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A KINETIC ANALYSIS OF HEAT TRANSFER BETWEEN TWO PARALLEL PLATES IN POLYATOMIC GAS

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Abstract

Heat transfer between two parallel plates in polyatomic gas was studied based on a linearized version of the Wang-Chang, Uhlenback, and de Boer equation with Gross-Jackson's type approximation. The analysis was carried out using phenomenological accommodation coefficients of translational and internal energies. Results showed that the net rate of heat transfer between two plates was less dependent upon such coefficients. On the otherhand distributions of number density and temperatures were dependent upon those coefficients. A comparison with the experiments using the electron beam fluorescence technique suggested that the accommodation coefficients for the rotational energy of nitrogen might be much less than unity.

1. Introduction

This paper is concerned with the one-dimensional heat transfer problem between two parallel plates in polyatomic gas. This study was motivated from the question whether or not the Kassem and Hickmann's postulation[1] is correct. They carried out experiments of heat transfer in polyatomic gas (nitrogen gas) in the transition regime and found that the gradient of the rotational temperature was much larger than the gradient of the translational temperature (in fact the negative gradient of number density). That is, the experimental results showed that a slip of the rotational temperature was much less than the one of the number density. This fact, they postulated, suggested that the rotational temperature accommodates itself completely to the temperature at the surface. But such postulation might be contradict to the conventional theoretical predictions[2),3)] of accommodation coefficients of internal energies.

Since the energy accommodation coefficients are substantial parameters for the evaluation of the thermal force[4)] on a small particle with high thermal conductivity, the Kassem and Hickman's postulation should be critically examined. This report presents a theoretical study of this problem based on the Gross-Jackson's model equation[5] to the Wang-Chang, Uhlenbeck, and de Boer (WCUB) equation,[6] which retains 17 moments of the WCUB equation correct. It is obvious that the previous studies[7),8] based on the Morse model equation[9] is inadequate because this model equation yielded incorrect Euken factors[6] for polyatomic gases. The effects of the internal energies, however, will be investigated introducing a macroscopic collision number Z_{in}[10] which is required to establish a thermal equilibrium between the translational mode and the internal mode. Boundary conditions for emitted molecules will be described using phenomenological accommodation coefficients.[11] A method of solution of the kinetic model equation used in this report will be found in the previous papers.[12]

2. Formulation of the Problem

Consider a stationary gas at a constant pressure p_0 and a temperature T_0 between two parallel plates at $x=0$ and $x=L$. The two plates are maintained at constant but slightly different temperatures $T_0+\frac{1}{2}\varDelta T$ and $T_0-\frac{1}{2}\varDelta T$. Let us $f^i(x, v, \epsilon_i)$ be the probability density (distribution function) of gas molecules in the ith internal energy state having velocity between v and $v+dv$. The WCUB equation is given by

$$v_x\frac{\partial f^i}{\partial x}=\sum_{j,k,\ell}\int\int(f'^{\ell}f'^{k}-f^i f^j)\,gI_{ij}^{k\ell}\,d\Omega\,d\mathbf{v} \tag{1}$$

where the primes refer to the distribution function after a two body collision. The scattering cross section, $I_{ij}^{k\ell}$ contains transition probabilities from the ith to kth and from jth to ℓth states. The solid angle of scattering is $d\Omega$ and the integral is summed over all states except i.

Macroscopic moments, the density n, flow velocity $\mathbf{u}$; $\mathbf{u}=(u,0,0)$, translational temperature T_{tr}, internal energy $\bar{E}$, and temperature T are given by

$$\begin{bmatrix} n \\ u \\ T_{tr} \\ \bar{E}\end{bmatrix}=\sum_i\int\int\int_{-\infty}^{\infty}\begin{bmatrix}1 \\ v_x \\ \frac{(\mathbf{v}-\mathbf{u})^2}{3nk} \\ \epsilon_i\end{bmatrix}f^i\,d\mathbf{v}, \tag{2}$$

$$T=\frac{\frac{3}{2}\frac{k}{m}T_{tr}+c_{vin}\bar{E}}{c_v}, \tag{3}$$

where k is the Boltzmann constant, m the mass of molecule, c_{vtr} the specific heat of the translational energy, c_{vin} the specific heats of the internal energy, and $c_v=c_{vtr}+c_{vin}$.

Using knowledges of moment equations of the WCUB equation,[13),14] a kinetic model equation that retains 17 moments of Eq. (1) correct is yielded by

$$\mathbf{v}\frac{\partial f^i}{\partial \mathbf{x}}=\beta[f_0^i(1+\Phi^i)-f^i] \tag{4}$$

where

$$f_0^i=\frac{n_i\exp(-E_i/kT)}{\Sigma_i\exp(-E_i/kT)}(2\pi RT)^{-\frac{3}{2}}\exp[-\frac{(\mathbf{v}-\mathbf{u})^2}{2RT}],$$

$$\Phi^i=[\tfrac{2}{3}(\frac{2nJ_1}{\beta})-\frac{c_{vin}}{c_v}](\frac{T_{tr}}{T}-\frac{T_{in}}{T})(\tfrac{3}{2}-\frac{c^2}{2RT})+\tfrac{3}{2}\frac{R}{c_{vin}}(\frac{E_i}{kT}-\frac{\bar{E}}{kT})$$

$$+(1-\frac{8nJ_4}{5\beta})p_{mn}(\frac{c_m c_n}{2RT}-\frac{1}{3}\frac{c^2}{2RT}\delta_{mn})$$

$$+(1-\frac{16nJ_5}{15\beta})\frac{4}{5}q_m^{tr}\frac{c_m}{\sqrt{2RT}}(\frac{c^2}{2RT}-\frac{5}{2})$$

$$+(1-\frac{R}{c_{vin}}\frac{2nJ_6}{3\beta})\frac{2R}{c_{vin}}q_m^{in}\frac{c_m}{\sqrt{2RT}}(\frac{E_i}{kT}-\frac{\bar{E}}{kT})$$

$$+(\frac{2nJ_7}{3\beta})\frac{2R}{c_{vin}}q_m^{in}\frac{c_m}{\sqrt{2RT}}(\frac{E_i}{kT}-\frac{\bar{E}}{kT})$$

$$+(\frac{5nJ_2}{3\beta})\frac{R}{c_{vin}}\frac{4}{5}q_m^{in}\frac{c_m}{\sqrt{2RT}}(\frac{c^2}{2RT}-\frac{5}{2})$$

where R is the gas constant, $R = k/m$. The dimensionless shear stress p_{mn} and the heat fluxes, q_m^{tr} and q_m^{in} are given by

$$p_{mn}=\frac{1}{mn}\sum_i\int\int\int_{-\infty}^{\infty}\frac{c_m c_n}{2RT}f^i\,d\mathbf{c},$$

$$q_m^{tr}=\frac{1}{mn}\sum_i\int\int\int_{-\infty}^{\infty}\frac{c_m}{\sqrt{2RT}}(\frac{c^2}{2RT}-\frac{5}{2})f^i\,d\mathbf{c},$$

$$q_m^{in}=\frac{1}{mn}\sum_i\int\int\int_{-\infty}^{\infty}\frac{c_m}{\sqrt{2RT}}(\frac{E_i}{kT}-\frac{\bar{E}}{kT})f^i\,d\mathbf{c}.$$

The parameters J_1 to J_7 denote collision integrals defined in Mason and Monchick[15] and are expressed in terms of conventional collision integrals $\Omega^{(2,2)}$, $\Omega^{(1,1)}$, and $\Omega^{(\tau)}$ [6),16)] as

$$J_1=J_2=J_7=\Omega^{(\tau)},\quad J_3=J_4-\tfrac{5}{6}J_1,\quad J_4=\Omega^{(2,2)},$$

$$J_5=\Omega^{(2,2)}-\tfrac{25}{24}\Omega^{(\tau)},\quad J_6=4(\frac{c_{vin}}{R})\Omega^{(1,1)}+\tfrac{3}{2}\Omega^{(\tau)},$$

where $\Omega^{(1,1)}$ and $\Omega^{(2,2)}$ are related to the shear viscosity μ and the self diffusion coefficient D,

$$\mu_1 = \frac{5kT}{8\Omega^{(2,2)}}, \quad mnD_1 = \frac{3kT}{8\Omega^{(1,1)}},$$

while $\Omega^{(\tau)}$ is related to the bulk viscosity κ

$$\kappa_1 = \left(\frac{kT}{2\Omega^{(\tau)}}\right)\left(\frac{c_{vin}}{c_v}\right)^2,$$

where the subscript 1 denotes the first approximation of the Chapmann-Enskog solution.[16)]

Let introduce a conventional collision number Z_{in}[10)] defined by

$$Z_{in} = 4p\tau / \pi\mu \tag{5}$$

where the acoustic relaxation time τ is given by

$$\tau^{-1} = \frac{4}{3}\left(\frac{c_{vtr}}{c_{vin}}\right) J_1,$$

A relaxation time required for inelastic collisions may be given by

$$\tau_{in}^{-1} = \left(\frac{c_v}{c_{vin}}\right)\left(\frac{4}{3} n J_1\right).$$

Since the total collision number ν is given by $\nu = p/\mu$, the ratio of the inelastic collision $\nu_{in} (= 1/\tau_{in})$ to the total collision ν is given by

$$\delta = \frac{\nu_{in}}{\nu} = \frac{5}{6} \frac{c_v}{c_{vin}} \frac{J_1}{J_4}$$

Then Eq. (5) is reduced to

$$Z_{in} = \frac{4}{\pi}\left(\frac{c_v}{c_{vtr}}\right)\delta^{-1}. \tag{6}$$

A distribution function of the emitted molecules with a velocity $\mathbf{c}(= \mathbf{v}/\sqrt{2RT})$ and with an internal energy state E_i is given by

$$c_x f^{i+}(\mathbf{c}, E_i) = \sum_{i'} \int_{c'_x < 0} |c'_x| f^{i'-}(\mathbf{c}', E_{i'}) W(\mathbf{c}', E_{i'}, \mathbf{c}, E_i) \, d\mathbf{c}' \tag{7}$$

in terms of the distribution function of the incident molecules $f^{i'-}$ where $W(\mathbf{c}', E_{i'}, \mathbf{c}, E_i)$ represents a differential reflection probability that a molecule of an incident velocity $\mathbf{c}'$ and an internal state $E_{i'}$ is reflected into the velocity space element $d\mathbf{c}$ in the vicinity of $\mathbf{c}$ and in the internal energy state E_i. If we assume a simple (Maxwell type) differential reflection probability in analogus to the monatomic gas case,[17]

$$W(\mathbf{c}', E_{i'}, \mathbf{c}, E_i) = [(1-\alpha_t)\,\delta(\mathbf{c}' - \mathbf{c}_r) + \alpha_t \frac{2}{\pi}\exp(-\mathbf{c}^2)]$$

$$\times [(1-\alpha_i)\,\delta(E_{i'} - E_i) + \alpha_i \frac{\exp(-E_i)}{Q_{in}(T)}],$$

where $\mathbf{c}_r = \mathbf{c} - 2\mathbf{n}\mathbf{c}_x$, $Q_{in}(T)$ the partition function, α_t the accommodation coefficient of the translational energy, α_i the accommodation coefficient of the internal energy, $\delta(x)$ the Dirac's delta function, and $\mathbf{n}$ the unit vector normal (outward) to the surface, Eq. (7) yields

$$f^{i+}(\mathbf{c}, E_i) = (1-\alpha_t)(1-\alpha_i) f^{i-}(\mathbf{c}_r, E_i) + (1-\alpha_t)\,\alpha_i \frac{\exp(-E_i)}{Q_{in}(T)}$$

$$\times \sum_{i'} f^{i'-}(\mathbf{c}_r, E_{i'}) + \alpha_t(1-\alpha_i)\frac{2}{\pi}\exp(-\mathbf{c}^2)\int_{c_x'<0} |c_x'|\, f^{i-}(\mathbf{c}', E_i)\, d\mathbf{c}'$$

$$+ \alpha_t \alpha_i \frac{2}{\pi}\exp(-\mathbf{c}^2)\frac{\exp(-E_i)}{Q_{in}(T)} \sum_{i'} \int_{c_x'<0} |c_x'|\, f^{i'-}(\mathbf{c}', E_{i'})\, d\mathbf{c}'. \qquad (8)$$

The energy accommodation coefficients are defined by

$$\alpha_t = \frac{E_t^- - E_t^+}{E_t^- - E_t^w}, \quad \alpha_i = \frac{E_i^- - E_i^+}{E_i^- - E_i^w},$$

where E^+ and E^- denote fluxes of energy of the emitted and impinged molecules, whereas E^w refers to the value when gas and the wall are in equilibrium. Detailed discussion on the accommodation coefficients were given by Kuscer.[11]

Outside the Knudsen layer the distribution function may approach the 17 moment solution to the WCUB equation and in one-dimensional problem the solution is given by

$$f^i = f_0^i\{1 - \frac{\kappa}{nkT}\frac{du}{dx}[(c^2 - \frac{3}{2}) + \frac{3}{2}(E_i - 1)] - \frac{4}{3}\frac{\mu}{nkT}\frac{du}{dx}(c_x^2 - \frac{1}{3}c^2)$$

$$- \frac{c_x}{nkT\sqrt{2RT}}[\frac{4}{5}\lambda_i(c^2 - \frac{5}{2}) + 2\lambda_i(E_i - 1)]\frac{dT}{dx}\}. \qquad (9)$$

where

$$f_0^i = n_0(2\pi RT_0)^{-\frac{3}{2}}\exp(-c^2)\frac{\exp(-E_i)}{Q_{in}(T_0)}.$$

3. Method of Solution

When the temperature difference between two plates is small, $\Delta T/T \ll 1$, Eq. (4) may be rewritten in a linearized version of it. Conveniently, we introduce the reduced distribution functions ϕ, θ, and ψ defined by

$$\begin{bmatrix} \phi(x, c_x) \\ \theta(x, c_x) \\ \psi(x, c_x) \end{bmatrix} = \sum_i \pi^{-1} \iint_{-\infty}^{\infty} \begin{bmatrix} 1 \\ c_y^2 + c_z^2 - 1 \\ E_i - 1 \end{bmatrix} \phi^i dc_y dc_z , \tag{10}$$

where $f^i = f_0^i(1 + \phi^i)$. Multiplying Eq. (4) by the weighting functions $1, 2RT(c_y^2 + c_z^2 - 1)$, and $RT(E_i - 1)$, integrating the results with respect c_y and c_z, and summing up them with respect to i in accordance with Eq. (10), we obtain

$$\begin{aligned} c_x \frac{\partial \Phi}{\partial x} &= \Phi_1 N + \Phi_2 U + (\Phi_{3t} + \frac{c_{vin}}{R} \Phi_{3r})\, t \\ &+ (1-\delta) \frac{c_{vin}}{c_v} (t_{tr} - t_{in}) (\Phi_{5t} + \Phi_{5r}) + \frac{4}{15} (1 - \frac{5}{2}\delta)\, q_x^{tr} \Phi_{4t} \\ &+ 2(1 - \frac{1}{2}\delta - \frac{5}{6A^*})\, q_x^{in} \Phi_{4r} - \Phi , \end{aligned} \tag{11}$$

where

$$\Phi = (\phi, \theta, \psi)^t, \quad \Phi_1 = (1, 0, 0)^t, \quad \Phi_2 = (2c_x, 0, 0)^t,$$

$$\Phi_{3t} = (c_x^2 - \frac{1}{2}, 1, 0)^t, \quad \Phi_{3r} = (0, 0, 1)^t, \quad \Phi_{4t} = (c_x^3 - \frac{3}{2} c_x, c_x, 0)^t,$$

$$\Phi_{4in} = (0, 0, c_x)^t, \quad \Phi_{5t} = \Phi_{3t}, \quad \Phi_{5in} = (0, 0, -\frac{3}{2})^t,$$

and the superscript t implies the transpose of a vector or a matrix. In Eq. (11) the distance x is nondimensionalized by a reduced mean free path ℓ_0

$$\ell_0 = \frac{\mu_0}{m n_0} \sqrt{\frac{2}{RT_0}} = \frac{\sqrt{2RT_0}}{\beta} .$$

The perturbed variables are defined by

$$\begin{aligned} &n = n_0(1 + N), \quad u = \sqrt{2RT_0}\, U, \quad T_{tr} = T_0(1 + t_{tr}), \\ &T_{in} = T_0(1 + t_{in}), \quad T = T_0(1 + t), \end{aligned} \tag{12}$$

while the energy conservation equation yields

$$q_x = q_x^{tr} + q_x^{in} = \text{constant}.$$

In terms of the reduced distribution functions these perturbed variables are expressed as

$$N = (1, \phi), \quad U = (c_x, \phi), \quad t_{tr} = \frac{2}{3}[(c_x^2 - \frac{1}{2}, \phi) + (1, \theta)],$$

$$t_{in} = (1, \psi), \quad t = c_v^{-1}(c_{vtr} t_{tr} + c_{vin} t_{in}),$$

$$q_{tr} = (c_x^3 - \frac{3}{2} c_x, \phi) + (c_x, \theta), \quad q_{in} = (c_x, \psi). \tag{13}$$

In Eq. (11) the following abbreviation is used

$$(A, B) = \pi^{-\frac{1}{2}} \int_{-\infty}^{\infty} AB \exp(-c_x^2)\, dc_x .$$

The boundary conditions (8) for $x = \pm\frac{L}{2}$ and (9) yield

$$\Phi^{\pm}(\mp\frac{L}{2}, c_x 0) = (1 - \alpha_t) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 - \alpha_i \end{bmatrix} \Phi^{\mp}(\mp\frac{L}{2}, -c_x)$$

$$-\Phi^{*\pm} \mp \Delta T \alpha_t \begin{bmatrix} c_x^2 - 1 \\ 1 \\ 0 \end{bmatrix} \mp \Delta T \alpha_i \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \tag{14}$$

$$\Phi^{*\pm} = 2\alpha_t \int_{\mp\infty}^{0} c_x [\Phi_1 \phi^{\mp}(\mp\frac{L}{2}) + (1 - \alpha_i)\Phi_{3in} \psi^{\mp}(\mp\frac{L}{2})] \exp(-c_x^2)\, dc_x ,$$

$$\Phi(\infty) = N_0 \Phi_1 + t_0 (\Phi_{3tr} + \frac{c_{vin}}{R}\Phi_{3in}) + (\frac{dt}{dx})_\infty x (\Phi_{3tr} - \Phi_1 + \frac{c_{vin}}{R}\Phi_{3in})$$

$$+ \frac{4}{5} q_x^{tr} \Phi_{1tr} + 2 q_x^{in} \Phi_{1in} . \tag{15}$$

In the Knudsen layer adjacent to the wall surface gas is not in thermal equilibrium in spite of the collisions of particles emitting from the surface with those impinging on to the surface. The distribution function Φ is accordingly divided into two parts, $\Phi^{\pm} = \Phi(c_x, 0)$ and these half-range distribution functions may be expanded using the half-range Hermite polynomials $H_\xi(\eta)$ [8] where $\eta=|c_x|$:

$$\Phi^{\pm}=\sum_{\xi=0}^{n}\begin{bmatrix} a_\xi^{\pm} \\ b_\xi^{\pm} \\ c_\xi^{\pm} \end{bmatrix} \quad ; \quad \begin{bmatrix} a_\xi^{\pm} \\ b_\xi^{\pm} \\ c_\xi^{\pm} \end{bmatrix}=\int_0^\infty H_\xi \Phi^{\pm}\exp(-\eta^2)\,d\eta, \tag{16}$$

Substituting the expansion form (16) into Eq. (11) and using the orthonormal relation of H_ξ [8], we obtain a set of differential equations of the coefficients $a_\xi^{\pm}$, $b_\xi^{\pm}$, and $c_\xi^{\pm}$

$$\frac{dX}{dx}=\Gamma X;\; X=\begin{bmatrix} X^+ \\ X^- \end{bmatrix}, \tag{17}$$

where the vectors $X^{\pm}$ are defined by

$$X^{\pm}=(a_1^{\pm},\ a_2^{\pm},\ \cdots,\ a_n^{\pm},\ b_1^{\pm},\ b_2^{\pm},\ \cdots,\ b_n^{\pm},\ c_1^{\pm},\ c_2^{\pm},\ \cdots,\ c_n^{\pm})^t,$$

and Γ is the matrix of order 6n when the first n terms of the expansions (16) are retained; the components of the matrix Γ can be obtained from the vectorial form of the righthand side of Eq. (11). Hereafter the superscript $+$ and $-$ denote respectively the upper and the lower halves of a vector or a matrix. In terms of the vector X, Eq. (11) is rewritten as

$$N=X_1\cdot X,\quad U=X_2\cdot X,\quad t_{tr}=\frac{2}{3}X_{3tr}\cdot X,$$

$$t_{in}=X_{3in}\cdot X,\quad q_{tr}=X_{4tr}\cdot X,\quad q_{in}=X_{4in}\cdot X, \tag{18}$$

where X_i is a constant vector (See Appendix).

A general solution of Eq. (17) is given as

$$\mathbf{X}=\sum_{\xi=1}^{3n-2}[P_\xi^p\exp(\lambda_\xi x)\mathbf{U}_\xi^p+P_\xi^n\exp(-\lambda_\xi x)\mathbf{U}_\xi^n]+\mathbf{X}_f \tag{19}$$

where $\lambda_\xi(>0)$ is the eigenvalue of the characteristic equation

$$[\Gamma-\lambda\mathbf{I}]=\lambda^4\prod_{\xi=1}^{3n-2}(\lambda^2-\lambda_\xi^2)=0, \tag{20}$$

where $\mathbf{I}$ is the unit matrix, $\mathbf{U}_\xi$ the eigenvector corresponding to the eigenvalue $\lambda_\xi(\neq 0)$, and $\mathbf{X}_f$ denotes the fluid dynamic solution corresponding to the eigenvalue $\lambda_\xi=0$, i.e., the 17 moment solution of Eq. (17) is given by

$$\mathbf{X}_f=N_0\mathbf{X}_1+t_0\mathbf{X}_3+\bar{Q}_\infty\mathbf{X}_4+(\mathbf{X}_1-\mathbf{X}_3)\bar{Q}_\infty x, \tag{21}$$

where $\mathbf{X}_3$ and $\mathbf{X}_4$ are constant vectors (See Appendix). Arbitrary parameters P_k^p and P_k^n as well as N_0, t_0, and $\bar{Q}_\infty$ will be determined from the boundary conditions.

The boundary condition (12) can be rewritten in terms of vectors as

$$\bar{\mathbf{X}}^{\pm}(\mp\frac{L}{2}) = \bar{W}\bar{\mathbf{X}} + 2\alpha_t \mathbf{X}_1^{+}(\mathbf{X}_2^{+})^t \bar{\mathbf{X}}^{\mp}, \tag{22}$$

where

$$\bar{W} = (1-\alpha_t)\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1-\alpha_i \end{bmatrix} + 2\alpha_t(1-\alpha_i)\mathbf{X}_{3in}^{+}(\mathbf{X}_{4in}^{+})^t,$$

$$\bar{\mathbf{X}}^{\pm} = \mathbf{X}^{\pm} \mp \frac{1}{2}\Delta T\mathbf{X}_3^{\pm}.$$

The second term in the righthand side of Eq. (22) is the vectorial form of

$$N_\omega(\mp\frac{L}{2}) = 2\alpha_t[\pm\int_{\mp\infty}^{0} |c_x|\,\phi^{\mp}\exp(-c_x^2)\,dc_x \mp \frac{1}{4}\Delta T],$$

that is, the number density of molecules which suffer diffuse reflection.

Since $t(0) = N(0) = 0$, we find $N_0 = t_0 = 0$. If $L/\ell_s \gg 1$, $-\bar{Q}_\infty$ denotes the temperature gradient. Imposing the boundary conditions (22) on the solution (19), we can determine the values of all unknowns, P_ξ^p, P_ξ^n ($k = 1, 3n-2$), and $\bar{Q}_\infty$. The antisymmetric nature of the problem results in $P_\xi^p/P_\xi^n = \pm 1$ where the positive or negative sign depends on the sign of the ratio $\mathbf{U}_\xi^p/\mathbf{U}_\xi^n$. The temperature slip coefficient d_t and the slip coefficient of number density d_n at $x = -L/2$ are formally obtained as

$$d_t\bar{Q}_\infty = -\frac{1}{2}\bar{Q}_\infty L - \frac{1}{2}\Delta T, \quad d_n\bar{Q}_\infty = \frac{1}{2}\bar{Q}_\infty L - N_\omega,$$

which yields

$$\bar{Q}_\infty = \frac{-\Delta T}{L + 2d_t}, \quad N_\omega = \frac{1}{2}\bar{Q}_\infty L - d_n\bar{Q}_\infty.$$

If $L/\ell_s \gg 1$, d_t and d_n yield the conventional slip coefficients (See Table 1). Since the distribution functions is thus determine, we can evaluate any necessary moments of the distribution function.

Table 1. Slip coefficients for diatomic gas; A* = 1.095, s = 2.

	Slip coefficients			
α_i	d_n	d_t	d_n	d_t
	(Zr = 3.0, α_t =1.0)		(Zr = 10.0, α_t = 1.0)	
1.0	0.99377	−1.75759	0.96767	−1.75200
0.8	0.96743	−1.72387	0.92940	−1.70479
0.6	0.94357	−1.69333	0.89280	−1.65912
0.4	0.92186	−1.66553	0.85672	−1.61491
0.2	0.90201	−1.64013	0.82206	−1.57208
0.0	0.88380	−1.61681	0.78847	−1.53057
	(Zr = 3.0, α_t =0.7)		(Zr = 10.0, α_t = 0.7)	
1.0	1.91733	−3.42206	1.89293	−3.41247
0.8	1.76885	−3.20950	1.73908	−3.20208
0.6	1.64268	−3.02859	1.59684	−3.00745
0.4	1.53402	−2.87251	1.46474	−2.82659
0.2	1.43935	−2.73625	1.34153	−2.65780
0.0	1.35601	−2.61608	1.22617	−2.49964

4. Results and Discussion

In the previous section we obtain the analytical solution of the one-dimensional heat transfer problem in polyatomic gases. The results depend on the five parameters, s, $Kn(=\ell_s/L)$, Z_{in}, α_t, and α_i, where s denotes the internal degree of freedom. If Kn tends to zero, the present problem reduced to the problem to evaluate the slips (jumps) of temperatures. As shown in Table 1 values of slip coefficients are less dependent on the collision number Z_{in} as long as α_i is close to unity: As $Z_{in} \to \infty$, for instance, $d_n \to 0.9651$ and $d_t \to -1.7688$ for $s = 2$ and $\alpha_t = \alpha_i = 1$. Even if α_i is close to zero, effects of Z_{in} on the slip coefficients are not so significant. As α_i decreases, values of the slip coefficients decrease. This trend is enforced as α_i decreases. The values of slip coefficients increase significantly as α_t decrease. In Fig. 1 the slip coefficients are shown against s for the case when $Z_{in} = 2.122$, $A* = 1.0$, and $\alpha_t = \alpha_i = 1.0$. The slip coefficients increase gradually as s increases. The results showed that the values of slip coefficients slightly depend on the value of $A*$. For the Lenard-Johns potential $A* = 1.095$ is a good estimations.[18]

As aforementioned Kassem and Hickmann measured rotational temperature and number density of nitrogen gas between two parallel plates with different temperature. Their results of number density measurements showed a good agreement with the results of previous works.[19] Their results are, however, convenient because they measured number density as well as rotational temperature: They assumed that under a constant pressure a reciprocal of number density yielded translational temperature. In Fig. 2 theoretical predictions of dimensionless heat flux in the transition regime are shown against the reciprocal of the Kundsen number. It is found that the heat flux is weakly dependent upon the accommodation

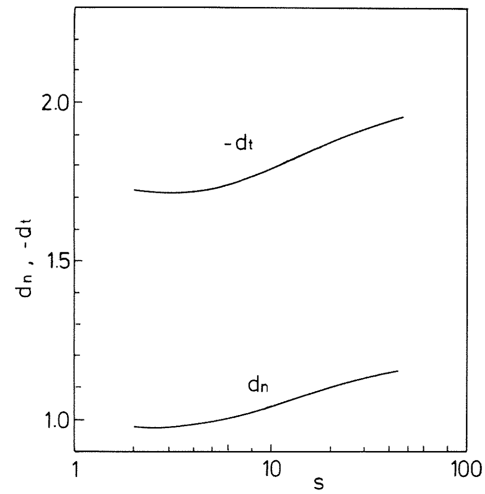

Fig. 1. Dependence of slip coefficients on the internal degree of freedom.

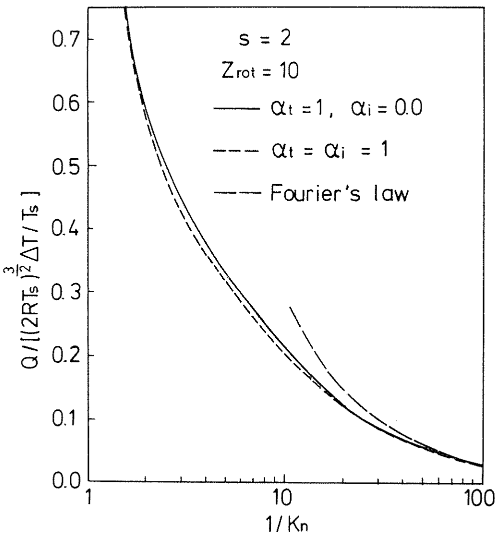

Fig. 2. Heat flux in the transition regime.

coefficient α_i. The dependence of the heat flux on α_i is shown in Fig. 3 for the case when $Kn = 0.19$. Distributions of rotational temperature and number density are shown in Fig. 4 (see

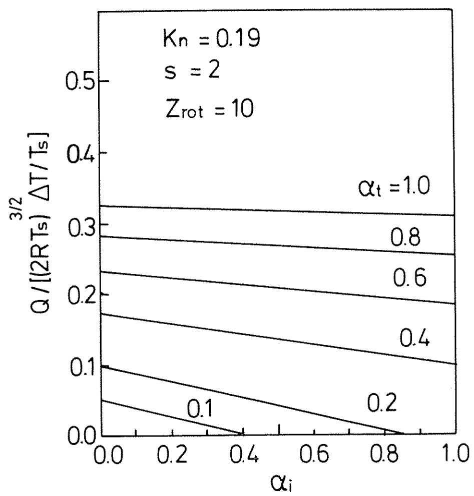

Fig. 3. Dependence of Heat flux on the accommodation coefficients: Kn = 0.19, s = 2, $Z_{in} = 10$, $A^* = 1.095$.

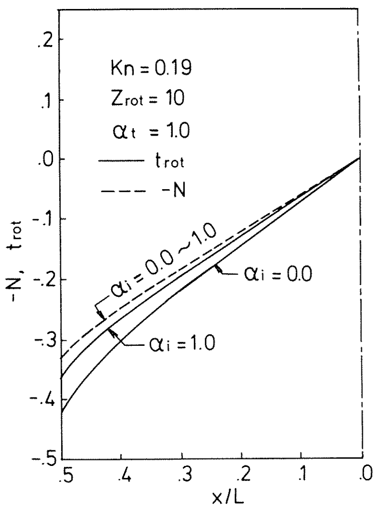

Fig. 4. Distributions of number density and rotational temperature: Kn = 0.19, s = 2, $Z_{in} = 10$, $A^* = 1.095$, $\alpha_t = 1.0$.

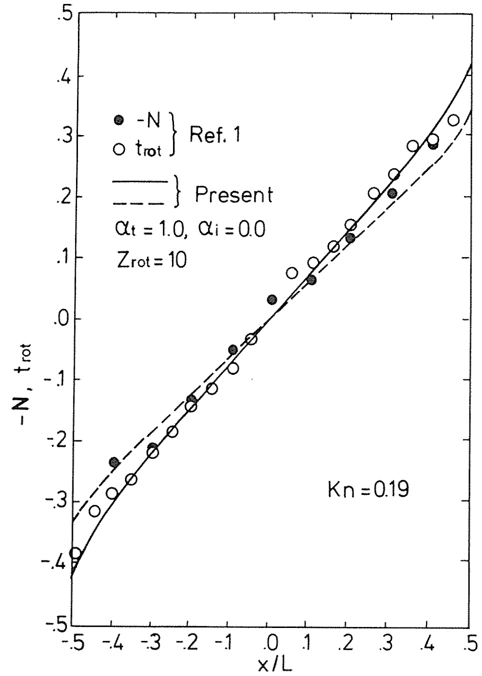

Fig. 5. Distributions of number density and rotational temperature for Kn = 0.19.

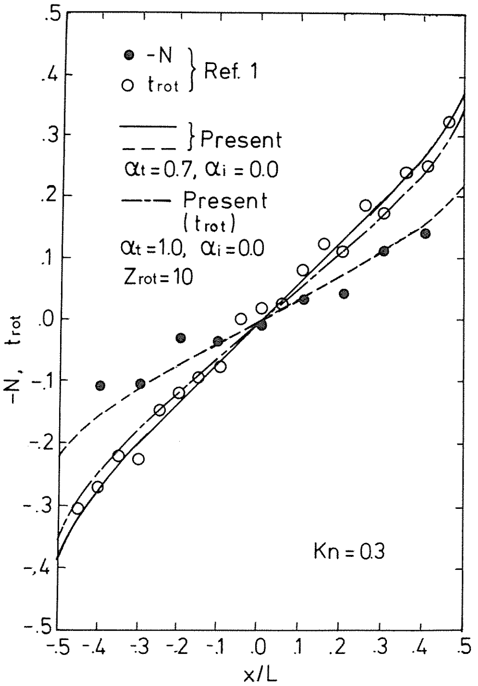

Fig. 6. Distributions of number density and rotational temperature for Kn = 0.3.

5. Appendix

The constant vectors $\mathbf{X}_1$, $\mathbf{X}_2$, $\mathbf{X}_{3tr}$, $\mathbf{X}_{3in}$, $\mathbf{X}_{4tr}$, and $\mathbf{X}_{4in}$ are the vectorial forms of Φ_1, Φ_2, Φ_{3t}, Φ_{3in}, $c_x(\Phi_{3t} - \Phi_1)$, and $c_x\Phi_{3in}$, respectively. The vectors $\mathbf{X}_3$ and $\mathbf{X}_4$ are given by

$$\mathbf{X}_3 = \mathbf{X}_{3tr} + \left(\frac{c_{vin}}{R}\right)^{\frac{1}{2}},$$

$$\mathbf{X}_4 = \left(\frac{2}{5}\frac{c_{vtr}}{R}f_t\right)^{\frac{1}{2}}\mathbf{X}_{4tr} + \left(\frac{C_{vin}}{R}f_i\right)^{\frac{1}{2}}\mathbf{X}_{4in},$$

where the Eucken factors are given by

$$f_t = \frac{2}{5}\left[1 - \frac{5}{4}\frac{c_{vin}}{c_v}\left(1 - \frac{12A^*}{25}\right)\delta\right],$$

$$f_i = \frac{6A^*}{5}\left[1 + \frac{c_{vtr}}{c_v}\left(\frac{5}{4} - \frac{3A^*}{5}\right)\delta\right].$$

also Table 2) for the same case as Fig. 3. In accordance with the change of slip coefficients in Table 1 the distribution of rotational temperature deviates downward as α_i decreases. In Figs. 5 and 6 present results are compared with the Kassem and Hickman's results. The present results for $\alpha_t = 1$ and $\alpha_i = 0$ show a good agreement with their results for $Kn = 0.19$, while the results for $\alpha_t = 0.7$ and $\alpha_i = 0$ agree at best with their results for $Kn = 0.3$. Since values of α_t of unbaked surfaces are expected to be close to unity,[20] the choice of $\alpha_t = 0.7$ is a little suspicious. On the other hand, innegligible uncertainties may be included in the measurements of number density of rarefied gas in contrast to the measurements of rotational temperature. In spite of such uncertainties it is confirmed through the present comparison that the significant deviation of the distribution of rotational temperature from that of number density is attributed to the smallness of α_i. If α_i is close to unity, the both distributions might be much closer than the measured distributions. More accrate measurements of number density and theoretical studies using nonlinear kinetic equations may be expected.

Table 2. Distributions of number density and temperatures; $s = 2$, $Kn = 0.19$, $Zr = 3.0$, $\alpha_t = 1.0$.

x/L	N	t_{tr}	t_{rot}
		($\alpha_i = 1.0$)	
−0.50	0.3399	−0.3563	−0.3540
−0.45	0.2870	−0.2988	−0.2972
−0.40	0.2500	−0.2590	−0.2581
−0.35	0.2165	−0.2234	−0.2228
−0.30	0.1842	−0.1895	−0.1892
−0.25	0.1526	−0.1567	−0.1565
−0.20	0.1215	−0.1246	−0.1245
−0.15	0.0909	−0.0931	−0.0930
−0.10	0.0604	−0.0619	−0.0618
−0.05	0.0302	−0.0309	−0.0309
0.00	0.0000	0.0000	0.0000
		($\alpha_i = 0.0$)	
−0.50	0.3418	−0.3591	−0.3847
−0.45	0.2905	−0.3036	−0.3185
−0.40	0.2539	−0.2641	−0.2749
−0.35	0.2203	−0.2283	−0.2363
−0.30	0.1877	−0.1940	−0.2000
−0.25	0.1557	−0.1606	−0.1651
−0.20	0.1242	−0.1278	−0.1311
−0.15	0.0929	−0.0955	−0.0978
−0.10	0.0618	−0.0635	−0.0650
−0.05	0.0309	−0.0317	−0.0324
0.00	0.0000	0.0000	0.0000

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