Memoirs of the Faculty of Engineering, Nagoya University
Vol. 37, No. 1 (1985) — Research Report —

COMPUTATIONAL CODE OF INTERPLANETARY TRAJECTORY OF A SPACECRAFT

—Application to Galileo Mission—

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(Received March 4, 1985)

1. Introduction

Numerical codes to analyze an interplanetary spacecraft trajectory have been made in many organizations including JPL, the code of which is composed of about 330 thousands lines. Recently our laboratory developed a program, aiming at its application to a variety of spacecraft missions, referring to the report of National Aerospace Laboratory.[1] In order to check this program, it was applied to the GALILEO MISSION[6~8] and attempted to yield a fitting with existing NASA trajectories.

2. Analytical method

In this analysis, the trajectory of a spacecraft is generated by the Cowell method. This method integrates the equations of motion in a rectangular coodinate system and obtains the axial components of the position and velocity vectors of the spacecraft under the influence of perturbed accelerations. The equations of motion are integrated by the Gauss-Jackson formula[1], where the initial values are given by the 8 th-order Runge-Kutta method.[1, 4]

The equations of motion are integrated in the following system given by Ref. 1:

(1) The system of time; modified Julian Date.

(2) Coodinate system; Earth 1950. 0 mean equinox and equatorial system.

  (i) Origin; the center of the mass of a celestial body under consideration (Sun).

  (ii) Principal axis; the direction of Earth 1950. 0 mean equinox.

  (iii) Fundamental plane; Earth 1950. 0 mean equator.

The other coodinate systems and their mutual transformations are used only to give output, as shown in the following:

(i) True equatorial coodinate system of date,
(ii) Earth-fixed coodinate system,
(iii) Moon-fixed coodinate system,
(iv) Coodinate system based on the equator of Sun,
(v) Coodinate system based on the planetary equator.

Since the data on the planetary equator and equinox are given only for Venus, Earth, Mars, and Jupiter, the equatorial systems for the other planets can not be excercised in a rigorous sense.

3. Equation of motion

The equation of motion of an interplanetary spacecraft is given in the report of NAL[1] as

$$\ddot{\boldsymbol{r}}_c=-Gm_s\frac{\boldsymbol{r}_c}{|\boldsymbol{r}_c|^3}+\boldsymbol{a}_{PG}+\boldsymbol{a}_{GR}+\boldsymbol{a}_{SR}+\boldsymbol{a}_{SJ}+\boldsymbol{a}_{CF}, \tag{1}$$

where $\boldsymbol{r}_c$ is the position vector of the spacecraft with origin at the mass center of Sun, G the Gaussian gravity constant, and m_s the mass of Sun.

Perturbed accelerations under consideration are:

(i) *The acceleration caused by the planetary gravitational force*;

$$\boldsymbol{a}_{PG}=Gm_s\sum_{n=1}^{9}\left(\frac{m_n}{m_s}\right)\left(\frac{\boldsymbol{r}_n-\boldsymbol{r}_c}{|\boldsymbol{r}_n-\boldsymbol{r}_c|^3}-\frac{\boldsymbol{r}_n}{|\boldsymbol{r}_n|^3}\right), \tag{2}$$

where $\boldsymbol{r}_n$ and m_n are the position vector and the mass of a planet with origin at Sun. In this analysis, the ephemeris of planets[2] is used to obtain $\boldsymbol{r}_n(t)$.

(ii) *The acceleration caused by the effect of spacecraft speed on general relativity*;

$$\boldsymbol{a}_{GR}=\frac{Gm_s}{C^2|\boldsymbol{r}_c|^2}\{[2(1+\gamma)\phi_c-\gamma|\dot{\boldsymbol{r}}_c|]\boldsymbol{r}_c+2(1+\gamma)(\boldsymbol{r}_c\cdot\dot{\boldsymbol{r}}_c)\dot{\boldsymbol{r}}_c\}, \tag{3}$$

where C is the speed of light, and $\phi_c=Gm_s/|\boldsymbol{r}_c|^2$ the Newton potential. This equation is based on the theory of Brans-Dicke where, in this analysis, the parameter $\gamma=1$ corresponding to the Einstein general relativity theory.

(iii) *The acceleration caused by solar radiation*;

$$\boldsymbol{a}_{SR}=\frac{P_0}{|\boldsymbol{r}_c|^2}\left(\frac{A_c}{m_c}\right)(1+\gamma_s+\gamma_d)F(s)\frac{\boldsymbol{r}_c}{|\boldsymbol{r}_c|}, \tag{4}$$

where A_c and m_c are the cross sectional area and the mass of the spacecraft. γ_s and γ_d are the specular and the diffusive reflectivity coefficients, and P_0 the solar radiation pressure constant. $F(s)$ is the function of shadow; if the spacecraft is in the shadow of a planet, we put $F(s)=0$, while $F(s)=1$ otherwise.

(iv) *The acceleration caused by J_2 term of Sun*;

$$\left.\begin{aligned}
\boldsymbol{a}_{SJx}&=\frac{3}{2}Gm_sJ_2R_s^2\left(\frac{x_c}{|\boldsymbol{r}_c|^5}\right)\left[5\left(\frac{z_c}{|\boldsymbol{r}_c|}\right)^2-1\right],\\
\boldsymbol{a}_{SJy}&=\frac{3}{2}Gm_sJ_2R_s^2\left(\frac{y_c}{|\boldsymbol{r}_c|^5}\right)\left[5\left(\frac{z_c}{|\boldsymbol{r}_c|}\right)^2-1\right],\\
\boldsymbol{a}_{SJz}&=\frac{3}{2}Gm_sJ_2R_s^2\left(\frac{z_c}{|\boldsymbol{r}_c|^5}\right)\left[5\left(\frac{z_c}{|\boldsymbol{r}_c|}\right)^2-3\right],
\end{aligned}\right\}\tag{5}$$

where J_2 is the J_2 term of the solar gravity potential and R_s the equatorial radius of Sun.

(v) *Acceleration caused by control thrust*;
(a) *general control force* is written as

$$\left.\begin{aligned}
\boldsymbol{a}_{CF}&=\boldsymbol{F}/\left[m(t)-\frac{1}{2}\dot{m}h_c\right],\\
m(t+h_c)&=m(t)-\dot{m}h_c,
\end{aligned}\right\}\tag{6}$$

where $\boldsymbol{F}$ is the thrust vector, $\dot{m}$ the rate of propellant consumption, and h_c the time step of numerical integration.
(b) *control thrust in the direction of tangent* is expressed as

$$\left.\begin{aligned}
&\boldsymbol{a}_{CF}=\boldsymbol{F}/\left[m(t)-\frac{1}{2}\dot{m}h_c\right],\\
&m(t+h_c)=m(t)-\dot{m}h_c,\\
&F_{cx}=|\boldsymbol{F}|\cos\alpha\cos\beta,\\
&F_{cy}=|\boldsymbol{F}|\cos\alpha\sin\beta,\\
&F_{cz}=|\boldsymbol{F}|\sin\alpha,\\
&\cos\alpha=\sqrt{\dot{x}_t^2+\dot{y}_t^2}/|\dot{\boldsymbol{r}}_t|,\quad \sin\alpha=\dot{z}_t/|\dot{\boldsymbol{r}}_t|,\\
&\cos\beta=\dot{x}_t/\sqrt{\dot{x}_t^2+\dot{y}_t^2},\quad \sin\beta=\dot{y}_t/\sqrt{\dot{x}_t^2+\dot{y}_t^2},\\
&\boldsymbol{F}=(F_{cx},\ F_{cy},\ F_{cz}),\quad |\dot{\boldsymbol{r}}_t|=(\dot{x}_t,\ \dot{y}_t,\ \dot{z}_t).
\end{aligned}\right\}\tag{7}$$

4. Time step and estimation of error

The time step for the numerical integration must be small enough to provide good accuracy, and simultaneously large enough to cover a long flight time. Then, there arises a problem on how to determine an appropriate time step.

Performing the numerical integration of Eq. (1) under the same initial conditions other than time step, the obtained position and velocity vectors $\boldsymbol{r}(t, h)$ and

$\dot{\boldsymbol{r}}(t, h)$ can be described as functions of time t and time step h. Then the errors with respect to position and velocity, $\Delta|\boldsymbol{r}|$ and $\Delta|\dot{\boldsymbol{r}}|$, can be defined, by assuming that $\boldsymbol{r}(t, h=1 \text{ min})$ and $\dot{\boldsymbol{r}}(t, h=1 \text{ min})$ are close to exact solutions, as

$$\Delta|\boldsymbol{r}|=[|\boldsymbol{r}(t, h)|-|\boldsymbol{r}(t, h=1 \text{ min})|]/t, \tag{8}$$

$$\Delta|\dot{\boldsymbol{r}}|=[|\dot{\boldsymbol{r}}(t, h)|-|\dot{\boldsymbol{r}}(t, h=1 \text{ min})|]/t. \tag{9}$$

Table 1 shows daily errors $\Delta|\boldsymbol{r}|$ and $\Delta|\dot{\boldsymbol{r}}|$ for a fixed $t=10$ days, while h is varied up to 2 days.

Table 1. The comparison between the actual errors $\Delta|\boldsymbol{r}|$, $\Delta|\dot{\boldsymbol{r}}|$ and the estimated errors $O(E_{GJ})$, $O(\dot{E}_{GJ})$ for different time steps h.

h	$\Delta\|\boldsymbol{r}\|$ (A.U.)	$\Delta\|\dot{\boldsymbol{r}}\|$ (A.U. /DAY)	$O(E_{GJ})$ (A.U.)	$O(\dot{E}_{GJ})$ (A.U. /DAY)
1 min	—	—	0.261×10^{-9}	0.864×10^{-13}
0.25 day	4×10^{-8}	7×10^{-7}	0.920×10^{-7}	1.092×10^{-7}
0.5 day	8×10^{-5}	1×10^{-3}	0.183×10^{-6}	0.432×10^{-6}
1.0 day	1×10^{-4}	0.5×10^{-3}	0.358×10^{-6}	0.170×10^{-5}
2.0 day	1.1×10^{-4}	2×10^{-3}	0.657×10^{-6}	0.685×10^{-5}

Stepwise roundoff errors generated by using m-th-order Gauss-Jackson formula are evaluated as in Ref. 1:

$$E_{GJ}=\sqrt{\sum_{i=1}^{3}(h^2 a_{m+1}\nabla^{m+1}f_{i,m})^2}, \tag{10}$$

$$\dot{E}_{GJ}=\sqrt{\sum_{i=1}^{3}(hc_{m+1}\nabla^{m+1}f_{i,m})^2}, \tag{11}$$

where a_{m+1} and c_{m+1} are the appropriate coefficients and $\nabla^{m+1}f_{i,m}$ the $(m+1)$-th-order difference in i-th direction $(i=x, y, z)$. In order to compare estimated roundoff errors with actual errors, the daily roundoff errors for the same period of time $(t=10$ days), $O(E_{GJ})$ and $O(\dot{E}_{GJ})$, are defined as follows:

$$O(E_{GJ})=\sum^{10\,days} E_{GJ}/10_{days}, \tag{12}$$

$$O(\dot{E}_{GJ})=\sum^{10\,days} \dot{E}_{GJ}/10_{days}. \tag{13}$$

Table 1 shows the comparison between the actual errors $\Delta|\boldsymbol{r}|$, $\Delta|\dot{\boldsymbol{r}}|$ and the estimated roundoff errors $O(E_{GJ})$, $O(\dot{E}_{GJ})$ for different time steps h, showing whether the main error source is the roundoff or not. According to Table 1, when $h=0.25$ day, $\Delta|\boldsymbol{r}|$ and $\Delta|\dot{\boldsymbol{r}}|$ are sufficiently small where $\Delta|\boldsymbol{r}|$ and $O(E_{GJ})$ as well as $\Delta|\dot{\boldsymbol{r}}|$ and $O(\dot{E}_{GJ})$ are of the same order. When $h=0.5$ day, however, $\Delta|\boldsymbol{r}|$ and $\Delta|\dot{\boldsymbol{r}}|$ become considerably large and, in addition, $\Delta|\boldsymbol{r}|$ and $O(E_{GJ})$ as well as $\Delta|\dot{\boldsymbol{r}}|$ and $O(\dot{E}_{GJ})$ are of different order. From these result, the time step h is chosen to be 0.25 day. And the criteria on how to determine subsequent h are chosen as

$$O(E_{GJ}) < 5.0 \times 10^{-7}, \tag{14}$$

$$O(\dot{E}_{GJ}) < 5.0 \times 10^{-7}. \tag{15}$$

As a result, if $O(E_{GJ})$ or $O(\dot{E}_{GJ})$ exceeds 5.0×10^{-7}, the time step h is reduced to a half of its previous value so that the roundoff error satisfies the criteria (14) and (15).

5. Numerical results

5. 1. Assumptions

Since concrete data on the GALILEO mission are not published, to our knowledge, anywhere including Refs. 6, 7 and 8, the following assumptions have to be introduced to apply this analysis to the mission:

(i) The position of a spacecraft on the launch date is on the line connecting Earth and Sun, while its altitude from the Earth surface is 2 million kilometers.

(ii) Launch energy C_3 is the sum of the spacecraft kinetic energy and the potential energy measured from the Earth surface.

(iii) The characteristics of the Broken Plane Maneuver (BPM) is described as follows:

Total mass; $m_0 = 2000\text{kg}$,

Propellant mass; $m_p = 200\text{kg}$,

The rate of propellant consumption; $\dot{m}_p = 0.04\text{kg/sec}$,

Velocity increment; $\Delta V = 231\text{m/sec}$,

Thrust; $|F| = \dot{m}_p |V_j| = \dot{m} \Delta V / ln\left(\dfrac{m_0}{m_0 - m_p}\right) = 0.0877\text{kg}\cdot\text{km/sec}^2$,

BPM velocity increment inclination with velocity vector; 82 degrees,

BPM is performed 240 days after launch.

(iv) We presume that only the sunshade of the spacecraft GALILEO contributes to the perturbed acceleration $\boldsymbol{a}_{SR}$ caused by solar radiation, where the specular and diffusive reflectivities γ_s and γ_d are considerably high to prevent radiative heating. Thus we assume that the cross sectional area A_c, the specular reflectivity coefficient γ_s and the diffusive reflectivity coefficient γ_d of the spacecraft are

$$A_c = 4\ \text{m}^2,$$

$$\gamma_s = \gamma_d = 0.9.$$

5. 2. Numerical results

Figs. 1~3 show the trajectories the GALILEO spacecraft leaving Earth with an ecliptic inclination 2.9 degrees on 1986/5/19, 21 and 23. 240 days after launch, BPM is performed to provide the inclination equal to that of Jupiter at the instant of arrival. After encounter with Jupiter, the trajectory of the spacecraft is changed by the acceleration caused by Jupiter, to an amount depending on the geometry of encounter.

Figs. 4~6 show the trajectories of the spacecraft using a coodinate system with origin at Jupiter. For the 1986/5/19 launch (Fig. 4), the trajectories of the

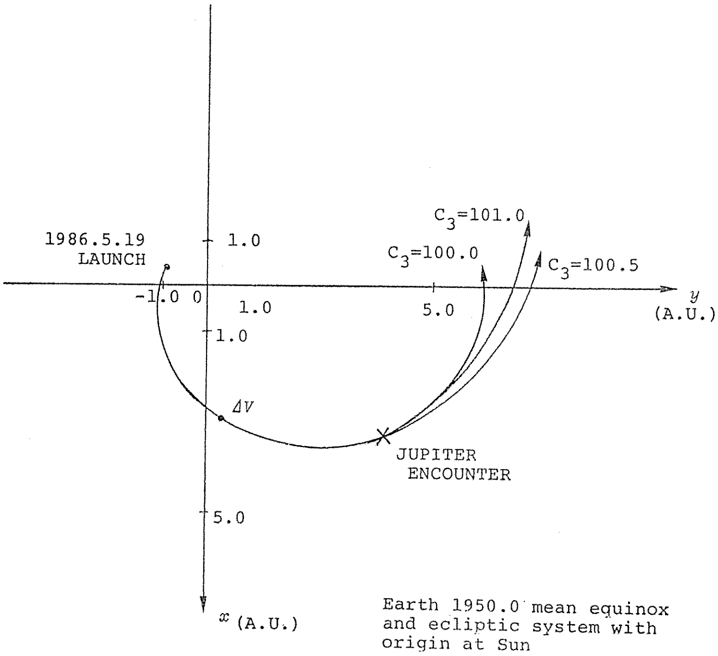

Fig. 1. The trajectories of the GALILEO spacecraft leaving Earth with an ecliptic inclination 2.9 degrees on 1986/5/19. Launch energy C_3=100.0, 100.5 and 101.0 km^2/sec^2. BPM (ΔV=231 m/sec) is performed 240 days after launch.

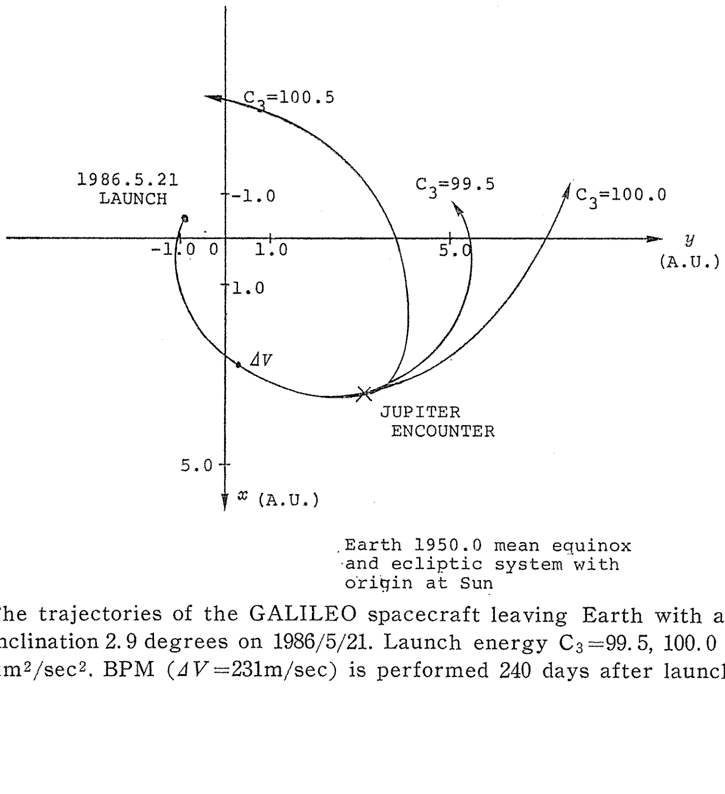

Fig. 2. The trajectories of the GALILEO spacecraft leaving Earth with an ecliptic inclination 2.9 degrees on 1986/5/21. Launch energy C_3=99.5, 100.0 and 100.5 km^2/sec^2. BPM (ΔV=231m/sec) is performed 240 days after launch.

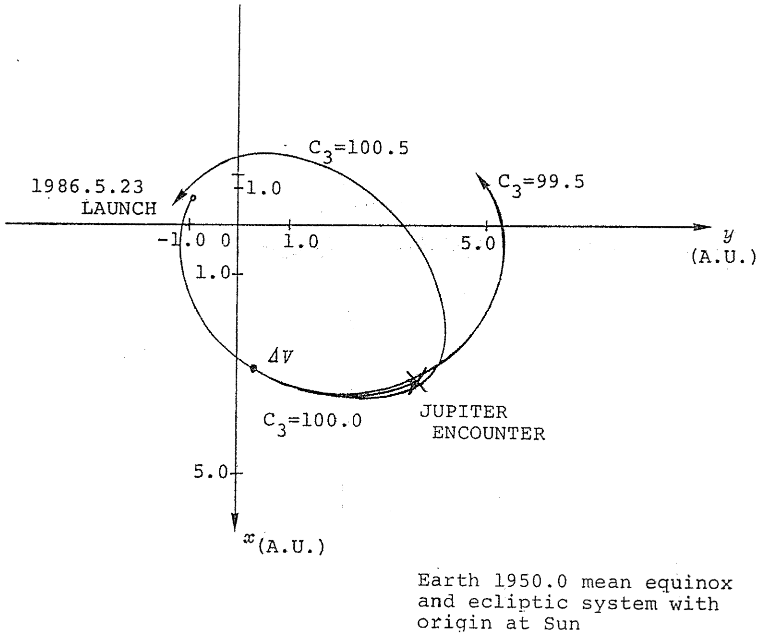

Fig. 3. The trajectories of the GALILEO spacecraft leaving Earth with an ecliptic inclination 2.9 degrees on 1986/5/23. Launch energy C_3=99.5, 100.0 and 100.5 km^2/sec^2. BPM ($\varDelta V$=231 m/sec) is performed 240 days after launch.

spacecraft approach Jupiter, when the launch energy C_3 is increased. Thus it is readily understood that in order to hit the spacecraft to Jupiter, the required launch energy C_3 will be more than 101 km^2/sec^2. Fig. 5 shows the trajectories of the 1986/5/21 launch. In the case of $C_3=100$, the spacecraft approaches Jupiter from the inside of the heliocentric trajectory of Jupiter and leaves to its outside, while in the case of $C_3=100.5$ the spacecraft approaches Jupiter from the outside of the heliocentric trajectory and leaves to the inside. These results indicate that in order to collide the spacecraft with Jupiter, the launch energy C_3 will be 100~100.5. For the 1986/5/23 launch (Fig. 6), it is evident that the spacecraft hits Jupiter when $C_3=100$ km^2/sec^2.

If we look at an aspect of gravity-assisted maneuver or swingby, heliocentric energy change due to planetary encounter is given by

$$\varDelta U = v_p v_h \hat{P} \cdot (\hat{O} - \hat{I}), \tag{16}$$

where v_p and v_h are the planet velocity and the spacecraft approach velocity, while $\hat{P}$, $\hat{O}$, and $\hat{I}$ are the unit vectors in the directions of the plant velocity, the spacecraft outgoing and incoming asymptotes, respectively, as shown in Fig. 7.

It is convenient to write the Eq (16) in the form

$$\varDelta U = E^* f, \tag{17}$$

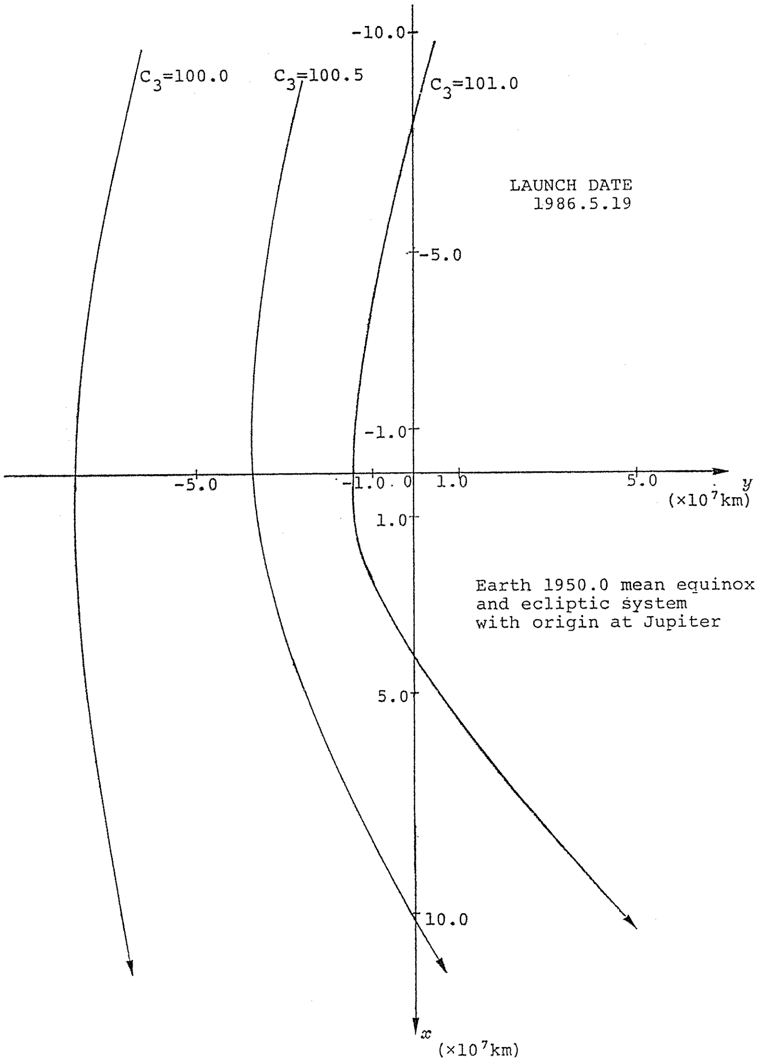

Fig. 4 The Jovicentric orbits of the GALILEO spacecraft leaving Earth with an ecliptic inclination 2.9 degrees on 1986/5/19. Launch energy C_3=100.0, 100.5 and 101.0 km²/sec². BPM (ΔV=231 m/sec) is performed 240 days after launch.

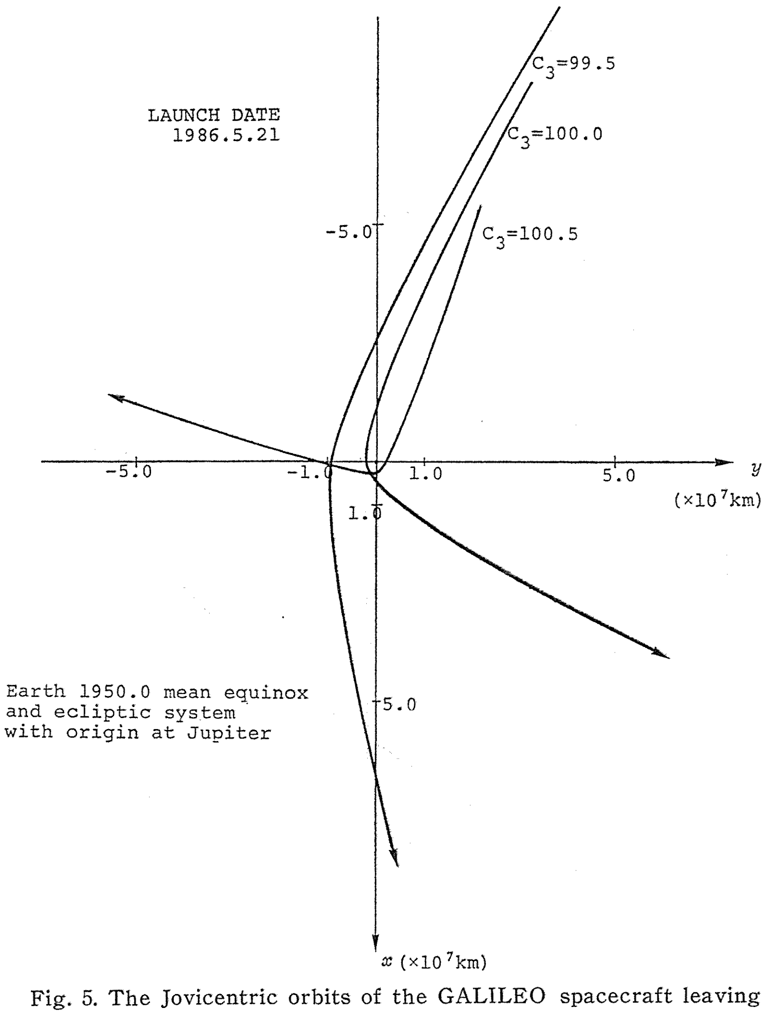

Fig. 5. The Jovicentric orbits of the GALILEO spacecraft leaving Earth with an ecliptic inclination 2.9 degrees on 1986/5/21. Launch energy $C_3=99.5$, 100.0 and 100.5 km^2/sec^2. BPM ($\Delta V=231$ m/sec) is performed 240 days after launch.

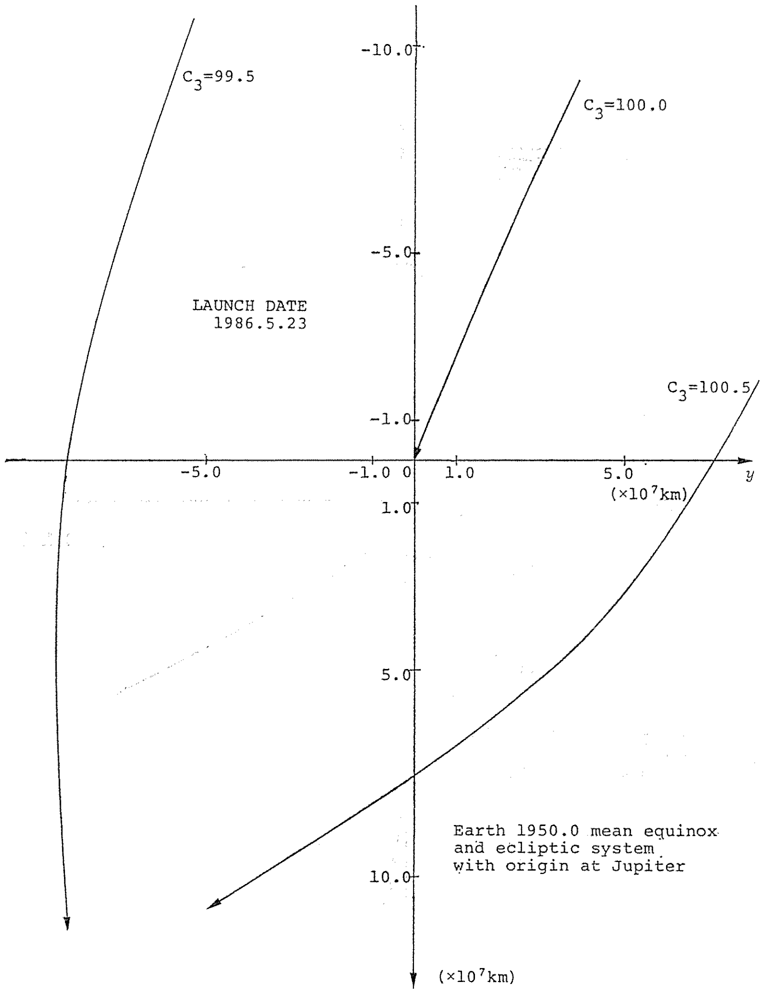

Fig. 6. The Jovicentric orbits of the GALILEO spacecraft leaving Earth with an ecliptic inclination 2.9 degrees on 1986/5/23. Launch energy C_3=99.5, 100.0 and 100.5 km^2/sec^2. BPM (ΔV=231 m/sec) is performed 240 days after launch.

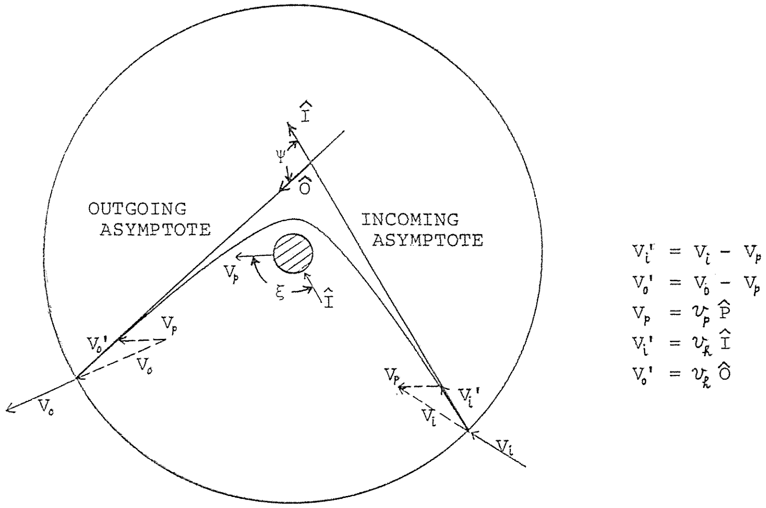

Fig. 7. Geometry of encounter.

where $E^* \equiv 2v_h v_p$ represents the maximum energy increase for the given planet and spacecraft velocities, and $f \equiv \hat{P} \cdot (\hat{O} - I)/2$ is an energy index determining the magnitude and sign of the energy change achieved in the encounter.

Heliocentric energy U of a spacecraft is given by

$$U = \frac{1}{2}v^2 - \frac{Gm_s}{r}, \tag{18}$$

where v is the velocity of the spacecraft, G the Gaussian gravity constant, m_s the mass of Sun, and r the distance between the spacecraft and Sun. Using Eq. (16), the heliocentric energy change ΔU is calculated by

$$\Delta U = U_{post} - U_{pre}, \tag{19}$$

where U_{pre} and U_{post} are the *pre-* and *post-*encounter heliocentric energies.

Table 2 shows the relations between f, U_{pre}, U_{post}, ΔU and the date of encounter for different launch dates and different launch energies. From the result that all U_{post} are less than zero, these trajectories are all elliptic and the spacecraft can not escape from the solar system. According to Table 2, Eq. (17) accurately holds between ΔU and f. Using Eq. (16), the spacecraft approach velocity v_h can be estimated as about 6 km/sec. *Pre-*encounter heliocentric energy U_{pre} varies with the launch energy C_3, but these values are $140 \sim 150$ km^2/sec^2. From the fact that Jupiter velocity $v_p \approx 13.6$ km/sec and $v_h \approx 6$ km/sec, the maximum energy increase $E^* \approx 162$ km^2/sec^2. Therefore, it is readily seen that if a small but appropriate Jupiter orbit insertion maneuver and a subsequent navigation orbit trim maneuver are performed, U_{post} becomes positive, and the spacecraft will escape from the solar system.

Table 2. The relations between f, U_{pre}, U_{post}, ΔU and the date of encounter for different launch dates and different launch energies.

LAUNCH DATE	C_3 (km²/sec²)	f	U_{post} (km²/sec²)	U_{pre} (km²/sec²)	ΔU (km²/sec²)	ARRIVAL DATE
1986. 5.19	100.0	0.23	−108.2	−146.5	38.3	1988. 6.18
1986. 5.19	100.5	0.38	−78.5	−144.1	65.6	1988. 6. 2
1986. 5.19	101.0	0.48	−59.4	−142.0	82.6	1988. 9.12
1986. 5.21	99.5	0.16	−122.6	−148.4	25.8	1988. 9.12
1986. 5.21	100.0	0.60	−53.1	−146.5	93.4	1988. 8.11
1986. 5.21	100.5	0.32	−95.7	−144.1	48.4	1988. 7.18
1986. 5.23	99.5	0.13	−124.7	−148.4	23.7	1988.11. 2
1986. 5.23	100.0	—	—	−146.3	—	1988. 9.25
1986. 5.23	100.5	0.04	−137.9	−144.1	6.2	1988. 9. 7

References

1) Koichi Matsushima, Toru Shiho, Masaaki Murata and Sumio Takeuchi; Space Trajectory and Mission Analysis Program "STANPS", National Aerospace Laboratory Report No. 359, 1978.
2) P. K. Seidermann, L. E. Doggot and M. R. DeLuccia; Mean Elements of the Principal Planets, The Astronomical Journal Vol. 79, No. 1, p. 57–60, 1974.
3) P. M. Fitzpatrick; Principles of Celestial Mechanics, Chaps. 2, 3 and 6, Academic Press, 1972.
4) E. Fehlberg; Classical Fifth-, Sixth-, Seventh-, and Eighth- Order Runge-Kutta Formula with Stepsize Control, NASA TR R-287, 1968.
5) Masahisa Torao; Mathmatics of Space Navigation, Chap. 11, Morikita Press, 1970.
6) W. J. O'Neil and R. T. Mitchell; Galileo Mission Overview, AIAA 21st Aerospace Sciences Meeting (Reno, 1983), Project Galileo Mission and Spacecraft Design (invited session), JPL D-0518.
7) L. J. Miller, J. K. Miller and W. E. Kirhofer; Navigation of Galileo Mission, ibid.
8) L. A. D'Amario and Dennis V. Byrnes; Interplanetary Trajectory Design for the Galileo Mission, ibid.
9) G. A. Frandro; Fast Reconnaissance Mission to the Outer Solar System Utilizing Energy Derived from the Gravitational Field of Jupiter, Astronautica Acta, Vol. 12, No. 4, p. 329–337, 1966.