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THERMOCONVECTIVE MOTION IN A TWO-LAYERED SYSTEM

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Abstract

The convection phenomena in a layered liquid system have been analyzed by a few papers[1-4]. At least two motivations for the activity in this area can be pointed out: 1) To intensify convective mixing in liquids (chemical engineering). 2) To reduce convective mixing in liquids (space technology). Covering main liquid by another one having definite properties, we can reduce or enlarge convective motion in the lower liquid. Hydrodynamical and thermal interaction changes the characteristics of convective motion in both layers. The change of physical parameters in one of the layers permits to influence the hydrodynamics and heat transfer in the other layer. In general, the structure of convective flow in the layers depends on the volumetric buoyancy force and the thermocapillary stress at the free surface and interface. In the case of reduced gravity the input of various components into the balance of forces acting in a liquid changes drastically.

The thermocapillary force begins to play an important, and in many cases, decisive role due to the existence of surface tension dependence on temperature. To investigate the influence of different factors such as gravity, viscosity, thermoconductivity and thermocapillarity upon main characteristics of velocity and temperature fields in a two-layered system, a multiparametric numerical study of a two-dimensional model on thermogravitational and thermocapillary convection is carried out.

1. STATEMENT OF PROBLEM

The physical and computational domain investigated in this study is shown schematically

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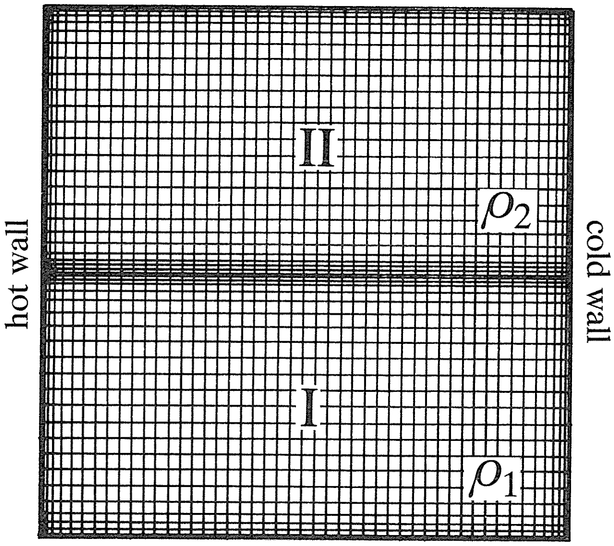


Fig. 1

in Fig. 1. The system consists of two immiscible fluids contained in a two-dimensional open cavity. The lighter fluid occupies the space on the top of the heavier fluid, thus forming a stable layered system. An external temperature gradient is imposed on the system by maintaining the side walls of the cavity at different isothermal conditions. The two-fluid interface and the upper free surface are assumed to be flat and undeformable in the present calculation.

The mathematical formulation of the problem includes both the Navier—Stokes equation under the Boussinesque approximation and the heat conduction equation written in the vorticity ω, stream function Ψ and temperature T.

The governing equations for the incompressible fluid in the lower layer are as follows:

$$U_1 \frac{\partial \omega_1}{\partial x} + V_1 \frac{\partial \omega_1}{\partial y} = Pr_1 \nabla^2 \omega_1 + Gr\,(Pr_1)^2 \frac{\partial_1 T}{\partial x},$$

$$U_1 \frac{\partial T_1}{\partial x} + V_1 \frac{\partial T_1}{\partial y} = \nabla^2 T_1, \tag{1}$$

$$\nabla^2 \Psi_1 = \omega_1 .$$

In the upper layer we have

$$U_2 \frac{\partial \omega_2}{\partial x} + V_2 \frac{\partial \omega_2}{\partial y} = \varkappa\, Pr_2 \nabla^2 \omega_2 + Gr\,(Pr_1)^2\, \beta \frac{\partial_2 T_2}{\partial x},$$

$$U_2 \frac{\partial T_2}{\partial x} + V_2 \frac{\partial T_2}{\partial y} = \varkappa\, \nabla^2 T_2, \tag{2}$$

$$\nabla^2 \Psi_2 = \omega_2 .$$

Here we have used

$$\omega = \frac{\partial U}{\partial y} - \frac{\partial V}{\partial x}, \quad U = \frac{\partial \Psi}{\partial y}, \quad V = \frac{\partial \Psi}{\partial x}, \quad \nabla^2 = \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial x^2}.$$

The boundary conditions at the bottom of the tank $y = 0$ and the lateral surfaces $x=0$, $x=1$ are nonslip ones:

$$\Psi = 0, \quad \partial \Psi / \partial n = 0 . \tag{3}$$

On the free surface there are a balance between viscous and thermocapillary forces (Marangoni effect) and impermeability conditions:

$$\Psi = 0, \quad \frac{\partial U_2}{\partial y} = Ma_2 \frac{\partial T_2}{\partial x}, \quad Ma_2 = \frac{d\sigma}{dT} \frac{H(T_1 - T_0)}{\varkappa_1 \rho_2 \upsilon_2} . \tag{4}$$

The bottom and upper free surfaces are assumed to be thermally insulated:

$$\partial T_i / \partial n = 0.$$

At the interface $y=1/2$ between the fluids, the following conditions are imposed:
Continuity of velocity and temperature;

$$(\partial \Psi / \partial y)_1 = (\partial \Psi / \partial y)_2 . \tag{5}$$

Impermeability condition where the interface is flat;

$$\Psi_1 = \Psi_2 . \tag{6}$$

Continuity of the heat flux;

$$k_1 \left(\frac{\partial T}{\partial y}\right)_1 = k_2 \left(\frac{\partial T}{\partial y}\right)_2 . \tag{7}$$

The balance of viscous forces;

$$\frac{\partial U_1}{\partial y} = \eta \frac{\partial U_2}{\partial y} + Ma_1 \frac{\partial T_1}{\partial x},$$

which is rewritten as

$$\omega_1 = \eta\, \omega_2 + Ma_1 \frac{\partial T_1}{\partial x} . \tag{8}$$

From the boundary conditions between the fluids written above, it follows that for the calculation of vorticity we need to have one more relation. This condition will be found in a way analogous to getting the boundary condition for vortex near solid surface. A Taylor series expansion for the stream function is used near both sides of the interface;

$$\Psi_i = \Psi_{i,o} + \frac{\partial \Psi_i}{\partial y} h_i + \frac{\partial^2 \Psi_i}{\partial y^2} h_i^2 + \dots . \tag{9}$$

Substituting the value of first derivative ($\partial\Psi/\partial y$) in Eq. (9) into Eq. (6) and taking into account that in the case of undeformable surface

$$\omega_i = \frac{\partial^2 \Psi_i}{\partial y^2} \text{ and } \Psi_i = 0 \text{ (for y=1/2)},$$

then the required relation is written as

$$\omega_1 + \omega_2 = \frac{2}{h^2} (\Psi_1 + \Psi_2) . \tag{10}$$

Here and below the subscript i = 1 corresponds to the lower layer, i = 2 to the upper layer. In Eqs. (1)—(10) we have used the dimensionless variables based on the physical properties of the heavier fluid: $x = x'/H$, $y = y'/H$ are the coordinates, $l = H/L$ is the aspect ratio, $U = U'H/\varkappa_1$ is the axial velocity, $V = V'H/\varkappa_1$ is the vertical velocity, $\omega = \omega' H^2/\varkappa_1$ is the vorticity, $\Psi = \Psi'\varkappa_1 H$ is the stream function, $T = (T'-T_h)/(T_h-T_o)$ is the temperature, T_h and T_o are the temperatures of the hot and cold walls, $\varkappa$ is the thermal diffusivity, K is the heat conductivity, β is the dynamic viscosity, $k = k_2/k_1$, $\varkappa = \varkappa_2/\varkappa_1$, $\beta = \beta_2/\beta_1$, $\eta = \eta_2/\eta_1$, $Pr = \nu/\varkappa$ is the Prandtl number, $Gr = \beta_1 g\Delta T L^3/\nu_1^2$ is the Grashof number.

For the numerical solution of Eqs. (1)—(10), an implicit finite difference second-order-accurate scheme with a nonuniform grid and monotonic approximation on the convective terms is used. The main results are obtained on the grid 51*51. The motionless condition is taken as the initial state. The solution procedure consists of the following sequence of events: (i) Solve the vorticity field. (ii) Solve the Poisson equation. (iii) Solve the temperature field. Proceed to the next step in time.

The development of vorticity and temperature fields toward the steady state is calculated for all the cases. Here only the steady-state regimes will be presented.

The mathematical modeling of the thermoconvection enables us to obtain a large volume of information about the motion of the liquids in a layered system and the associated temperature distribution.

In solving Eqs. (1)—(10), the numerical values of the dimensionless parameters are chosen so that Pr of the lighter fluid is twice as large as that of the heavier fluid (Pr = 1) and the both fluids have the same dynamic viscosities. The other parameters of the system are as follows: $k = 0.5$, $\rho = 0.8$, $\beta = 2$, $\varkappa = 0.625$.

2. RESULTS AND DISCUSSIONS

As a first step in numerical simulations, the problem of natural convection in an enclosure is investigated. For all the cases in Fig.2, the fluid system is bounded by nonslip walls on

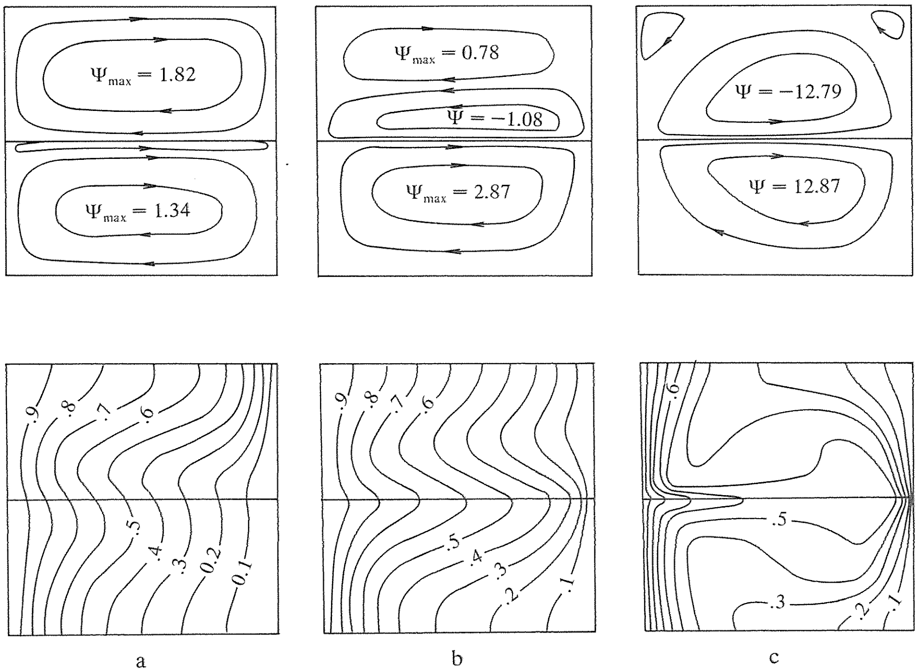


Fig.2

all the four sides. All the calculations are done for the aspect ratio $l = 1$.

The steady-state stream functions and isotherm distribution for buoyancy convection are shown in Fig.2-a (Gr $= 10^4$, $Ma_1 = 0$, $Ma_2 = 0$). Two large convective cells, one in each fluid, are formed in the system. Both cells are driven in the clockwise direction by thermal buoyancy and this causes a conflicting situation in the vicinity of the interface. The upper liquid wins this conflict and drives a small counter clockwise cell in the lower liquid. The thermal buoyancy force in the lighter liquid is stronger than that in the heavier liquid ($\beta = 2$) and the maximum stream function value Ψ_{max} characterizing the intensity of the circulation in liquid 2 is about 50% higher than that in liquid 1. The magnitude of this value is written inside each cell. The isotherms are distorted in accordance with the structure of vortices. The convective cells in the system draw the heat away from the hot wall and the maximal temperature occurs in the upper region of system.

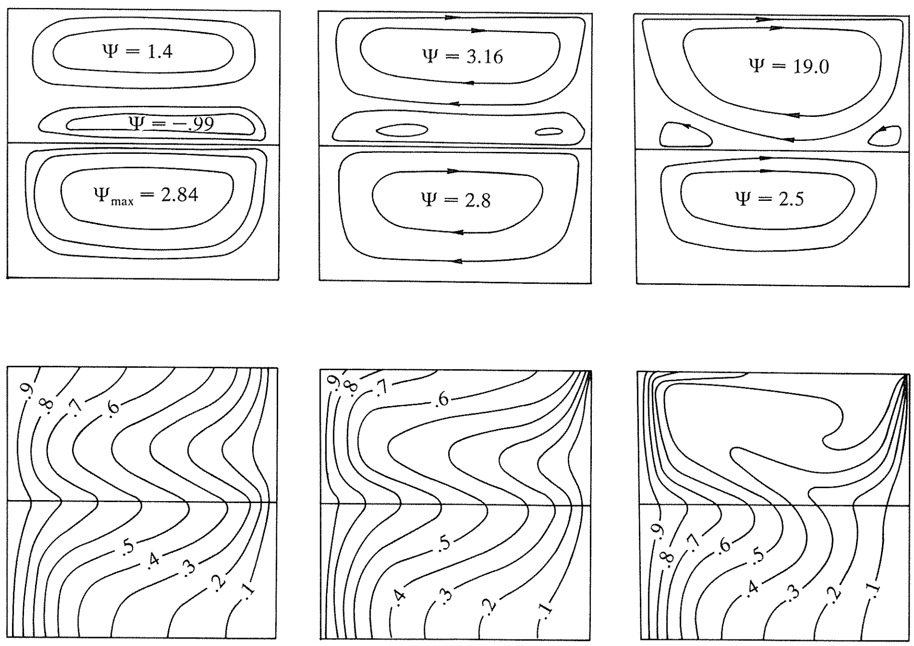
It is found out that for the given set of parameters the small secondary recirculation moves to the upper layer when the weak Marangoni force appears at the interface between liquids. It was observed for the $Ma_1 = 10^2$; this picture is not presented here. With increasing Marangoni force the counter clockwise cell grows up and the positively-directed cell is compressed to have decreasing intensity. Fig.2-b corresponds to the case $Ma_1 = 10^3$, $Ma_2 = 0$ and Gr $= 10^4$. As seen in the isotherm picture, Marangoni force enlarges the heat transfer along the interface and reduces it in the upper part of the system. When Marangoni number Ma_1 is 10^4, the convection in the layered system is dominated by thermocapillary force. Fig.2-c corresponds to the case $Ma_1 = 10^4$, Gr $= 10^4$ and $Ma_2 = 0$. The motion pattern becomes almost symmetrical with respect to the plane of interface. The large oppositely-directed cells which have approximately the same strength are formed in the system. At the upper corners there are two small positive cells that are the result of the buoyancy force and the difference of the physical properties of liquids. The maximum velocity at the interface is two-orders-of-magnitude larger than that in the case 2-a. The isotherm picture also shows the dominating effect of Marangoni convection. The heat from the hot side is quickly transported by convective flux along the interface and it creates a "local" storage of the heat near the center of the cold wall. The central part of the system $X \simeq 0.5$ has an approximately uniform temperature $0.4 < T < 0.5$. It may be mentioned that a stable flow picture is formed in a short time, but as far as the thermal field is concerned, the achievement of equality between the average Nusselt numbers at hot and cold walls demands enough time.

The next step of this paper is to examine the influence of Marangoni force at the upper surface.

First a zero-shear surface ($U_y = 0$) is treated where there is no Marangoni force at the upper boundary. Fig.3-a corresponds to an open cavity where $Ma_1 = 10^3$, $Ma_2 = 0$ and $Gr = 10^4$. The flow and temperature fields for this and 2-b cases look similar and only a small difference is noticed in the upper layer. The intensity of the primary cell is approximately 40% greater than the one in the case 2-b. In the stream function picture, the boundary between the cells is moved slightly upwards.

In the open cavity case the isotherms are slightly more shifted toward the cold wall in comparison with the case 2-b.

Next calculation is done taking account of the surface tension effect at the upper boundary. Fig.3-c corresponds to the case $Ma_1 = 10^3$, $Ma_2 = 10^3$ and $Gr = 10^4$. In this case, the free-surface Marangoni force enlarges the size of the upper primary cell and its intensity.



a b c

Fig.3

This circulation occupies more than half of the layer. With regard to the secondary cell the strength of motion becomes a little (10—15%) weaker. The presence of the interface Marangoni force Ma_1 protects the heavier liquid from the influence of the upper capillary force Ma_2, and as a result the intensity of motion in liquid 1 does not change essentially. The strong convective cell in the liquid 2 draws the isotherms away from the hot wall and tends to bunch them together into the above-right corner.

By further increasing Ma_2 to 10^4, Fig.3-c shows that there is a high-intensity vortex in the lighter liquid and the interface capillary force now does not save the motion in the lower fluid. It is seen from the isotherm structure that the temperature distribution changes drastically. In all the previous cases the process of flowfield development from motionless to the steady state is similar; first, one or two vortices in every layer, not more than three altogether appear near the hot wall, and then they grow and fill the full space. In this case the process is very quick and two vortices are formed at every layer and subsequently the vortex with negative circulation in the lower liquid is suppressed. When the free surface capillary force is essentially stronger than that one, the isothermal core exists in the lighter liquid.

The problem of convection in a layered system was investigated in the paper [2], using another numerical method and the variables U, V, P. There is a qualitative similarity between the results. As for the direct comparison, the result of the case 3-b is completely similar to his case 17, but interestingly they correspond to the different values of interface Marangoni number Ma_1.

The physical quantities of interest determining the system heat transfer are the local and average Nusselt numbers which are given by the following expressions:

Local Nusselt number $Nu = -(\partial T/\partial X)_{x=o}$,

average Nusselt number $Nu = \int Nu\, dx$.

The local Nusselt number distribution for the hot wall is shown in Fig.4. Curves 1-6 correspond to the cases 2-a, 2-b, 2-c, 3-a, 3-b, 3-c. For the enclosure problem (Curve 1), the maximum local heat transfer rate is located near the bottom of the system. With appearance

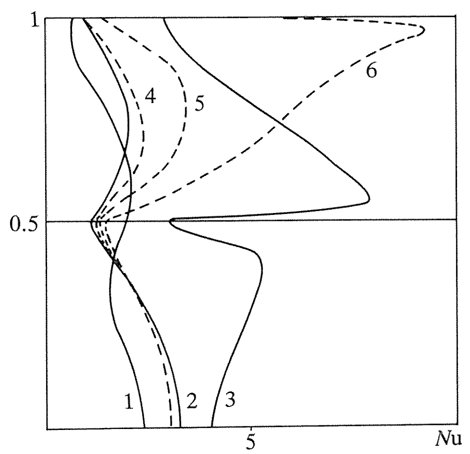


Fig.4

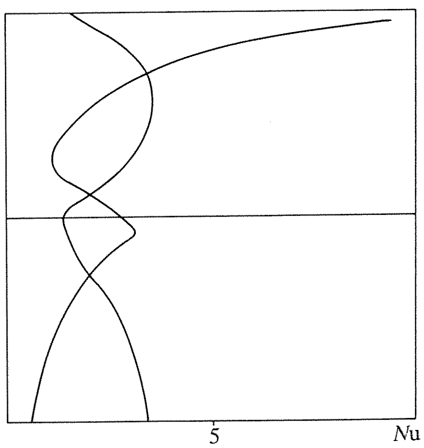


Fig.5

of the weak interface force (Curve 2), the curve shape is changed. The additional minimum in heat transfer rate appears in the vicinity of interface. As was mentioned, the motion in an open cavity (3-a) does not change too much in comparison with the enclosure (2-b). Curves 2 and 4 have practically the same shape.

By adding Maragoni force to the free surface (Curve 5), the heat exchange becomes more active only in the upper layer. In the case 2-c (Curve 3) the strong Marangoni convection at the interface creates flowfield and causes significant heat transfer enhancement near the liquids boundary.

In the case of the dominant free-surface capillary force (Curve 6), the maximum velocity takes place near the upper boundary. This intense flow causes local maximum in Nu near the surface. The heat transfer rate in the heavier liquid is reduced due to the secondary cell.

Fig.5 compares the distribution of Nusselt number between the hot and cold walls for the case 3-b.

3. CONCLUSIONS

The thermal buoyancy and capillary convection in the system of two immiscible fluids have been numerically investigated. The results of calculation show that the existence of capillary force changes the flowfield drastically. In the case of dominant interface capillary force in both liquids, there is a strong counter-directed motion. In the case of dominant Marangoni force a strong motion exists only in the upper liquid.

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