Memoirs of the Faculty of Engineering, Nagoya University
Vol. 38, No. 2 (1986) —Research Report—

ANALYTICAL CONSIDERATIONS OF CHATTER VIBRATION OF LATHE TOOLS

SHINOBU KATO, ETSUO MARUI*,
MASATOSHI HASHIMOTO** and TOSHIKATSU YAMADA**

Department of Electronic-Mechanical Engineering

(Received October 20, 1986)

Abstract

Chatter vibration occurring in the spindle-workpiece system of a lathe is treated theoretically, considering the phase lag of cutting force and chatter marks, the dynamic variation effects on cutting force of cutting velocity, and the rake angle under vibration. As a result, it is clarified that the chatter vibration is mainly induced by the phase lag of cutting force (primary chatter) and by the phase lag of chatter marks in successive cutting (regenerative chatter). The dynamic variations of the cutting velocity and rake angle make the spindle-workpiece system more unstable under vibration. The effect of cutting velocity variation is more remarkable than that of rake angle in both types of chatter. Chatter vibration can effectively be suppressed by enlarging the damping capacity of the system.

1. Introduction

In the previous studies[1~2], the primary and regenerative chatter vibrations occurring in a lathe spindle-workpiece system were experimentally observed, and the effects of the vibratory characteristics (i. e., equivalent mass, spring constant, damping coefficient, etc.) or cutting conditions on chatter vibration were examined. Then the dynamic characteristics of cutting force variation due to the continuous variations of cutting depth, cutting velocity and rake angle as in the chatter vibration state are obtained by the systematic experiment. The obtained experimental data are important for the fundamental consideration on chatter vibration in the

* Department of Mechanical Engineering, Gifu University
** Department of Mechanical Engineering, Toyota College of Technology

spindle-workpiece system.

In the present study, theoretical investigations are carried out on the chatter behavior of the spindle-workpiece system. The spindle-workpiece system is regarded as a vibratory system with two degrees of freedom, which deflects in horizontal (thrust cutting force) and vertical (main cutting force) directions. The dynamic characteristics of the cutting force is introduced in the equation of motion. The effects of various parameters on the chatter initiation are estimated theoretically, and compared with the experimental results.

2. Stability Boundary of Chatter Vibration

2. 1. Equation of Motion of Spindle-Workpiece System

Figure 1 shows the coordinate system determined on the workpiece[2]. The spindle-workpiece system is a vibratory system having two degrees of freedom, the vibratory characteristics of which are as follows: The equivalent mass is m, the viscous damping coefficient is c, and the spring constant is k. These characteristics are identical in both horizontal and vertical directions. The cutting force acting on the workpiece can be derived from equations (1)′, (2) and (6) in the previous study[2]. Using the coordinate system shown in Fig. 1, the equation of motion of the spindle-workpiece system is written as follows:

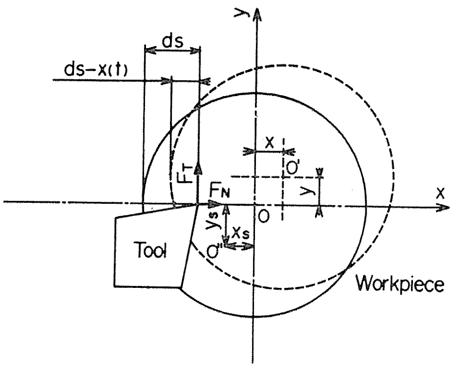

Fig. 1. Workpiece coordinate system during vibration.

$$\left.\begin{aligned} & m\ddot{x}+c\dot{x}+k(x+x_s) \\ & =b\left\{K_{N0}-a_{Nv}\cdot\dot{y}-a_{N\alpha}\cdot\tan^{-1}\left(\frac{\dot{x}}{V_0-\dot{y}}\right)\right\} \\ & \quad\times\left\{d_s-x\left(t-\frac{\theta}{\beta}\right)+\mu\cdot x\left(t-\frac{\theta}{\beta}+\frac{\theta^*}{\beta}\right)\right\} \\ & m\ddot{y}+c\dot{y}+k(y+y_s) \\ & =b\left\{K_{T0}-a_{Tv}\cdot\dot{y}-a_{T\alpha}\cdot\tan^{-1}\left(\frac{\dot{x}}{V_0-\dot{y}}\right)\right\} \\ & \quad\times\left\{d_s-x\left(t-\frac{\theta}{\beta}\right)+\mu\cdot x\left(t-\frac{\theta}{\beta}+\frac{\theta^*}{\beta}\right)\right\} \end{aligned}\right\} \qquad (1)$$

where the following notations are used: b is the cutting width, d_s is the prescribed cutting depth, V_0 is the cutting velocity, K_{N0} and K_{T0} are the specific cutting resistances of thrust and main cutting forces, a_{Nv} and a_{Tv} are the velocity coefficients, $a_{N\alpha}$ and $a_{T\alpha}$ are the rake angle coefficients, Θ and θ are the phase lags of the thrust and main cutting forces behind the cutting depth variation, Θ^* is the phase lag of the chatter marks, μ is the overlap factor and β is the chatter frequency.

The following relations exist among the steady terms in equation (1).

$$\left.\begin{aligned} kx_s &= K_{N0}\cdot b\cdot d_s \\ ky_s &= K_{T0}\cdot b\cdot d_s \end{aligned}\right\} \tag{2}$$

The quantities $\dot{x}$, x and $\dot{y}$ are considered to be infinitesimal. Disregarding the terms of higher order in equation (1), and using the notations of equation (3), the equation of motion is rewritten as follows:

$$\frac{c}{m}=2n,\ \frac{k}{m}=p^2,\ \frac{b}{m}=\lambda \tag{3}$$

$$\left.\begin{aligned} &\ddot{x}+\left(2n+\frac{\lambda\cdot a_{N\alpha}\cdot d_s}{V_0}\right)\dot{x}+p^2x \\ &\quad+\lambda\cdot K_{N0}\left\{x\left(t-\frac{\Theta}{\beta}\right)-\mu\cdot x\left(t-\frac{\Theta}{\beta}+\frac{\Theta^*}{\beta}\right)\right\}+\lambda\cdot a_{Nv}\cdot d_s\dot{y}=0 \\ &\lambda\cdot K_{T0}\left\{x\left(t-\frac{\theta}{\beta}\right)-\mu\cdot x\left(t-\frac{\theta}{\beta}+\frac{\Theta^*}{\beta}\right)\right\} \\ &\quad+\frac{\lambda\cdot a_{T\alpha}\cdot d_s}{V_0}\dot{x}+\ddot{y}+(2n+\lambda\cdot a_{Tv}\cdot d_s)\dot{y}+p^2y=0 \end{aligned}\right\} \tag{4}$$

Putting the overlap factor $\mu=0$, the equation of motion for the primary chatter vibration is obtained.

2. 2. Stability Boundary

The steady state solutions of equation (4) are assumed as follows:

$$\left.\begin{aligned} x &= x_0 e^{st} \\ y &= y_0 e^{st} \end{aligned}\right\} \tag{5}$$

The characteristic equation can be obtained by substituting equation (5) into equation (4) as follows:

$$\begin{vmatrix} s^2+\left(2n+\frac{\lambda\cdot a_{N\alpha}\cdot d_s}{V_0}\right)s+p^2 + \lambda\cdot K_{N0}\left(e^{-\frac{s\Theta}{\beta}}-\mu\cdot e^{-\frac{\Theta-\Theta^*}{\beta}s}\right) & \lambda\cdot a_{Nv}\cdot d_s\cdot s \\ \frac{\lambda\cdot a_{T\alpha}\cdot d_s}{V_0}s+\lambda\cdot K_{T0}\left(e^{-\frac{s\theta}{\beta}}-\mu\cdot e^{-\frac{\theta-\Theta^*}{\beta}s}\right) & s^2+(2n+\lambda\cdot a_{Tv}\cdot d_s)s+p^2 \end{vmatrix}=0 \tag{6}$$

When steady state chatter vibration occurs,

$$s = j\beta \quad (j = \sqrt{-1})$$

This relation is substituted in equation (6), and the result is divided into real part and imaginary part. The relation of the real part may be stated,

$$A_1\lambda^2 + A_2\lambda + A_3 = 0 \tag{7}$$

And the relation of the imaginary part may be stated,

$$B_1\lambda^2 + B_2\lambda + B_3 = 0 \tag{8}$$

where

$$\left.\begin{aligned}
A_1 &= -\frac{d_s^2(a_{Tv}\cdot a_{N\alpha} - a_{Nv}\cdot a_{T\alpha})\beta^2}{V_0} \\
&\quad + d_s[a_{Tv}\cdot K_{N0}\{\sin\theta - \mu\sin(\theta-\theta^*)\} \\
&\quad - a_{Nv}\cdot K_{T0}\{\sin\theta - \mu\cdot\sin(\theta-\theta^*)\}]\beta \\
A_2 &= \left[-\frac{2n\cdot a_{N\alpha}\cdot d_s}{V_0} - 2n\cdot a_{Tv}\cdot d_s \right. \\
&\quad \left. - K_{N0}\{\cos\theta - \mu\cdot\cos(\theta-\theta^*)\}\right]\beta^2 \\
&\quad + 2n\beta\cdot K_{N0}\{\sin\theta - \mu\cdot\sin(\theta-\theta^*)\} \\
&\quad + K_{N0}\cdot p^2\{\cos\theta - \mu\cdot\cos(\theta-\theta^*)\} \\
A_3 &= (p^2-\beta^2)^2 - 4n^2\beta^2
\end{aligned}\right\} \tag{9}$$

$$\left.\begin{aligned}
B_1 &= -d_s\cdot\beta[a_{Nv}\cdot K_{T0}\{\cos\theta - \mu\cdot\cos(\theta-\theta^*)\} \\
&\quad - a_{Tv}\cdot K_{N0}\{\cos\theta - \mu\cdot\cos(\theta-\theta^*)\}] \\
B_2 &= (p^2-\beta^2)\beta\left(a_{Tv}\cdot d_s + \frac{a_{N\alpha}\cdot d_s}{V_0}\right) \\
&\quad - (p^2-\beta^2)\cdot K_{N0}\{\sin\theta - \mu\cdot\sin(\theta-\theta^*)\} \\
&\quad + 2n\beta\cdot K_{N0}\{\cos\theta - \mu\cdot\cos(\theta-\theta^*)\} \\
B_3 &= 4n\beta(p^2-\beta^2)
\end{aligned}\right\} \tag{10}$$

Equations (7) and (8) are the simultaneous equations for the unknowns λ and β. By eliminating the unknown λ from both equations, then the following relation is obtained.

$$\begin{aligned}
&A_1^2B_3^2 + A_1(A_3B_2^2 - 2A_3B_1B_3 - A_2B_2B_3) \\
&\qquad + (A_3^2B_1^2 - A_2A_3B_1B_2 + A_2^2B_1B_3) = 0
\end{aligned} \tag{11}$$

The unknown λ is obtained as follows:

$$\lambda = -\frac{A_3B_1 - A_1B_3}{A_2B_1 - A_1B_2} \tag{12}$$

When the system parameters satisfy the relations described by equations (11) and (12), the steady state chatter vibration can occur in the spindle-workpiece system. The chatter frequency and the stability boundary λ_{cr} can be calculated by these equations.

3. Calculation of Stability Boundary and Comparison with Experimental Result

In this section, the stability boundaries of the primary and the regenerative chatter vibrations are calculated and compared with the experimental result. The influencing degree of dynamic cutting force variation on stability boundary is discussed.

The factors representing the dynamic variation effects are given in Table 1, which are determined from the experiments of the previous reports[1~2]. The vibratory characteristics of the spindleworkpiece system is shown in Table 1 of the previous study[1].

Table 1. Parameters influencing dynamic variations of cutting force.

A		Primary	Regenerative
ds	mm	0.010	0.025
V_0	m/s	0.265	0.265
K_{NO}	N/m²	1.79 ×10⁹	1.23 ×10⁹
K_{TO}	N/m²	4.28 ×10⁹	3.32 ×10⁹
Θ	deg	16	16
θ	deg	12	12
Θ^*	deg	———	85
a_{NV}	N·S/m³	-0.64 ×10⁹	-0.64 ×10⁹
a_{TV}	N·S/m³	-2.75 ×10⁹	-2.75 ×10⁹
$a_{N\alpha}$	N/m²	-1.7 ×10⁹	-1.7 ×10⁹
$a_{T\alpha}$	N/m²	-1.1 ×10⁹	-1.1 ×10⁹

3. 1. *Primary Chatter Vibration*

In Fig. 2, the stability boundary λ_{cr} of the primary chatter vibration calculated by equation (12) is compared with the experimental result. In the figure, the calculated stability boundary λ_{cr} is converted to the stability boundary expressed by cutting force using the following equations.

$$\left.\begin{aligned} K_{INth} &= \lambda_{cr} \cdot m \cdot K_{NO} \\ K_{ITth} &= \lambda_{cr} \cdot m \cdot K_{TO} \end{aligned}\right\} \tag{13}$$

The calculated result agrees well with the experimental result. Then, it is ascertained that the proposed mechanism for chatter vibration occurring is reasonable.

The effects of dynamic characteristics are investigated in the following.

Figure 3 shows the effect of the phase lag θ of thrust cutting force. The calculated results (λ_s, β_s) considering only the phase lag of the cutting force are shown by dotted lines in the figure for reference.

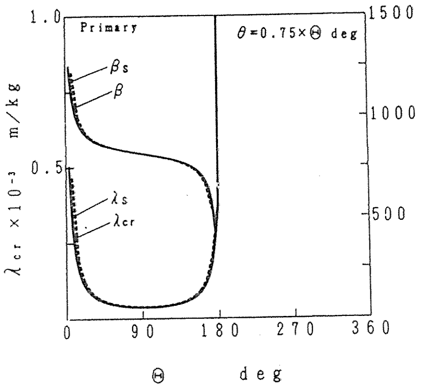

Fig. 2. Comparison of experimental result with calculated result on stability boundary.

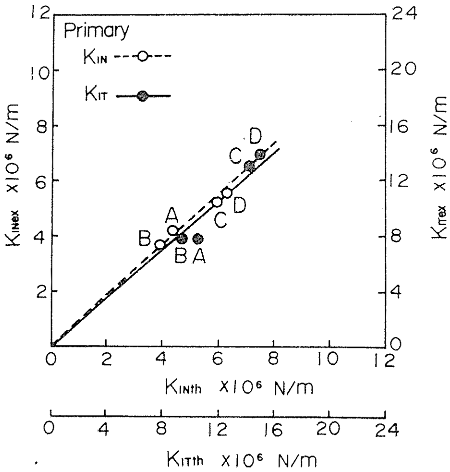

Fig. 3. Relations between stability boundary, frequency and phase lag of thrust cutting force.

It is understood from the figure that the stability boundary λ_{cr} calculated considering all of the dynamic cutting force characteristics is a little lower than λ_s. Hence, it is clear that the dynamic variations of cutting velocity and rake angle make the vibratory system unstable. The vibratory system becomes most unstable, when $\Theta=90°$. Figure 3 is the case of $\theta=0.75\,\Theta$. The stability boundary for other values of θ is calculated, but the result is almost the same as shown in Fig. 3. Hence, the phase lag of the main cutting force does not affect the stability boundary. The chatter frequency β, given in the same figure, decreases with the increase of the phase lag Θ. At the range of phase lag Θ observed in the experiment, β is approximately constant and is almost equal to the natural frequency of the system.

From the above considerations, it is clarified that the phase lag of the trust cutting force plays an important role in the primary chatter vibration, and the phase lag of the main cutting force does not affect the chatter initiation.

Figure 4 shows the effect of the cutting velocity coefficient in thrust force. The value of λ_{cr} is small and the chatter vibration occurs easily, when the cutting velocity coefficient a_{Nv} is small. The effect of a_{Nv} is remarkable in the case of small phase lag of cutting force.

Figure 5 shows the effect of the rake angle coefficient in the thrust cutting

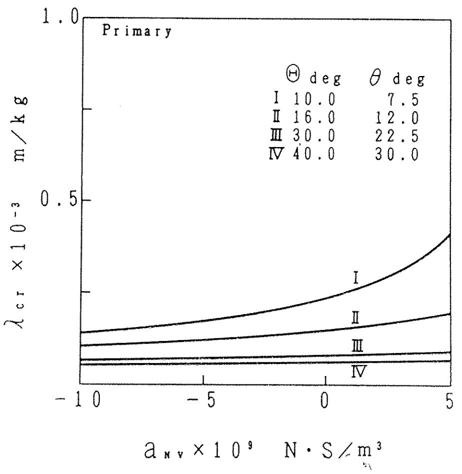

Fig. 4. Relation between stability boundary and velocity coefficient of thrust cutting force.

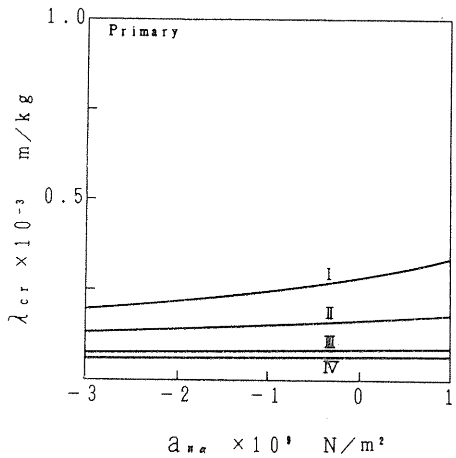

Fig. 5. Relation between stability boundary and rake angle coefficient of thrust cutting force.

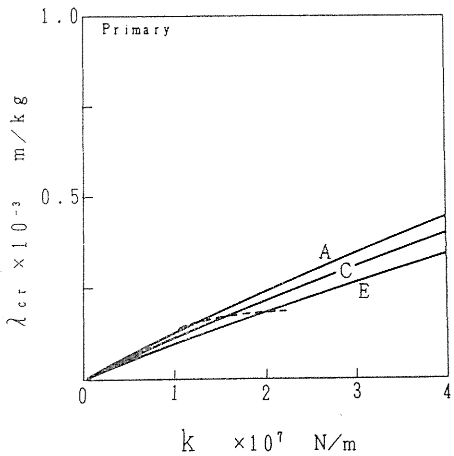

Fig. 6. Relation between stability boundary and spring constant.

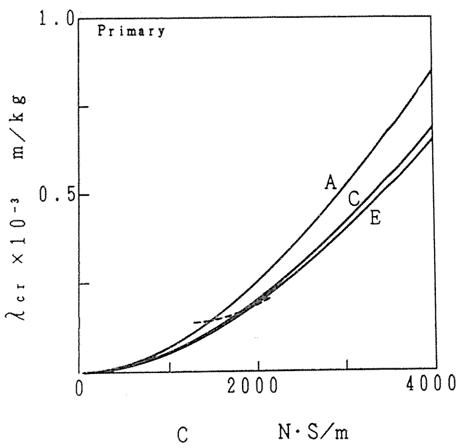

Fig. 7. Relation between stability boundary and damping coefficient.

force. Chatter vibration easily occurs when $a_{N\alpha}<0$. On the contrary, the rake angle coefficient $a_{T\alpha}$ in main cutting force does not affect the stability boundary.

Now, the effects of vibratory characteristics are given in Figs. 6 and 7. One of the vibratory characteristics of the systems A, C and E used in the experiment is changed independently, in this calculation.

It is understood from Fig. 6 that the stability boundary increases almost linearly with the increase of the spring constant k. The dotted line in the figure shows the relation in the actual spindle-workpiece system used in the experiment.

From Fig. 7, it is clear that the stability boundary increases abruptly with the increase of the damping coefficient, and chatter vibration hardly occurs. The effect of damping coefficient is stronger than the spring constant. It is difficult for chatter vibration to occur in the actual lathe as shown by the dotted line, when the damping coefficient is large.

From the above considerations, it is most effective to increase the damping coefficient for the prevention of primary chatter vibration.

Lastly, the quantitative effects of three factors which affect the dynamic cutting force characteristics, i. e., (1) the phase lag of cutting force, (2) cutting velocity variation and (3) rake angle variation, are treated here. Figure 8 shows the relation between the stability boundary λ_s obtained by considering the factor (1) and the stability boundary λ_{cr} obtained by considering the factors (1) ~ (3). The symbol ● and $\lambda_{cr}(\alpha)$ are the stability boundary when the factors (1) and (3) are

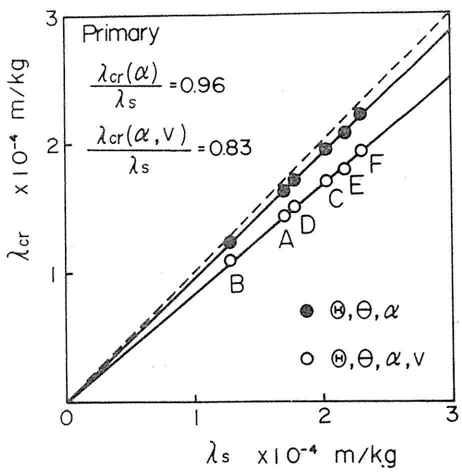

Fig. 8. Influencing degree of various parameters on stability boundary.

considered in the calculation. And the symbol ○ and λ_{cr} (α, v) are the stability boundary when all factors are considered in the calculation. It is recognized in the figure that the stability boundary is decreased by the factors (2) and (3). The stability boundary $\lambda_{cr}(\alpha)$ is decreased from λ_s by 4 % owing to the dynamic variation of rake angle. It is also decreased by 13 % owing to the dynamic variation of cutting velocity.

As a result, the phase lag of cutting force has the greatest influence on the stability boundary of the primary chatter vibration. The effect of other factors decreases in the order of cutting velocity variation and rake angle variation.

3. 2. Regenerative Chatter Vibration

The calculated stability boundary is compared with the experimental one in Fig. 9. The overlap factor in this case is $\mu=1.0$. The calculated result agrees well with the experimental result qualitatively. However, the calculated one is greater than the experimental one for each system.

The effects of the dynamic cutting force on the stability boundary λ_{cr} are shown in Figs. 10 and 11.

Figure 10 shows the relations between the stability boundary λ_{cr}, chatter frequency β and the phase lag Θ of the thrust cutting force. The qualitative tendencies of λ_{cr} and β are the same as the primary chatter vibration. However, in the regenerative chatter vibration, the stability boundary is decreased owing to the phase lag of chatter marks and the curve is shifted to the left side of the figure. This fact corresponds to the apparent increase

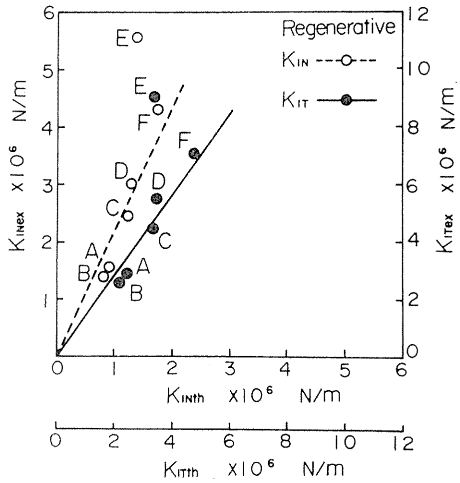

Fig. 9. Comparison of experimental result with calculated result on stability boundary.

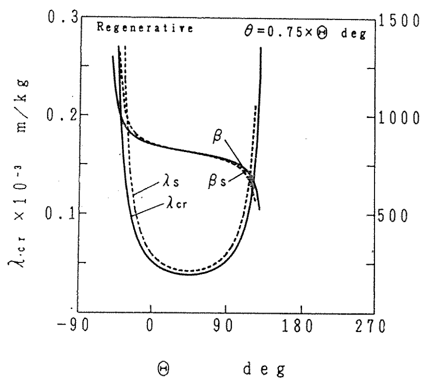

Fig. 10. Relations between stability boundary, frequency and phase lag of thrust cutting force.

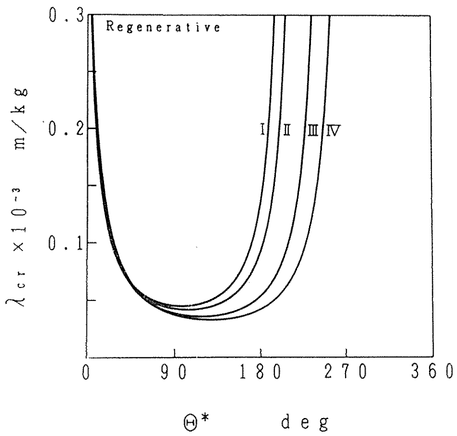

Fig. 11. Relation between stability boundary and phase lag of chatter marks.

of phase lag in the cutting force. The stability boundary is not affected by the change of phase lag in the main cutting force.

Figure 11 shows the effect of the phase lag of chatter marks, the parameter in the figure is the phase lag of the cutting force. It is clear in the figure that there exists a phase lag Θ^* where the chatter vibration is most likely to occur and that this lag Θ^* is more than 90^c in the case of large cutting force phase lag.

The qualitative effects of the dynamic variations of cutting velocity and rake angle are the same as in the primary chatter vibration.

The qualitative effect of each vibratory characteristic in regenerative chatter is similar to that in the primary chatter vibration.

Lastly, the effect of three factors influencing the dynamic cutting force is investigated quantitatively. Figure 12 shows the relations between the stability boundary λ_s obtained by considering the phase lags of cutting force and the stability boundary λ_{cr} obtained by considering all factors. The symbols ● and $\lambda_{cr}(\alpha)$ are the stability boundary when the factors (i.e., phase lag of cutting force and rake angle variation) are considered in the calculation. And other symbols ○ and $\lambda_{cr}(\alpha, v)$ are the stability boundary when all factors are considered. It is recognized from the figure that the stability boundary is decreased by all factors, as in the primary chatter vibration. The effects of these factors increase in the order of phase lag of cutting force, cutting velocity variation and rake angle variation. The effects of the cutting velocity variation and rake angle variation are weak in the regenerative chatter vibration in comparison with the case of the primary chatter vibration, because of the phase lag of chatter marks.

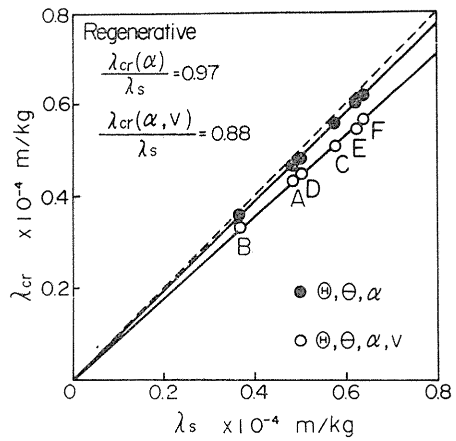

Fig. 12. Influencing degree of various parameters on stability boundary.

4. Concluding Remarks

The chatter vibration occurring in the spindle-workpiece system is treated theoretically, where the vibratory system is regarded as the system having two degrees of freedom and deflects in the thrust and the main cutting force directions. The cutting force acting on the workpiece is affected by the phase lag of the cutting force behind the cutting depth variation, phase lag of chatter marks and the dynamic variations of cutting velocity and rake angle. As a result, it is ascertained that the chatter vibration of the spindle-workpiece system is induced by the phase lag of cutting force behind the cutting depth variation (in the primary chatter vibration) and the phase lag of chatter marks (in the regenerative chatter vibration). The stability boundary is lowered by the dynamic variation effect of cutting velocity and rake angle on the cutting force. Among various vibratory characteristics of the system, the damping coefficient has the strongest influence on the chatter vibration.

References

1) Kato, S., et al., "Characteristics of Chatter Vibration of Lathe Tools," Memoirs of the Faculty of Engineering, Nagoya University, Vol. 38, 1986, p. 208.
2) Kato, S., et al., "Characteristics of Dynamic Cutting Force and Mechanism of Chatter Vibration," Memoirs of the Faculty of Engineering, Nagoya University, Vol. 38, 1986, p. 216.