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A STRONG SHOCK WAVE SUPPORTED BY THE ABSORPTION OF LASER

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Abstract

Basically laser propulsion can be achieved either by LSC (Laser Supported Combustion Wave) or by LSD (Laser Supported Detonation Wave) as a mechanism for a propellant gas to absorb laser radiation. Although LSC has been studied by Keefer, Kemp and others [2, 3, 5,] since laser propulsion was proposed in the beginning of 1970's, virtually no analyses on LSD have been attempted since Raizer [1] gave a full description of laser nuclear fusion where he presented several attempts to analyze and found out possible mechanisms on laser absorption by strongly heated gas.

In the present analysis the structure of a LSD is shown by solving one-dimensional gasdynamic equations taking account of inverse bremsstrahlung absorption of laser energy incident on the front shock wave. The structure of the detonation consists of (i) a shock wave heating the low-temperature non-absorbing propellant gas up to a very high temperature enabling it to absorb laser radiation, (ii) followed by a thick absorption region where the subsonic flow is accelerated by exothermicity to the sonic velocity. Virtually all the laser energy is utilized to raize the temperature of this region until an equilibrium state is established between the radiation at incident laser wavelength and the bremsstrahlung radiation emission from the heated gas.

The Chapman-Jouguet condition is imposed to determine the propagation velocity of the detonation as an eigen value for a given laser intensity; radiation equilibrium is achieved at the sonic state. In practice, the calculation is performed in a manner that an eigen-value laser intensity is searched for a given detonation velocity to satisfy the C-J condition. Out of four conservation equations, the energy and radiative transfer relations contain radiation terms in differential forms which are integrated using the RK method.

The results show that the thickness of a detonation wave is several mm through several microns and the propagation velocity D_s satisfies a relation given by Raizer (Ref. 1);

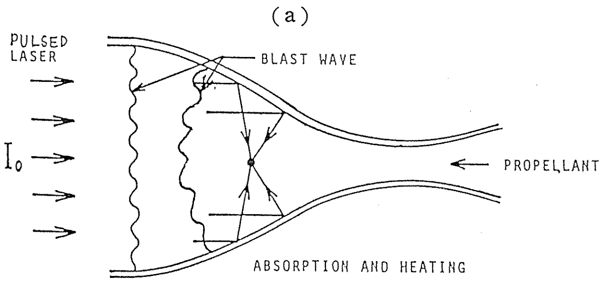
$$D_s=[2(\gamma^2-1)I_0/\rho_0]^{1/3},$$

which gives the propagation velocity of the order of 50 km/sec for the laser intensity $I_0\cong 10^8$ w/cm^2.

A multi-dimensional laser detonation using the spherical coordinate is partly analyzed as well.

1. Introduction

As shown in Fig. 1(a), the laser is thrown into a laser engine system on a pulsed mode, successively repeating a series of the processes; (i) breakdown of gas, (ii) laser absorption, (iii) generation of a shock wave and (iv) exhaust of a high-temperature gas. In the present analysis, the process is modeled into a plane steady shock wave generated by an incident laser energy, as shown in Fig. 1(b). Prior to the arrival of a strong shock wave, a laser radiation is propagating through a low-temperature gas that is assumed transparent. Once the incident laser penetrates the shock wave, it is absorbed by the gaseous medium due to the high temperature generated by the shock wave heating. The mechanism of laser absorption is inverse bremsstrahlung under the temperature range of the present problem (much lower than nuclear fusion temperatures).



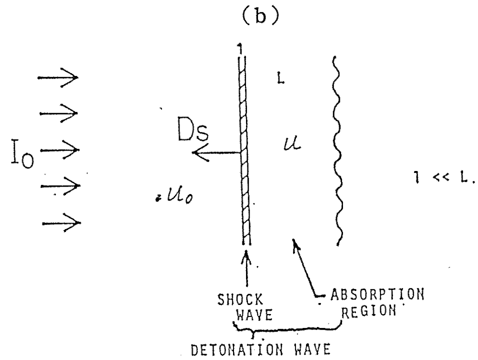


Fig. 1 (a) Concept of propulsion using Laser-Supported Detonation.
(b) Wave Coordinates in One-Dimensional Plane Laser Detonation.

The shock wave in turn is supported by the exothermicity caused by the absorption of the laser. This is fully identical to a shock wave supported by the energy released from the chemical reaction behind the shock wave. Thus a combined shock wave and exothermic laser absorption can be called as a laser-supported detonation.

If we are interested only in the propagation velocity of the laser detonation, the application of Chapman-Jouguet condition would be enough to yield the velocity. However, it is also important (i) to discover the structure of the absorbing region behind the leading shock wave, (ii) to find out the practical lower limit of the laser intensity that can support a detonation, and (iii) to acquire an equilibrium state between laser absorption and bremsstrahlung radiation emission at the Chapman-Jouguet point, which actually determines the accurate exothermicity and as a result the accurate propagation velocity.

Therefore we solved an eigen-value problem for a set of onedimensional conservation equations containing the terms of radiation absorption and emission. Unlike the conventional gasdynamic equations, we had to include one more equation for the radiation intensity at the laser wave length.

2. Mathematical Formulation of Plane Detonation

First, a physical model is introduced for a laser-supported detonation. A detonation consists of a leading shock wave followed by a laser absorption region. We consider that the present propellant is a purely atomic hydrogen gas, in front of the detonation, which is partly ionized after the passage of the leading shock wave and then immediately reaches ionization equilibrium throughout the absorption region. Therefore, the Rankine-Hugoniot relations exclude the heat of dissociation and the chemistry behind the leading shock wave is controlled only by the Saha equation and the neutral plasma assumption. In addition, the ionization energy is not taken into the equation of energy conservation. Other than equilibrium ionization chemistry, all the real gas effects are excluded from the analysis where a constant value is assumed for the specific heat ratio $\gamma=1.4$. Note here that the present rather naive physical model can easily be augmented by implementing all the real gas effects like equilibrium or non-equilibrium chemistry (dissociation, ionization and other reactions) and temperature dependency of each enthalpies.

The conservation of mass, momentum and energy can be written in the plane one-dimensional coordinate system as [4]

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho u)=0, \tag{1}$$

$$\frac{\partial \rho u}{\partial t}+\frac{\partial}{\partial x}(\rho u^2)=-\frac{\partial p}{\partial x}, \tag{2}$$

$$\frac{\partial \rho E_s}{\partial t}+\frac{\partial}{\partial x}[\rho u(E_s+p)]=q_{abs}-J, \tag{3}$$

where ρ, u, p and E_s denote the mass density, gas velocity, pressure and specific internal energy. q_{abs} indicates the energy absorption caused by inverse bremsstrahlung (watt/m^3) while J shows the emission due to bremsstrahlung (watt/m^3). It is noted here that the shock wave is considered infinitesimally thin and thus all the transport phenomena are neglected. The calculated results indicate that even the case of weak shock waves yields only 100 microns of absorption region and therefore the precursor phenomena as well as finite shock wave thickness may need to be taken into account in the future.

The above three equations are reduced to the following forms after integrating once and applying the boundary conditions in the uniform flow in front of the leading shock wave:

$$\rho_0(D_s-u_0)=\rho(D_s-u), \tag{4}$$

$$\rho_0[(D_s-u_0)^2+RT_0]=\rho[(D_s-u)^2+RT], \tag{5}$$

$$\rho_0(D_s-u_0)C_pT_0+\frac{1}{2}\rho_0(D_s-u_0)^2+\int_0^x q_{abs}dx$$
$$-\int_0^x Jdx=\rho(D_s-u)C_pT+\frac{1}{2}\rho(D_s-u)^2, \tag{6}$$

where the flow is assumed steady and the gas is perfect. As will be stated later, the concentrations of all the existing chemical species are calculated from the equilibrium conditions. In other words, we assume here that the flow is in both thermodynamic and chemical equilibrium. Non-equilibrium cases can be analyzed by solving the gasdynamic and radiation intensity equations along with the species conservation and internal energy transfer equations.

3. Bremsstrahlung

The bremsstrahlung is an important mechanism of energy loss from a plasma, releasing electromagnetic waves during the collision between an electron and an ion. In contrast, there is the absorption of this J_ν and in addition the induced emission from the absorption. The overall emission from the bremsstrahlung is obtained from the detailed balance among these phenomena as

$$J_\nu=cU_{\nu p}K_\nu\ \ (1-e^{-h\nu/kT_e}), \tag{7}$$

where $J_\nu\, d\nu$ denotes the emission power density at frequency interval ν, $\nu+d\nu$, while $U_{\nu p}$ the energy density of the electromagnetic waves with the absorption coefficient K_ν. The reciprocal of K_ν, i. e. the average mean free path of photon re-absorption, is usually longer than 1 cm that is much greater than the thickness of the absorption region in the present laser detonation; thus, the emission has only to be considered from the bremsstrahlung.

Integrating the power density in terms of entire frequency range, we obtain the following power density appearing in Eqs. (3) and (6):

$$J=\frac{32\pi}{3}\left(\frac{2\pi kT_e}{3m_e}\right)^{1/2}\frac{Z^2e^6}{m_ec^3h}n_in_e$$
$$=1.42\times10^{-40}\sqrt{T_e}n_in_e \text{ (watt/m}^3\text{)}, \tag{8}$$

for a hydrogen plasma consisting of neutral, ion and electron species. Here T, n_i and n_e indicate the temperature, ion number density and electron number density.

4. Inverse Bremsstrahlung

In contrast with the previous bremsstrahlung, the translational energy of an electron oscillated by the incoming radiation can convert to the translational energy of ions and neutral species due to the collision with the electron. This is called the inverse bremsstrahlung or classical absorption.

Since the mechanism of absorption of an incoming laser is the inverse bremsstrahlung, the laser intensity can be determined by the following equation:

$$q_{abs}=\frac{dI}{dx}=-K_aI, \tag{9}$$

which can be integrated, after taking account of the boundary condition $I=I_0$ at $x=0$ (at the shock wave), into

$$I=I_0e^{-\tau},\quad \tau=\int_0^x K_a dx. \tag{10}$$

Assuming that a CO_2 laser (10.6 μm) is irradiated into a hydrogen gas, the absorption coefficient K_{aen} caused by the collision between an electron and a neutral atom, and K_{aei} by the collision between an electron and an ion, are summed up to provide the total absorption coefficient K_a:

$$K_a=K_{aen}+K_{aei}, \tag{11}$$

$$K_{aen}=1.6\times10^{-48}\sqrt{T_e}n_en_n$$
$$\exp[-0.0122\sqrt{T_e}(1-5.28\times10^{-4}\sqrt{T})], \tag{12}$$

$$K_{aei}=(2.445\times10^{-42}/\sqrt{T})n_en_i(e^{1357/T}-1). \tag{13}$$

Once we have obtained the distribution of the number densities of existing chemical species, it becomes possible to evaluate the absorption coefficients, enabling us to integrate the gasdynamic equations (4) through (6).

5. Number Density

Since we have assumed an ionization equilibrium state throughout the flow field, the number densities of the electron, ion and neutral species are solved from the Saha equation:

$$\frac{n_i n_e}{n_n}=\frac{2g_i}{g_n}\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2}\exp\left(-\frac{E_1}{kT}\right),\ 2g_i=g_n, \tag{14}$$

and the neutral plasma assumption $n_e=n_i$ and the equation of state

$$p=(n_e+n_i+n_n)kT. \tag{15}$$

As to the temperatures, the equilibrium condition postulates

$$T_e=T_i=T_n=T. \tag{16}$$

Here E_1, g_i and g_n are the ionization energy and the statistical weights. In addition, h is the Planck constant, k the Boltzmann constant, m_e electron mass, e electron charge, Z degree of ionization, ν laser frequency and c light speed.

6. Method of Calculation and Discussion of Results

For a given propagation velocity of a Chapman-Jouguet detonation an appropriate laser intensity is solved as an eigen value using an iteration method shown below.

(i) First, the condition in front of the leading shock wave is set: T_0, $u_0=0$, R_0, P_0, ρ_0, n_{e0}, n_{i0}, n_{n0}.

(ii) The propagation velocity is given: D_s.

(iii) The Rankine-Hugoniot relations and the Saha equation give the conditions immediately behind the leading shock wave, since chemical equilibrium is assumed throughout the region behind the shock wave. This procedure needs iterations: T_1, u_1, ρ_1, P_1, R_1, n_{e1}, n_{i1}. n_{n1}.

(iv) The Chapman-Jouguet point where $M=1$ holds is determined from the continuity, momentum and Saha equations, needing iterations: T_2, u_2, ρ_2, R_2, p_2, n_{e2}, n_{i2}, n_{n2}.

(v) Since $q_{abs}=J$ holds at Chapman-Jouguet point ($M=1$), we can determine I_2.

(vi) Determine an optimal grid size for the integration, in other words, in order to be proportional to $1/K_\nu$.

(vii) Derive $u(x)$ by integrating the energy equation.

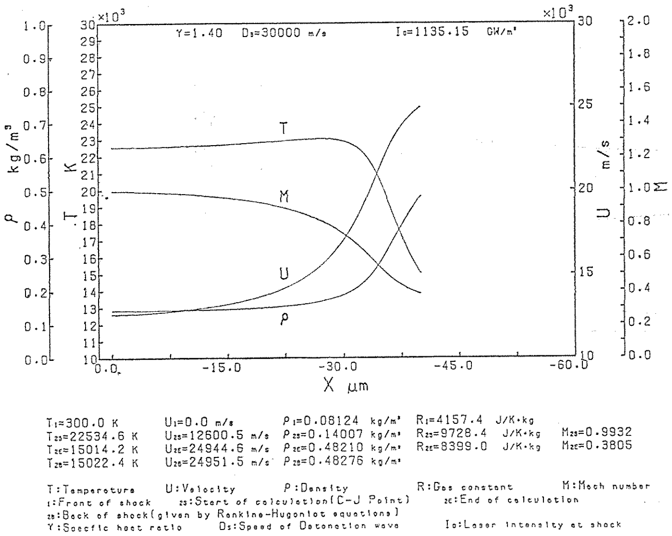
(viii) Derive T, R, n_e, n_i, n_n (x) using the Saha equation and iteration.

(ix) Derive I, ρ, p (x).

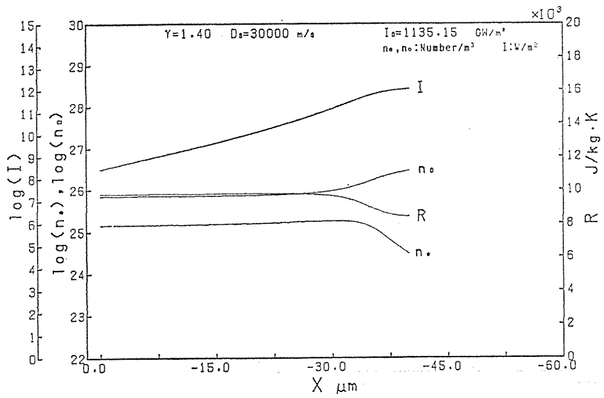
(x) If T and u both satisfy the Rankine-Hugoniot relations, then the location x is the position of the leading shock wave. The quantity I at this location is I_0 (the eigen value of the laser intensity). Otherwise, we go back to (vi) and repeat integration one step further.

We note here that the integration is performed from the downstream Chapman-Jouguet point up to the upstream shock wave; reverse integration. Using this procedure, there is no need for searching the Chapman-Jouguet value for the laser intensity I_0.

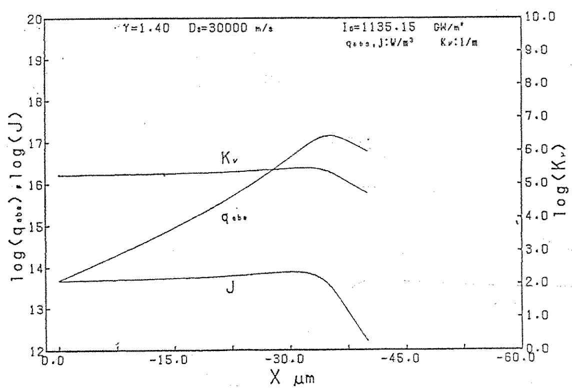
The distributions of the local temperature, velocity, radiation intensity etc. are shown in Fig. 2 through 9 for the propagation velocity 20~70 km/sec. The results illustrate the thickness of the detonation wave of the order of 10 microns for the



(a) The distributions of temperature T, flow Mach number M, flow volocity U and mass density ρ.



(b) The distributions of radiation intensity I at laser wavelength, number density n_n of neutral species, gas constant R per mass and number density n_e of electron.



(c) The distributions of absorption coefficient K_ν, absorbed energy intensity q_{abs} and emitted energy intensity J. At Chapman-Jouguet point(x=0.0), the absorption and emission balance exactly, leaving zero net energy transfer in the gas.

Fig. 2. Structure of a one-dimensional steady Chapman-Jouguet detonation propagating at a velocity D_s=30 km/sec under the laser intensity I_o=1135.15 gigawatt/m².

(a)

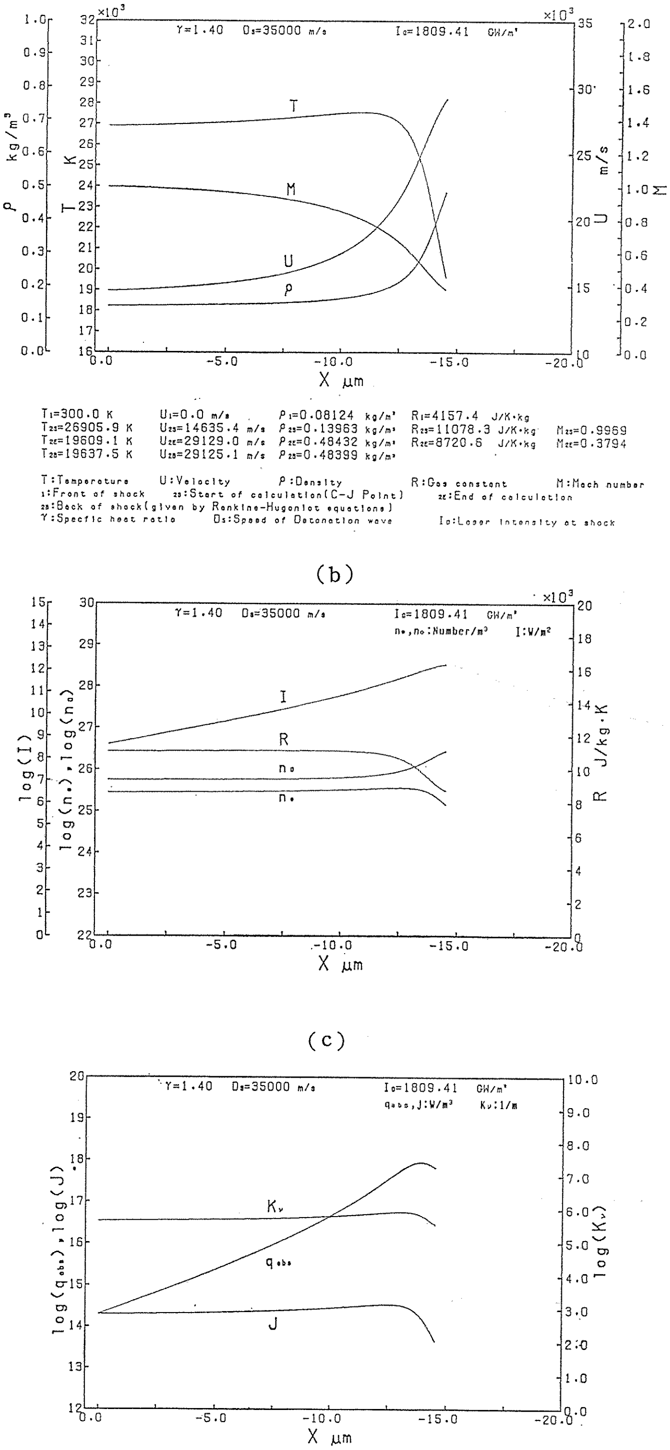
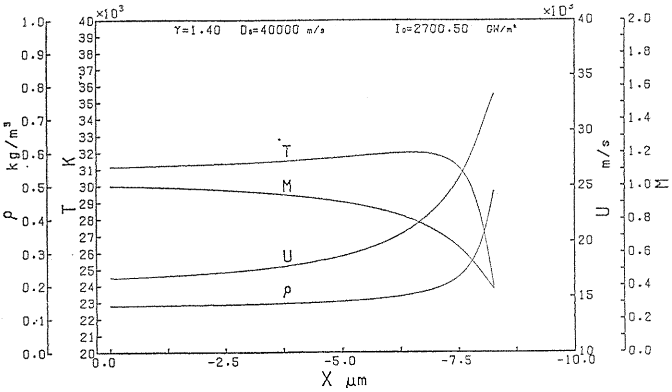


Fig. 3. (a)∼(c). Ibid for D_s=35 km/sec.

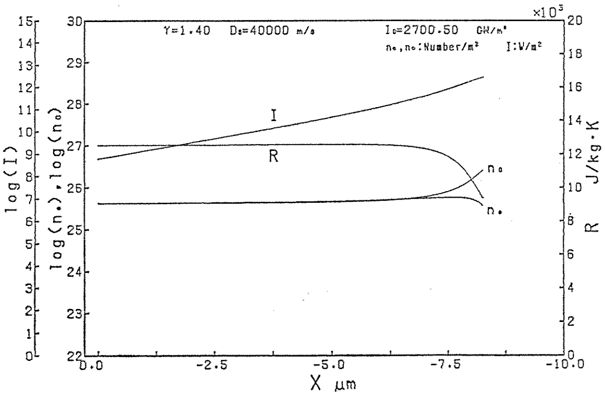
(a)



T_1=300.0 K U_1=0.0 m/s ρ_1=0.08124 kg/m³ R_1=4157.4 J/K·kg
T_{2S}=31139.9 K U_{2S}=16698.4 m/s ρ_{2S}=0.13946 kg/m³ R_{2S}=12496.5 J/K·kg M_{2S}=0.9983
T_{2E}=23913.2 K U_{2E}=33289.0 m/s ρ_{2E}=0.48422 kg/m³ R_{2E}=9341.5 J/K·kg M_{2E}=0.3795
T_{2B}=23914.7 K U_{2B}=33297.0 m/s ρ_{2B}=0.48480 kg/m³

T:Temperature U:Velocity ρ:Density R:Gas constant M:Mach number
1:Front of shock 2S:Start of calculation(C-J Point) 2E:End of calculation
2B:Back of shock(given by Rankine-Hugoniot equations)
γ:Specfic heat ratio Ds:Speed of Detonation wave Io:Laser intensity at shock

(b)



(c)

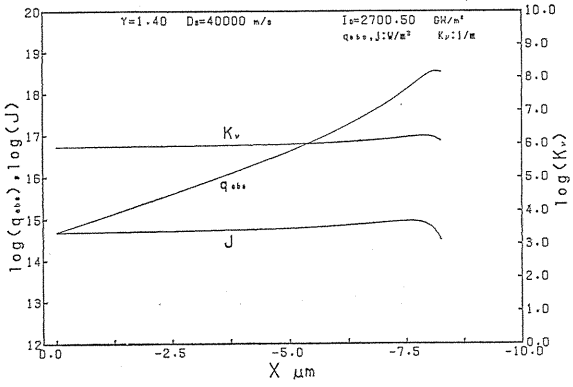
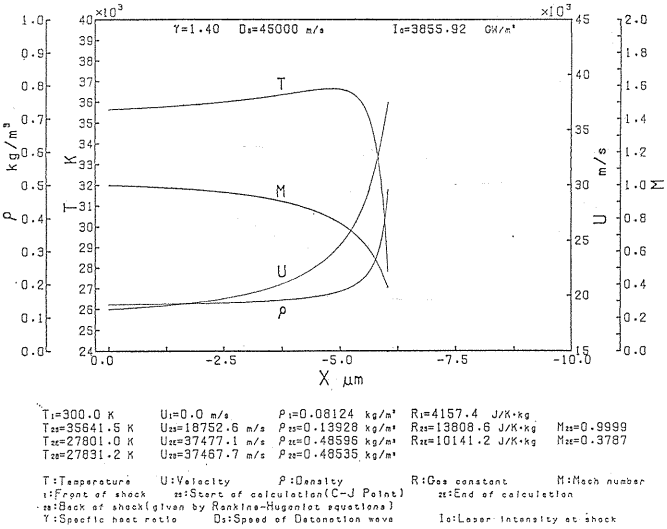


Fig. 4. (a)~(c). Ibid for D_s=40 km/sec.

(a)



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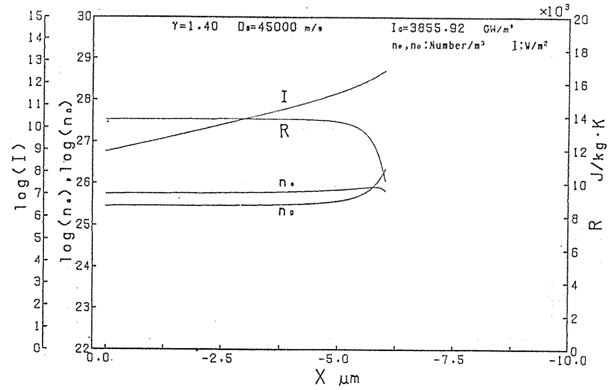

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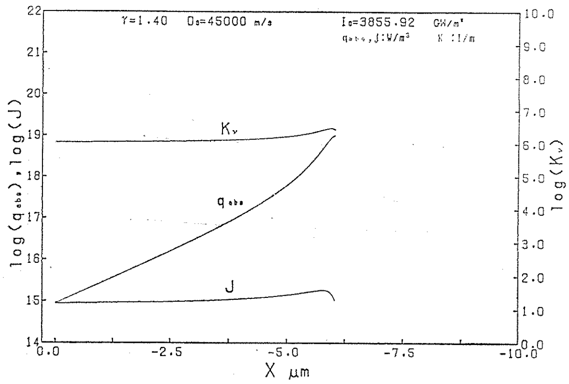


Fig. 5. (a)~(c). Ibid for $D_s=45$ km/sec.

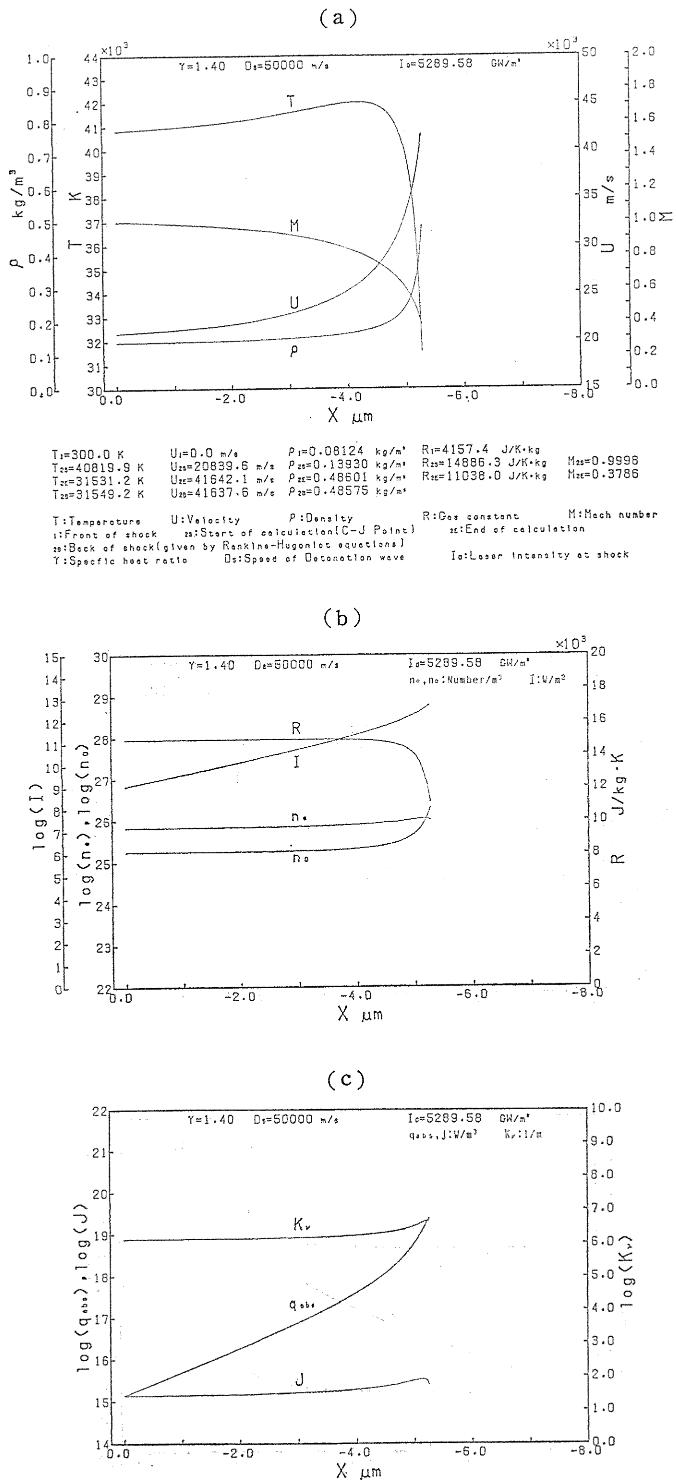


Fig. 6. (a)~(c). Ibid for D_s=50 km/sec.

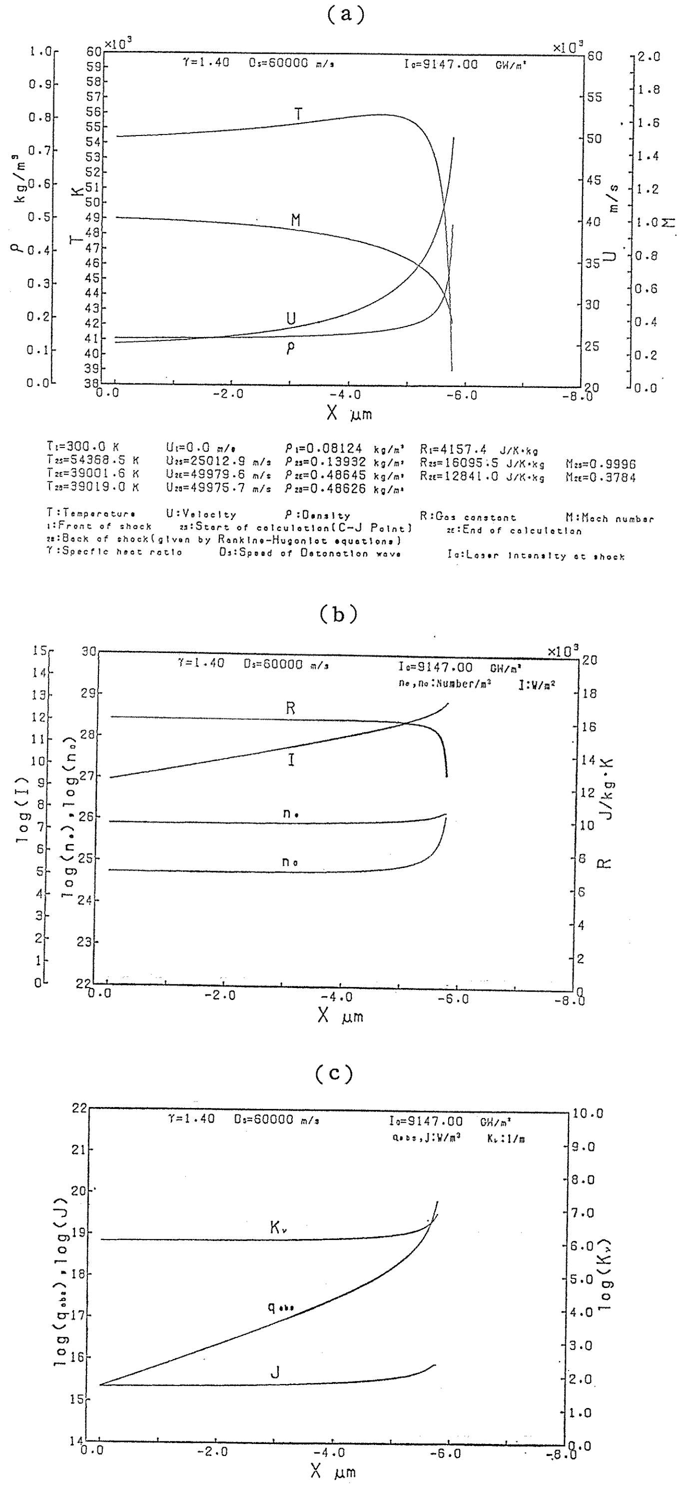
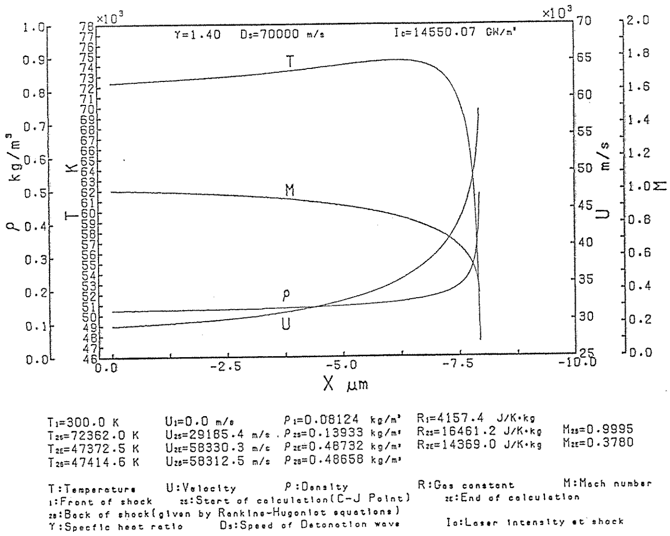


Fig. 7. (a)~(c). Ibid for D_s=60 km/sec.

(a)



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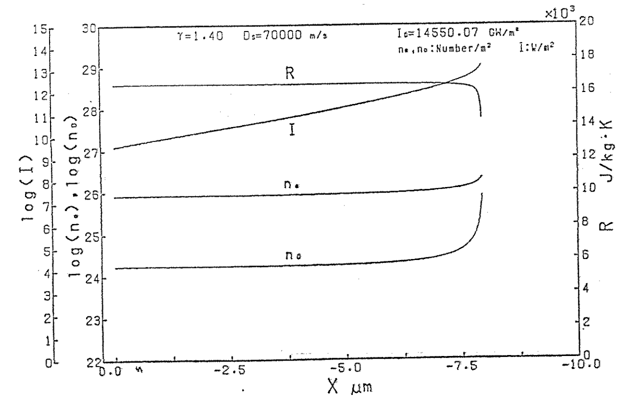

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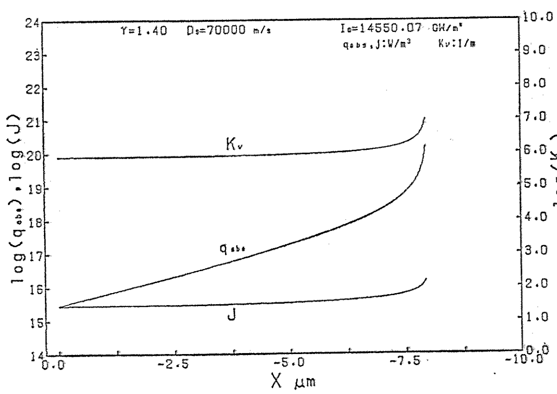
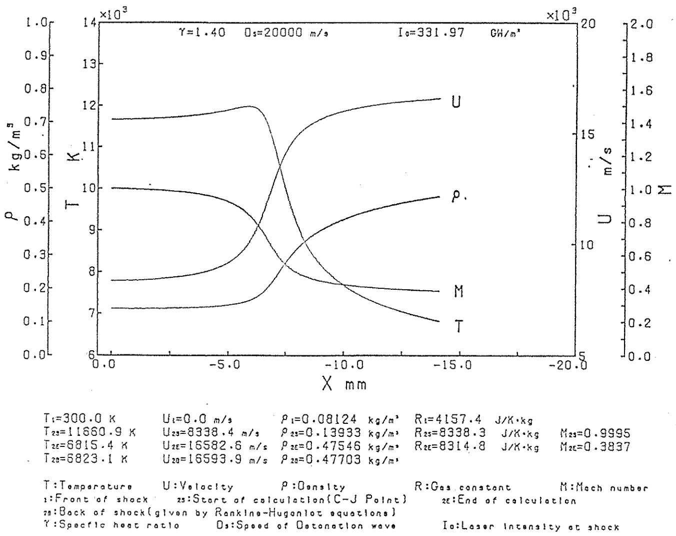


Fig. 8. (a)∼(c). Ibid for D_s=70 km/sec.

(a)



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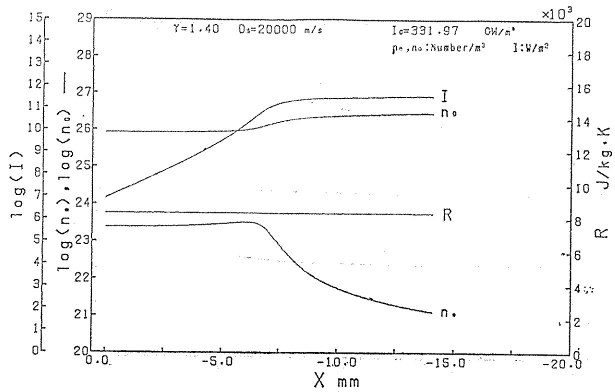

(c)

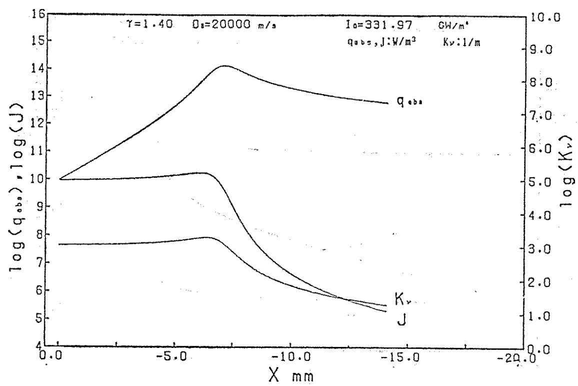

Fig. 9. (a)~(c). Ibid for D_s=20 km/sec.

laser intensity $I_0=(1\sim10)\times10^{12}$ w/m².

The calculated results show the following characters with respect to the solution:

(i) As a function of the laser intensity, the obtained propagation velocity shows the same tendency with the result of Raizer, as shown in Fig. 10. However, the observed quantitative deviations between the two results are as high as 20 %; the present results are lower than the curve of Raizer:

$$D_s=[2(\gamma^2-1)I_0/\rho_0]^{1/3}. \tag{17}$$

This may be attributed to the neglect of the dissociation and ionization energies of the propellant gas in the present numerical analysis.

(ii) At any detonation velocities, there is an equilibrium between bremsstrahlung emission and inverse bremsstrahlung absorption, leading to zero net radiation heat transfer ($q_{abs}=J$) at the Chapman-Jouguet point, as shown in all the detonation profiles in Figs. 2 through 9;

(iii) Below D_s=30 km/sec, it is interesting to see that the thickness of the detonation (from the shock wave to the C-J point) suddenly starts increasing. Although a steady detonation profile is calculated without difficulty, for example, at D_s=20 km/sec shown in Fig. 9, the detonation might be difficult to generate in reality; due to multi-dimensional instabilities, heat loss to lateral directions and different absorptioin mechanisms (inverse bremsstrahlung is not effective any more). From the order of tens of microns at $D_s\geqq30$ km/sec, the thickness of the detonation jumps to 15 mm at D_s=20 km/sec, three orders of magnitude higher. This shows that the limit of laser detonation is close to D_s=20 km/sec and I_0=332 GW/m²=0.332 MW/mm².

(iv) For a low propagation velocity (D_s=20 km/sec), the absorption gradually occurs downstream of the leading shock wave. The temperature immediately behind the leading shock is only 7000 k, since the Mach number corresponding to D_s=20 km /sec is only 15 in the present atomic hydrogen gas and, in addition, the increase in the particle numbers (neutral atom into ion and electron) due to ionization.

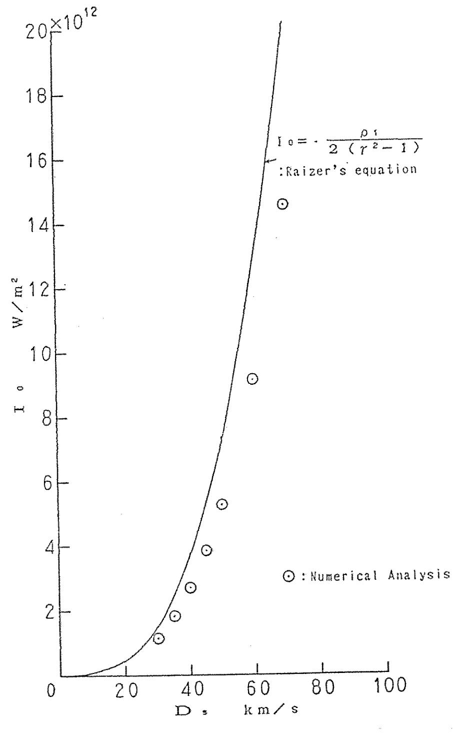

Fig. 10. Calculated detonation velocity D_s as a function of laser intensity I_0. Comparison is made with Raizer's theoretical results.

7. Analysis for Detonation of Spherical Symmetry

Now we consider a case where at time $t=0$ (initially) a laser is focused onto a point of spherical symmetry, generating a spherical blast wave behind which the laser absorption occurs due to the above-mentioned mechanism.

This is different from the above plane one-dimensional problem in the following respects:

(i) A nonsteady problem starting from the point of symmetry. In the actual calculation, the initial condition is set up for a wave which has slightly travelled from the center. We can assume a finite-size region in the center that has known physical properties.

(ii) There are two radiation intensities, one I_ν^- directing inward and the other I_ν^+ directing outward. The former is the incoming laser intensity whereas the latter corresponds to the reflected and emitted part of the radiation.

(iii) Since the entire process is strongly nonsteady, it would be better to allow non-equilibrium chemistry.

Based on such considerations, we utilize the following fundamental equations to do numerical analysis:

Mass;

$$\frac{\partial r^2\rho}{\partial t}+\frac{\partial r^2\rho u}{\partial r}=0, \tag{18}$$

Momentum;

$$\frac{\partial r\rho u}{\partial t}+\frac{\partial r^2\rho(u^2+RT)}{\partial r}-2r\rho RT=0, \tag{19}$$

Energy;

$$\frac{\partial r^2\left(\frac{1}{2}\rho u^2+\rho\frac{R}{\gamma-1}T\right)}{\partial t}+\frac{\partial r^2\left(\frac{1}{2}\rho u^3+\rho\frac{\gamma R}{\gamma-1}uT\right)}{\partial r}$$
$$+r^2\left\{\int_0^\infty(-K_\nu I_\nu^- -K_\nu I_\nu^+ +2J_\nu)d\nu\right\}$$
$$+r^2E_1(k_{ne}+k_e)\left(n_e n_n-\frac{1}{K}n_e^2 n_i\right)=0, \tag{20}$$

Intensity;

$$\frac{\partial r^2 I_\nu^-}{\partial t}+\frac{\partial(-r^2 c I_\nu^-)}{\partial r}-cr^2(-K_\nu I_\nu^- +J_\nu)=0, \tag{21}$$

Intensity;

$$\frac{\partial r^2 I_\nu^+}{\partial t}+\frac{\partial(r^2 c I_\nu^+)}{\partial r}-cr^2(-K_\nu I_\nu^+ +J_\nu)=0, \tag{22}$$

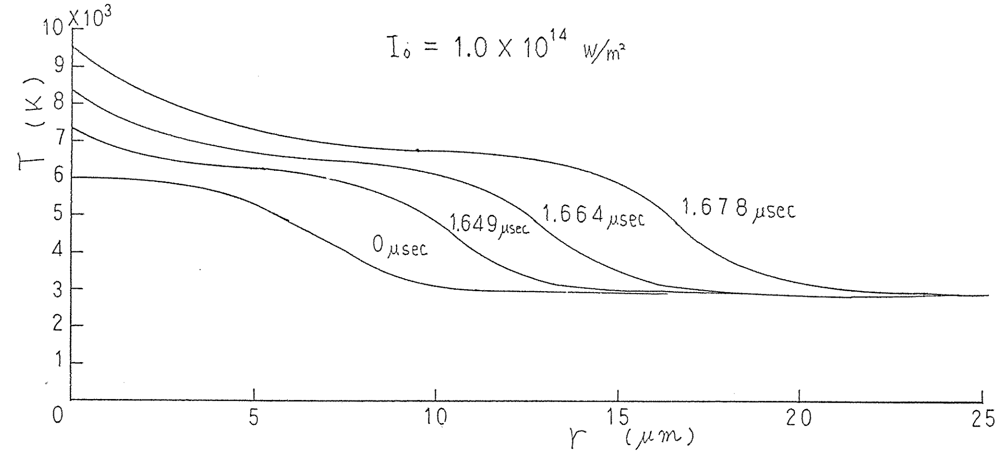

Fig. 11. History of temperature distribution in laser detonation during the initial phase of propagation. Formation of a shock wave and temperature rise due to laser absorption are observed.

Electron conservation;

$$\frac{\partial r^2 n_e}{\partial t}+\frac{\partial r^2 u n_e}{\partial r}$$

$$-r^2(k_{ne}+k_e)\Big((n_e n_n-\frac{1}{K}n_e^2 n_i\Big)=0 \tag{23}$$

Here the rate constants k_{ne} and k_e, the heat of reaction E_1 and equilibrium constant K are introduced.

An example calculation was perfomed for a set of parameters shown below: The temperature at the center $T_c=6000$ k, $p_c=1$ atm, the velocity $u_c=0$ and the laser intensity $I_0=10^{14}$ watt/m². The initial phase of the solution is shown in Fig. 11. As a numerical scheme, we used the MacCormack 2nd-order explicit method. While a shock wave is being formed, the temperature is increasing behind the shock wave due to the absorption of the incident laser. Further study is under progress.

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