first one is a business stealing effect ( $\alpha$ ) in which firm A deprives firm C of demand for the peripheral component. The second one is an inframarginal gain ( $\beta$ ) by which firm A increases the profit from consumers who purchase AC due to the increased price of its main component.<sup>9</sup> The last one is an erosion effect ( $\gamma$ ) in which firm A loses demand

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<sup>9</sup>Firm A cannot gain additional profit from consumers who buy AA because the total price of



for the main component because some of the consumers who would buy AC switch to BC due to the increased price of the main component of firm A. Each of these elements in the full-coverage case is represented by

$$\begin{aligned}
 \text{gain} & \quad \underbrace{\frac{\epsilon}{2t} \cdot \frac{1}{2} \cdot P_p^A}_{(\alpha)} + \underbrace{\frac{1}{2} \cdot \frac{1}{4} \cdot \epsilon}_{(\beta)}, \\
 \text{loss} & \quad \underbrace{\frac{\epsilon}{2} \cdot \frac{1}{4} \cdot P_m^A}_{(\gamma)},
 \end{aligned}$$

and Figure 2.3 gives a graphical interpretation. In equilibrium, the gain and loss must be balanced so that  $\alpha + \beta = \gamma$  must hold. From this condition,  $P_m^A \geq 1$  holds because otherwise the inframarginal gain exceeds the erosion effect, and therefore the loss is smaller than the gain. Then the price of the peripheral component must be less than or equal to zero since I assume the total price to be one. If the price of the peripheral component is below zero, then the gain from this price change is smaller than the loss. Thus, the first parties charge the main-component price at one and the peripheral-component price at zero.

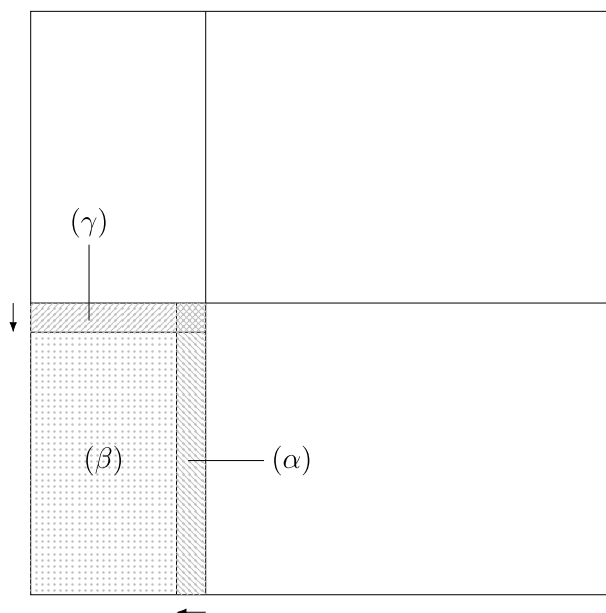


Figure 2.3: Effect of the price manipulation under full coverage.

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the system is constant.

The solution derived under the assumption of full coverage is not valid if  $V$  and  $t$  do not satisfy  $V \geq 3/2 + (3/4)t$ . In such a case — that is,  $V < 3/2 + (3/4)t$  — the market is partially covered. Since deriving the result in this case is intricate, I provide a detailed analysis in Appendix 2.A. The optimal prices when  $V = 2$  are shown in Figure 2.4 along with the prices under full coverage.

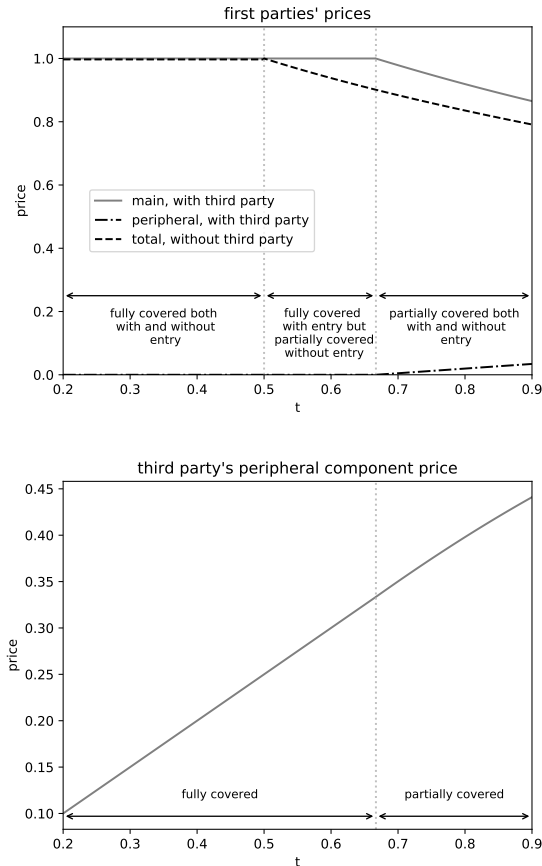


Figure 2.4: Equilibrium prices with respect to the transportation cost.

Similar to the full-coverage case, the first parties set the prices of their main components to be high and those of the peripheral components to be low though their peripheral-component prices are not zero. The price manipulation used to check the pricing under full coverage is also applicable for the partial-coverage case, and it shows that the mechanism of the firms' pricing is qualitatively the same to that of the full-coverage case. As shown in Figure 2.5, the three effects also appear under partial coverage. The business stealing effect ( $\alpha$ ), which represents gain, is

smaller in the partially covered market than in the fully covered one because fewer consumers buy AC ex-ante in this case and therefore fewer consumers switch to AA. The inframarginal gain ( $\beta$ ) is also smaller because again fewer consumers buy AC ex-ante in this case. The erosion effect ( $\gamma$ ), which represents loss, is larger under partial coverage because the choice of the threshold consumer is whether to buy the system or not.<sup>10</sup> By combining these three effects, the net benefit of this gain is smaller in the partial-coverage case and thus the first parties do not decrease their peripheral-component prices to be as low as those in the fully covered case.

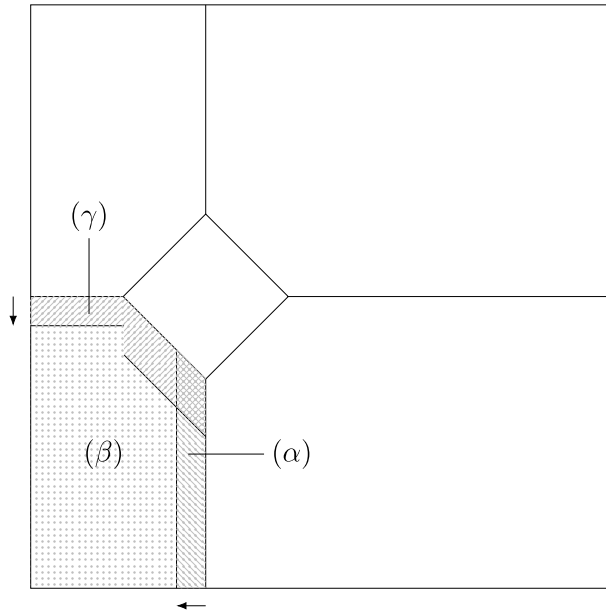


Figure 2.5: Effect of the price manipulation under partial coverage.

**No Third Party Entry** The profits when the market is fully covered are expressed by

$$\pi_i = \left( \frac{P^j - P^i}{2} + \frac{1}{2} \right) P^i,$$

where  $i, j = A, B$ ;  $i \neq j$  and  $P^i = P_m^i + P_p^i$ ,  $P^j = P_m^j + P_p^j$ . A Nash equilibrium in this subgame is a price  $P^{i*}$  satisfying

$$\frac{\partial \pi_i}{\partial P^i} = -\frac{1}{2} P^{i*} + \left( \frac{P^{j*} - P^{i*}}{2} + \frac{1}{2} \right) = 0.$$

<sup>10</sup>A detailed explanation is in Appendix 2.A.

Solving these equations I obtain

$$P^{i*} = 1,$$

inducing the profit

$$\pi_i^* = \frac{1}{2}.$$

This solution is valid when the consumer who is the most distant from the firms,  $(0, 1/2)$ , receives a non-negative payoff, that is,  $V - t - 1/2 - P^{A*} \geq 0$ . This boils down to  $V \geq 3/2 + t$ .

The full-coverage case without the third party in my model is the same as in the one-dimensional Hotelling model. Since the peripheral components of the first parties are identical in terms of differentiation, only the main components are differentiated. The first parties compete as if they competed over the main components in the one-dimensional Hotelling model. Accordingly, it turns out that the total price equals one, which is the degree of differentiation of the main component as the price in the one-dimensional model shows.

When  $V < 3/2 + t$ , the market is partially covered and I show the analysis in Appendix 2.A. The price in the market without the third party is shown in Figure 2.4 as “total, without third party”.

**Comparison** The graph at the top of Figure 2.6 shows the relation between the transportation cost and the profits of the first parties in each subgame when  $V = 2$ . The horizontal axis is the transportation cost, and the vertical one is the profit.

As the transportation cost goes down, the profits of the first parties increase until  $t = 0.5$  when the third party is in the market and until  $t = 0.667$  when it is not. The increase in profits and these values of  $t$  correspond to market coverage. Actually,  $t = 0.5$  and  $t = 0.667$  are the points at which the market shifts to full coverage from partial coverage. A decrease in the transportation cost widens the share of consumers served and increases the profits while the market is partially covered. Once the market becomes fully covered, the demand expansion stops and the profits

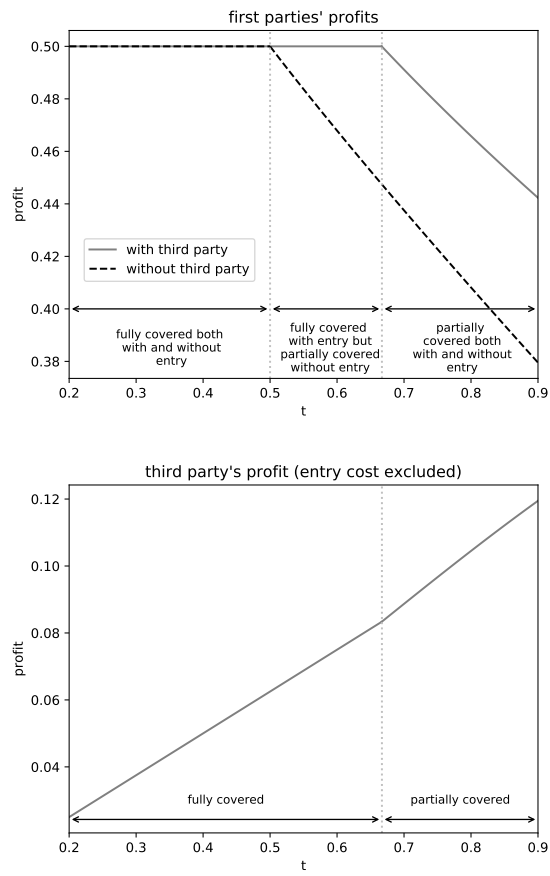


Figure 2.6: Equilibrium profits with respect to the transportation cost.

do not increase anymore even if the transportation cost drops. More specifically, the third party has two roles. One is to make a partially covered market become fully covered (that is when  $0.5 < t \leq 0.667$ ), and the other is to serve the system to more consumers even if the market is not fully covered under both regimes (that is when  $t > 0.667$ ). The important difference between the two cases, when the third party enters the market and when it does not, is that the threshold value of  $t$  at which the market shifts to full coverage from partial coverage is higher when the third party is in the market. In other words, the first parties can get the “maximum” profits even with a relatively high transportation cost under the presence of the third party.

It is observed that the entry does not damage the profits of the first parties. Moreover, the first parties are better off when  $t > 0.5$ . The reason for this is similar to that found in Whinston (1990) and Carlton and Waldman (2002), though the first party in their model is a monopolist and they discuss it without specifying the firms’ optimal behavior. The logic is explained as follows. The decision-making of the consumers is based on the integrated price of the main component and the peripheral component, not on each price. The consumers choose the best system out of AA, AC, BB and BC in the presence of the third party and out of AA and BB in its absence. The first parties determine the price of each component, which is equivalent to determining the prices of the bundles (AA and BB) and a main-component price if the third party is in the market. Otherwise, it is considered that each first party supplies only one bundled good because the components of the different first parties are not compatible. Now, suppose firm A sets its bundle price at  $P$  and earns profit  $\pi$  when the third party does not enter. Further suppose that the first party sets the price of its main component at  $P$  and that of its peripheral one at zero when the third party enters the market. Then it can gain not less than  $\pi$  when the third party is in the market and possibly more if firm A sets those prices rationally because demand for the main components never reduces by the entry. Actually, the optimal price of the main components when the third party is in the

market is higher than the optimal bundle price as shown in Figure 2.4.

It is interesting to look specifically into the case where the profits of the first parties increase by the entry, which corresponds to the range where the market gets covered more widely (when  $t > 0.5$ ) by the entrant. When the third party enters the market, the potential consumers who locate near the third party start to buy the system consisting of the main component of either first party and the peripheral component of the third party. These consumers would not buy the system without the entry. The first parties gain demand for the main component because the third party spurs demand for the system, while demand for the peripheral component declines to some extent. However, the loss from the decline is small if the price of the peripheral component is sufficiently low. The first parties gain more profit by the increased demand for the system, which entails the expansion of demand for the main components. Next, consider the case in which the market is fully covered regardless of the entry, which corresponds to  $t \leq 0.5$ . The first parties cannot gain additional demand for the main component in this case, as all of the consumers already consume the system, whereas the third party deprives the first parties of demand for the peripheral component. Also in this case, however, the first parties do not lose their profits because of the price manipulation.

While the first parties provide bundles (AA and BB) for the original consumers at the same price as in the case without the entry, they provide the main component at a high price for the switching consumers — shifting the loss made by the entry onto the main-component price. Thus, they do not earn less profits.

*Proposition 1.* The entry of the third party does not damage the profits of the first parties. Furthermore, when  $t > 0.5$ , entry increases the degree of market coverage and the profits of the first parties.

## 2.4.2 First Stage

In the first stage, firm C decides whether to enter the market. The bottom graph in Figure 2.6 shows the profit without the entry cost. As the transportation cost goes down, the profit declines accordingly. The decrease in transportation cost intensifies the competition in both the cases of full and partial coverage. Therefore, if the transportation cost is sufficiently low (the entry cost is sufficiently high), the third party does not enter the market.

*Proposition 2.* When the transportation cost of peripheral components is sufficiently high and/or the entry cost is sufficiently low, the third party enters the market.

The result provides a way to interpret real world markets. In fact, third parties are observed in the market of lenses for SLR cameras. This market is regarded as more differentiated. Third parties are not observed in the market of razors, which is thought to be less differentiated. In addition, as I state in the Introduction, it was considered to be less costly to develop games for PlayStation than for Nintendo 64 because PlayStation used CD-ROMs, which had more data capacity than Nintendo 64's ROM cartridges. The lower development cost of PlayStation software could be interpreted as a lower entry cost and this suggests why more third parties entered its software market and provided games for it than for Nintendo 64.

## 2.5 Social Welfare

Define social welfare as  $CS + \pi_A + \pi_B + \pi_C$ , where  $CS$  is the consumer surplus and is shown in Appendix 2.B. Let  $\pi_A^E$  denote the profit of firm A when the third party is in the market and  $\pi_A^{NE}$  when it is not. The difference between these two is expressed by  $\Delta\pi_A$ . Define  $\Delta\pi_B$  and  $\Delta CS$  in an identical way. The graph “with third party” in Figure 2.7 shows the social welfare without the entry cost when



$V = 2$ . The horizontal axis is the transportation cost. The social welfare is higher when the third party is in the market. The first parties do not lose their profits even if the third party enters the market, and the third party makes a positive profit. The consumers who are far from the first parties can get positive utility by consuming the system of which peripheral component is produced by the third party. Consumers have better options thanks to a wider variety of the goods and can save some transportation costs.

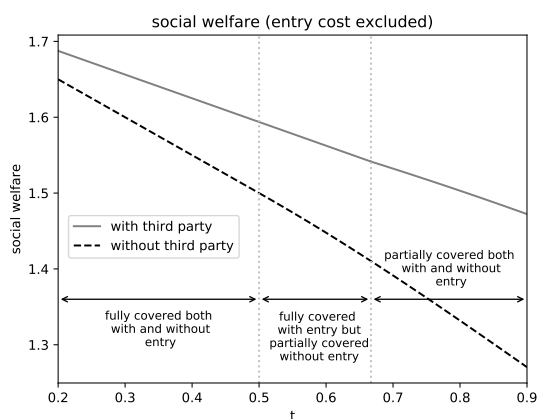


Figure 2.7: Social welfare.

A sufficiently high entry cost prevents the third party from entry even though it might raise the social welfare. The government should encourage entry if the increase in social welfare obtained by the entry is large enough, formally if  $\Delta CS + \Delta\pi_A + \Delta\pi_B + \pi_C > F$ .

The first parties might assist the third party by bearing some of the entry cost if the entry yields higher profits for the first parties. However, even though the entry is efficient for society, the first parties might not subsidize it because they do not take consumer surplus into account. This case arises when  $\Delta\pi_A + \Delta\pi_B + \pi_C < F$ . Combining these two conditions, intervention by the government is justified when  $\Delta\pi_A + \Delta\pi_B + \pi_C < F < \Delta CS + \Delta\pi_A + \Delta\pi_B + \pi_C$ .

*Proposition 3.* The government should encourage the entry of the third party when

$$\Delta\pi_A + \Delta\pi_B + \pi_C < F < \Delta CS + \Delta\pi_A + \Delta\pi_B + \pi_C.$$

## 2.6 Conclusion

In this chapter I discuss what determines the entry of a third party in a horizontally differentiated market where the good is comprised of two components. This analysis clarifies the following points. First, the degree of peripheral-component differentiation determines whether the third party enters. The entry occurs if and only if the peripheral components are highly differentiated. Since less differentiation causes more intense competition, the price of the peripheral component that the third party produces falls, which decreases the profit. Second, the entry of the third party may increase the profits of the first parties. This is because more consumers buy the system if the third party enters the market, which entails increased demand for the main component. Although the third party deprives the first parties of the peripheral-component demand, the first parties can alleviate this loss by setting the peripheral-component price at a significantly low level. Third, the policy to promote entry of the third party might increase the social welfare. The first parties may make extra profits due to the increased system demand, and some consumers can save the transportation cost.

It is observed that third parties enter the markets that are more differentiated and not the ones that are less differentiated. As for SLR cameras, third parties are in the market of producing only lenses. In the video game software market, which is considered to be highly differentiated, many third parties produce software for game consoles.

For the sake of simplicity, some topics are not included in this chapter. For example, the location choice of the firms is not discussed. In this chapter, the third party is assumed to exogenously locate away from the first parties to represent the uniqueness of its product. Now, one may question where a third party is likely

to locate if it could choose its location. It is easy to see that the third party will not locate near the first parties. The reason is simple. The third party wants to avoid competition.<sup>11</sup> The first parties may also find the remote location of the third party profitable because the third party effectively increases the demand for the main component when the location is away from the first parties to some extent. It is crucial for both parties to be distant with regard to peripheral-component differentiation.

It might be profitable for the first parties to produce on their own the peripheral component that the third party will produce. Producing multiple types of the peripheral components could be possible if the cost of producing additional varieties is low enough and few components are necessary to activate the main component. Some systems, however, need many components to function. First parties cannot cover all markets of the components needed to use the system. Another reason for letting third parties enter is that third parties might have better information about some markets than first parties do. Thus, even if first parties can produce several types of the peripheral components in several markets, a number of component markets are left for third parties to enter.

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<sup>11</sup>Suppose the first parties choose their locations in the first stage. Then the third party chooses its location in the second stage and all of the firms choose their prices in the third stage in the full-coverage case. The third party does not approach the first parties beyond a particular point on the Hotelling line in order to avoid price competition. Actually, it does not affect the results qualitatively as long as the third party does not locate exactly where the first parties are even if maximal differentiation is not assumed. This is because they earn profits mostly (entirely when the market is fully covered) from their main components. The first parties do not have an incentive to deter the entry of the third party since it does not harm them.

## Appendix 2.A Partially Covered Market Analysis

**Third Party Entry** In the case of the partial coverage market, there are five options for consumers: AA, BB, AC, BC, and do not buy. I compare each utility and derive the demands for them. For example, to derive the demand for AA, first compare the utility of buying AA with that of BB. Second, compare that of AA with that of AC. Lastly, compare the utility of AA with that of do not buy. I solve the equations numerically, since it is not possible to get a closed-form solution.

When the third party enters in the market under partial coverage, the profits are expressed as follows.

$$\begin{aligned}
 \pi_i = & \left[ \left( \frac{2V - P_m^j - P_m^i - 2P_p^C - 1}{2t} \right) \left( \frac{P_m^j - P_m^i}{2} + \frac{1}{2} \right) \right. \\
 & + \left( \frac{P_m^j - P_m^i}{2} + \frac{1}{2} + V - \frac{P_p^i}{2} - \frac{P_p^C}{2} - P_m^i - \frac{1}{2}t \right) \\
 & \left. \left( \frac{-2V + P_p^i + P_m^i + P_m^j + P_p^C + 1}{2t} + \frac{1}{2} \right) \frac{1}{2} \right] P_m^i \\
 & + \left[ \frac{1}{2} \left( V - \frac{P_p^i}{2} - \frac{P_p^C}{2} - P_m^i - \frac{1}{2}t + \frac{P_p^j + P_m^j - P_p^i - P_m^i}{2} + \frac{1}{2} \right) \right. \\
 & \left. \left( -\frac{V}{t} + \frac{P_p^j + P_m^j + P_m^i + P_p^C + 1 + t}{2t} \right) \right. \\
 & \left. + \left( \frac{V}{t} - \frac{P_p^j + P_m^j + P_p^i + P_m^i + 1}{2t} \right) \left( \frac{P_p^j + P_m^j - P_p^i - P_m^i}{2} + \frac{1}{2} \right) \right] (P_m^i + P_p^i),
 \end{aligned}$$

$$\begin{aligned}
\pi_c = & \left[ \left( \frac{2V - P_m^j - P_m^i - 2P_p^C - 1}{2t} \right) \left( \frac{P_m^j - P_m^i}{2} + \frac{1}{2} \right) \right. \\
& + \left( \frac{P_m^j - P_m^i}{2} + \frac{1}{2} + V - \frac{P_p^i}{2} - \frac{P_p^C}{2} - P_m^i - \frac{1}{2}t \right) \\
& \left. \left( \frac{-2V + P_p^i + P_m^i + P_m^j + P_p^C + 1}{2t} + \frac{1}{2} \right) \frac{1}{2} \right] P_p^C \\
& + \left[ \left( \frac{2V - P_m^j - P_m^i - 2P_p^C - 1}{2t} \right) \left( \frac{P_m^i - P_m^j}{2} + \frac{1}{2} \right) \right. \\
& + \left( \frac{P_m^i - P_m^j}{2} + \frac{1}{2} + V - \frac{P_p^j}{2} - \frac{P_p^C}{2} - P_m^j - \frac{1}{2}t \right) \\
& \left. \left( \frac{-2V + P_p^i + P_m^i + P_m^j + P_p^C + 1}{2t} + \frac{1}{2} \right) \frac{1}{2} \right] P_p^C \\
& - F.
\end{aligned}$$

As shown in Figure 2.5, the marginal effects in the partial-coverage case are obtained through an identical manipulation as the ones considered in the analysis of the full-coverage case. Although all the three effects appear, as in the full-coverage situation, their levels are different. In particular, the business stealing effect ( $\alpha$ ) is smaller, the inframarginal gain ( $\beta$ ) is smaller and the erosion effect ( $\gamma$ ) is greater than in the full-coverage case. For the business stealing effect, since fewer consumers purchase AC under the partial coverage in the first place, the consumers who switch to AA are also fewer. This gain is smaller than that in the full-coverage case. The inframarginal gain is smaller because fewer consumers purchase AC, and thus the gain from raising the price of the main component is also smaller. The erosion effect is greater in the partial-coverage case. To begin with, a price increase results in a greater demand decrease if the firm is a local monopolist than if the firm competes with another firm in the one-dimensional Hotelling model. The reason is as follows. Suppose a firm increases the price by  $\epsilon$  in the one-dimensional case, which is given graphically in Figure 2.8. The new threshold consumer in a local monopoly case is the one who locates  $\epsilon$  nearer to the firm than the former one. Could the new thresh-

old type in the local monopoly case also be the new threshold in the competitive case? The answer is no. The consumer who is  $\epsilon$  nearer to the firm receives the same payoff as the former threshold one from buying the product of the firm. However, she prefers buying from the firm that raises the price since she receives less utility from buying the product of the other firm. Thus, the threshold consumer in the competitive case does not locate as near as that in the local monopoly case. Analogous to the one-dimensional Hotelling model, the loss from the price increase of the main component in the model here is larger under the partial-coverage case, where each firm has consumers for whom it acts as a local monopolist (those consumers at the edge of buying and not buying). Putting all of the effects together, the net payoff obtained from this manipulation is smaller in the partial-coverage case than that in the full-coverage case. Therefore, the peripheral-component prices of the first parties do not decrease as much as those in the full-coverage case.

**No Third Party Entry** I analyze the partially covered market in this section. The profits are expressed by

$$\begin{aligned} \pi_i = & \left[ \frac{1}{2} \left( V - t - P^i + \frac{P^j - P^i}{2} + \frac{1}{2} \right) \left( -\frac{V}{t} + \frac{P^j + P^i + 1}{2t} + 1 \right) \right. \\ & \left. + \left( \frac{P^j - P^i}{2} + \frac{1}{2} \right) \left( \frac{V}{t} - \frac{P^i + P^j + 1}{2t} \right) \right] P^i, \end{aligned}$$

where  $i, j = A, B$ ;  $i \neq j$ . A Nash equilibrium in this subgame is a price  $P^{i*}$  satisfying

$$\begin{aligned} \frac{\partial \pi_i}{\partial P^i} = & P^{i*} \left[ -\frac{3 \left( \frac{P^{i*} + P^{j*} + 1}{2t} - \frac{V}{t} + 1 \right)}{4} - \frac{\frac{V}{t} - \frac{P^{i*} + P^{j*} + 1}{2t}}{2} \right. \\ & \left. + \frac{-P^{i*} + \frac{P^{j*} - P^{i*}}{2} + V - t + \frac{1}{2}}{4t} - \frac{\frac{P^{j*} - P^{i*}}{2} + \frac{1}{2}}{2t} \right] \\ & + \frac{\left( -P^{i*} + \frac{P^{j*} - P^{i*}}{2} + V - t + \frac{1}{2} \right) \left( \frac{P^{i*} + P^{j*} + 1}{2t} - \frac{V}{t} + 1 \right)}{2} \\ & + \left( \frac{P^{j*} - P^{i*}}{2} + \frac{1}{2} \right) \left( \frac{V}{t} - \frac{P^{i*} + P^{j*} + 1}{2t} \right) = 0. \end{aligned}$$

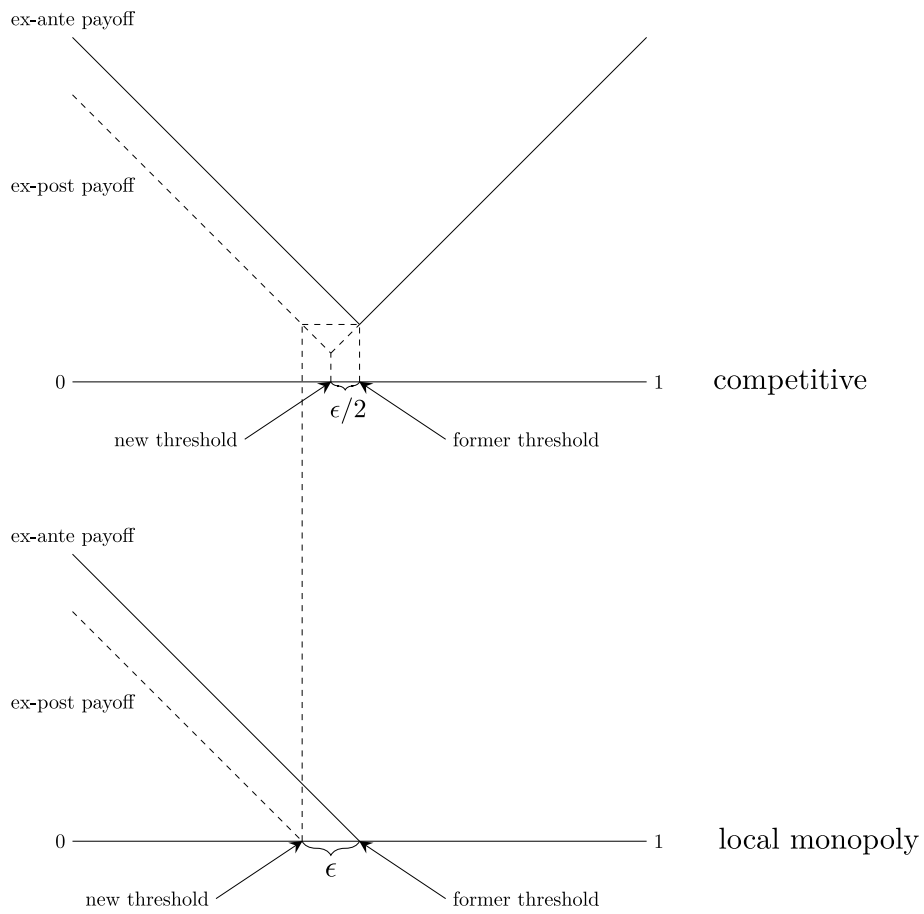


Figure 2.8: Threshold consumer in the Hotelling model.

Solving these equations I obtain

$$P^{i*} = \frac{\sqrt{4V^2 + (-32t - 4)V + 32t^2 + 48t + 1} + 6V - 8t - 3}{8},$$

inducing the profit

$$\begin{aligned} \pi_i^* = & -\frac{(4V^2 - 4V - 8t^2 + 8t + 1)\sqrt{4V^2 - (32t + 4)V + 32t^2 + 48t + 1}}{256t} \\ & + \frac{8V^3 - (32t + 12)V^2 + (80t^2 - 48t + 6)V - 64t^3 + 24t^2 + 32t - 1}{256t}. \end{aligned}$$

## Appendix 2.B Consumer Surplus Analysis

The consumer surplus in each case is expressed as follows.

### Third Party Entry

Full coverage:

$$\begin{aligned} CS = & 2 \left( \int_0^{\frac{P_p^{A*} - P_p^{C*}}{2t} + \frac{1}{2}} \int_0^{\frac{P_m^{B*} - P_m^{A*}}{2}} [V - g_m - tg_p - P_p^{C*} - P_m^{A*}] dg_m dg_p \right. \\ & \left. + \int_{\frac{P_m^{B*} - P_m^{A*}}{2}}^1 \int_0^{\frac{P_p^{B*} - P_p^{A*} + P_m^{B*} - P_m^{A*}}{2} + \frac{1}{2}} [V - g_m - t(1 - g_p) - P_p^{A*} - P_m^{A*}] dg_m dg_p \right). \end{aligned}$$



Partial coverage:

$$\begin{aligned}
CS = & 2 \left( \int_0^{\frac{P_p^{A^*} - P_p^{C^*}}{2t} + \frac{1}{2}} \int_0^{\frac{V - P_p^{A^*}}{2} - \frac{P_p^{C^*}}{2} - P_m^{A^*} - \frac{1}{2t}} [V - g_m - tg_p - P_p^{C^*} - P_m^{A^*}] dg_m dg_p \right. \\
& + \int_0^{\frac{2V - P_m^{B^*} - P_m^{A^*} - 2P_p^{C^*} - 1}{2t}} \int_{V - \frac{P_p^{A^*}}{2} - \frac{P_p^{C^*}}{2} - P_m^{A^*} - \frac{1}{2t}}^{\frac{P_m^{B^*} - P_m^{A^*}}{2} + \frac{1}{2}} [V - g_m - tg_p - P_p^{C^*} - P_m^{A^*}] dg_m dg_p \\
& + \int_{\frac{P_p^{A^*} - P_p^{C^*}}{2t} + \frac{1}{2}}^{\frac{2V - P_m^{B^*} - P_m^{A^*} - 2P_p^{C^*} - 1}{2t}} \int_{V - \frac{P_p^{A^*}}{2} - \frac{P_p^{C^*}}{2} - P_m^{A^*} - \frac{1}{2t}}^{V - tg_p - P_p^{C^*} - P_m^{A^*}} [V - g_m - tg_p - P_p^{C^*} - P_m^{A^*}] dg_m dg_p \\
& + \int_{\frac{P_p^{A^*} - P_p^{C^*}}{2t} + \frac{1}{2}}^1 \int_0^{V - \frac{P_p^{A^*}}{2} - \frac{P_p^{C^*}}{2} - P_m^{A^*} - \frac{1}{2t}} [V - g_m - t(1 - g_p) - P_p^{A^*} - P_m^{A^*}] dg_m dg_p \\
& + \int_{\frac{P_p^{A^*} - P_p^{C^*}}{2t} + \frac{1}{2}}^{-\frac{V}{t} + \frac{P_p^{B^*} + P_m^{B^*} + P_p^{A^*} + P_m^{A^*}}{2t} + \frac{1}{2t} + 1} \int_{V - \frac{P_p^{A^*}}{2} - \frac{P_p^{C^*}}{2} - P_m^{A^*} - \frac{1}{2t}}^{V - t(1 - g_p) - P_p^{A^*} - P_m^{A^*}} [V - g_m - t(1 - g_p) \\
& \quad \quad \quad - P_p^{A^*} - P_m^{A^*}] dg_m dg_p \\
& + \int_{-\frac{V}{t} + \frac{P_p^{B^*} + P_m^{B^*} + P_p^{A^*} + P_m^{A^*}}{2t} + \frac{1}{2t} + 1}^1 \int_{V - \frac{P_p^{A^*}}{2} - \frac{P_p^{C^*}}{2} - P_m^{A^*} - \frac{1}{2t}}^{\frac{P_p^{B^*} + P_m^{B^*} - P_p^{A^*} - P_m^{A^*}}{2} + \frac{1}{2}} [V - g_m - t(1 - g_p) \\
& \quad \quad \quad - P_p^{A^*} - P_m^{A^*}] dg_m dg_p \Big).
\end{aligned}$$

### No Third Party Entry

Full coverage:

$$CS = 2 \left( \int_0^1 \int_0^{\frac{P_p^{B^*} - P_p^{A^*}}{2} + \frac{1}{2}} [V - g_m - t(1 - g_p) - P_p^{A^*}] dg_m dg_p \right).$$

Partial coverage:

$$\begin{aligned}
CS = & 2 \left( \int_0^{-\frac{V}{t} + \frac{PB^* + PA^* + 1}{2t} + 1} \int_0^{V - t - PA^*} [V - g_m - t(1 - g_p) - P^{A^*}] dg_m dg_p \right. \\
& + \int_0^{-\frac{V}{t} + \frac{PB^* + PA^* + 1}{2t} + 1} \int_{V - t - PA^*}^{V - t(1 - g_p) - PA^*} [V - g_m - t(1 - g_p) - P^{A^*}] dg_m dg_p \\
& \left. + \int_{-\frac{V}{t} + \frac{PB^* + PA^* + 1}{2t} + 1}^1 \int_0^{\frac{PB^* - PA^*}{2} + \frac{1}{2}} [V - g_m - t(1 - g_p) - P^{A^*}] dg_m dg_p \right).
\end{aligned}$$

## Chapter 3

# Consumer Poaching in Two-Sided Markets

### 3.1 Introduction

The internet has significantly increased the amount of available information. Because of the immense quantities of information existing online, however, it is often hard to find what is really needed. Many online two-sided platforms are prospering since they solve this kind of problems as intermediaries. As a result, platforms have been accumulating their participants' information and might be using it to increase profits. Recently, regulatory authorities have been concerned about platforms' behavior including personalized pricing, which is enabled by revealed information about customers.<sup>1</sup> One possible type of personalized pricing is behavior-based price discrimination, based on the information about customers' purchasing histories. The progress in information technology and the rise of internet commerce have facilitated behavior-based price discrimination. Companies could poach customers from their rivals by setting lower prices to those who make a transition since they can easily

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<sup>1</sup>See [http://www.oecd.org/officialdocuments/publicdisplaydocumentpdf/?cote=DAF/COMP\(2018\)13&docLanguage=En](http://www.oecd.org/officialdocuments/publicdisplaydocumentpdf/?cote=DAF/COMP(2018)13&docLanguage=En) and <https://publications.parliament.uk/pa/ld201516/ldselect/lducom/129/129.pdf>

distinguish whether the customers have bought from them or not. Baker (2019) discusses predatory pricing and targeted discounting adopted by online platforms, and argues that customer data could be used to exclude rivals through targeted discounting because a potential predator may think it is less costly than undercutting its price uniformly (p. 130). He also states that “While antitrust law has historically adopted special rules for evaluating predatory pricing, targeted discounting involves a new mechanism that should be analyzed differently. [...] courts should assimilate the analysis of both forms of exclusionary conduct [...]” (p. 143).

To evaluate the economic consequence concerning online platforms which use targeted discounting, a model which includes both below-cost pricing and price discrimination should be analyzed. Although it is well known that two-sided platforms may set their prices below their marginal costs, customer poaching, a type of targeted discounting, in two-sided markets has not been explored widely.<sup>2</sup>

This chapter investigates pricing and considers whether or not behavior-based price discrimination is harmful in terms of social and consumer surplus when platforms poach their customers in a two-sided market. The modeling is as follows. Following Armstrong (2006), there are two platforms on the Hotelling line in a two-sided market. Consumers are uniformly distributed on the line and participate in either platform (single-homing). There are two periods. In the first period, the platforms choose their subscription fees. In the second period, they determine discriminated prices based on their customers’ purchase history; one for their loyal customers and the other for the switchers.

The main results of this chapter are twofold. First, when the market is two-sided and indirect network externalities are large enough, the competition at the first stage is intensified as the discount factor on the second-stage profit increases. This result cannot be obtained when the market is one-sided. In the one-sided poaching model, the demand at the first period becomes less responsive to first-period price changes

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<sup>2</sup>Lam (2017), footnote 6.

as the discount factor gets larger and this makes the first-period price higher. When the indirect network externalities in the two-sided market are large, the magnitude of the decrease in the responsiveness is small, which implies that this effect is small. The important factor is that the price for switchers is below the marginal costs in the two-sided market when the indirect network externalities are large. In addition to that, as the first period price increases so that the number of the participants of the platform in the first period decreases, the number of switchers in the second period rises. Thus, when the discount factor is high so that the platforms are farsighted, raising the first-period price is not beneficial for the platforms. If they do so, they need to sell their products to more customers at the below-cost price. Second, in the two-sided markets where platforms conduct customer poaching, both targeted discounting and below-cost pricing can be adopted in equilibrium and the below-cost price in the second stage is more likely to be set than in one-sided markets. The discriminated price offered to the rival's customers could be interpreted as predatory pricing in the sense that the price for the customers of the rival is lower than that for the loyal customers and can be lower than the marginal cost. However, the targeted discounting and below-cost pricing adopted by the platforms are not intended to exclude their rivals and therefore should not be considered as anti-competitive conduct. Since these pricing even raise consumer surplus, regulatory authorities should be careful about evaluating the pricing of two-sided platforms which use poaching strategies.

This chapter is related to the literature of customer poaching and two-sided markets. Fudenberg and Tirole (2000) analyze customer poaching in a one-sided market and show that poaching intensifies the competition in the second period while the competition in the first period is alleviated as the discount factor on the second-stage increases. This is because the price elasticity of demand in the first period decreases when the discount factor increases. Although this effect exists also in the two-sided model that I analyze, the effect is dominating that the platforms

prevent second-period consumers from switching, resulting in a decrease in the first-period prices when the indirect network externalities are high. The analysis of two-sided markets is pioneered by Caillaud and Jullien (2003), Rochet and Tirole (2003) and Armstrong (2006). The two-sidedness modeled in this chapter is related to Armstrong (2006). He shows that the price which a platform imposes on the consumer on one side is decreasing in the indirect network externalities of the other side and can even be below the marginal cost. The novel result in this chapter, which is opposite to the one-sided model, relies on this below-cost pricing in the presence of indirect network externalities.<sup>3</sup>

Although this chapter is the first one concerned with customer poaching by two-sided platforms, there are analyses on perfect price discrimination (Liu and Serfes (2013) and Kodera (2015)) and second-degree price discrimination (Böhme (2017) and Jeon et al. (2016)). In the context of two-period two-sided markets, Lam (2017) studies the effect of switching costs in two-sided markets and finds that the first period participation fee can be decreasing in switching costs, which would be U-shaped in the one-sided model. As in the switching-cost literature, consumers in the first period are randomly relocated on the Hotelling line in the second period in her paper. The assumption that consumers have the same preferences over periods may be more appropriate in some cases, however. In this chapter, I analyze a model where consumers have consistent preferences over periods and may switch without costs focusing on discriminatory prices offered by platforms.

This chapter is organized as follows. Section 2 provides the model. Section 3 analyzes platforms' pricing. Section 4 presents the welfare analysis. Section 5 concludes.

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<sup>3</sup>Choe et al. (2018) also obtain the same result in a one-sided setting by introducing asymmetric information acquisition about customers' preferences.

## 3.2 Model

Consider two platforms X and Y which operate in a two-sided market with sides A and B. They are located at both ends of a Hotelling line, 0 and 1 respectively. The marginal costs are normalized to zero. There are two periods with discount factor  $\delta \in (0,1)$ . In the first period, the platforms maximize the sum of their first and expected second period profits by choosing their first period subscription fees: platform X offers  $p_{A1}^X$  and  $p_{B1}^X$  to the consumers on side A and side B, whose populations are represented by  $n_{A1}^X$  and  $n_{B1}^X$  respectively. Platform Y offers  $p_{A1}^Y$  and  $p_{B1}^Y$  to the consumers on side A and side B, whose populations are represented by  $n_{A1}^Y$  and  $n_{B1}^Y$ . In the second period, the platforms choose their second period subscription fees given the prices in the first period. The platforms can poach the customers of their competitors on side A by offering discriminated prices to new customers in the second period as analyzed by Belleflamme and Peitz (2015), p. 259, Section 10.3. Accordingly, in the second period, platform X offers  $p_{A2}^{XX}$  to the consumers who participated in platform X in the first period, whose population is represented by  $n_{A2}^{XX}$ , and offers  $p_{A2}^{YX}$  to the customers who participated in platform Y in the first period, whose population is represented by  $n_{A2}^{YX}$ . Analogously, platform Y offers  $p_{A2}^{YY}$  and  $p_{A2}^{XY}$  to its loyal and switching consumers, whose populations are represented by  $n_{A2}^{YY}$  and  $n_{A2}^{XY}$ . Since the platforms do not poach their customers on side B, the prices offered by platform X and Y are  $p_{B2}^X$  and  $p_{B2}^Y$  respectively. The number of the consumers on side B who participate in X and Y are  $n_{B2}^X$  and  $n_{B2}^Y$ .

Consumers on sides A and B are uniformly located on the Hotelling line and they join either platform X or Y (single-homing). The per period utility of side A consumer  $\theta_A$  joining a platform is

$$V + e_A n_B - d_A^\theta - p_A,$$

where  $V > 0$  is the intrinsic value of participating in either platform. The parameter

$e_A \in [0, 1)$  measures the benefit obtained by one more participant on the other side and  $n_B$  represents the number of the consumers on side B participating in the same platform. The distance between consumer  $\theta_A$  and the platform he participates in is represented by  $d_A^\theta$ , where  $d_A^\theta = \theta_A$  if the consumer participates in X and  $d_A^\theta = 1 - \theta_A$  if the consumer participates in Y. The transportation cost is assumed to be one. Let  $p_A$  represent the fee which the consumer pays. Therefore, assuming side A consumers are forward-looking and the discount factor is the same with the one which the platforms have, the expected utility of side A consumer  $\theta_A$  in the first period has four cases: (i)  $V + e_A n_{B1}^X - \theta_A - p_{A1}^X + \delta[V + e_A n_{B2}^X - \theta_A - p_{A2}^{XX}]$  if  $\theta_A$  participates in X in both periods, (ii)  $V + e_A n_{B1}^X - \theta_A - p_{A1}^X + \delta[V + e_A n_{B2}^Y - (1 - \theta_A) - p_{A2}^{XY}]$  if  $\theta_A$  participates in X in the first period and switches to Y in the second period, (iii)  $V + e_A n_{B1}^Y - (1 - \theta_A) - p_{A1}^Y + \delta[V + e_A n_{B2}^Y - (1 - \theta_A) - p_{A2}^{YY}]$  if  $\theta_A$  participates in Y in both periods, and (iv)  $V + e_A n_{B1}^Y - (1 - \theta_A) - p_{A1}^Y + \delta[V + e_A n_{B2}^X - \theta_A - p_{A2}^{YX}]$  if  $\theta_A$  participates in Y in the first period and switches to X in the second period. The per period utility of side B consumer  $\theta_B$  joining a platform is defined analogously as follows

$$V + e_B n_A - d_B^\theta - p_B. \quad (3.1)$$

To clarify the effect of poaching in two-sided markets, the consumers on side B are assumed to be myopic in the sense that they maximize their utility given by (3.1) every period independently. In period 1, consumer  $\theta_B$  therefore chooses which platform to join by comparing  $V + e_B n_{A1}^X - \theta_B - p_{B1}^X$  and  $V + e_B n_{A1}^Y - (1 - \theta_B) - p_{B1}^Y$ . In period 2, she compares  $V + e_B (n_{A2}^{XX} + n_{A2}^{YX}) - \theta_B - p_{B2}^X$  and  $V + e_B (n_{A2}^{YY} + n_{A2}^{XY}) - (1 - \theta_B) - p_{B2}^Y$ .

### 3.3 Equilibrium

The intrinsic value  $V$  is assumed to be sufficiently high so that the market is fully covered.



### 3.3.1 Second Stage

First, the second stage is analyzed to derive the symmetric subgame perfect Nash equilibrium. The side A consumer who participated in platform X in the first period and is indifferent between staying in X and switching to Y in the second period, is calculated by solving

$$V + e_A n_{B2}^X - \theta_{A2}^X - p_{A2}^{XX} = V + e_A n_{B2}^Y - (1 - \theta_{A2}^X) - p_{A2}^{XY},$$

for threshold  $\theta_{A2}^X$  (see figure 3.1). The side A consumer who participated in platform Y in the first period and is indifferent between staying in Y and switching to X in the second period, is calculated by solving

$$V + e_A n_{B2}^Y - (1 - \theta_{A2}^Y) - p_{A2}^{YY} = V + e_A n_{B2}^X - \theta_{A2}^Y - p_{A2}^{YX},$$

for threshold  $\theta_{A2}^Y$ . Given the threshold in the first period,  $\theta_{A1}$ , the demands are represented by

$$\begin{aligned} n_{A2}^{XX} &= \theta_{A2}^X, \\ n_{A2}^{YX} &= \theta_{A2}^Y - \theta_{A1}, \\ n_{A2}^{YY} &= 1 - \theta_{A2}^Y, \\ n_{A2}^{XY} &= \theta_{A1} - \theta_{A2}^X. \end{aligned}$$

The demands on side B are derived also by calculating the threshold consumer. The consumer who is indifferent between joining platform X and Y is calculated by solving

$$V + e_B(n_{A2}^{XX} + n_{A2}^{YX}) - \theta_{B2} - p_{B2}^X = V + e_B(n_{A2}^{YY} + n_{A2}^{XY}) - (1 - \theta_{B2}) - p_{B2}^Y,$$

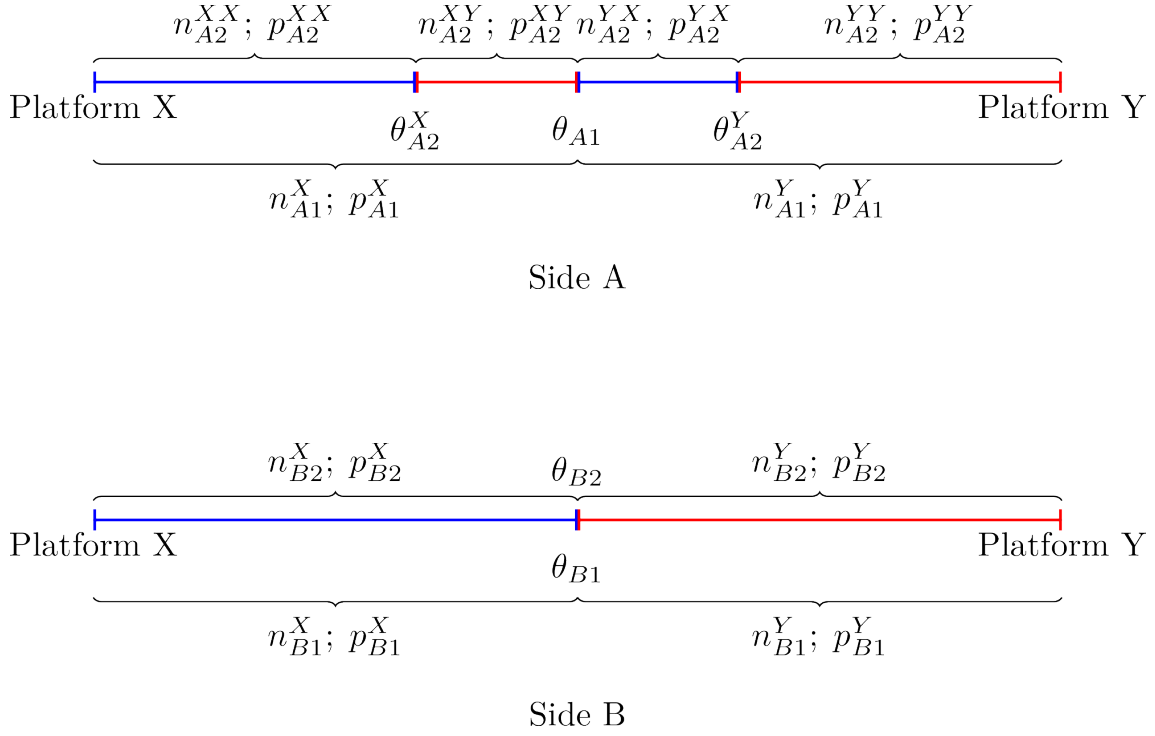


Figure 3.1: Locations and populations of loyal and switching consumers and the prices they pay.

for  $\theta_{B2}$ . The demands are represented by

$$\begin{aligned} n_{B2}^X &= \theta_{B2}, \\ n_{B2}^Y &= 1 - \theta_{B2}. \end{aligned}$$

Note that the platforms do not poach their customers, and consumers are myopic on side B. Platform X maximizes

$$\max_{p_{A2}^{XX}, p_{A2}^{YX}, p_{B2}^X} p_{A2}^{XX} n_{A2}^{XX} + p_{A2}^{YX} n_{A2}^{YX} + p_{B2}^X n_{B2}^X.$$

Because this is a symmetric game, I will focus only on platform X. The best responses are given by

$$p_{A2}^{XX} = \frac{(6e_B^2 + 18e_Ae_B + 12e_A^2 - 18)\theta_{A1}}{12e_B^2 + 30e_Ae_B + 12e_A^2 - 27} + \frac{-12e_B^3 + (5 - 30e_A)e_B^2 + (-12e_A^2 + 11e_A + 27)e_B + 2e_A^2 - 9}{12e_B^2 + 30e_Ae_B + 12e_A^2 - 27},$$

$$p_{A2}^{YX} = \frac{-(18e_B^2 + 42e_Ae_B + 12e_A^2 - 36)\theta_{A1}}{12e_B^2 + 30e_Ae_B + 12e_A^2 - 27} + \frac{-12e_B^3 - (30e_A - 13)e_B^2 - (12e_A^2 - 31e_A - 27)e_B + 10e_A^2 - 27}{12e_B^2 + 30e_Ae_B + 12e_A^2 - 27},$$

$$p_{B2}^X = \frac{(2e_B - 2e_A)\theta_{A1} + (4 - 4e_A)e_B^2 + (-10e_A^2 + 10e_A - 1)e_B - 4e_A^3 + 4e_A^2 + 10e_A - 9}{4e_B^2 + 10e_Ae_B + 4e_A^2 - 9}.$$

The fee for switchers,  $p_{A2}^{YX}$ , is lower than that for the customers who stay in the same platform,  $p_{A2}^{XX}$ , for any  $e_A$  and  $e_B$  when  $\theta_{A1} = 1/2$ .

### 3.3.2 First Stage

In symmetric equilibrium,  $\theta_{A1} = 1/2$  is assumed since the first-period demand on side A should be divided evenly by the two platforms. The cutoff consumer on side A in the first period always switches to the other platform in the second period because the fee for switchers is lower than that for the loyals. The side A threshold consumer is calculated by solving

$$\begin{aligned} & V + e_A n_{B1}^X - \theta_{A1} - p_{A1}^X + \delta[V + e_A n_{B2}^Y - (1 - \theta_{A1}) - p_{A2}^{XY}] \\ & = V + e_A n_{B1}^Y - (1 - \theta_{A1}) - p_{A1}^Y + \delta[V + e_A n_{B2}^X - \theta_{A1} - p_{A2}^{YX}], \end{aligned}$$

for  $\theta_{A1}$ . The demands on side A are represented by

$$\begin{aligned} n_{A1}^X &= \theta_{A1}, \\ n_{A1}^Y &= 1 - \theta_{A1}. \end{aligned}$$

The side B threshold consumer is calculated by solving

$$V + e_B n_{A1}^X - \theta_{B1} - p_{B1}^X = V + e_B n_{A1}^Y - (1 - \theta_{B1}) - p_{B1}^Y,$$

for  $\theta_{B1}$  as in the first period because the consumers on side B are myopic. The demands are represented by

$$\begin{aligned} n_{B1}^X &= \theta_{B1}, \\ n_{B1}^Y &= 1 - \theta_{B1}. \end{aligned}$$

Platform X chooses  $p_{A1}^X$  and  $p_{B1}^X$  to maximize

$$\max_{p_{A1}^X, p_{B1}^X} p_{A1}^X n_{A1}^X + p_{B1}^X n_{B1}^X + \delta \pi_2^X, \quad (3.2)$$

where  $\pi_2^X$  is the maximized second-period profit.

### 3.3.3 Equilibrium Outcome

The second-period outcome of the symmetric equilibrium is summarized as follows:

$$\begin{aligned} p_{A2}^{XX} &= 2/3 - e_B, & p_{A2}^{YX} &= 1/3 - e_B, & p_{B2}^X &= 1 - e_A, \\ n_{A2}^{XX} &= 1/3, & n_{A2}^{YX} &= 1/6, & n_{B2}^X &= 1/2. \end{aligned}$$

The outcome in the first period is

$$\begin{aligned}
p_{A1}^X &= \frac{(5e_B^2 + (14e_A + 3)e_B + 8e_A^2 - 3e_A - 9)\delta}{(12e_B^2 + 30e_Ae_B + 12e_A^2 - 27)} \\
&\quad + \frac{-12e_B^3 + (12 - 30e_A)e_B^2 + (-12e_A^2 + 30e_A + 27)e_B + 12e_A^2 - 27}{(12e_B^2 + 30e_Ae_B + 12e_A^2 - 27)}, \\
p_{B1}^X &= 1 - e_A, \\
n_{A1}^X &= 1/2, \\
n_{B1}^X &= 1/2.
\end{aligned}$$

This outcome is compatible with Fudenberg and Tirole (2000) and Armstrong (2006). When the indirect network externalities are absent, that is,  $e_A = e_B = 0$ , the first-period fee on side A,  $p_{A1}^X$ , is  $1 + \delta/3$ , which would be of course derived under the one-sided customer poaching model. Without poaching, the second-period fees on side A would be  $p_{A2}^{XX} = p_{A2}^{YX} = 1 - e_B$ . These are equivalent to the result in Armstrong (2006) just as the fee on side B, where the platforms do not poach their customers. The first terms become  $2/3$  and  $1/3$  because of poaching.

The platforms poach their customers in equilibrium and at the same time they may set their second-period price below their marginal costs depending on the values of indirect network externalities. It is noteworthy that the platforms offer below-cost prices for switchers and above-cost prices for the loyal in the second period when  $1/3 < e_B < 2/3$ . Although they set lower prices for switchers than for the loyals and the prices can be lower than their marginal costs, this pricing should not be considered as predatory pricing. The platforms do not intend to exclude their rivals by setting predatory prices and then recoup in the subsequent stage. The below-cost pricing and behavior-based price discrimination here are the consequence of market competition. Regulatory authorities therefore should be careful about evaluating the below-cost pricing for switchers in two-sided markets because it may not be actually predatory pricing even if it looks so.

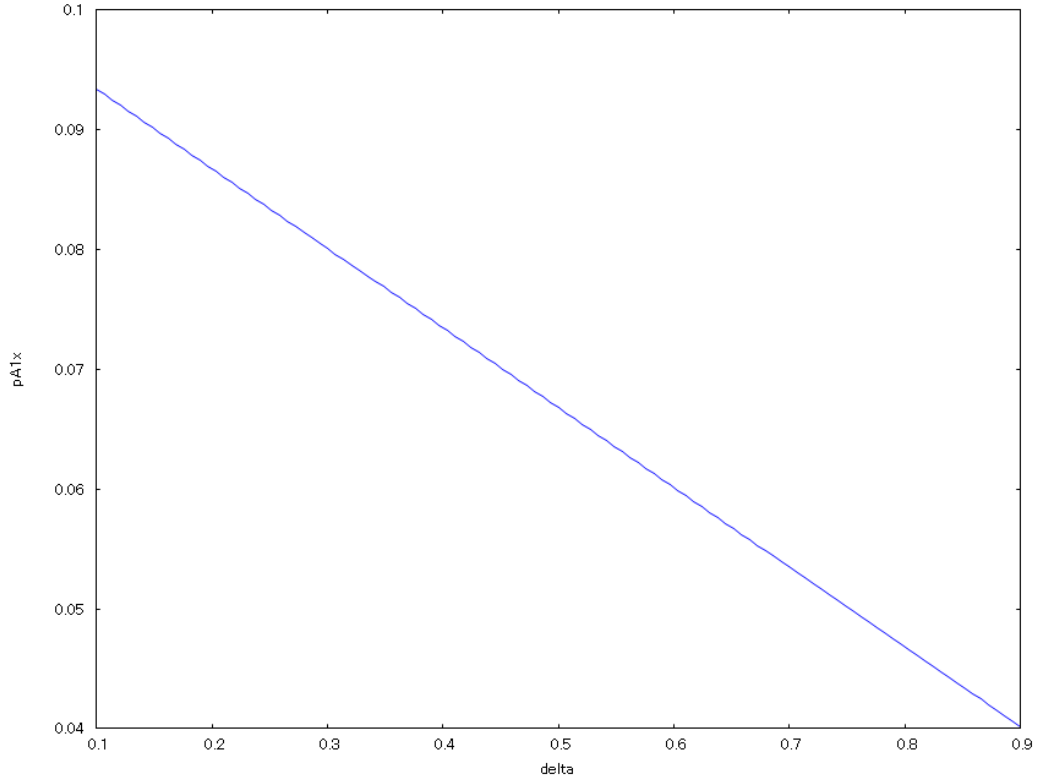


Figure 3.2: First-period price with respect to  $\delta$  when  $e_A = 0.25, e_B = 0.9$ .

*Proposition 1.* Two-sided platforms exercise below-cost pricing and targeted discounting in equilibrium without the intension of having an anti-competitive conduct.

In the one-sided poaching model,  $p_{A1}^X$  would be always increasing in  $\delta$  because the responsiveness of the first-period demand to its price change decreases as  $\delta$  increases. When the market is two-sided, however,  $p_{A1}^X$  can be decreasing in  $\delta$  as in figure 3.2, which plots the price against  $\delta$  under the case of  $e_A = 0.25, e_B = 0.9$ . Generally, the first-period price can be decreasing in  $\delta$  when the indirect network externalities are high (see figure 3.3) so that the prices for the switchers in the second stage are below the marginal costs. This does not happen in the one-sided poaching model.

*Proposition 2.* In the customer poaching model in a two-sided market, the first-period price is decreasing in  $\delta$  if indirect network externalities are strong.

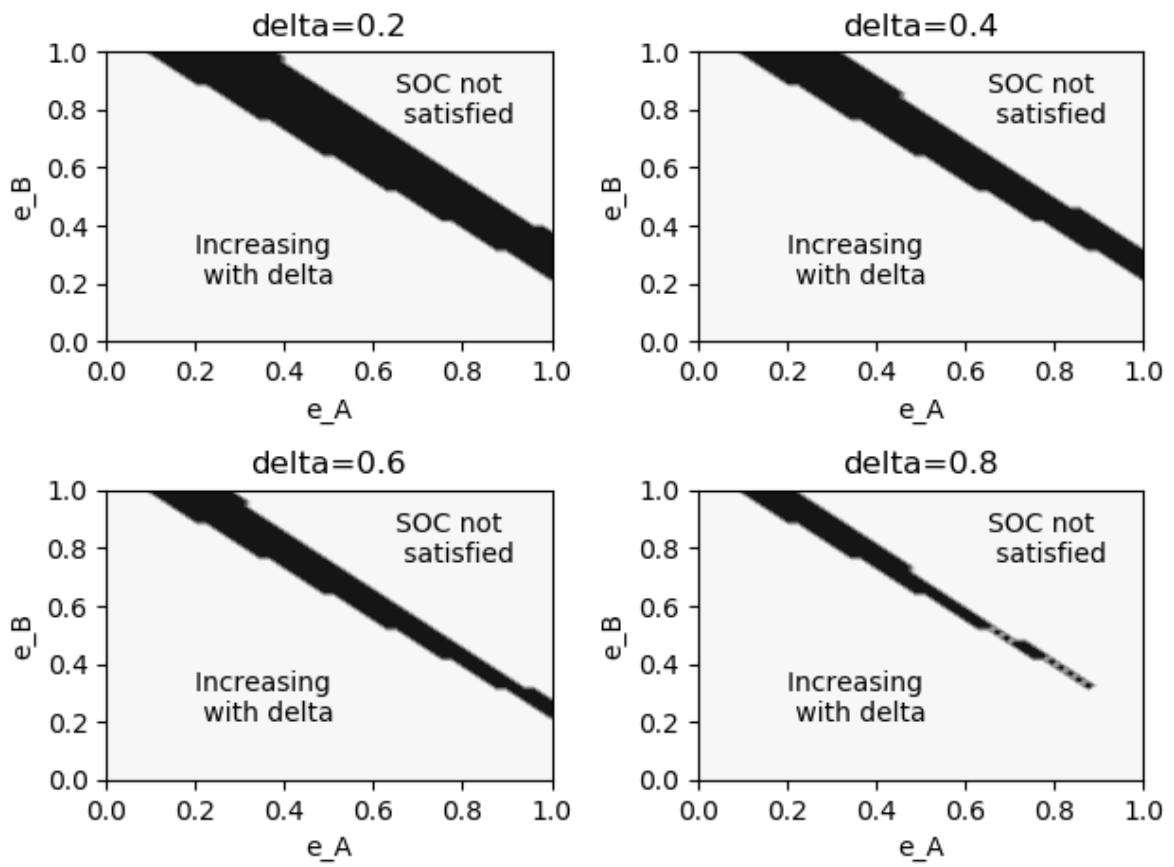


Figure 3.3: Regions of  $e_A$  and  $e_B$  where  $\partial p_{A1}^X / \partial \delta < 0$  and the S.O.C. is satisfied are shown in dark grey.

To obtain an insight of why an increase in  $\delta$  intensifies the competition in the first period, consider the decision making of platform X in the first period when  $e_A = 0.25$  and  $e_B = 0.9$ , under which  $p_{A1}^X$  is decreasing in  $\delta$ . Platform X maximizes its profit  $\pi^X$ , which is given by expression (3.2). Suppose platform X raised  $p_{A1}^X$ . Then, the marginal gain and loss are expressed by

$$\frac{\partial \pi^X}{\partial p_{A1}^X} = \underbrace{n_{A1}^X}_{+} + p_{A1}^X \underbrace{\frac{\partial n_{A1}^X}{\partial p_{A1}^X}}_{-} + p_{B1}^X \underbrace{\frac{\partial n_{B1}^X}{\partial p_{A1}^X}}_{-} + \delta \left( \underbrace{\frac{\partial p_{A2}^{XX}}{\partial p_{A1}^X}}_{-} n_{A2}^{XX} + \underbrace{p_{A2}^{XX}}_{-} \underbrace{\frac{\partial n_{A2}^{XX}}{\partial p_{A1}^X}}_{-} \right. \\ \left. + \underbrace{\frac{\partial p_{A2}^{YX}}{\partial p_{A1}^X}}_{+} n_{A2}^{YX} + \underbrace{p_{A2}^{YX}}_{-} \underbrace{\frac{\partial n_{A2}^{YX}}{\partial p_{A1}^X}}_{+} \right. \\ \left. + \underbrace{\frac{\partial p_{B2}^X}{\partial p_{A1}^X}}_{+} n_{B2}^X + p_{B2}^X \underbrace{\frac{\partial n_{B2}^X}{\partial p_{A1}^X}}_{+} \right).$$

Summarizing the above equation,

$$gain : n_{A1}^X + \delta \underbrace{\left( p_{A2}^{XX} \frac{\partial n_{A2}^{XX}}{\partial p_{A1}^X} + \frac{\partial p_{A2}^{YX}}{\partial p_{A1}^X} n_{A2}^{YX} + \frac{\partial p_{B2}^X}{\partial p_{A1}^X} n_{B2}^X + p_{B2}^X \frac{\partial n_{B2}^X}{\partial p_{A1}^X} \right)}_{(*)},$$

$$loss : p_{A1}^X \frac{\partial n_{A1}^X}{\partial p_{A1}^X} + p_{B1}^X \frac{\partial n_{B1}^X}{\partial p_{A1}^X} + \delta \underbrace{\left( \frac{\partial p_{A2}^{XX}}{\partial p_{A1}^X} n_{A2}^{XX} + p_{A2}^{YX} \frac{\partial n_{A2}^{YX}}{\partial p_{A1}^X} \right)}_{(**)}.$$

Table 3.1 lists the terms of the expression above when  $e_A = 0.25, e_B = 0.9$  and  $e_A = e_B = 0$ . First, the terms in table 3.1 are less sensitive to  $\delta$  when  $e_A = 0.25, e_B = 0.9$  than when  $e_A = e_B = 0$ . This implies that the effect is weak, which makes the first-period price increasing in  $\delta$  when the market is one-sided. Because these terms per se are not changed by  $\delta$  very much in the two-sided market,



	$e_A = 0.25, e_B = 0.9$	$e_A = e_B = 0$
$\frac{\partial n_{A1}^X}{\partial p_{A1}^X}$	$\frac{-3.26}{0.26\delta + 5.05}$	$\frac{-9}{6\delta + 18}$
$\frac{\partial n_{B1}^X}{\partial p_{A1}^X}$	$\frac{-2.93}{0.26\delta + 5.05}$	0
$\frac{\partial n_{A2}^{XX}}{\partial p_{A1}^X}$	$\frac{-0.13}{0.26\delta + 5.05}$	$\frac{-3}{6\delta + 18}$
$\frac{\partial n_{A2}^{YX}}{\partial p_{A1}^X}$	$\frac{3.13}{0.26\delta + 5.05}$	$\frac{6}{6\delta + 18}$
$\frac{\partial n_{B2}^X}{\partial p_{A1}^X}$	$\frac{1.4}{0.26\delta + 5.05}$	0
$\frac{\partial p_{A2}^{XX}}{\partial p_{A1}^X}$	$\frac{-2.78}{0.26\delta + 5.05}$	$\frac{-6}{6\delta + 18}$
$\frac{\partial p_{A2}^{YX}}{\partial p_{A1}^X}$	$\frac{3.74}{0.26\delta + 5.05}$	$\frac{12}{6\delta + 18}$
$\frac{\partial p_{B2}^X}{\partial p_{A1}^X}$	$\frac{1.3}{0.26\delta + 5.05}$	0

Table 3.1: Values of the terms when platform X raises  $p_{A1}^X$ .

the terms (\*) and (\*\*) should be highlighted. When  $e_B$  is large,  $p_{A2}^{XX}$  and  $p_{A2}^{YX}$  are negative but they would be positive if the indirect network externalities were absent. The term  $p_{A2}^{XX}(\partial n_{A2}^{XX}/\partial p_{A1}^X)$  is *gain* when  $e_B$  is large but this would be *loss* when  $e_B = 0$ . On the other hand,  $p_{A2}^{YX}(\partial n_{A2}^{YX}/\partial p_{A1}^X)$  is *loss* when  $e_B$  is large but would be *gain* when  $e_B = 0$ . As shown in table 3.1,  $\partial n_{A2}^{YX}/\partial p_{A1}^X$  is larger than  $\partial n_{A2}^{XX}/\partial p_{A1}^X$  seeing their absolute values. Thus, the absolute value of (\*\*) becomes large when  $p_{A2}^{YX}$  is negative due to large a  $e_B$ . Additionally, when  $e_A$  is large,  $p_{B2}^X$  is small too. This makes  $p_{B2}^X(\partial n_{B2}^X/\partial p_{A1}^X)$  small and eventually implies that (\*) is smaller. Combining the two effects, (\*) becomes smaller and (\*\*) becomes larger when the indirect network externalities are large. As  $\delta$  increases, the marginal *loss* part becomes larger than the marginal *gain* part. When  $\delta$  is large, raising  $p_{A1}^X$  is therefore not profitable for platform X.

Intuitively, the platforms decrease their fees and expand their turf in the first period when indirect network externalities are high. This reduces the number of switchers in the second period and cuts down the loss caused by the fees below their marginal costs in the second period.

### 3.4 Welfare Analysis

Define consumer surplus of side A as

$$\begin{aligned}
CS_A = & \int_0^{\frac{1}{3}} [V + e_A n_{B1}^X - \theta - p_{A1}^X + \delta(V + e_A n_{B2}^X - \theta - p_{A2}^{XX})] d\theta \\
& + \int_{\frac{1}{3}}^{\frac{1}{2}} [V + e_A n_{B1}^X - \theta - p_{A1}^X + \delta(V + e_A n_{B2}^Y - (1 - \theta) - p_{A2}^{XY})] d\theta \\
& + \int_{\frac{1}{2}}^{\frac{2}{3}} [V + e_A n_{B1}^Y - (1 - \theta) - p_{A1}^Y + \delta(V + e_A n_{B2}^X - \theta - p_{A2}^{YX})] d\theta \\
& + \int_{\frac{2}{3}}^1 [V + e_A n_{B1}^Y - (1 - \theta) - p_{A1}^Y + \delta(V + e_A n_{B2}^Y - (1 - \theta) - p_{A2}^{YY})] d\theta,
\end{aligned}$$

and side B as

$$\begin{aligned}
CS_B = & \int_0^{\frac{1}{2}} [V + e_B n_{A1}^X - \theta - p_{B1}^X + \delta(V + e_B (n_{A2}^{XX} + n_{A2}^{YX}) - \theta - p_{B2}^X)] d\theta \\
& + \int_{\frac{1}{2}}^1 [V + e_B n_{A1}^Y - (1 - \theta) - p_{B1}^Y + \delta(V + e_B (n_{A2}^{YY} + n_{A2}^{XY}) - (1 - \theta) - p_{B2}^Y)] d\theta.
\end{aligned}$$

Define producer surplus as

$$PS = \pi^X + \pi^Y.$$

Social surplus is defined as

$$SS = (CS_A + CS_B) + PS.$$

*Proposition 3.* Consumers are better-off when platforms poach their customers while social surplus is deteriorated.

Compared to the case where the platforms set their prices statistically and do not poach their customers, the consumer surplus is improved and the producer surplus is reduced for any values of indirect network externalities, which means that this result also holds when the market is one-sided. In one-sided markets, the first-period price is always higher when the firms poach their customers while in two-sided markets, it is lower when the platforms exercise customer poaching for the range where the first-period price is decreasing in  $\delta$ . Consumers are thus better off in both periods when the indirect network externalities are in this range. The social surplus is deteriorated when the platforms poach their customers. This is because the consumers who switch in the second period pay avoidable transportation costs. They could save transportation costs if they stayed in the platforms that they joined in the first period. Regulatory authorities should take into account that consumers are better off when platforms exercise customer poaching even when below-cost prices for switchers are observed because platforms do not intend to increase their market power.

### **3.5 Conclusion**

This chapter examines the pricing of two-sided platforms and its influence on social and consumer surplus when they can poach their customers. I show that in equilibrium targeted discounting as a form of behavior-based price discrimination and below-cost pricing are used by platforms in a two-sided market. However, the platforms do not have the intension to exclude their rivals, and thus regulatory authorities should be careful even when below-cost pricing for switchers is observed.

While social surplus is deteriorated due to increased transportation costs, consumers are better-off under customer poaching.

It is widely known that in the one-side model of poaching, the price in the first period always increases as the discount factor on the second-period profit increases. In this chapter, I show that the price in the first period decreases as the discount factor increases in the two-sided model with large indirect network externalities. The reason is as follows. The platforms have to sell their products at below-cost prices in the second period when indirect network externalities are large. Raising the first-period fee increases the number of switchers in the second period, and thus the platforms have to sell more customers at the below-cost prices. This makes it less beneficial for platforms to raise the price in the first period when they are more farsighted.

# Chapter 4

## Patent Value and Network

### Centrality

#### 4.1 Introduction

Innovation is a major driving force of economic growth and hence it is required to encourage innovation in order to stimulate economic growth. However, a natural question arises: how can we measure innovation? This is a fundamental question for governmental organizations and all other economic agents since we can gain little knowledge about it if we cannot measure and observe it. It is not possible to fully understand what drives innovation and to what extent it contributes to innovation without seeing it. The first thing to do is to make innovation observable by measuring it. However, it is not clear how to do it.

One proxy of innovation among others is how many patents are applied and granted. Patent counts are observable and are thought to be applied and granted more if the rate of inventive activities rises. Moreover the value of patents can be estimated by focusing on the market value of the company which acquired patents. The background idea is that when a patent is granted to a company, the market assesses the value of the patent and it is incorporated to the market value of the

company. Using this idea, a number of studies have estimated how valuable a patent is. While the idea is useful that patents increase the market value of the holders, one issue is that patents are heterogeneous. Only a small portion of patents are valuable while the majority are of little importance and as a result using only patent counts equalizes the value of patents regardless of their significance. Some indices which reflect the quality should be included to analyze the value of them.

It is suggested that patent citations involve the information about the quality of patents (Trajtenberg (1990); Albert et al. (1991)). If a patent has high quality, it delivers many subsequent inventions from it and therefore is cited by many other patents. However, does the number of citations always work properly as an index of patent quality? Consider the following example, which is provided in figure 4.1. The

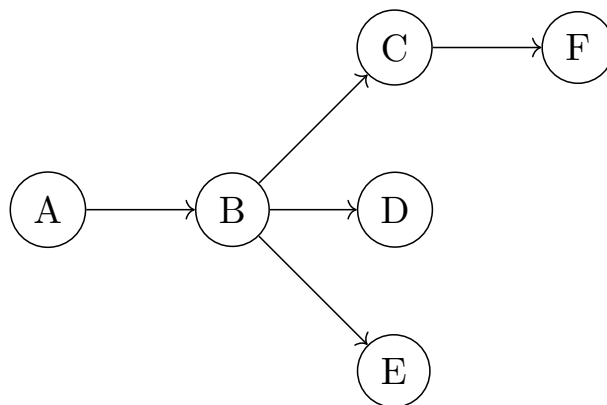


Figure 4.1: Example of a patent citation network.

circles are patents and if a patent cites another one, the arrow points at the citing patent from the cited one. For example, patent A is cited by patent B and patent B is cited by patent C, D and E. It is doubtless that patent B is important because it is cited by the other three patents. Likewise, patent A is cited by patent B and patent C is cited by patent F, which means that both have the same number of citations. It is, however, reasonable to consider that patent A is more important than patent C because patent A is cited by the important one, patent B; whereas patent C is cited by the one which does not produce any subsequent inventions. In other words, patent A has one direct citation from patent B and four indirect citations

from patent C, D, E and F while patent C has one direct citation from patent C only. Patent A delivers not only one “child” but also three “grandchildren” and one “great-grandchild”, while patent C has one “child” only. Counting the number of citations does not capture high-degree citations, or indirect citations.

In this chapter, I introduce the Bonacich centrality proposed by Bonacich (1987) as an indicator of the quality of patents in order to incorporate the importance of indirect citations and estimate the value of patents. Bonacich centrality is a measure used in social network analysis which evaluates the prestige of agents in the network. The score of the Bonacich centrality is high if an agent has relationships with those who themselves have high scores of Bonacich centrality. In general, its concept embodies the view that “those who have relationships with prestigious people have power”; in the context of patents, “those which are cited by important patents are important”. Applying the methodology of social network analysis, the Bonacich centralities of the patents in the example of figure 4.1 are calculated in table 4.1 with some parameter value. Because it takes indirect citations into account, the

	A	B	C	D	E	F
citations	1	3	1	0	0	0
centrality	1.28	3.09	1	0	0	0

Table 4.1: Citations and centralities of the patents in figure 4.1.

score of patent A is higher than patent C.

Incorporating the Bonacich centrality, I estimate that a one percent point increase in the ratio of indirect citations to direct citation increases the market value by 0.6% approximately. Also, R&D expenditures per asset and citations counts per patent are statistically significant. Although indirect citations do not provide patent royalties to the holders<sup>1</sup>, the market assesses them positively. Having many indirect

<sup>1</sup>It should be noted that patent royalties are not always paid to the directly cited patents. If

citations imply that it is a breakthrough invention and innovative products will be developed by the patentee. Additionally, it is suggested that the patents with many indirect citations increase the market value due to the fact that patents are used for cross-licensing and negotiations. The patents having many indirect citations give the holders an advantageous position in the negotiations.

This article is organized as follows. Section 4.2 reviews the related literature. Section 4.3 summarizes the basic concepts of social network analysis. Section 4.4 provides the data and the variables used in the analysis. Section 4.5 specifies the estimating equation. Section 4.6 presents the results of this analysis. Section ?? concludes.

## 4.2 Related Literature

It is known that patents are enormously skewed in terms of their quality and that some indicators which reflect the importance of patents are needed. Trajtenberg (1990) and Albert et al. (1991) propose that citation counts that a patent receives should be indicators of the quality of the patent. Using patent renewal data, Harhoff et al. (1999) figure out that the patents which renewed to full-term expiration are more highly cited, implying that the patents which the patentee considers valuable have more citations. Harhoff et al. (2003) use the patent value directly derived from a survey of patentees and suggest that the patent value is positively related to the number of both forward and backward citations.

Hall et al. (2005) use patent citations as indicators of patent quality and with the market value of listed companies, they estimate that an extra direct citation per patents increases the market value of the holder by 3%. However, as I mention in section 4.1, direct citations do not capture indirect citations. It is expected that the impact of an extra direct citation becomes smaller when indirect citations

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the new patent infringes the previous patent inventions, the new one needs to pay royalties to the cited ones.



are incorporated, since direct citations entail indirect ones provided that the stock market participants consider them valuable.

Many studies apply the methodology of social network analysis to analyze patents by constructing citation networks (see Jaffe and de Rassenfosse (2017) for survey). Mariani et al. (2019) use a time-rescaled PageRank to identify important patents with the list of the patents that experts selected as significant inventions. Google's PageRank (Brin and Page (1998)) is also a centrality measure which has similar characteristics that as those of the Bonacich centrality. Additionally the time-rescaled PageRank is an indicator developed from Google's PageRank to deal with the time bias that is generated by older patents having a longer time period to receive citations than newer ones. In this chapter, the time bias is handled by limiting the citation span to five years.<sup>2</sup>

The difference between the PageRank and Bonacich centralities is that PageRank takes into account the number of backward citations. If a patent with forward citations, say patent A, cites some patents, the Bonacich centrality gives those cited patents the scores which reflects the number of the forward citations that patent A has regardless of how many patents it cites. The idea of PageRank is that if patent A cites more patents, the relative importance of each cited patent becomes smaller. The PageRank scores of the cited patents go down as the number of cited patents increases.

Bonacich centrality may be more appropriate for assessing the value of patents due to the following reason, however. The market evaluates patents positively because they bring cash flow to the patentee. Patents are used to develop new products and sell them exclusively, which generate cash flow but they are also used for licensing. Therefore, direct citations are valuable also because of the royalties that the patentee gets. While obtaining more indirect citations imply that the patent is of high quality, they do not provide royalties to the patentee in principle, however.

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<sup>2</sup>A patent gains more than 50% of the citations, which it receives by the expiration date within five years for the US patents according to Nagaoka et al. (2010).

Indirect citations are also thought to be appraised positively by the stock market because it provides more bargaining power to the patentee in the negotiations of licensing. Suppose an important patent is granted to a company and it infringes and cites another patent. The company which has the important patent needs to negotiate with the one which has the cited patent. The opportunity cost of the breakdown of the negotiation is high for the company with the important patent because its patent is important and thus the breakdown would take away quite a lot of cash flow. This gives bargaining power to the company which has the cited patent. This bargaining power is not thought to change even if there are more cited patents given that all the cited patents are necessary.

### 4.3 Network and Centrality

Before constructing a patent citation network, a brief summary of social network analysis is provided below. For more information, see Jackson (2008) and Newman (2018).

A directed network consists of a set of nodes  $N$  with  $|N| = n$  and links denoted by a real-valued  $n \times n$  matrix  $\mathbf{g}$  where its element  $(\mathbf{g})_{ij}$  is 1 if there is a one-way relation from  $i$  to  $j$  and 0 if not. This matrix is called the adjacency matrix of the network. The in-degree of a node is the number of incoming links to the node and the out-degree is the number of outgoing links from a given node. A  $i - j$  walk  $W$  is a sequence of links which begins with  $i$  and ends at  $j$ . The length of  $W$  is the number of nodes encountered in  $W$ , allowing multiplicity. Entry  $(\mathbf{g}^k)_{ij}$  of the adjacency matrix to the power  $k$ ,  $\mathbf{g}^k$ , shows the number of walks of length  $k$  from  $i$  to  $j$ .

To capture how important a node is, a number of different measures have been developed each evaluating different aspects of the importance which the node has. In this analysis, a measure called Bonacich centrality is used which takes the neigh-

bors' importance into account when a node is evaluated. Basically, the concept of Bonacich centrality captures the idea that “those who have powerful friends are powerful”. Formally, the Bonacich centrality is defined as follows (Bonacich (1987)),

$$\mathbf{C}(b, \mathbf{g}) = \mathbf{g}\mathbf{1} + b\mathbf{g}^2\mathbf{1} + b^2\mathbf{g}^3\mathbf{1} + b^3\mathbf{g}^4\mathbf{1} + \dots, \quad (4.1)$$

where  $\mathbf{1}$  denotes the  $n$ -length column vector whose entry is all 1 and  $b$  is a discount factor, which satisfies  $0 < b < 1$ . The left-hand side is the  $n \times 1$  column vector which denotes the Bonacich centralities of node  $1, 2, \dots, n$ . The first term of the right-hand side  $\mathbf{g}\mathbf{1}$  represents how many walks of length 1 there are from each node to any of the other nodes in the network, that is,  $(\mathbf{g}\mathbf{1})_i$  denotes the out-degree of node  $i$ . Similarly,  $\mathbf{g}^2\mathbf{1}$  in the second term denotes the number of walks of length 2 from each node to any of the nodes and is multiplied by discount factor  $b$  because it needs two steps to reach it. The subsequent terms  $b^{k-1}\mathbf{g}^k\mathbf{1}$  are interpreted analogously. I specify that those who have more out-degree are more prestigious.<sup>3</sup> When  $b$  is less than  $1/\lambda_{max}$ , where  $\lambda_{max}$  denotes the largest eigenvalue of  $\mathbf{g}$ , equation (4.1) is expressed as<sup>4</sup>

$$\mathbf{C}(b, \mathbf{g}) = (I - b\mathbf{g})^{-1}\mathbf{g}\mathbf{1}. \quad (4.2)$$

Note that node  $i$ 's Bonacich centrality is derived as well by solving the system of equations below

$$C_i(b, \mathbf{g}) = b \sum_j g_{ij} C_j(b, \mathbf{g}) + \sum_j g_{ij}, \quad \forall i. \quad (4.3)$$

This expression clearly shows that the Bonacich centrality grasps the idea of “those

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<sup>3</sup>In many cases such as the network analysis of the World Wide Web, those who have more in-degree are important. This depends solely on modelling strategies, however. If the ones with many in-degree are prestigious, taking the transpose of the adjacency matrix results in the same centrality scores.

<sup>4</sup>Debreu and Herstein (1953).

who have powerful friends are powerful” as node  $i$ ’s power,  $C_i(b, \mathbf{g})$  is high when the sum of the power of its friends,  $\sum_j g_{ij}C_j(b, \mathbf{g})$  is also high.

Using the concept of social network analysis, a patent citation network is constructed in the following way. Patents and citation relationships are described as nodes and directed links. If patent  $i$  is cited by patent  $j$ ,  $i$  has an outward link to  $j$ , which is described graphically in figure 4.2. The out-degree of node  $i$  thus

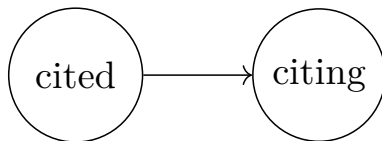


Figure 4.2: Patent citation.

represents the number of citations that patent  $i$  has. Then, the Bonacich centrality written in equation (4.1) is directly applicable to the patent citation network. The first term  $\mathbf{g}\mathbf{1}$  represents the number of citations each patent has. The second term is the score which evaluates the (indirectly) citing patents of walk 2. The following terms assess the (indirectly) citing patents of walk  $k$  analogously. Another way of interpreting the Bonacich centrality in the patent citation network is that summing up the out-degree (i.e., direct citations) and the centralities that the directly citing patents have, when the centrality of a patent is calculated as is given in equation (4.3).

## 4.4 Data

The data used in this analysis has two sources; one is a database of patent information applied in Japan and the other is the financial data of the companies listed in Japanese stock markets. The patent data is brought from the Institute of Intellectual Property Patent Database (IIP-DB), which provides the information about the patents applied in Japan between 1964 and 2017 including the citation relationships of the patents. Nikkei NEEDS provides the financial data of Japanese listed com-

panies such as market capitalization, total assets, liabilities and R&D expenditures disclosed every March, the end of the financial year.

The market value is defined as the sum of market capitalization and total liabilities net of cash and cash equivalents. The R&D expenditures stock of firm  $k$  in year  $t$  is calculated by summing up the past 10 year expenditures of firm  $k$  using 15% depreciation rate as the literature in this field. For instance, the R&D expenditures stock of firm  $k$  in year 2009 is the sum of firm  $k$ 's expenditures in year 2000, 2001, 2002,  $\dots$ , and 2009 each multiplied by the depreciation rate  $0.85^9$ ,  $0.85^8$ ,  $0.85^7$ ,  $\dots$ , and  $0.85^0$ .

Patent counts stock is calculated as follows. First, define the patent counts of firm  $k$  in year  $t$  as the number of patents granted to  $k$  between April 1st,  $t - 1$  and March 31st,  $t$ . The patent counts stock of firm  $k$  in year  $t$  is constructed by aggregating  $k$ 's past ten year patent counts using a 15% depreciation rate in the same way as the R&D expenditures stock.

Since patents need time to receive citations, next five years are considered in this analysis as with Lanjouw and Schankerman (2004). First, consider firm  $k$ 's citations in year  $t$  as follows. Pick up the patents that  $k$  acquired between April 1st,  $t - 1$  and March 31st,  $t$ . Then,  $k$ 's citations are defined as the number of the citations that those patents obtain by March 31st,  $t + 4$ . For example, the citations in year 2001 are calculated by counting the number of citations which the patents granted between April 1st, 2000 and March 31st, 2001 obtain by March 31st, 2005. The citations stock of  $k$  in year  $t$  is constructed by summing up past ten year citations of  $k$  using a 15% depreciation rate.

The construction of centrality stocks requires the specification of patent citation networks. As the citations need the next five years to be accumulated, the centrality also involves the next five years after a patent is granted. In order to derive the centrality of firm  $k$  in year  $t$ , a patent citation network is constructed by listing any patents cited between April 1st,  $t - 1$  and March 31st,  $t + 4$  and the ones which

cite them in the same period. Then, the centralities of  $k$ 's patents granted between April 1st,  $t - 1$  and March 31st,  $t$  are calculated<sup>5</sup> using a discount factor  $b = 0.09$ . By aggregating the past ten year centralities of  $k$ , the centrality stock of  $k$  in year  $t$  is derived using a 15% depreciation rate.<sup>6</sup>

While IIP-DB has the information about the patents applied between 1964 and 2017, the data about R&D expenditures are available from 2000 to 2019. Only the R&D expenditures stocks in 2009 and afterwards can be obtained since the stocks need the past ten years. Because IIP-DB has the patent information granted only before March 17, 2017 and constructing the citations and centrality stocks requires the next five years, these stocks are available only until 2012.<sup>7</sup> Therefore, I used data of market capitalization, assets, liabilities, R&D expenditures, patent, citations and centrality stocks between 2009 and 2012 for estimation. The number of records is about 1600 whose financial data i.e., market cap, cash and cash equivalents, assets, liabilities and R&D expenditures stock, are available every year. When the patent stock data is merged by using the names of companies, the number of feasible records are approximately 1000 every year. The firms with zero R&D expenditures, patent or direct citations stocks are taken out from this dataset since these values are also the denominators of some variables used in the analysis and the sample size of the dataset becomes  $N = 3360$ . Table 4.2 summarizes the variables used in this analysis. As is said in the literature, the variables made from the account items and the patent data are skewed greatly looking at the difference between the means and medians. Direct citations stock per patents stock is about 0.9, which is far less than the US data Hall et al. (2005) collected from 1979 to 1988. This may be due to the different time periods of the data and the patenting propensity in Japan and the US. Nagaoka et al. (2010) report that the number of applications in the US around the

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<sup>5</sup>Technically speaking, it is not possible to calculate only the centralities of firm  $k$  or only the patents granted between  $t - 1$  and  $t$ . The centralities of all the patents in the network are calculated and the ones granted between  $t - 1$  and  $t$  are extracted.

<sup>6</sup>The validity of choosing 0.09 for the discount factor is discussed later.

<sup>7</sup>Many companies disclose their financial statements after March 17.

Variable	Mean	Median	Minimum	Maximum	Standard Deviation
Market value (¥M)	323836	45838.24	32.02	16582654	989730.9
Assets (¥M)	283930.2	56605	184	10612765	808923.3
Market value/Assets	0.912	0.837	0.006	8.079	0.441
R&D stock (¥M)	37289.9	3543.0	0.695	3679655	174436
Patent stock	432.1666	53.93431	0.23162	18981.6	1632.69
Direct citations stock	491.308	43.390	0.232	25243.79	2069.332
Centrality stock	526.504	45.47849	0.23162	27503.63	2221.005
R&D stock /Assets	0.11374	0.07243	0.00002	1.29489	0.12533
Patent stock /R&D stock	0.07098	0.01553	0.00002	72.31424	1.25893
Direct citations stock /Patents stock	0.90178	0.81287	0.06408	8.52100	0.55547
Indirect citations stock /Direct citations stock	0.05087	0.04540	0	1.64344	0.04687

Table 4.2: Sample statistics, 2009-2012 with  $N = 3360$  observations.

1980's was 100,000 whereas four times more applications were submitted in Japan around the 2000's. It is more difficult to gain a citation in Japan around the 2000's and thus getting an extra citation is expected to be more valuable.

## 4.5 Model

In this analysis, the firm level market value function is used which is based on Griliches (1981) and employed frequently in the literature:

$$V_{it} = q_t(A_{it} + \gamma K_{it}), \quad (4.4)$$

where  $V_{it}$  denotes the market value of firm  $i$  in year  $t$ ,  $A_{it}$  denotes the assets recorded in firm  $i$ 's balance sheet in year  $t$ ,  $K_{it}$  denotes the intangible knowledge assets which firm  $i$  has in year  $t$ ,  $q_t$  absorbs market fluctuation and  $\gamma$  represents the shadow price of  $K$  to  $A$ . Dividing both sides by  $A_{it}$  and taking logarithm, equation (4.4) becomes

$$\log Q_{it} = \log \left( \frac{V_{it}}{A_{it}} \right) = \log q_t + \log \left( 1 + \gamma \frac{K_{it}}{A_{it}} \right), \quad (4.5)$$

where  $Q_{it} = V_{it}/A_{it}$  is called Tobin's  $Q$ .

While knowledge stock  $K$  is thought to incorporate R&D expenditures, patents, citations and other intangible assets, it is not obvious how to specify knowledge stocks. Hall et al. (2005) consider it as a stream of informational disclosure; R&D expenditures reveal the commitment to a R&D project, patent counts show how successful the project is and the number of citations reveals the quality or importance of the invention. In light of this view, they estimate the equation below,

$$\log Q_{it} = \log q_t + \log \left( 1 + \gamma_1 \frac{R\&D_{it}}{A_{it}} + \gamma_2 \frac{PAT_{it}}{R\&D_{it}} + \gamma_3 \frac{CITES_{it}}{PAT_{it}} \right), \quad (4.6)$$

where  $R\&D$ ,  $PAT$  and  $CITES$  stand for the stocks of R&D expenditures, patents



and direct citations.

Following their point of view, I include the stock of the Bonacich centrality into equation (4.6). Note that the Bonacich centrality can be divided into the direct and indirect citation terms:

$$C(b, \mathbf{g}) = \underbrace{\mathbf{g}\mathbf{1}}_{\text{direct citations}} + \underbrace{b\mathbf{g}^2\mathbf{1} + b^2\mathbf{g}^3\mathbf{1} + b^3\mathbf{g}^4\mathbf{1} + \dots}_{\text{indirect citations}}, \quad (4.7)$$

where the direct citations' term above is the same value as  $CITES$  in equation (4.6). Separating the Bonacich centrality into the direct and indirect parts, the estimating equation becomes

$$\log Q_{it} = \log q_t + \log \left( 1 + \gamma_1 \frac{R\&D_{it}}{A_{it}} + \gamma_2 \frac{PAT_{it}}{R\&D_{it}} + \gamma_3 \frac{DCITES_{it}}{PAT_{it}} + \gamma_4 \frac{ICITES_{it}}{DCITES_{it}} \right), \quad (4.8)$$

where  $DCITES$  and  $ICITES$  denote the stocks of the direct and indirect citations as in equation (4.7). Once the stock of citations is factored in, the market presumes how many indirect citations the companies obtain. However, some companies gain more indirect citations than expected and some have less. Therefore, the amount of indirect citations per direct citation is incorporated in the equation to be estimated.

## 4.6 Results

Equation (4.8) and its variants are estimated using non-linear least squares with year dummies, not including fixed firm effects as in Hall et al. (2005). Table 4.3 presents the results of the estimation, where  $CENT$  represents the centrality stock.

The variables except for  $PAT/R\&D$  are statistically significant. The coefficient of  $CENT/PAT$  is smaller than  $DCITES/PAT$  since  $CENT$  includes both direct and indirect citations while obtaining a direct one is more valuable than an indirect one. The coefficient of  $DCITES/PAT$  is larger in column 3 than that in column

	1	2	3
<i>R&amp;D/A</i>	1.064*** (.103)	1.073*** (.101)	1.077*** (.102)
<i>PAT/R&amp;D</i>	-.001 (.006)	-.002 (.006)	-.002 (.006)
<i>DCITES/PAT</i>	.107*** (.023)		.133*** (.022)
<i>CENT/PAT</i>		.112*** (.020)	
<i>ICITES/DCITES</i>	.759*** (.283)		
<i>N</i> = 3330			

Table 4.3: Summary of estimation with dependent variable log Tobin's  $Q$ . \* $p < .1$ ; \*\* $p < .05$ ; \*\*\* $p < .01$ .

1 because direct citations entail indirect ones. Incorporating the indirect citations variable thus decreases the magnitude of the impact on Tobin's  $Q$ . While Hall et al. (2005) point out that patents per R&D expenditure are significant using the US data, it is not so in Japan. Cohen et al. (2002) report that companies have higher propensity to patent in Japan than in the US, and Japanese companies tend to apply patents for cross-licensing and negotiations rather than achieving exclusivity. This may suggest that since only high quality patents can be used to gain advantage in negotiation, the number of patent counts itself is not relevant but the quality of patents is important and is reflected on the stock prices.

The quantitative impact of  $R\&D/A$  on Tobin's  $Q$  is calculated by the following log-linear elasticities,

$$\frac{\partial \log Q}{\partial (R\&D/A)} = \hat{\gamma}_1 \left( 1 + \hat{\gamma}_1 \frac{R\&D}{A} + \hat{\gamma}_2 \frac{PAT}{R\&D} + \hat{\gamma}_3 \frac{DCITES}{PAT} + \hat{\gamma}_4 \frac{ICITES}{DCITES} \right)^{-1}. \quad (4.9)$$

The impacts of the other explanatory variables are calculated in the same way as

summarized in table 4.4, evaluating them at both means and medians.

Ratios	Mean	Median
$\frac{\partial \log Q}{\partial(R\&D/A)}$	.847	.888
$\frac{\partial \log Q}{\partial(PAT/R\&D)}$	-	-
$\frac{\partial \log Q}{\partial(DCITES/PAT)}$	.085	.089
$\frac{\partial \log Q}{\partial(ICITES/DCITES)}$	.604	.633

Table 4.4: Impact of the ratios on Tobin's  $Q$ .

A one percent point increase in the ratio of R&D to assets raises the Tobin's  $Q$  by 0.85%, an extra direct citation per patent boosts 8%, and a one percent point increase in the ratio of indirect to direct citations expands it by 0.6%. Note that the indirect citations stock is calculated using the discount factor  $b = 0.09$  and the value of the coefficient changes by using different discount factors. It is suggested, however, that indirect citations have influence on the market value of the patent holders. This is a notable result in the sense that indirect citations themselves do not give patent royalties to the patentee in principle, and the patentee of the original patent does not have the right to exclude the use of the indirect citing patents.<sup>8</sup> One possible explanation is that the invention is significant and the patentee may have the opportunities to develop innovative products from it, which profits the patentee. Another explanation is that the patentees gain bargaining power in negotiations, which is based on the fact Cohen et al. (2002) point out that patents are likely to be used to induce cross-licensing and negotiations in Japan. If an important invention has been built and it uses some other patents, the developer needs to negotiate with the companies which have the cited patents. Because the developer and the original patent holder know that the new invention is important, the developer's opportunity

<sup>8</sup>Note that royalty payment systems depend on the private contract the parties agreed on and they could incorporate the conditions when the citing patents are cited.

cost of disagreement in the negotiation is high and thus the patentee of the cited patents enjoy high bargaining power. The stock market considers that the firms which have those cited patents gain an advantageous position in the negotiations and assesses them positively.

## 4.7 Conclusion

In this chapter, I introduce the Bonacich centrality as a quality index of patents and estimate how much the indirect citations derived from the centrality affects the market value of the patent holders. A one percent point increase in the ratio of indirect citations per direct citation, raises the market value by 0.6% with the discount factor  $b = 0.09$  as R&D expenditures per asset and citations per patent are also statistically significant and positively correlated. This implies that the stock market considers the indirect citations valuable while they do not directly give cash flow to the patentee. Having many indirect citations, however, gives the patentee bargaining power in the negotiation of licensing since it reveals that those patents are core inventions.

However, what makes indirect citations increase the Tobin's  $Q$  of the patentee is not obvious; it helps the patentee apply the new technology, gain bargaining power in negotiations or other channels affecting the market value. Furthermore, patents cite others to show that the new inventions have novelty compared to the previous ones. In this case, the more citations a patent receives, the less significance it has. It is necessary to adopt the data which has the information about what reason the patent is cited for.

The validity of setting the discount factor of the Bonacich centrality at  $b = 0.09$  is not clear. This value is closest to the largest discount factor which ensures equation (4.1) to converge. However, to estimate the impact of indirect citations on the market value of companies, the cardinal value is the one which should be employed.

Moreover even if the Bonacich centrality is used as an ordinal number, the ranking changes depending on what level of a discount factor is adopted and thus it should be chosen carefully. Finding the appropriate level of the rate is necessary for further research.

# Chapter 5

## Concluding Remarks

### 5.1 Conclusion

This dissertation discusses technological relationships focusing on patent citation and standardization. The technologies nowadays are deeply related to each other in the sense that a single product contains many components and this tendency will be further increased. The product to be developed may contain multiple patented technologies. Also, those components need to coordinate to work as one product. In economies with complex technologies, how do companies disentangle the relationships of those patents and assess the value of them to develop a new product? How and why do companies make their components compatible with the other components? Why in some industries, are there firms which provide only some of the necessary components? How do companies try to expand their standards? In order to answer these questions, chapter 5 discusses the patent citation, chapter 3 analyzes entry and complements of a third party and chapter 4 studies platforms' behavior-based price discrimination in two-sided markets.

By reviewing the results and presenting the limitation of each study, this dissertation will be closed.

**Chapter 2** This chapter considers the pricing of first parties and the entry of a third party which produces a peripheral component. It is clarified that the differentiation of the peripheral component determines whether or not the third party enters the market. The entry of the third party may raise the profits of the first parties because the latter can capture the consumers which would not buy the system without the former. The entry also may increase the social welfare since consumers can save their transportation costs.

There are limitations, however, because some topics are not discussed. Compatibility between the first parties is not taken into account in this model. Actually, consumers would not choose a hybrid system of the first-party components even if they were compatible in my model. This is because the first parties supply their peripheral components at the same location. Second, what if the peripheral components of the first parties and the third party are vertically differentiated? Though the peripheral components themselves are not inferior, the quality of peripheral components of third parties would be relatively low in terms of warranties or malfunction which occurs when they are attached to the main components. In this situation, the first parties make decision in the market which is horizontally differentiated with respect to the main components and is vertically differentiated with respect to the peripheral components.

**Chapter 3** This chapter analyzes customer poaching in two-sided markets focusing on the pricing of platforms and its influence on welfare. In equilibrium, two-sided platforms may use behavior-based price discrimination and below-cost pricing in a two-sided market while the platforms do not intend to eliminate their rivals. Regulatory authorities should be careful even when below-cost pricing for switchers is observed. Additionally, it is shown that the price in the first period decreases as the discount factor increases with strong indirect network externalities. This result is not obtained in one-sided markets. When the indirect network externalities

are strong, the platforms are supposed to set their fees for switchers below their marginal costs. Platforms thus try to expand their turf by setting lower prices in the first period and reduce the number of switchers in order to decrease the loss from the below-cost prices in the second period.

**Chapter 4** Using patent data as an innovation indicator, the citation among patents is interpreted as the relationship among those innovations. While citation counts are used as the importance of patents in order to deal with the heterogeneity of patents, they are not sufficient indicators. In chapter 5, the heterogeneity of patent quality is taken into account by using the Bonacich centrality in the patent citation network. Bonacich centrality is a centrality measure which counts walks emanating from a given node. This analysis finds that a one percent point increase in the Bonacich centrality raises the market value of the patentee by 0.6%.

From the data, it is observed that indirect citations raise the market value of the patent holders. It is not obvious what increases the market value, however. Some studies suggest that patents are used in order to gain an advantageous position in negotiations. In other words, patents are used to secure the market access of technologies. Confirming this hypothesis needs further data about the negotiations of technology transfer and further case studies about actual negotiation processes.



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