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SUPPLEMENTS TO "THEORY OF SECONDARY FLOW IN CASCADES"

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Abstract

On the previous report "Theory of Secondary Flow in Cascades"[1], some matters to be supplemented, improved or revised are explained, together with the further results which were obtained since then.

The reason why the theory of secondary flow in cascades has not yet fully succeeded in explaining the three-dimensional character of cascade flows, the fact that we should notice the vortex metamorphosis and the trailing shed vortex being matters which have no direct connection each other, and the wake of cascade blade is the key to solve the relation between the axisymmetric flow and the actual three-dimensional flow in turbo-machinery, are stated in this report.

CONTENTS

1. Prologue

On the previous report 'Theory of Secondary Flow in Cascades"[1], some matters to be supplemented, improved or revised will be explained in this report, together with the further results which were obtained since then.

One of the reason why the theory of secondary flow in cascades has not yet fully succeeded in explaining the three-dimensional character of cascade flows, is of course in the assumption of inviscid perfect fluid disregarding viscosity. But there exists another important point to be noticed, that this theory is a first approximation theory and there are often carelessnesses in the treatment of small quantities of first order. It is logically well understood that the secondary flow theory has the destiny that the secondary flow is the difference between a large quantity and another large quantity and therefore we need the greatest prudence in the treatment of small quantities, but unfortunately almost all reports published in the past had defects in this point. Especially in spite of the variation of blade circulation and the trailing shed vortex ensued on it being the most important part of the secondary flow, the consideration on the variation of blade circulation (small quantity of first order) was missing in these reports, therefore the stories became ambiguous and we got shady impressions.

2. On the Definition of Secondary Flows

The secondary flow is ordinarily understood being the difference between the ideal flow and the actual flow which is the consequence of existence of boundary layers, but this type of expression of definition has defects that the answer differs according as the definition of ideal flow differs, and besides there may be problems because it is the difference between a large quantity and another large quantity as mentioned above.

The more intelligible way to define the secondary flow is that which uses the idea of vortex. Replacing the inflow boundary layer of cascade with the vortex, we consider the latter to be a quantity of $O(\varepsilon)$ [Order ε]. To get the secondary flow we examine how this vortex is carried by the flow to transform (metamorphose) into new vortices, and what sort of vortices will newly be shed from blades, and all these vortices are expressed by quantities of $O(\varepsilon)$. The flow carrying the vortex to metamorphose is a quantity of $O(1)$, therefore even if there exists an error of $O(\varepsilon)$ on the carrying flow the effect on metamorphosed vortices will be of $O(\varepsilon^2)$ which may be negligible. This is the reason why we use the potential flow etc. as a carrier of vortices. The consideration of flow from the standpoint of vortex is a good way, as mentioned above, to treat quantities of small order without any confusion.

W. R. Hawthorne[2] made a definition that the streamwise component of the vortex is the secondary vortex, and the flow induced by this vortex component is the secondary flow. This is a very clear and good definition. Therefore we must recognize that there may be cases in which the carrier flow itself containes secondary vortices or secondary flows in it.

The definition differs from an author to another in some cases, for an example the definition published by L. H. Smith[3] caused discussions between him and the Cambridge group the chief of which is Hawthorne.[4,5] Because the definition by Smith contains obscure points the author thinks it is better to employ the definition by Hawthorne. (see Appendix A-I) (The normal component to the flow of passage vortex is included in the secondary vortex family in this report. This is the vortex corresponding to the boundary layer. See the next chapter.)

3. On the Vortex System of Secondary Flow in Cascades

The vortex system in cascade is illustrated in Fig. 1. $A_2'B_2$ and $A_2'A_2$ are called passage vortex and trailing filament vortex respectively. Secondary vortices contained in the downstream of cascade are not only these two but the trailing shed vortex, which corresponds to the variation of blade circulation if it varies along the blade span, is in the blade wake together with the trailing filament vortex. The author understands that these names of vortices were introduced by Hawthorne.

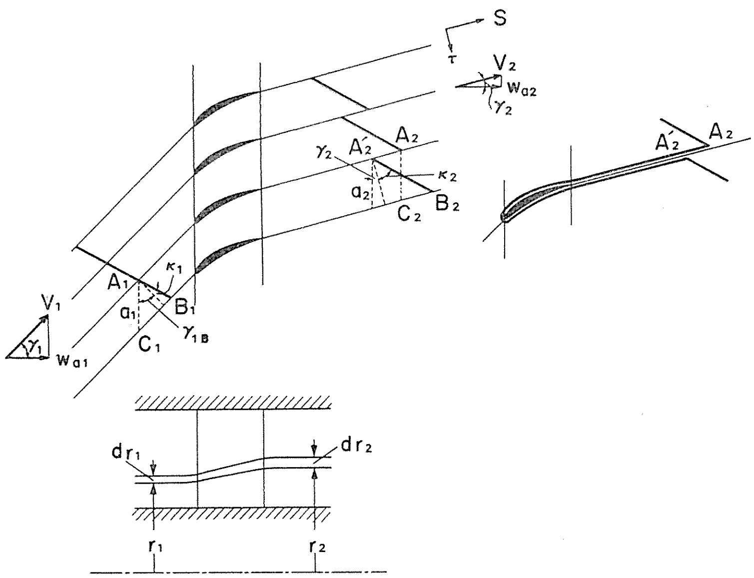

Fig. 1. Vortex system of cascade.

Because the passage vortex and the trailing filament vortex are criated by the metamorphosis of vortex A_1B_1 in the upstream the author named them "metamorphic vortex". (In the previous report[1] he used a name 'quasi vortex" because these vortices are not the entire of secondary vortices. But "metamorphic vortex" is regarded to express the entity more correctly.)

The relation between the strength of trailing vortex (sum of trailing filament vortex and trailing shed vortex) Γ (strength of vortex contained in unit span) and the spanwise velocity at blade trailing edge Δw_T is[2]

$$\Gamma = 2\Delta w_T \tag{1}$$

(see Fig. 2)

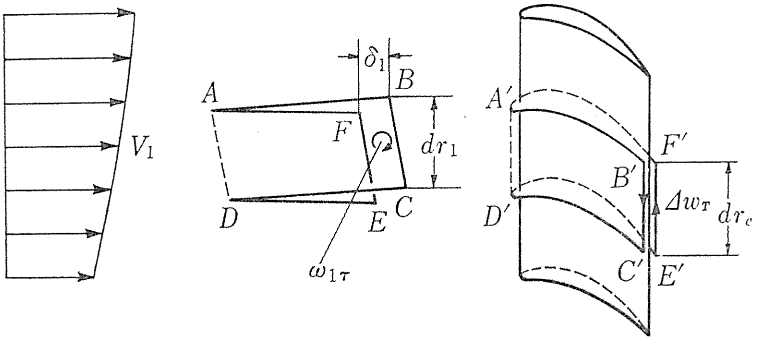

Fig. 2. The flow containing vortex passing through the blade.

Summing up the vortices in downstream of cascade, we have Table 1. The secondary vortices which have deep relations to the secondary flow are vortices parallel to the flow and asterisk marks * are attached in the table. Only vortex having a component normal to the flow is passage vortex and represents the boundary layer in exit flow.

- secondary vortex family
  - metamorphic vortex
    - passage vortex
      - normal component to the flow
      - parallel component* to the flow
    - trailing filament* vortex (parallel to the flow) — trailing vortex
  - shed vortex
    - trailing shed* vortex (parallel to the flow) — trailing vortex

Table 1. Secondary vortex family.

Expressing both components of the passage vortex (vorticity) by $\omega_{2p\tau}$ (normal component) and ω_{2ps} (streamwise component) we have[1)]

$$\omega_{2p\tau} = \omega_{1\tau} \frac{\cos\gamma_2}{\cos\gamma_1} \frac{r_2}{r_1} = \omega_{2\tau} \tag{2}$$

This can be obtained merely from inlet and exit flow conditions. Where $\omega_{2\tau}$ is the vortex component normal to flow and $\omega_{2\tau} = \omega_{2p\tau}$ as explained in the above. Integrating equation (2) we can consider the boundary layer growth in cascade. [7. 4. 5 (p. 225) in the previous report[1)]]

ω_{2ps} has relations to the passage configuration, but if there is no streamwise vorticity (secondary vorticity) in upstream we often use the following equation

$$\omega_{2ps} \doteqdot 2\omega_{1\tau}(\gamma_1 - \gamma_2) \tag{3}$$

There is a tendency of the theory to estimate the value of ω_{2ps} too small by this equation as explained later in Appendix II, and this tendency is supposed to depend on the deceleration rate or stagger angle of cascades. Suitable empirical coefficient may be needed to this equation.

Let us consider the passage vortex and the trailing filament vortex which are constituents of the metamorphic vortex, the secondary flow induced by each of them has a value corresponding to the configuration of blade passage respectively, but if we put these two together and consider the entire induced flow or mean value [this means that we consider the trailing (filament) vortex which exists in a limited region to be distributed as vorticity] we have a value which is the function of only inlet and outlet flow conditions (for example, inlet and outlet flow angles) regardless of cascade conditions or blade profiles etc..

Furthermore, if we calculate the blade circulation, we find that the secondary flow induced by metamorphic vortex produces no blade circulation change. We must be aware that if there exists any actual blade circulation change, this has 1 : 1 correspondence to the trailing shed vortex and has no relation to the metamorphic vortex.

The upshot is that the blade circulation is not decided by the secondary flow considerations which deal with the vortex metamorphosis, but we must consider, for example, a matter such as Kutta's condition in non-uniform stream which determines the blade circulation.

4. On the Axisymmetric Flow Theory

The reason why the axisymmetric flow becomes a problem of discussion is that there is an idea in which the difference between the actual flow in a turbo-machinery with finite blade spacing and the axisymmetric flow is the secondary flow. In other words we shall be able to get the actual turbo-machinery flow if we add the secondary flow in the blade passage of finite spacing to the axisymmetric flow.

The first advocate of this idea was L. H. Smith[3], but the practical technique written in his report was not clear. Hawthorne[4] and the author[1] pointed out that there might be misunderstandings in his report. Smith's opinions were that the discrepancy between Hawthorne's theory and Smith's is only the difference of definition of secondary flow, and the author's idea has an error in the definition of axisymmetric flow. Author's interpretation is explained in Appendix I, and there still remain questions on Smith's practical technique represented by the equation [9] in Appendix I.

The authr wishes the reader not to judge that theories proposed by Cambridge group headed by Hawthorne are perfect. There are questions on the treatment of trailing shed vortex being imperfect. (The strength of trailing shed vortex is decided by the Kutta's condition in a non-uniform flow, and not by the consideration of metamorphic vortex.)

We may understand from descriptions explained in 2. that the axisymmetric flow is useful as an approximate flow (base flow). And what is important in this instance is that we must be aware the vortex being contained in the axisymmetric flow i.e. base flow itself, and its component parallel to the flow must be regarded as the secondary vortex. The upshot is that the secondary vortex in axisymmetric flow is indispensable to get secondary flows in actual turbo-machinery of finite blade spacing.

There was no one who got the trailing shed vortex in axisymmetric flow for long time. Reports published, in which many authors expressed that they got it, were doubtful. In fine, if we regard the axisymmetric flow being the case with infinitesimal spacing of the finite spacing cascade, and try to get the limiting value of vortex system of secondary flows, we always find the trailing vortex to be disappeared by this approach. Furthermore the axisymmetric theory itself has substantially no way to get it. (See chapter 4. in the previous report.[1])

The circumstances how the trailing shed vortex was obtained in axisymmetric flow by the author was explained in the previous report[1]. If the exit flow angle γ_{2e} satisfies the relation $\tan\gamma_{2e}=K/r_2$ (K: constant), that is the exit flow being of free vortex type, there is no streamwise vortex in the exit flow (including trailing shed vortex) whatever vortices are contained in the inflow. This is an important result obtained from the author's study. (This result is on the case of axisymmetric flow, but of course can be applied to the linear cascade of infinitesimal spacing.) In fine we can regard that there exists the vortex rectification process in the cascade of large aspect ratio.

5. On the Secondary Flow in Linear Cascades

Vortices in the flow of linear cascade can be regarded as a special case of that of turbomachine cascade. But we must pay attention to the expectation that we can calculate the trailing shed vortex under the idea mentioned below.

The author explained in the previous report[1] that if we consider a Trefftz plane normal to the flow at the exit of linear cascade shown in Fig. 3, the secondary vortex is the one normal to this plane and the secondary flow is regarded to

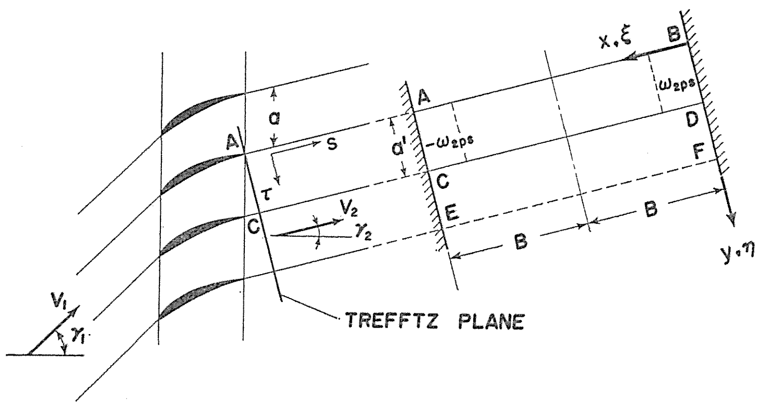

Fig. 3. Trefftz plane.

be calculated as the two-dimensional flow in this plane induced by this vortex. (Hawthorne's proposal[2]) The two-dimensional flow velocities induced at boundaries AB and CD correspond to Δw_T in equation (1), and therefore we can get the sum of trailing filament and trailing shed vortices.

The author also explained in the previous report[1] that concerning the form of boundary ABDC (same on CDFE, ... etc.) induced velocities in y-direction at trailing vortex planes CD, EF etc. are 0, and therefore these planes may be kept flat. At the same time we can say from the fact mentioned above that these boundaries (viz. wakes) will be kept at the same positions regardless of the secondary vortices existing or not. (Non-existence of secondary vortex means two-dimensional cascade flow.)

We must be aware that a large assumption is included in the base of these results. The fact, that we consider the flow in a rectangle under the condition that there is no induced velocity in y-direction at AB, CD ...... etc., means that the Kutta's condition in the flow containing secondary flows is completed by the non-existance of flow in y-direction. But it is not clear that if this way of thinking is good. It is supposed that if the blade is very thin or its trailing edge has cusped form this idea may be correct. Then assuming the idea being correct, we arrive at an attractive result that because sides AB, CD ...... are not deformed by the secondary flow as mentioned above the direction of wake of cascade must point out the direction of wake or the direction of exit flow of two-dimensional cascade (because there is no secondary flow or flow in y-direction in the two-dimensional cascade). We can say eventually that the exit flow direction of two-dimensional cascade can be obtained easily and directly from the direction of wake of non-two-dimensional cascade without complicated experiments such as boundary layer suctions in the hope to actualize the two-dimensional cascade flow.

The experiment[7] to verify the idea mentioned above is explained in the Appendix II.

Fig. 5 is the example of results of exit flow measurements. (Symbols are illustrated in Fig. 4.) If we assume the position of wake being that of the maximum loss (indicated in the figure by dotted line ......), we can say that the non-deformation of wake is realized near the center of span. Although the dotted line indicating maximum loss position is bent at the vicinity of side walls because inlet side wall boundary layers are forced to turn into the blade wake by the secondary flow, if we select the line indicating $\varsigma_2 = 0.1$ (where ς_2 is total pressure loss coefficient[6]) (pointed out by ↑ in the figure) to point out the wake position we find the wake being almost non-deformed.

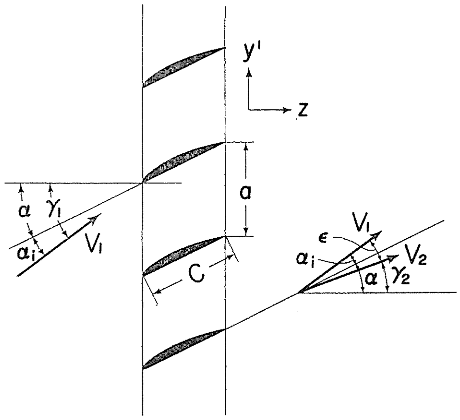

Fig. 4. Symbols

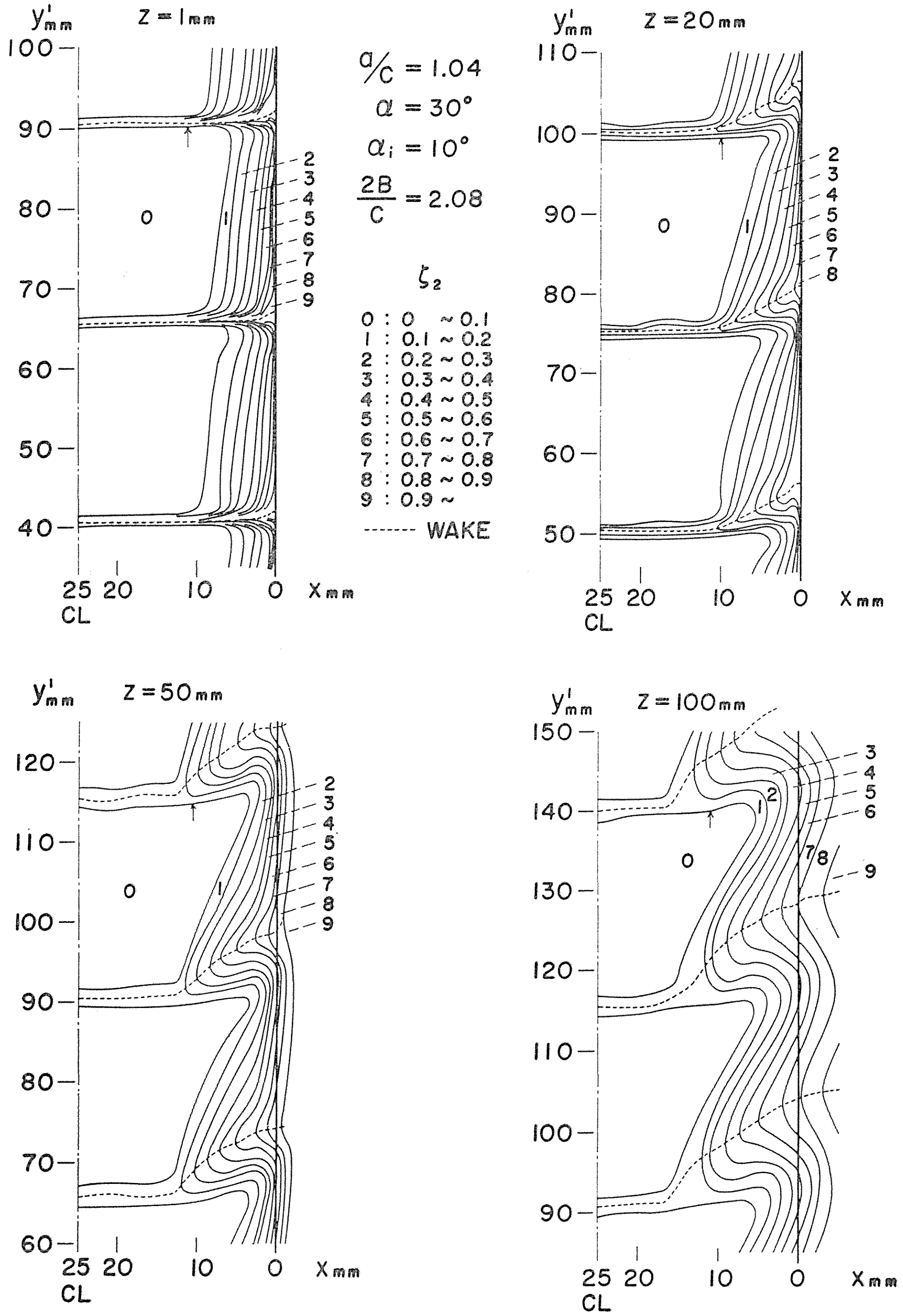

Fig. 5. Loss coefficient distribution in exit flow.

6. Epilogue with the Forecast of the Relation between Axisymmetric Flow and Secondary Flow

Let us now consider a case in which preserving the similarity of sectional geometry of linear cascade, the blade spacing is brought to the infinitesimal. This corresponds to the axisymmetric flow of free vortex type turbo-machine and therefore there exists no secondary vortex in the exit flow in accordance with the result mentioned in Chapter 4. The infinitesimal spacing means the aspect ratio being infinite and the flow in cascade is same as that in two-dimensional cascade.

In other words, as mentioned above, provided the direction of wake of cascade having secondary flow in it coincides with that of two-dimensional cascade, we can say that the former coincides with the exit flow direction of the cascade of infinitesimal blade spacing. Or if we extend our idea to the turbo-machinery the former coincides with the axisymmetric exit flow direction of the turbo-machinery.

Let us add a few comments that, assuming the axisymmetric flow is known, if we employ the cascade blade arrangement (of finite spacing) which has the same two-dimensional exit flow direction as the axisymmetric exit flow one, what kind of flow will be developed? Probably the direction of wake of that cascade (of finite spacing) will be same as the direction of axisymmetric flow. There may exist secondary flows induced by secondary vortices between these wakes. The circumstances are illustrated in Fig. 6 (a), (b), and (c).

(a) is a sketch showing the exit flow of turbo-machine cascade. AE, BF, CG, …… in the figure indicate trailing edges of blades. Planes AETP, BFUQ, etc. are ones of blade wakes. PQUT is one section of the flow between two adjoining wakes, and secondary vortices (passage vortices) are included in it.

(b) is the case in which the blade segment is same as that of case (a) but its size is reduced to infinitesimally small keeping the solidity constant. This corresponds to the actuator disk model. All small blades have wakes and the form of these wakes is expected to be same as wakes of case (a).

(c) is the axisymmetric cascade which has the same blade axial length as case (a) but its spacing and blade thickness are infinitesimal. We make the exit flow direction of this axisymmetric cascade being same as the two-dimensional exit flow direction of the blade segment of case (a). The form of each wake in this case is expected being same as that of case (a).

The difference between (b) and (c) is the difference of axial dimensions, and when the radial displacement of stream line is large, (c) is supposed being better model than (b).

The idea mentioned above is passably a forecast and we need a verification for it. But, provided this idea comes true, the wake direction of turbo-machine cascade can be calculated by the axisymmetric theory, and what we should do in the next place is the correction on induced velocities by passage vortices. Because this induced velocity has no character to deform the wake, the calculation is expected to be simple.

Fig. 7 is the reproduction of the induced velocity at the center of span of linear cascade printed in the previous report[1)] at Fig. 7-12, and this suggests us that if the aspect ratio of blade passage (let's consider a section perpendicular to

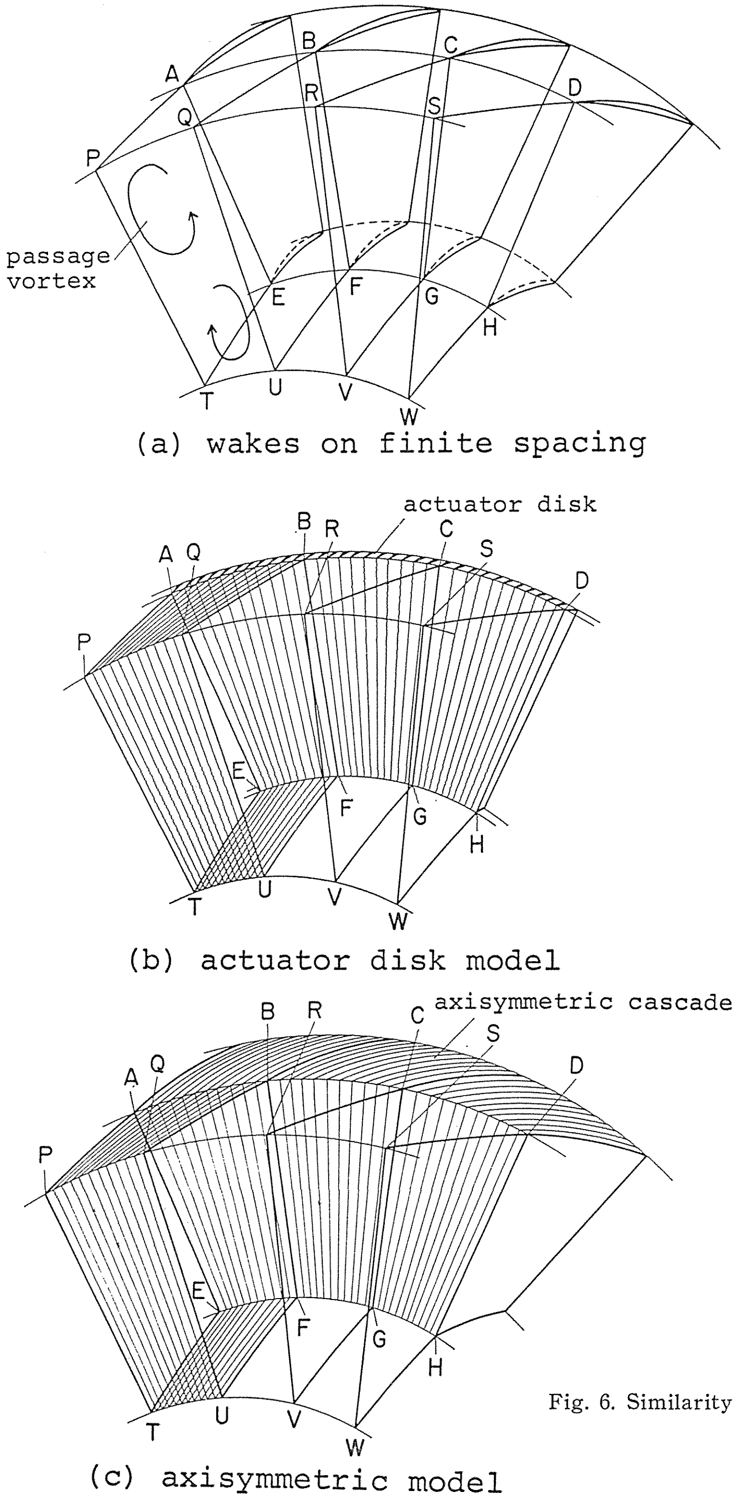

Fig. 6. Similarity of wakes.

the flow) is larger than 4 this induced velocity is rather small, and therefore we can expect to get pretty good estimations only from the solution of axisymmetric theory without any correction on secondary flows.

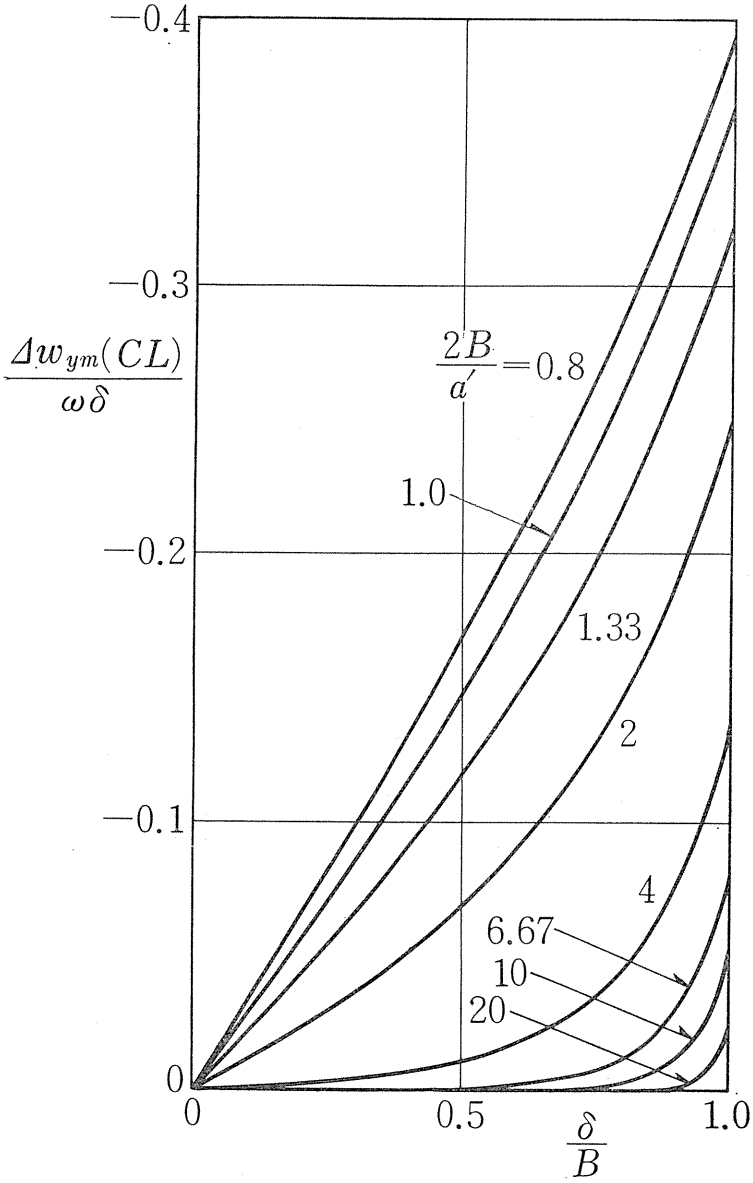

Fig. 7. Mean induced velocity at the center of span.

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Appendices

A-I On Smith's Definition

From Smith's report[3] his definition on the secondary vorticity is the difference between the actual vorticity and the axisymmetric vorticity as expressed by the following equation

$$\overleftarrow{\zeta}_s=\overleftarrow{\zeta}_{b2}-\overleftarrow{\zeta}_2 \qquad [9]$$

(p. 1069 in the ASME paper 1955. ρ_2 is removed for the sake of simplicity.)

Where ζ_s : secondary vorticity

ζ_b : vorticity obtained by the blade-to-blade solution which is the best approximation of the actual flow

ζ : vorticity of the axisymmetric flow

subscript 2 : after a blade row

If we express afresh this by Hawthorne's expressions, we have

$$[\text{secondary vorticity}]^{(a)}=[\text{actual passage vorticity}]^{(b)} - [\text{axisymmetric vorticity}]^{(c)} \qquad [A]$$

The expression (b) or ζ_{b2} comes from what he is saying in his report.

If we think that the axisymmetric blade row is the one having the infinitesimally small blade spacing but its configuration is similar to the actual one, we recognize that [axisymmetric vorticity]$^{(c)}$ is consisted of [axisymmetric passage vorticity]$^{(c-1)}$ and [axisymmetric trailing (filament and shed) vorticity]$^{(c-2)}$. Then equation [A] becomes

$$[\quad]^{(a)}=[\quad]^{(b)}-[\quad]^{(c-1)}-[\quad]^{(c-2)} \qquad [B]$$

But [actual passage vorticity]$^{(b)}$ is equal to [axisymmetric passage vorticity]$^{(c-1)}$ because of the similarity of configuration in the sense of first approximation. (Passage vorticity is the function only of turning angle provided $\varsigma_{\perp 1}$ is same. Where $\varsigma_{\perp 1}$ is the vorticity component perpendicular to the flow before a blade row.) Therefore we have

$$[\text{secondary vorticity}]^{(a)}= -[\text{axisymmetric tailing vorticity}]^{(c-2)} \qquad [C]$$

This is queer!?

The followings are the author's opinions: In the treatment of secondary flow problems, the largest problem is the trailing vortex, especially the trailing shed vortex. Of course the latter has close relation to the problem of blade circulation and the Kutta's condition to decide the circulation. From this reason the approach made by Smith (p. 1067 ASME paper 1955) was very good and reliable, and also his expressions "actual vane circulation" and mere "vane circulation". The difference of these two is most important, and is always the point of question of almost all reports published. In Hawthorne's or Horlock's reports care was not paid on this point and the author feels doubt to them. We must be aware of that the indiscrimination of the both will result in the disappearance of the trailing shed vortex.

From the idea mentioned above, the author thinks, the comparison of any two theories (for example Smith's with Hawthorne's et al.)[4),5),8)] should be done on cases which contain the trailing shed vortices in them, otherwise they have less meanings because the most important point is neglected.

The author's comment on Smith's definition in the previous report[1)] (p. 195) must be revised as mentioned above, in which there was a misunderstanding on the axisymmetric flow.

A-II A Method of Consolidating The Exit Flow Angle of Cascade[7)]

1. Apparatus and Procedure

High speed cascade wind tunnel which belongs to the author's laboratory was used. The detailed exposition is found in the Reference 6), and the sketch is shown in Fig. A-1. Symbols are illustrated in Fig. 4. The blade profile used was RAF-6. Inflow velocity was about 70m/s.

In the report of Reference 6) the exit flow was measured by the yaw-meter, and in the present experiment the direction of wake is measured under completely same cascade conditions as the former. Cascade conditions and inlet boundary layer displacement thicknesses are shown in Table A-1. (Blade span is 50 mm, chord lengths are 24, 36, 48 and 60 mm.) A sample of inlet boundary layer is shown in Fig. A-2.

a/C	AR	$Re \times 10^{-5}$	α	α_t	γ_1	δ^*/B (%)
1.04	0.83	2.7	0°	0°	0°	6.0
				5	5	6.8
				10	10	6.3
				15	15	6.7
			30°	0	30	5.9
				5	35	5.5
				10	40	5.2
				15	45	5.1
			50°	0	50	5.0
				5	55	5.0
				10	60	4.6
				15	65	4.2
			60°	0	60	4.4
				5	65	4.1
				10	70	4.0
	1.04	2.1	0°	0°	0°	5.3
				5	5	5.3
				10	10	5.3
				15	15	5.2
			30°	0	30	4.9
				5	35	5.3
				10	40	4.3
				15	45	4.1
			50°	0	50	3.9
				5	55	3.9
				10	60	3.9
				15	65	4.0
			60°	0	60	3.7
				5	65	3.5
				10	70	3.7
0.83	0.83	2.8	0°	10°	10°	6.2
			60°	5	65	3.8
				10	70	3.8
	1.04	2.2	0°	10°	10°	6.2
			60°	5	65	4.0
				10	70	4.3
1.25	0.83	2.7	60°	5°	65°	4.1
				10	70	4.0
	1.04	2.1		5°	65°	4.0
				10	70	3.6
1.67	1.04	2.1	60°	5°	65°	4.2
	1.39	1.6				3.9
	2.08	1.1				4.1

a/C	AR	$Re \times 10^{-5}$	α	α_t	γ_1	δ^*/B (%)
1.04	1.39	1.5	0°	0°	0°	5.8
				5	5	5.9
				10	10	6.1
				15	15	5.9
			30°	0	30	5.8
				5	35	5.1
				10	40	5.4
				15	45	5.3
			50°	0	50	5.1
				5	55	4.7
				10	60	4.0
				15	65	4.1
			60°	0	60	4.2
				5	65	3.8
				10	70	3.8
	2.08	1.0	0°	0°	0°	5.3
				5	5	5.1
				10	10	5.2
				15	15	5.1
			30°	0	30	5.2
				5	35	5.0
				10	40	4.9
				15	45	4.7
			50°	0	50	4.9
				5	55	4.6
				10	60	4.3
				15	65	4.2
			60°	0	60	4.2
				5	65	3.7
				10	70	3.7
0.83	1.39	1.6	0°	10°	10°	6.3
			60°	5	65	3.7
				10	70	3.7
	2.08	1.1	0°	10°	10°	5.3
			60°	5	65	4.2
				10	70	5.9
1.25	1.39	1.6	60°	5°	65°	4.5
				10	70	3.8
	2.08	1.1		5°	65°	3.9
				10	70	3.5

Table A-1 Experimental conditions.

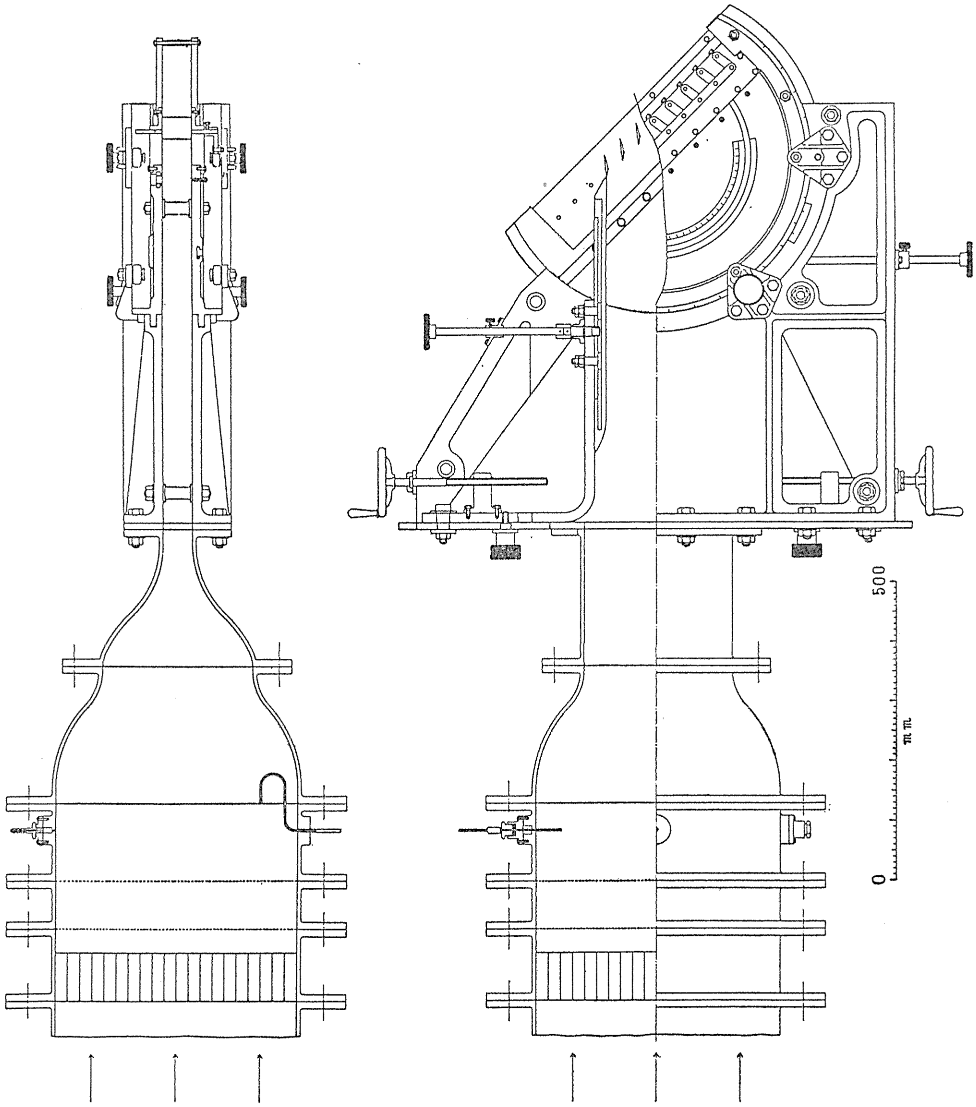

Fig. A-1 High speed cascade tunnel.

Exit flow was measured by the total pressure tube composed in the arrow-head yawmeter used in the previous experiment[6], and it was fixed in the mean direction of flow at a certain z-position, traversed in y-direction to find a minimum total pressure position which was regarded as being the position of wake. The z-positions, of measurements are 1, 20, 50 and 80 mm from the blade trailing edge.

There may be a question wheather the wake center position can be indicated by the minimum total pressure position or not, but from the standpoint of convenience in measurement this was regarded as a practical way to find the wake center position. Therefore the author decided to define this position being the wake position and try to do further treatment, and to regard that if this gives us good

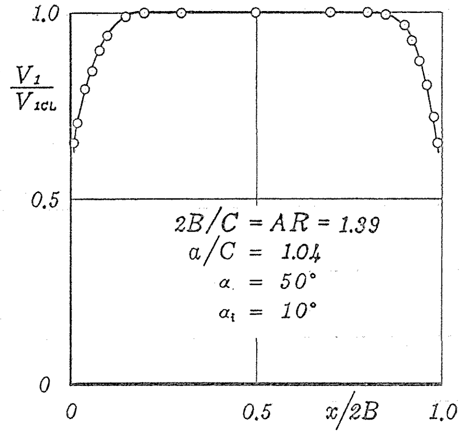

Fig. A-2 Inlet flow velocity. (one example)

conclusions the process will be proved being good.

Because the boundary layer of blade upper surface is generally thicker than that of lower, the minimum total pressure point is supposed being slightly in upper surface side of the wake center. But considering what we want to get is the wake direction and not the wake position itself, the error produced by the scheme mentioned above will be hoped to be small.

2. Results and Considerations

The direction of wake (which is expressed by the turning angle ε_w) as the result of experiment at a/C=1.04 is shown in Fig. A-3. Comparing this result with the turning angle ε measured by yaw-meter and shown in Fig. A-4 which is

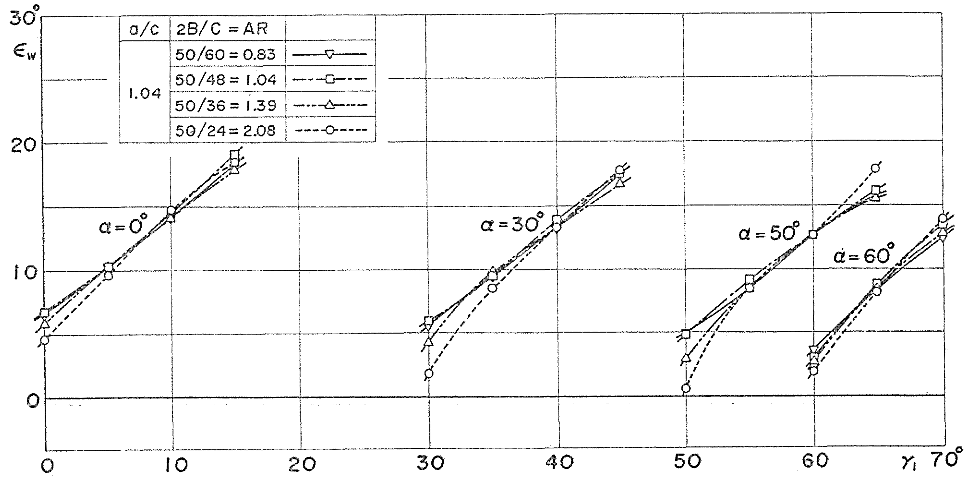

Fig. A-3 Turning angle based on wake.

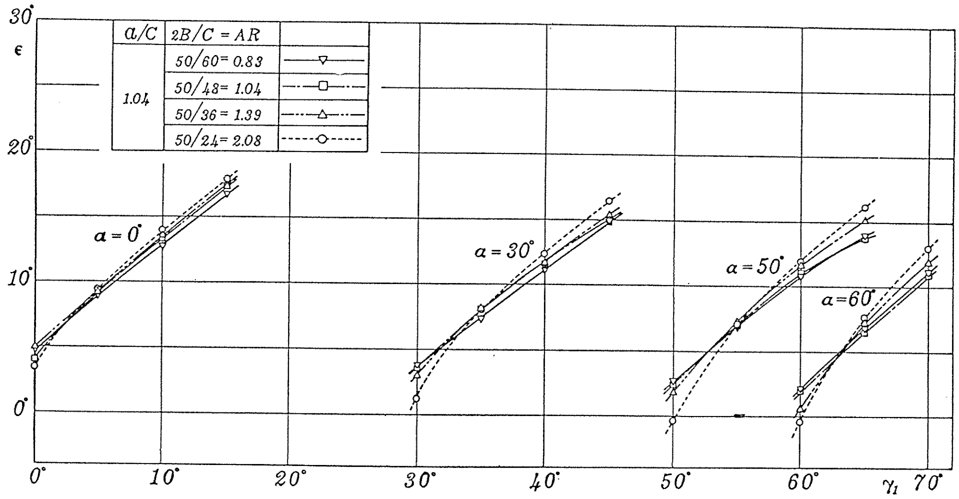

Fig. A-4 Turning angle measured by yaw-meter.

taken from the previous report[6], we can find that the variation of ε_w with respect to aspect ratio AR is apparently smaller than that of ε. The expectation that the value of ε_w is almost constant regardless of AR seems to be supported in the range of attack angle $\alpha_i=5\sim10°$. Bnt there exist rather large discrepancies of curves at larger and smaller α_i. The reason of this circumstance is not clear. But consulting

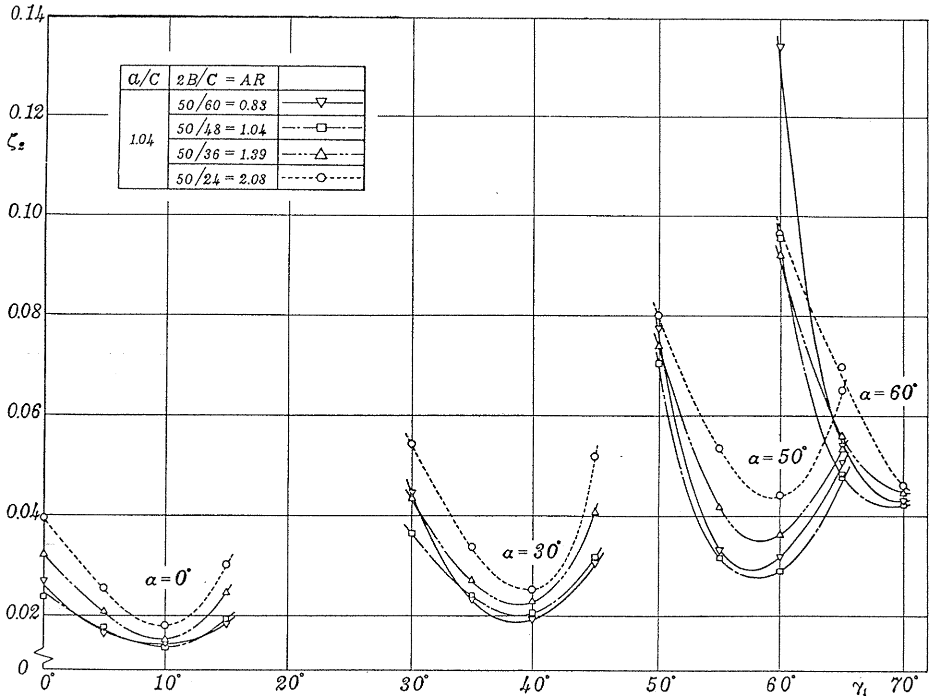

Fig. A-5 Loss coefficient.

ζ_2-curves shown in Fig. A-5, they suggest us that discrepancies between ε_w and ε are larger at large ζ_2 and very small at small ζ_2. Because we require small ζ_2 for the design work of turbo-machinery, the result mentioned above is very advantageous for us since the value of ε_w can be obtained from only one experiment of any AR.

Fig. A-6 is the result at different a/C and shows us ε_w is more coherent than ε illustrated in Fig. A-7. (If the curve of ε_w is horizontal straight line, the expectation is perfect.)

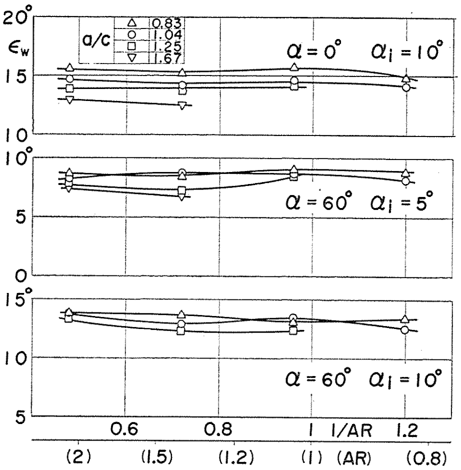

Fig. A-6 Turning angle based on wake.

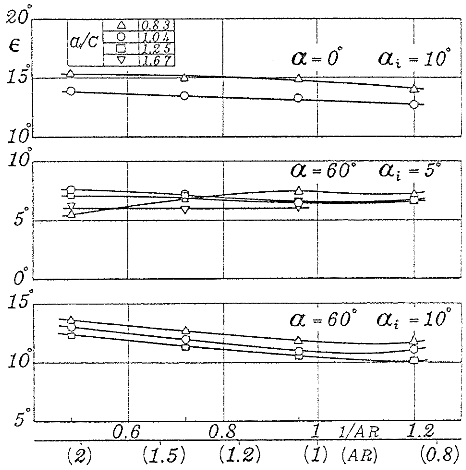

Fig. A-7 Turning angle measured by yaw-meter.

As stated above we may get a definite turning angle of cascade regardless of aspect ratio by the method measuring the direction of wake excepting cases of large loss coefficients. This means we need not care the aspect ratio of cascade and gives us the simplicity on accumulation of data on cascade performances.

Because this wake direction is expected by the theory to indicate that of two-dimensional cascade, to get the wake direction means to get the exit flow angle of two-dimensional cascade. Furthermore when we want to consider the secondary flow in cascade this can be treated as the secondary flow in rectangular region surrounded by these wakes and side walls. This means that the difference of angles between the wake direction and exit flow direction measured by yaw-meter should show the effect of secondary flow in rectangular region.

In Fig. A-8 values of ε_w and ε are indicated by □ and ○ respectively. These were selected on cases shown in Fig. A-3and Fig. A-6 in which ζ_2 is small so as ε_w is almost constant. Although experimental points are linked almost as they are in Fig. A-6 and A-7, but ε_w's are shown by horizontal lines (thick solid lines) in Fig. A-8. ε's can be also shown in the same manner but by straight lines falling toward the right (thick broken lines). The discrepancy between the line and symbol □ or ○ is supposed to be the experimental error. Because the magnitude of this error is comparable to the magnitude of deviation angle ($\varepsilon_w - \varepsilon$) produced by the secondary flow, discussions by the direct comparison of values of □ and ○ are unreasonable. Therefore the comparison between the solid line and the broken line may be a good way to consider the difference of turning (or exit flow) angles produced by the secondary flow.

Fine solid lines in Fig. A-8 indicate values of ε which are expected by the secondary flow theory (secondary flow in the rectangular region,) (calculated by the method explained in the previous report[1]) in which the passage vorticity ω_{2ps} was assumed constant in the region $0<x<\delta$. The detailed method is explained in Reference 7)). Because of not so much examples, we must not jump to a conclusion, but the author regards that the coincidence of experimental and theoretical values

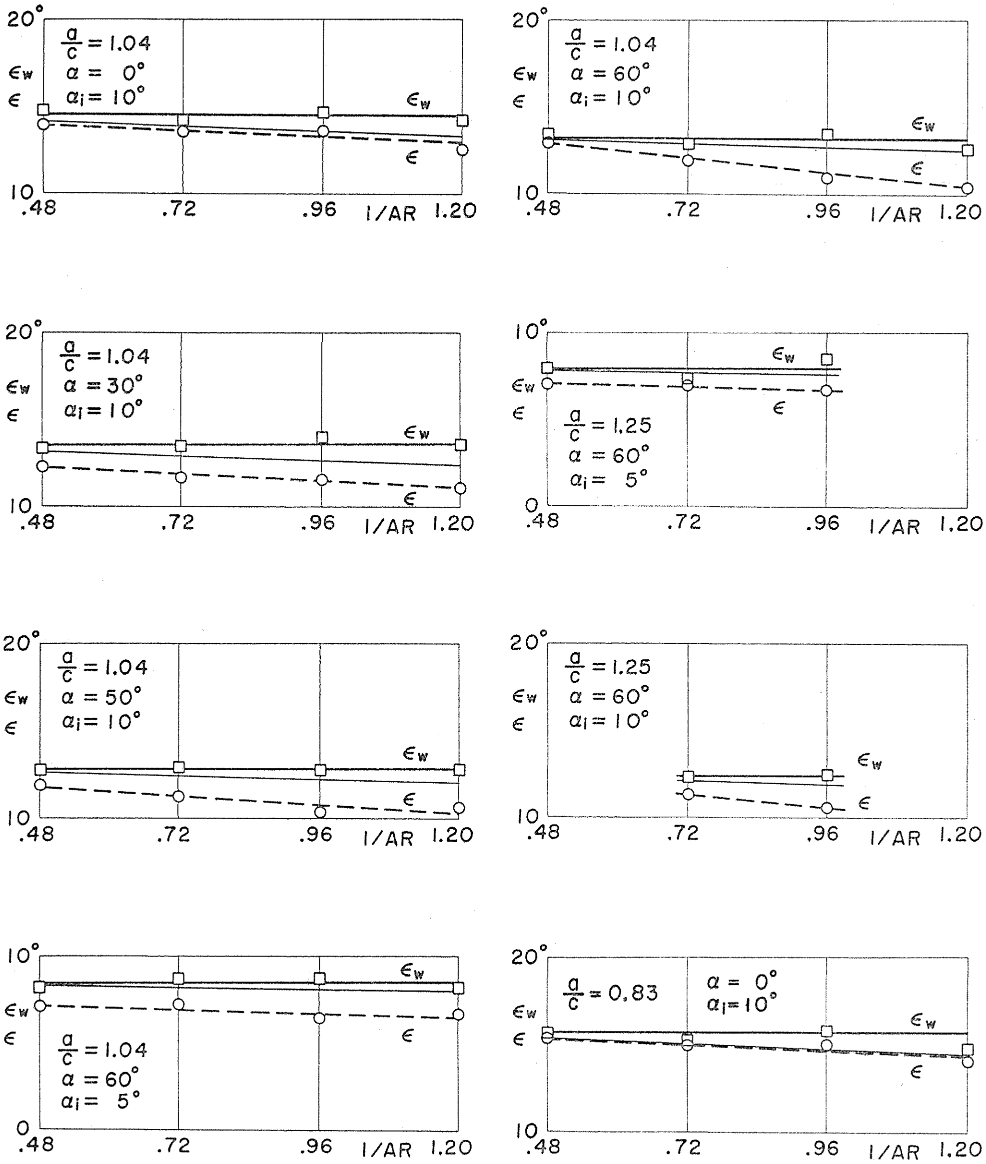

Fig. A-8 Turning angles. (comparison with theory) is good at the case $\alpha=0°$. At $\alpha=30°$ the experimental value of $(\varepsilon_w-\varepsilon)$ is about twice of the theoretical one, about three times at $\alpha=50°$ and four~five times, at $\alpha=60°$. The discrepancy is supposed to be attributed to the boundary layer growth* which is not included in the theory. (*At $\alpha=0°$ the flow is an accelerating one and there may be almost no boundary layer growth. At large α the boundary layer growth is dominant.) In this connection, equation (3) was used for the theoretical calculation of secondary vorticity, and results mentioned above suggest us that by using a suitable coefficient multiplied to this equation we shall be able to get much better agreement between theoretical and experimental values.

It may be another good suggestion that the fine thread, attached to the blade trailing edge will give more convenient way to find the direction of wake than the troublesome measurement for the minimum total pressure position.

3. Conclusions

As a means to approach the exit flow direction of two-dimensional cascade, a method in which the direction of wake of cascade is observed was tested. The result proved that the direction of wake has a definite value regardless of the aspect ratio of cascade as far as the loss coefficient is low, and furthermore there exists a reasonable relation from the standpoint of secondary flow between the above direction and the mean exit flow direction measured by an ordinary yaw meter method. Therefore, we can assert that the description of the wake direction as a characteristic of cascade is a good way to consolidate the characteristics of cascade.