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主　論　文　の　要　旨

論文題目　Simple-minded objects in triangulated categories and Cohen-Macaulay differential graded modules
（三角圏における単純対象の類似物と Cohen-Macaulay DG 加群）

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論　文　内　容　の　要　旨

Part 1. Derived categories and triangulated categories appear in many areas of mathematics, such as algebraic geometry, representation theory and algebraic topology. An important way to construct a new triangulated category is the Verdier quotient. But usually the morphisms in the quotient category are too complicated to understand. It is known in some nice cases, Verdier quotient can be realized as a reduction process, which is another way to construct a new triangulated category. The reduction can be realized as a certain sub (or subfactor) category of the given one and is easier to study.

The first part of this thesis is devoted to introducing a new reduction process of triangulated categories with respect to simple-minded collections (or SMC for short). Simple modules are one of the most basic and important objects in the representation theory of algebras. There have been many generalizations and studies of simple modules. Among them, SMC in derived categories and SMS (=simple-minded system) in singularity categories have particular importance. We show that SMC behaves very nice under reduction process and there is a bijection between SMCs in the new triangulated category and SMCs in the original one containing certain objects. A similar process called the SMS reduction has been studied by Coelho Simoes and Pauksztello.

Moreover, we consider the pair $(\mathcal{T},\mathcal{T}^p)$ of a triangulated category $\mathcal{T}$ and its thick subcategory $\mathcal{T}^p$ satisfying certain assumptions, and also the singularity category $\mathcal{T}_{sg}:=\mathcal{T}/\mathcal{T}^p$. We show the quotient map sends SMCs in $\mathcal{T}$ to SMSs in $\mathcal{T}_{sg}$, which is highly non-trivial. Further we prove that the SMS reduction is the shadow of the SMC reduction, that is, the following diagram of operations commutes.

$$\begin{tikzcd}
\mathcal{T} \arrow[r, "\text{sing. category}"] \arrow[d, "\text{SMC reduction}"'] & \mathcal{T}_{\mathrm{sg}} \arrow[d, squiggly, "\text{SMS reduction}"] \\
\mathcal{U} \arrow[r, "\text{sing. category}"] & \mathcal{U}_{\mathrm{sg}} \cong (\mathcal{T}_{\mathrm{sg}})_{\mathcal{R}}
\end{tikzcd}$$

This is parallel to a result obtained by Iyama and Yang that Calabi-Yau reduction is the shadow of silting reduction.

Part 2. The second part of this thesis is devoted to introducing Cohen-Macaulay (CM) differential graded (dg) modules and study their representation theory. The notion of Cohen-Macaulay (CM) modules is classical in commutative algebras, and has natural generalizations for non-commutative algebras. The category of CM modules has been studied by many researchers in representation theory. On the other hand, the derived categories of differential graded (dg) categories introduced by Bondal-Kapranov and Keller is an active subject appearing in various areas of mathematics. We are aimed to introduce CM dg modules to connect these two subjects.

We consider a nice class of dg algebras A called Gorenstein and introduce the category $\mathbf{CM}\,A$ of CM dg A-modules. For the case of Gorenstein algebras, the category of CM modules is a Frobenius exact category, and Buchweitz found a triangle equivalence between the stable category of CM modules and certain Verdier quotient of the derived category called the singularity category by Orlov. However, $\mathbf{CM}\,A$ does not have a natural structure of Frobenius exact category in our setting. Surprisingly we find that it has a structure of Frobenius extriangulated category, which is introduced by Nakaoka and Palu as a common generalization of exact category and triangulated category. As an analogue of Buchweitz's equivalence, we show that the stable category $\underline{\boldsymbol{CM}}\,A$ is triangle equivalent to the singularity category. We study $\mathbf{CM}\,A$ from a point of view of representation theory, and show that it admits Auslander-Reiten extensions. One of the most important problems in CM representation is to understand CM-finite Gorenstein (dg) algebras, in the sense that they have only finitely many indecomposable CM (dg) modules. We give a complete classification of Auslander-Reiten quivers of CM-finite d-symmetric dg algebras for $d \geq 1$, which is one of the most basic cases of CM-finite Gorenstein dg algebras.

Part 3. The third part is the continuation of the first two parts. Recently there is increasing interest in negative Calabi-Yau (CY) triangulated categories, including $\underline{\boldsymbol{CM}}\,A$ for d −symmetric dg algebras A (which are $(-d)-CY$). These categories often contain d-SMS, which plays a key role in the study. We show that there is a bijection between d-SMSs in $(-d)-$CY cluster category and maximal simplices in $\Delta^d(\Phi)$ without negative simple roots, where $\Delta^d(\Phi)$ is introduced by Fomin and Reading as a generalization of the combinatorial structure of cluster algebras of finite type.