Memoirs of the Faculty of Engineering, Nagoya University
Vol. 32, No. 2 (1980)

CONTROL APPROACH TO UNKNOWN OBJECTS

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(Received November 18, 1980)

Abstract

The control scheme introducing artificial intelligence is named by the author as "Intelligent Control" and is expected to realize sophisticated control technologies for unknown objects. In this paper, it is emphersized that intelligent control machines are classified into two kinds of (1) synthetic- and (2) heuristic control machines. The overt behaviors of both machines look very human-like, but their inner mechanisms and therefore the design principles are much different.

Author's research group has been developed various kinds of intelligent control machines. The aim of this paper is to survey those developed machines and is especially (1) to present an interesting synthetic approach which is a recursive learning control scheme to be designed in sequential vector space, and (2) to note briefly on the developing research of heuristic searching algorithms. The control machines named as LCDS and Learndynatrol are both new schemes developed in the synthetic design principle. While, the shifting basepoint method and the figure-ground search policy are also new developments in the research of heuristic control scheme.

CONTENTS

1. Introduction

Natural begins are to be considered essentially as unknown objects for human observer, in the sense that human can observe only the external behavior of the objects. Even human itself is an unknown natural being for the external observer. Scientists and engineers have been naturally endeavored to identify the objects or to make proper models whose external behavior are equal or similar to the original natural objects. Afterwards, they have intended to control and/or tried to synthesize control strategies for the artificial models. On the other hand, artificial systems which are designed by engineers are composed of, in usual, only well-known subsystems. However, some of artificial systems are constructed as a combination of well-known and unknown subsystems. Such systems as the industrial plants operating in a complex natural environmet and the man-machine systems which operate in interactive relation between artificial machines and human are involved in the latter category.

The conventional control theory has aimed at the development of control scheme for the objects which are described by mathematical tools. In other words, almost all the results of control theory are usable, in principle, for well-known or well-described systems.

However, it has been often required, in practice, to control the unknown objects or the unknown-wellknown combined objects. Much effort have been devoted in development of the control scheme for unknown plants. The PID control plan representing the conventional feedback process control is esteemed the excellent control sheme for unknown or partially unknown plants, because of its excellent suppression function for any disturbances regardless of the class of type and character of plants.

General plans for the control of unknown objects are classified essentially in two categories; one is the pursuit to make models of the unknown objects and then to seek control schemes for the identified or estimated models. The other is a control policy called as intelligent control which is the scheme to introduce the natural intelligent functions of human for the direct control of unknown objects.

The two fundamental approaches mensioned above have their proper features. The features of the former is that the excellent fruits of advanced estimation- and control theories can be used effectively. On the other hand, the latter lies in modern developments of try and error technologies, convergent recursive approaches and learning and heuristic control schemes which are the basic modes of control searching developed by human who is facing to the control of unknown objects. The decision problem to adopt which of the above two approaches should depend on the judgement of design engineers.

The two streams of research on the control of unknown objects have grown up rapidly and become very interesting fields in the systems and control engineering. In the former group, there are many important theories such as estimatieon of stochatic process, state- and parameter estimation, identification and realization of unknown systems, and moreover the dual- or hybrid control theories such as state-estimation and control, parameter-estimation and control, state-estimation parameter-estimation and control etc. In the latter group, there are many up-to-date growing theories and technologies such as optimalizing control, hill-climbing or peak searching, adaptive and learning control, game and conflict theories etc. The author and his research group has been engaged in the two research streams mentioned above, especially in identification and realization theories of linear and nonlinear systems, hill-climbing and optimization technologies, adaptive and learning control schemes, fuzzy theoretical decision making process and so on.

This paper is a survey summerizing some presented and unpresented works of author's research group, focusing in the control of unknown objects.

2. Control Methodology for unknown Objects

2. 1. Philosophical considerations towards Intelligent Control

The philosophy of automatic control has its origin in human's control concept appearing in the man-machine control systems, in which controlled object is an unknown process and human acts as a controller. Various arts and methodologies of control have been developed along the philosophy that automatic control means is originally a technology to replace some of human's control functions by artificial machines.

The substitition arts of human's control functions by machines are devided into the following two categories. One is the reasonable mechanization of control behavior irrespective of the essential feature of human's control function. It is noticeable, in this case, that the artificial control machine is designed on theoretical basis of control theory. Thus, this method is applicable to the theoretric control planning of well known or well-described objects. On the other hand, the other is an imitation- or simulation technology of human intelligence, in which human intelligence such as optimization, adaptation, learning, heuristics and so on may be introduced into the essential functions in control systems such as cognition, decision, action and so on.

Therefore, the art of automatic control belonging to the latter category is generally to be developed refering to human's mental and / or biological control functions. So, it is natural that the functional and / or conceptual approaches of automatic control are to be progressed on the view points of psycho-engineering and / or bio-engineering. Along this philosophy of automatic control, there have been proposed various technologies and strategies of automatic control, such as control schemes of optimalizing control, adaptive control, learning control etc.

Now the art is progressing to the more sophisticated scheme of automatic control by introducing more complex psychological functions such as heuristics, recognition, association, concept-formation, decision-making etc. The author call the group of these sophisticated arts of control as "*Intelligent control*".[1,2,3] Thus, the latter approach is considered to be very useful for the control of unknown and / or complex objects.

2. 2. Concept and Methodology of Intelligent Control

Now, the natural intelligences of human are considered to have the following four features:

1. Synthetic and harmonic function
2. Subjective or self-decisive function
3. Intuitive function
4. Fuzziness property

Accordingly, the artificial intelligent machine is desired to have these complex functions as much as possible. For example, the sophisticated intelligent machine is desired to have integrated intelligences composed of such functions, for example, as sensing, congnizing, judging, critisizing and so on. "*Fuzziness*" is also a noticeable property which is essencially accompanied with all mental actions of human. Anyway, it is believed commonly that the more complete realization of such intelligent machines is quite indebted to the active application of conventional logical computer, and new-sense computer which will be invented in future.

The positive introduction of functions of optimization and adaptation into automatic control scheme over twenty years ago is the start towards intelligent control. The technology of optimal control or optimization is aimed the realization of the best situation, which is consistent to the basic principle of human control action. Adaptive control aims to realize the eternal preservation of optimality or the searching pursuit of optimality, by readjusting parameters of the main or low-level controller, in order to minimize the performance function through the command of higher-level adaptive controller. The advanced control scheme which accumulates the searching data obtained by adaptive control process and uses positively the accumulated experiences to counteract to the environmental changes, is called as "*Learning control*".

Futhermore, the control strategy introducing an interesting function of heuristics which is one of human intelligences is called as "*Heuristic control*". And also the control introducing the pattern recognizing function is called as "*Pattern recoqnizing control*". The group of these intelligent control schmes are considered to be powerful for the control of unknown, variable parameter linear and / or nonlinear systems.

2. 3. Synthetic and Heuristic Approaches to Intelligent Contol Machine

We discussed in *2. 1* that the art of automatic control have developed pointing to the mechanization of human control functions. Of course, some of intelligent machines which are artificial products in the engineering of automatic control are not developed by the above approach, but by pure theoretical approaches. However, it should be remarked that those machines seem equivalent as far as concerned to the apparent behavior. Here, we make a temporary definition of artificial intelligent control machine as:

[Definition] "*Artificial intelligent control machine*" is meant by a black box system whose external behavior is similar to the control action of human.

The inner algorithm by which the machine operates is not necessary to be the same as real inner mechanism of human. Depending upon the form of inner algorithm, the intelligent control machine can be classified into the following two categories :

1. Synthetic intelligent machine :
   This machine behaves according to the algorithm which assures optimality and/or convergency of the recursive behavior by the results of theoretical pursuits.
2. Heuristic intelligent machine :
   The machine behaves in trial and error mode according to the repetion of human-like inference and judgement. The conditions of optimality, convergency, uniqueness etc. on the behavior of this machine are not always assured.

The difference between the two machines mentioned above lies in the difference of simulation-or realization form of human's intelligent functions. The former is a simulation algorithm obtained as a conclusion of mathematical persuit. While, the latter is an algorithm which is obtained by mere imitation or analogy of human behavior. However the apparent behaviors of the two machines are similar in the point that the behavior modes of both machines are the recursive processes which appear generally in learning-or heuristic process of human. The problem that which of the two types should be selected in the practical design of intelligent machine is a case-by-case one. The former is to be selected positively for the case that the mathematical-or theoretical tool being suitable to analyse and to synthesize the intelligent function seems possible to be found easily. The latter is the opposite case. It will not be preferable that system designer selects the latter easily without enough theoretical pursuit by mathematical approach. Anyway, the final decision in practical design should be done considering many conditions from various view points of technology, realizability, economic and energy saving and so on.

The arthor's research group has developed, various kinds of intelligent machines (schemes) for the past about thirty years. Researches on a system self-improving its control dynamics,[4] a system learning optimal control policy,[5] Learndynatrol,[6] recursive identification,[7, 8] recursive realization, dual control system under stochastic environment[9,10] and so on are involved in the category of synthetic control. While, researches on Learntrol (*Learn*ing Con*trol*),[11~13] Heuristic control,[14, 1, 2] Hill-climbing algorithm,[15~18] Fuzzy, theoretic approach to humans decision making,[19~21] etc. are included in the group of heuristic approach. Main focus of this paper is to present a new technology of asymptotic recursive control process for unknown plant,[4, 6] and to survey several results of researches on peak-searching technologies[17,18] which belong to heuristic approach.

3. Synthetic Approach to the Control for Unknown Objects

In this chapter, is presented a new means to describe input-output time relation of a system in sequential vector space (S. V. S.),[6, 22] and is proposed a recursive synthesis method of the control for unknown objects basing on the S. V. S. control theory.

3. 1. Sequential Vector Space Theory for Control Approach to Unknown Objects

"*Sequential Vector* or S. V." is meant here by a vector composed of number sequence or discrete values or discrete functions which are sampled from a continuous time function. We will call especially the sequential vector composed of sampled (clipped) functions as "*modified sequential vector*." Thus, the space composed of sequential vectors is called as the sequential vector space. The sequential vector space (S. V. S.) control theory is based on a new system theory in which the description of input-, output functions and the analysis and syntesis of system response are all treated in the sequential vector space. Thus, this theory is very powerful for the treatment of time-variable and / or non-linear systems, so that is very useful to the design of control scheme for unknown system.

In the following sections, the outline of the S. V. S. control theory is firstly described and afterwards two new control systems LCDS and Leardynatrol are presented.

3. 2. Outline of S. V. S. Control Theory

3. 2. 1. Fundamental Concept

The state-space system theory has made a great contribution to the recent development of control theory. However, the treatment of non-liner systems is still hard and complex, and its relationship to the classical control system is not clarified so sufficiently. The theory in sequential vector space which is outlined in this chapter has a hope of bringing a considerable contribution to these remained fields.

The S. V. S. control theory is a humble system theory of automatic control based on a discrete system theory in which a time-function is expressed by the sequential vector and the input-output vector relation of a system is described through a matrix of Sequential Transfer Matrix (c. f. *3. 2. 2*). The sequential transfer matrix is a matrix composed of sampled values or sampled functions of impulse response of the system. The concept of sequential vector and sequential transfer matrix are not both new. Sequential vector has often been used in vectoral treatment of a time function or time series, and the sequential tranfer matrix has also appeared in the discrete system theory.[23] However, the author's attempt to make a control theory by means of these two basic concepts is fairly new.

The control theory in sequential vector space is available for almost all kinds of systems. That is, regardless of variety and complexity of system structure, the computation of system response and the synthesis of control can be pursued for the linear and non-linear, the time-invariable and time-variable and their combined systems. Furthermore, although this theory is essentially effective for the discrete system, it is also available for the continuous system with slight modification, therefore for the continuous-discrete combined system. The remarkable features appear in the treatment of the time-variable and non-linear system. The possibility of analysis and synthesis for the complex non-linear system is a big expectation of this theory.

The automatic- or real-time design of control for poorly known object is an interesting subject in adaptive and learning control. The S. V. S. control theory is also very useful for the research in this field. That is, the development of real-time design of control for unknown plant through S. T. M. (c. f. *3. 4* $\sim$ *3. 5*) is a very contributive progress in the field of adaptive and learning control. The basic procedure for this development is a composition of convergently recursive processes which are consisted of the computation of linearized equivalent S. T. M. and the determination of incremental control in each stage. We have acquired two interesting methods of approach. One is a system of LCDS and the other is Leardynatrol. The former is a real-time designing system which learns the system dynamics (or S. T. M.) at each stage by applying small test step signal and decides the control by which the system can secure the subgoal in that stage by using the learned S. T. M.. The latter of Learndynatrol is an advanced Learntrol with an ability of self-improvement of its dynamic behavior, which firstly stores the data obtained in step-by-step search of Learntrol[11~13, 24~27)] and in its succeeding period, secondly computes the linearized equivalent S. T. M. and thirdly determines the incremental control in each stage.

3. 2. 2. Definition of Sequential Transfer Matrix (*S. T. M.*)

Now, let the sequential vectors of input-, output functions and the modified sequential vectors of output function be as

$$\text{S. V. of input function } \boldsymbol{f}=\{f(jT)\}^T=\{f_j\}^T=(f_0\ f_1\cdots f_n)^T \tag{3.1}$$

$$\text{S. V. of output function } \boldsymbol{c}=\{c(jT)\}^T=\{c_j\}^T=(c_0\ c_1\cdots c_n)^T \tag{3.2}$$

Modified S. V. of output function

$$\boldsymbol{c}(\tau)=\{c(jT+\tau)\}^T=(c_0(\tau)\ \ c_1(\tau)\cdots c_n(\tau))^T \tag{3.3}$$

where τ is a time variable defined only in a sampling interval $[jT,\ (j+1)T]$, $j=0$, 1, 2, $\cdots$ n, then $0\leq\tau<T$.

The input-out relation of a linear system can be written as

$$\boldsymbol{c}=H\boldsymbol{f} \tag{3.4}$$

and

$$\boldsymbol{c}(\tau)=H(\tau)\boldsymbol{f} \tag{3.5}$$

where H and $H(\tau)$ are paired Matrices composed of S. V. and modified S. V. of impulse response $g(t)$ of linear system respectively, and are written as follows:

$$H=\begin{pmatrix} g_1 & 0 & 0 & \cdot & \cdot & \cdot & \cdot \\ g_1 & g_0 & 0 & \cdot & \cdot & \cdot & \cdot \\ g_2 & g_1 & g_0 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & & & \cdot \\ \cdot & \cdot & \cdot & & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & & \cdot & \\ g_n & g_{n-1} & \cdot & \cdot & \cdot & \cdot & g_0 \end{pmatrix} \tag{3.6}$$

$$H=(\boldsymbol{g}\ S\boldsymbol{g}\ S^2\boldsymbol{g}\cdots S^n\boldsymbol{g}) \tag{3.7}$$

$$H(\tau)=(\boldsymbol{g}(\tau)\ \ S\boldsymbol{g}(\tau)\ \ S^2\boldsymbol{g}(\tau)\cdots S^n\boldsymbol{g}(\tau)) \tag{3.8}$$

Where $\boldsymbol{g}=(g_0\ g_1\ g_2\cdots g_n)^T$, $\boldsymbol{g}(\tau)=(g_0(\tau)\ g_1(\tau)\ g_2(\tau)\cdots g_n(\tau))^T$ and S is a shift matrix defined as

$$S=\begin{pmatrix} 0 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ 1 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & & & & & & \cdot \\ \cdot & \cdot & & & & & & \cdot \\ \cdot & \cdot & & & & & & \cdot \\ 0 & 0 & \cdot & \cdot & \cdot & \cdot & 1 & 0 \end{pmatrix} \quad (n\times n) \tag{3.9}$$

We call H and $H(\tau)$ as Sequential Transfer Matrix or S. T. M. and modified S. T. M. respectively. It becomes clear by the following discussion that S. T. M. H and modified S. T. M. $H(\tau)$ are derived from Z-Transform and modified Z-Transform of impulse response of the linear system. Applying the expression of Eq. (3.7) and Eq. (3.8), the output function Eq. (3.4), and Eq. (3.5) becomes as:

$$\boldsymbol{c}=(\boldsymbol{g}\ S\boldsymbol{g}\ S^2\boldsymbol{g}\cdots S^n\boldsymbol{g})\boldsymbol{f}=(\sum_{i=0}^{n} f_iS^i)\boldsymbol{g} \tag{3.10}$$

or

$$\boldsymbol{c}=(\sum_{i=0}^{n} g_iS^i)\boldsymbol{f} \tag{3.11}$$

and

$$\boldsymbol{c}(\tau)=(\sum_{i=0}^{n} g_i(\tau)S^i)\boldsymbol{f} \tag{3.12}$$

Here, let

$$G(S)\triangleq \sum_{i=0}^{n} g_iS^i \tag{3.13}$$

$$G(S,\tau)\triangleq \sum_{i=0}^{n} g_i(\tau)S^i \tag{3.14}$$

then

$$\boldsymbol{c}=G(S)\boldsymbol{f} \tag{3.15}$$

$$\boldsymbol{c}(\tau)=G(S,\ \tau)\boldsymbol{f} \tag{3.16}$$

The S. T. M. $G(S)$ and the modified S. T. M. $G(S,\ \tau)$ are derived, as can be guessed easily from their function form of Eq. (3.12) and Eq. (3.13), from pulse transfer function or modified pulse transfer function of the system. That is, considering $S^i=0$ for $i>n$,

$$G(S)=\sum_{i=0}^{\infty} g_iS^i=[G(z)]_{\substack{z\to S\\ 1\to I}} \tag{3.17}$$

$$G(S,\ \tau)=\sum_{i=0}^{\infty} g_i(\tau)S^i=[G(z,\ \tau)]_{\substack{z\to S\\ 1\to I}} \tag{3.18}$$

3. 2. 3. System Response of Linear System Subjected to Sampled-Functions Input

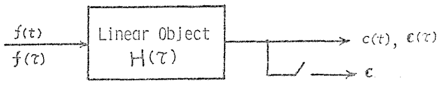


Fig. 3. 1. Linear objects subjected to sampled-functions input

The continuous function or the output of sampler-holding system is considered as a sampled-functions input for the subjected system. The system response of linear system being subject to a sampled-functions input is described as

$$\boldsymbol{c}(\tau)=[H(\tau)*\boldsymbol{f}(\tau)] \tag{3.19}$$

where the symbol $*$ means the comvolution defined by Eq. (3.20), Eq. (3.21).

$$[H(\tau)*\boldsymbol{f}(\tau)]=\{[h_{ij}(\tau)*f_i(\tau)]\} \tag{3.20}$$

$$[h_{ij}(\tau)*f_i(\tau)]=\int_0^T h_{ij}(\tau-\nu)f_i(\nu)d\nu \tag{3.21}$$

The detail about the definition and computation process of the convoltion is presented in Ref. (22).

3. 2. 4. System Response of Non-linear Systems

Any non-linear system can be described by the combination of linear elements and zero-memory non-linear elements.[28] The derivation of dynamical behavior of this system is very difficult even by the conventional analytical method. Author's procedure described here is a new method to secure the transient response of such complex non-linear systems by applying the sequential vector space approach. The very merit of sequential vector space method is based on the matter that the output sequential vector of zero-memory non-linear element can be easily obtained by executing the corresponding non-linear operation refering to each component of the input sequential vector.

Eor example, the performance computation of the open loop non-linear system shown in Fig. 3. 2 is executed by the next relations.

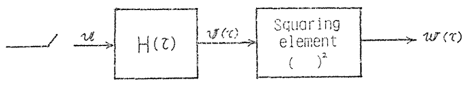


Fig. 3. 2. Open loop non-linear system with squaring element

$$\boldsymbol{v}(\tau)=H(\tau)\boldsymbol{u} \tag{3.22}$$

$$\boldsymbol{w}(\tau)=\boldsymbol{v}(\tau)\times\boldsymbol{v}(\tau)\triangleq(v_0^2(\tau)\ \ v_1^2(\tau)\cdots v_n^2(\tau)) \tag{3.23}$$

By the same way, the behavior of closed-loop non-linear system can be secured.

3. 2. 5. System Response of Time Variable Systems

The output response of time-variable system can be also secured by the same way as the procedure which has been described in the preceding section.

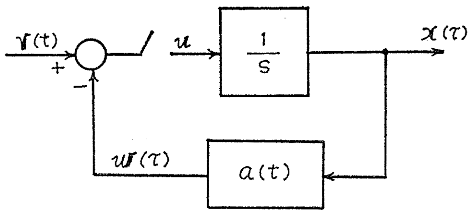


Fig. 3. 3. A simple time-variable system

Now, we will consider the simple time-variable system of Fig. 3. 3. The system equation can be expressed by

$$\boldsymbol{x}(\tau)=\frac{1(\tau)\mathbf{1}}{\mathbf{1}-S}\boldsymbol{u} \tag{3.24}$$

$$\boldsymbol{w}(\tau)=A(\tau)\boldsymbol{x}(\tau) \tag{3.25}$$

where $A(\tau)$ is the S. T. M. of time variable element $a(t)$, which is described as

$$A(\tau)=\begin{pmatrix} a_0(\tau) & 0 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & a_1(\tau) & & & & & \cdot \\ 0 & 0 & a_2(\tau) & & & & \cdot \\ \cdot & \cdot & \cdot & \cdot & & & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdot & \cdot & \cdot & a_n(\tau) \end{pmatrix} \tag{3.26}$$

$$a_j(\tau)=a(jT+\tau), \quad j=0,\ 1,\ 2,\ \cdots,\ n.$$

From Fq. (3. 24) and Eq. (3. 25), Eq. (3. 27) and Eq. (3. 28) can be derived.

$$x_j(\tau)=u_0 1(jT+\tau)+u_1 1((j-1)T+\tau)+\cdots+u_j 1(\tau),$$
$$j=0,\ 1,\ 2,\ \cdots,\ n \tag{3.27}$$

$$w_j(\tau)=a_j(\tau)x_j(\tau), \quad j=0,\ 1,\ 2,\ \cdots,\ n \tag{3.28}$$

Then components of the error sequential vector $\boldsymbol{u}$ are calculated by the next relation.

$$u_j=r_j(0)-w_j(0)=r_j(0)-a_j(0)x_j(0)=r_j(0)-a_j(0)\sum_{l=0}^{j}u_l \tag{3.29}$$

Then

$$u_j=\frac{1}{1+a_j(0)}[r_j(0)-a_j(0)\sum_{l=0}^{i-1}u_l] \tag{3.30}$$

Applying Eq. (3. 30) to Eq. (3. 27), the components of $\boldsymbol{x}(\tau)$ can be secured.

$$x_j(\tau)=\sum_{j=0}^{n}\left(\frac{1}{1+a_j(0)}\right)[r_j(0)-a_j(0)\sum_{j=0}^{j-1}u_l]1(jT+\tau) \tag{3.31}$$

The above procedure can be straightly extended to the general time-variable systems such as the higher order time-variable system or the non-linear multiloop time-variable discrete system.

3. 2. 6. System Synthesis Through S. T. M.

The system synthesis through S. T. M. is applicable to the various systems such as linear, non-linear and time-variable system which operate in the discrete or discrete-continuous combined modes. The fundamental features of this synthesis method are:

(1) The method is a unique one available to almost all kinds of systems.

(2) Being possible to process in quite mechanical manner, the method is very suitable to the computer processing, even if it will usually require the operation and computation of matrices and determinants.

We have two methods of synthesis in sequential vector space. One is a direct method and the other is an convergent method. The former is suitable for the design of control for well-known objects and the latter is for extremely complex well-known, poorly-known and unknown objects. And, the former is very convenient for the general off-line design, while the latter may be a powerful approach for the on-line or real-time designing of the sophisticated adaptive or learning control. Since the direct method is useful for well-known system, the utility of this method has been explained, in this survey, only on the simple examples in *3. 2. 2～3. 2. 5.* While, the convergent method is a very useful recursive approach for the synthesis of intelligent control for unknown system, so we will treat it in detail hereafter.

3. 3. General Description of Convergent Recursive Approach

3. 3. 1. Object and Aspect of the Method

As described in Chap. 1, there are two basic approaches to the control of unknown plant. One is the method in which the control or the control strategy is decided after the plant dynamics has been clarified by system identification. While the other is the real-time recursive method to improve the control dynamics convergently by learning how to control the object directly under the condition of poor a priori knowledge about the object. The most important subject in the former is the identification technology. However, the identification for non-linear, complex systems is very difficult or impossible. Therefore the former method will be only applicable, in usual, to linear and comparatively simple objects. Since unknown plants are to be considered complex and non-linear, the latter method may be the most effective approach to the control of non-linear, complex unknown plant. Moreover, in the latter case, learning approach will be introduced to develop methodologies seeking not only how to control but also how to estimate system parameters or system transfer characteristics.

The behavior pattern of human learning is the trial and error mode with insight. Hence the convergent and speedy acquistion of optimum control by the recursive approach is based on this essential learning function. That is, the objective of the learning controller is to search the best dynamic behavior when it is required to shift from a steady state (arbitrary fixed starting state) to another steady state (given final state) by the repetitive searching of control for the object which is to be considered as unknown non-linear system.

All the signals such as control input, plant output, and desired output are expressed by sequential vectors and the learning control process is described as an iterative trajectory in the sequential vector space. Then, the criterion function is the Euclid distance between the current point and the goal in the vector space. The unknown controlled object which is essentially non-linear can be linearized approximately in each small scale step of the learning process. Repeating the determination of subgoal, the estimation of unknown S. T. M. and the evaluation of the increment of control in each stage of the iterative procedure, the system approaches gradually to the aimed goal and finally reaches to the sub-optimal control which exists in the range with spedified small distance from the desired final goal. Since the control experience in each stage is utilized in the linealizing operation in the succeeding stages, this procedure is worthy to call learning process. This means that the learning controller have learned the way to improve the dynamic (transient) response of the original control system progressively.

3. 3. 2. Problem Statement

Let the desired performance of the control system or the goal point in $\boldsymbol{x}$-spece (sequential vector space of plant output) be $\boldsymbol{x}_d$, and the pre-fixed staring point be $\boldsymbol{x}^0$. The criterion function in quadratic form is expressed as $J=\|\boldsymbol{x}_d-\boldsymbol{x}\|$, which is the Euclid distance from $\boldsymbol{x}$ to $\boldsymbol{x}_d$. Then the present problem is to search the best control from the admissible controls in $\boldsymbol{u}$-space (sequential vector space of control), causing the plant output colsest to the goal $\boldsymbol{x}_d$, in other words, is to get the control which minimizes J under specified constraint on control, for example saturation character, $u<M$, M is positive finite number, through the recursive linearization operation in each small-scale stage of the iterative procedure.

3. 3. 3. Description of Search Principle

In this section, the convergent recursive approach proposed in *3. 3. 2.* is explained refering to Fig. 3. 4.

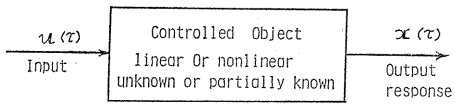


Fig. 3. 4. Unknown controlled object

The optimum synthesis of control in sequential vector space by the recursive approach can be stated as folows:

The problem is to find the sequential vector of optimal control $\mathbf{u}_{opt}$ which satisfies the condition to minimize $J=\|\boldsymbol{x}_d-\boldsymbol{x}\|$ under the specified constraint, for example, $u_j\leq M$, M is possible funite value, by a convergent recursive procedure, by which the object output convergently approaches step by step to the desired output vector $\boldsymbol{x}^d$.

The method requests the following assumptions.

(1) Controlled object operates rereatedly as in the cases of start up control, batch process control or regulater subjected to repeating disturbances.
(2) Impulse response of the object settles or can be considered to settle in finite time.
(3) Dynamic characteristics of the object in each stage* is piece-wise linear.

*) In order to distinghush the step in this recursive control procedure from that in the sampling process of a control response, we refer to the former as "stage" and the latter as "step".

In other words, the increment of control Δu^i and the corresponding variation of output Δx^i in the i-th stage are both so small that they are in an approximately linear relation through H^i, that is,

$$\Delta x^i = H^i \Delta u^i \tag{3.32}$$

Now, we explain the searching principle of the convergent recursive procedure, refering to the vector trajectory in the sequential vector space in Fig. 3. 5.

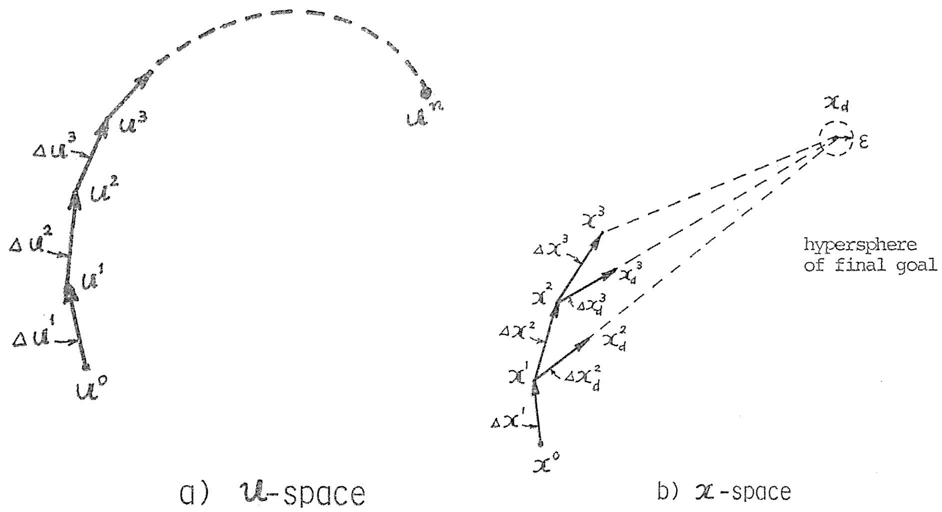


Fig. 3. 5. Sequential vector space description of convergent recursive approach

Initial Situation: Initially assume that the arbitrary selected control u^0 (for instance step function control with the height to cause the desired steady state of object output) causes the output x^0. We call the association of (u^0, x^0) as the initial situation of the iterative improving approach.

First Stage: The operations in first stage are shown in four terms (1-1) ~ (1-4).

(1-1) Select an appropriate increment of control in the first stage Δu^1.

(1-2) Compute the resultant control u^1 by using the next relaiton.

$$u^1 = u^0 + \Delta u^1 \tag{3.33}$$

(1-3) Measure the output vector x^1 corresponding to the resultant control u^1. The association (u^1, x^1) is the control situation which the system has reached in the first stage.

(1-4) Compute the Euclid distance $d^1 = \|x_d - x^1\|$, and check that d^1 is smaller than or equal to the specified threshold ε. If $d^1 > \varepsilon$, proceed to the second stage.

Second Stage: The operations in the second stage are composed of the computation of S. T. M. of the first stage, the selection of subgoal, and the determination of control. They are shown in the following eight terms of (2–1) ~ (2–8).

(2–1) Compute the incremental variation $\Delta \boldsymbol{x}^1$ corresponding to the incremental control $\Delta \boldsymbol{u}^1$ by the following formula.

$$\Delta \boldsymbol{x}^1 = \boldsymbol{x}^1 - \boldsymbol{x}^0 \tag{3.34}$$

(2–2) Compute the each component of the linearized S. T. M. of the first stage $H^1(\boldsymbol{x}^0)$, by applying numerical data of $\Delta \boldsymbol{u}^1$ and $\Delta \boldsymbol{x}^1$ to Eq. (3.35).

$$\Delta \boldsymbol{x}^1 = H^1(\boldsymbol{x}^0) \Delta \boldsymbol{u}^1 \tag{3.35}$$

(2–3) Decide the subgoal in the second stage $\boldsymbol{x}_d^2$ by the relation of

$$\left.\begin{aligned} \boldsymbol{x}_d^2 &= \boldsymbol{x}^1 + \alpha^2(\boldsymbol{x}_d - \boldsymbol{x}^1) \\ &= (1-\alpha^2)\boldsymbol{x}^1 + \alpha^2 \boldsymbol{x}_d \end{aligned}\right\} \tag{3.36}$$

α^2 : a prescribed constant.

(2–4) Secure the desired output variation in second stage by the next relation.

$$\Delta \boldsymbol{x}_d^2 = \boldsymbol{x}_d^2 - \boldsymbol{x}^1 \tag{3.37}$$

(2–5) Compute the incremental control in the second stage $\Delta \boldsymbol{u}^2$ which causes the expected output variation $\Delta \boldsymbol{x}_d^2$, by using the S. T. M. in the first stage $H^1(\boldsymbol{x}^0)$ given in the step (2–2). Computation formulas for the components of $\Delta \boldsymbol{u}^2$ are derived from the next vector equation.

$$\Delta \boldsymbol{x}_d^2 = H^1(\boldsymbol{x}^0) \Delta \boldsymbol{u}^2 \tag{3.38}$$

Notice that the relation of Eq. (3.38) is based on the assumption that H^1 is approximately available for the second stage.

(2–6) Sequre the practical control in the second stage by the relation of Eq. (3.39).

$$\boldsymbol{u}^2 = \boldsymbol{u}^1 + \Delta \boldsymbol{u}^2 \tag{3.39}$$

If there are any specified constraints on $\boldsymbol{u}^2$ such as the saturation, each component of $\Delta \boldsymbol{u}^2$ must be selected so as to satisfy the constraints.

(2–7) Measure the output $\boldsymbol{x}^2$ corresponding to the actual control $\boldsymbol{u}^2$. The association $(\boldsymbol{u}^2, \boldsymbol{x}^2)$ is the control situation acquired in the second stage.

(2–8) Compute the distance $d^2 = \|\boldsymbol{x}_d - \boldsymbol{x}^2\|$ and check that $d^2 \leq \varepsilon$. If $d^2 > \varepsilon$, the procedure proceeds to the next stage.

Succeeding Stages: The same procedures as the second stage are repeated in the succeeding stages. Passing through $\boldsymbol{x}^3$, $\boldsymbol{x}^4$, $\cdots$ which are located near $\boldsymbol{x}_d^3$, $\boldsymbol{x}_d^4$, $\cdots$ respectively, the system approaches to the final goal $\boldsymbol{x}_d$. This iterative process finishes when it entered in the domain with small distance ε from $\boldsymbol{x}_d$. Fig. 3. 6 is the block diagram for the above iterative procedure. And Fig. 3.7 shows the converging aspect of system situation in the convergent recursive process.

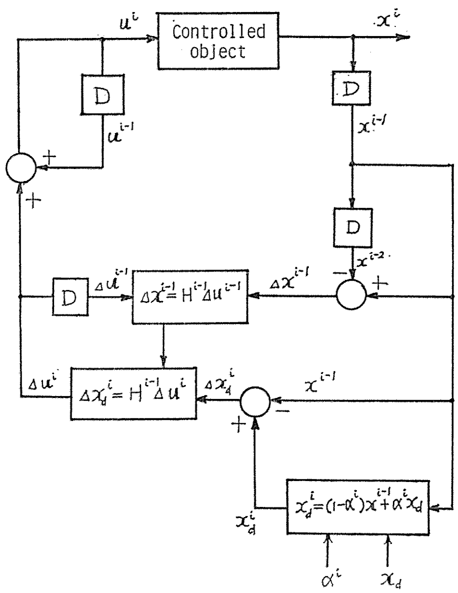


Fig. 3. 6. Block diagram for convergent recursive procedure ($i \geq 2$)
D : one-stage delay or memory element

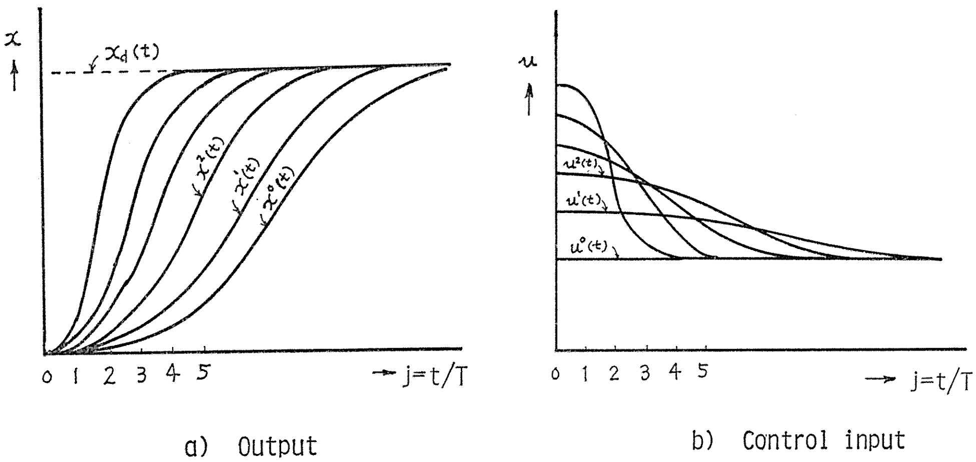


Fig. 3. 7. Graphical explanation of converging process by the recursive procedure

3. 3. 4. Convergency of Recursive Process

Convergency of the output vector series $\boldsymbol{x}^0$, $\boldsymbol{x}^1$, $\boldsymbol{x}^2$, $\cdots$ is dependent upon the approximation degree of the evaluated H^{i-1}, $i=2, 3, 4, \cdots$. If H^{i-1} is related to the real S. T. M. in i-th stage G^i by

$$G^i = [\boldsymbol{I} - \Delta^i] H^{i-1}, \tag{3.40}$$

then the following expression is true.

$$x^i = x^{i-1} + G^i \Delta u^i = x^{i-1} + [I - \Delta^i] H^{i-1} \Delta u^i$$

$$= x^{i-1} + [I - \Delta^i] \Delta x_d^i = x^{i-1} + [I - \Delta^i] \alpha^i [x^d - x^{i-1}] \quad (3.41)$$

Subtracting both sides of Eq. (3.41) from x_d,

$$x^d - x^i = [I - \alpha^i (I - \Delta^i)] (x^d - x^{i-1})$$

$$= M^i (x^d - x^{i-1}) \quad (3.42)$$

where

$$M^i \triangleq [I - \alpha^i (I - \Delta^i)] \quad (3.43)$$

since

$$\| x_d - x^i \| \leq \| M^i \| \, \| x_d - x^{i-1} \|,$$

the convergency condition of this recursive process becomes as:

$$\| M^i \| = \sqrt{\lambda_{max} (M^{i\,T} M^i)} < 1 \quad (3.44)$$

or

$$\sqrt{\sum | M^i_{k\,l} |^2} < 1 \quad (3.45)$$

The real S. T. M. G^i is calculated by applying experimental data (Δu^i, Δx^i) into the relation of Eq. (3.46).

$$\Delta x^i = G^i \Delta u^i \quad (3.46)$$

Hence, Δ^i can be computed by applying G^i and H^{i-1} into Eq. (3.40). Thus, it is possible for the recursive process to be convergent by selecting α^i to satisfy the condition of Eq. (3.44) or (3.45) in each stage.

3. 3. 5. Notation on Control Determination

The desired output x_d can take any form which is, for example, the ideal form [0, 1, 1, …] for the unit step input or [0, kT, $2kT$, …] for the ramp input kt. However, there may happen several troublesome problems in the actual planning of the recursive procedure.

(1) According to the linear sampled-data control theory, it is clear that the settling time of dead-beat response is dependent upon both the type of input funct-ion and the number of degree of the differential equation of controlled object, and also is longer than the settling time of the prototype response (Raggazzini)[29]. Thus, the determination of desired output must be preceded by the pre-estimation of the degree of lag of the augumented system (the whole system including the original system of the input function). If the output x is forced to access the ideal form of x_d which is selected independently of the above property, there will appear several unwanted phenomena such as the discontinuous jump at sampling points, the considerable deviation during sampling points, and moreover the diver-gent response in the worst case.

(2) For non-linear system the settling time will depend on not only the

characteristics of linear dynamical part but also that of separable non-linear unit, and therefore becomes longer, in general, than that of the system excluding the non-linear part. This is caused by the fact that several new frequencies (higher and lower order harmonics) should generate in the non-linear system. Thus, there may occur, in general sense, the case in which dead-beat response is unrealizable due to the generation of infinite parasitic frequencies in the non-linear system. Then the control must be determined under the pre-estimation of generated frequencies.

3. 4. Real-Time Iterativ Synthesis of Control for Unknown Linear Objects

In this and next sections, a real-time iterative design of control for the linear and non-linear objects whose character are unknown or poorly-known will be discussed. The author proposes here an automatic on-line redesigning system which is avilable for any of linear and non-linear, time-variable and time-invariable, and continuous and discrete systems. The basic principle of the procedure is to design or redesign the system in real time by systematic method to simulate the object by an equivalent model of S. T. M..

Now, we will consider a system which is operating in a good condition. When the system is disturbed by a sudden change of the environment, the system must be automatically redesigned so that it attains to a new optimal control condition corresponding to that external disturbance. In this case, if the designer has no information about the system character, he will be forced to redesign the system under the assumption that the system may have the most general structure. If he has only a certain knowledge about the system of being linear or linear time-variable, it becomes possible for him to redesign by an easier procedure, by which the optimal control condition may be straightly secured through actual measurement of S. T. M.. However, when he must consider that the system may be non-linear or time-variable non-linear, he must design it by probably only one approach of learning or asymptotic recursive procedure of design.

In this section, the real-time design for the linear time-invariable or linear time-variable system will be developed. And in next section, the real-time design for the unknown non-linear object will be presented.

3. 4. 1. Real-Time Design of Control for Unkown Time-invariable Linear Objects

For linear system it is quite possible to find an optimal control to get the desired output, only if S. T. M. or impulse response of the object can be given once by any way. Thus, the design procedure is nonrecursive.

3. 4. 1. 1. Computation of S. T. M.

The system relation of linear system is obviously described by Eq. (3. 47).

$$\boldsymbol{x}=H\boldsymbol{u} \tag{3.47}$$

or

$$x_j=g_ju_0+g_{j-1}u_1+\cdots+g_0u_j, \quad j=0,\ 1,\ 2,\ \cdots,\ n \tag{3.48}$$

As far as the system is noise-free, the weighting sequence g_0, g_1, g_2, $\cdots$ is easily obtained by applying a test step input $u(t)$. Calculating equations are

$$g_j=(\Delta x_j-\Delta x_{j-1})/\Delta u, \quad j=0, 1, 2, \cdots, n \tag{3.49}$$

where, Δx_j is the step response sequence corresponding to $\Delta u(t)$. The S. T. M. of the linear system is evaluated by applying g_j, $j=0, 1, \cdots, n$ of Eq. (3.49) to Eq. (3.6).

3. 4. 1. 2. Determination of Optimum Control

The optimal control $\mathring{\boldsymbol{u}}$ causing the desired output $\boldsymbol{x}_d$ can be derived directly from the vector relation of Eq. (3.50).

$$\boldsymbol{x}_d=H\mathring{\boldsymbol{u}} \tag{3.50}$$

or

$$\left.\begin{aligned}\mathring{u}_j&=(x_{dj}-g_j\mathring{u}_0-g_{j-1}\mathring{u}_1-\cdots-g_1\mathring{u}_{j-1})/g_0, \quad g_0\neq 0\\ \mathring{u}_j&=(x_{d,\,j+1}-g_{j+1}\mathring{u}_0-g_j\mathring{u}_1-\cdots-g_2\mathring{u}_{j-1})/g_1, \quad g_0=0, \quad g_1\neq 0\end{aligned}\right\} \tag{3.51}$$

$$j=0, 1, 2, \cdots, n$$

where g_j, $j=0, 1, 2, \cdots, n$ are, of course, those which have been secured by Eq. (3.49).

Although we can get $\mathring{u}_j$, $j=0, 1, 2, \cdots, n$ by iterative evaluation of Eq. (3.51), the evaluated solution of control may cause to appear the hidden deviation between sampling instants, even if the behavior at the sampling timh is coincident with the component of $\boldsymbol{x}_d$. If the degree of system lag s_p and that of input function s_i are both obtained by some procedure, dead-beat response can be realized as follows: Under the dead-beat condition, $\mathring{u}_j=\mathring{u}_{s_0-1}$, $j=s_0$, s_0+1, $\cdots$, $s_0=s_p+s_i$. Hence the optimal control sequence $\mathring{u}_j$, $j=0, 1, 2, \cdots, s_0-1$ can be calculated by solving the following simultaneous equations.

$$\left.\begin{aligned}&x_{d,\,s_0+j}=g_{s_0+j}u_0+g_{s_0+j-1}u_1+\cdots+(g_{j+1}+\cdots+g_0)u_{s_0-1}\\ &\quad j=-1, 0, 1, \cdots, s_0-2 \quad \text{for} \quad g_0\neq 0\\ &\quad j=0, 1, 2, \cdots, s_0-1 \quad \text{for} \quad g_0=0, \; g_1\neq 0\end{aligned}\right\} \tag{3.52}$$

The succeeding controls $\mathring{u}_{s_0}$, $\mathring{u}_{s_0+1}$, $\cdots$ should be $\mathring{u}_{s_0-1}$, as far as the estimated value of s_0 is correct.

If the application of the test step signal forces the system to shift to the state $(\boldsymbol{u}^1, \boldsymbol{x}^1)$ from the initial condition $(\boldsymbol{u}^0, \boldsymbol{x}^0)$, the incremental control $\Delta\boldsymbol{u}^2$ compensating the incremental output $(\boldsymbol{x}_d-\boldsymbol{x}^1)$ is calculated by Eq. (3.53).

$$\Delta\boldsymbol{x}_d^2=\boldsymbol{x}_d-\boldsymbol{x}^1=H\Delta\boldsymbol{u}^2 \tag{3.53}$$

Then

$$\boldsymbol{u}^2=\boldsymbol{u}^1+\Delta\boldsymbol{u}^2 \tag{3.54}$$

Thus, by applying the second stage control $\boldsymbol{u}^2$, the system reaches at $\boldsymbol{x}_d$ by two stages. Fig. 3.8 shows the basic flow diagram of the real-time recursive design procedure for noiseiess, unknown time-invariant object.

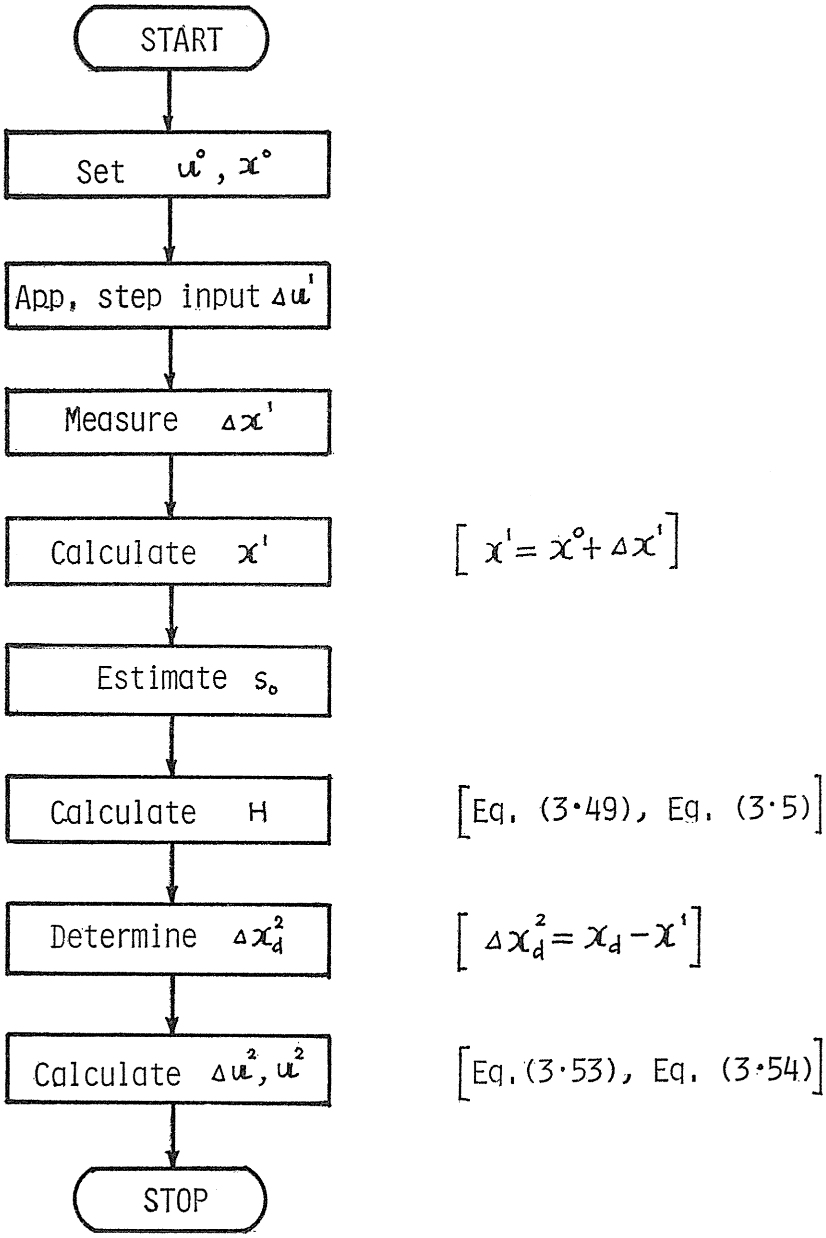


Fig. 3. 8 Computer flow diagram of real-time design for time-invariable control system

3. 4. 2. Real-Time Design of Control for Unknown Time-Variable Linear Objects

In this section, we will discuss on the real-time optimal control for the linear plant whose properties excepting the quality of time variation of the system parameters are quite unknown. The basic principle and the main algorithm are almost the same as those of the time-invariable linear system in *3. 4. 1.* However, the important difference appears in the measurement of equivalent S. T. M. and in the computation of control. These subjects will be discussed in the following sections.

3. 4. 2. 1. Equivalent S. T. M. of Time-Variable Linear System

For convenience, we will now consider a simple, second order, time-variable system given by Eq. (3. 55).

$$\ddot{x}(t)+a(t)\dot{x}(t)+b(t)x(t)=u(t) \tag{3.55}$$

Let the coefficients $a(t)$, $b(t)$ varies around a relevant constant value $\hat{a}$, $\hat{b}$ and be expressed by Eq. (3. 56).

$$\left.\begin{aligned} a(t)&=\hat{a}+\Delta a(t) \\ b(t)&=\hat{b}+\Delta b(t) \end{aligned}\right\} \tag{3.56}$$

We can rewrite Eq. (3. 55) to Eq. (3. 57).

$$\ddot{x}(t)+\hat{a}\dot{x}(t)+\hat{b}x(t)=u(t)-\{\Delta a(t)\dot{x}(t)+b(t)x(t)\} \tag{3.57}$$

The system equation of Eq. (3. 55) is linear, then the second term in right side of Eq. (3. 57) is proportional to the input u. Expressing that term by Eq. (3. 58),

$$\Delta a(t)\dot{x}(t)+\Delta b(t)x=k(t)u(t) \tag{3.58}$$

we can get next equation.

$$\ddot{x}(t)+\hat{a}\dot{x}(t)+\hat{b}x(t)=u(t)\{1-k(t)\} \tag{3.59}$$

Where the proportional coefficient $k(t)$ is a time function representing the parameter-variation of the system. Referring to Eq. (3. 59), the following two items can be derived:

(a) System response of the time-variable system caused by the input u is the same as that of a time-invariable system with coefficients of $\hat{a}$, $\hat{b}$ subjected by $u\{1-k(t)\}$. The function of $u\{1-k(t)\}$ is an equivalent input.

(b) $u\cdot k(t)$ is a component of the input which represents the effect of parameter-variation on the system response.

Referring to Eq. (3. 59), the vector equation of Eq. (3. 60) can be directly derived.

$$\boldsymbol{x}(\tau)=G(\tau)*[I-K(\tau)\boldsymbol{u}(\tau)] \tag{3.60}$$

When the input is a discrete value input, Eq. (3. 60) comes to Eq. (3. 61).

$$\boldsymbol{x}(\tau)=[G(\tau)*(I-K(\tau))]\boldsymbol{u} \tag{3.61}$$

Where, in Eq. (3. 60) and Eq. (3. 61),

$G(\tau)$ = Modified S. T. M. of the time-invariable system with coefficients of $\hat{a}$ and $\hat{b}$.

I = Unit matrix

$$K(\tau)=\begin{pmatrix} k_0(\tau) & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & k_1(\tau) & & & & \cdot \\ \cdot & \cdot & \cdot & & & \cdot \\ \cdot & \cdot & & \cdot & & \cdot \\ \cdot & \cdot & & & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & \cdot & k_n(\tau) \end{pmatrix} \tag{3.62}$$

$$k_j(\tau)=k(jT+\tau), \quad j=0, 1, 2, \cdots, n$$

Now, if the parameter variation is approximated by a step-wise curve, the expression of Eq. (3.63) is correct.

$$K(\tau)=\boldsymbol{I}(\tau)K(0)=\boldsymbol{I}(\tau)K \tag{3.63}$$

where, $\boldsymbol{I}(\tau)$ is a diagonal matrix whose component is the zero-order holding function $1(\tau)$. That is

$$\boldsymbol{I}(\tau)=1(\tau)I. \tag{3.64}$$

Then Eq. (3.61) can be modified as follows:

$$\left.\begin{aligned} \boldsymbol{x}(\tau) &= [G(\tau) * \boldsymbol{I}(\tau)](I-K)\boldsymbol{u} \\ &= H(\tau)(I-K)\boldsymbol{u} \\ &= H_{eq}(\tau)\boldsymbol{u} \end{aligned}\right\} \tag{3.65}$$

where

$H(\tau)$ = Modified S. T. M. of the time-variable system preceded by the zero-order holder.

$$=\begin{pmatrix} h_0(\tau) & 0 & \cdot & \cdot & \cdot & \cdot & 0 \\ h_1(\tau) & h_0(\tau) & \cdot & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & & \cdot & & & \cdot \\ \cdot & \cdot & & & \cdot & & \cdot \\ \cdot & \cdot & & & & \cdot & \cdot \\ h_n(\tau) & h_{n-1}(\tau) & & \cdot & \cdot & \cdot & h_0(\tau) \end{pmatrix} \tag{3.66}$$

$$H_{eq}(\tau)=\{(h_{eq})_{jl}(\tau)\}=H(\tau)[I-K]$$

$$=\begin{pmatrix} h_0(\tau)(1-k_0) & 0 & \cdot & \cdot & \cdot & 0 \\ h_1(\tau)(1-k_0) & h_0(\tau)(1-k_1) & & & & 0 \\ \cdot & \cdot & \cdot & & & \cdot \\ \cdot & \cdot & & \cdot & & \cdot \\ \cdot & \cdot & & & \cdot & \cdot \\ h_n(\tau)(1-k_0) & h_{n-1}(\tau)(1-k_1) & & \cdot & \cdot & h_0(\tau)(1-k_n) \end{pmatrix} \tag{3.67}$$

or

$$(h_{eq})_{jl}(\tau)=\begin{cases} h_{j-l}(\tau)(1-k_l), & j\geq l \\ 0, & j<l \end{cases}, \quad j,\ l=0,\ 1,\ 2,\ \cdots,\ n \tag{3.68}$$

$H_{eq}(\tau)$ is the equivalent modified S. T. M. of the original time-variable system.

The output vector at sampling instants can be obtained by putting $\tau=0$ in Eq. (3.65).

$$\boldsymbol{x}=H[I-K]\boldsymbol{u}=H_{eq}\boldsymbol{u} \tag{3.69}$$

where

$$\left.\begin{aligned} &H_{eq}=H[I-K] \\ &(h_{eq})_{jl}=\begin{cases} h_{j-l}(1-k_l), & j\geq l \\ 0, & j<l \end{cases} \end{aligned}\right\} \tag{3.70}$$

And next relations follow.

$$\begin{aligned} x_j=h_j(1-k_0)u_0+h_{j-1}(1-k_1)u_1+\cdots+ \\ +h_1(1-k_{j-1})u_{j-1}+h_0(1-k_j)u_j, \quad j=0,\ 1,\ 2,\ \cdots,\ n \end{aligned} \tag{3.71}$$

3. 4. 2. 2. Computation of Equivalent S. T. M.

In this paragraph, we will explain the computing procedure of equivalent S. T. M. of the time-variable system only at the points of $\tau=0$. When the parameter variation is repetitive, the elements of H_{eq} can be calculated by Eq. (3.71) applying the data of the responses which are caused by impulsive inputs starting at $t/T_s=0$, 1, $\cdots$. However, this procedure requires too many times of measurements of impulse response to be adopted.

The author now proposes a better method by which all the elements of H_{eq} can be measured by applying step signal, only two times, if the ocurrence of parameter variation are controllable. In this case, the measurement of equivalent S. T. M. is performed by the following two step.

(a) Measurement of step response of the time-invariable system with $\hat{a}$ and $\hat{b}$.

Stop the parameter-variation and measure the step response. Let the test signal be unit step function and the corresponding output be $\Delta\boldsymbol{x}$. The components of H can be derived by putting $k_0=k_1=\cdots=0$ and $u_0=u_1=\cdots=1$ in Eq. (3.71), or by putting $\Delta u=1$ in Eq. (3.49) which is replaced g by h. That is,

$$h_j = \Delta x_j - \Delta x_{j-1}, \quad j=0,\ 1,\ 2,\ \cdots,\ n \tag{3.72}$$

(b) Meaurement of step response of the time-variable system

Measure the unit step response $\Delta \boldsymbol{x}$ of the system with variable coefficients of $a(t)$ and $b(t)$. Putting $u_0 = u_1 = \cdots = 1$ the recursive equation of Eq. (3.73) to calculate the components of matrix $(I-K)$ can be derived from Eq. (3.71).

That is,

$$1-k_j = \begin{cases} \dfrac{1}{h_0}\left[\Delta x_j - \sum_{l=1}^{j} h_l (1-k_{j-l})\right], & h_0 \neq 0 \\ \dfrac{1}{h_1}\left[\Delta x_{j+1} - \sum_{l=2}^{j+1} h_l (1-k_{j-l+1})\right], & h_0 = 0,\ h_1 \neq 0 \end{cases} \tag{3.73}$$

The h_j, $j=0, 1, \cdots, n$ in Eq. (3.73) are the ones secured in the previous step (a). After the calculation of h_j and $1-k_j$, $j=0, 1, \cdots, n$, the components of H_{eq} is computed by Eq. (3.70).

3. 4. 2. 3. Determination of Optimal Control

The optimal control $\mathring{\boldsymbol{u}}$ causing the desired output $\boldsymbol{x}_d$ is determined by applying the S. T. M. H_{eq} which has been secured by the procedure explained in *3. 4. 2. 2.* to Eq. (3.74) ~ Eq. (3.76).

$$\boldsymbol{x}_d = H_{eq} \mathring{\boldsymbol{u}} \tag{3.74}$$

or

$$\mathring{u}_j = \frac{x_{d,\,j} - \sum_{l=0}^{j-1} (h_{eq})_{jl} \mathring{u}_l}{(h_{eq})_{jj}}, \quad j=0,\ 1,\ \cdots,\ n \tag{3.75}$$

If $h_0=0$ and then $h_{00}=h_{11}=\cdots=h_{nn}=0$, then Eq. (3.75) must be substituted by Eq. (3.76).

$$\mathring{u}_j = \frac{x_{d,\,j} - \sum_{l=0}^{j-2} (h_{eq})_{jl} \mathring{u}_l}{(h_{eq})_{j,\,j-1}}, \quad j=0,\ 1,\ \cdots,\ n \tag{3.76}$$

when $(h_{eq})_{j,\,j-1}$ may be zero even if $h_1 \neq 0$, $\mathring{u}_j$ must be approximated by $\mathring{u}_{j-1}$.

In the computing process of components of $\mathring{\boldsymbol{u}}$ by Eq. (3.75) or Eq. (3.76), the following two notices should be taken.

(a) After application of the test step input, the system control situation will be moved from $(\boldsymbol{u}^0, \boldsymbol{x}^0)$ to $(\boldsymbol{u}^1, \boldsymbol{x}^1)$. Then the control in second stage comes to $(\mathring{\boldsymbol{u}} - \boldsymbol{u}^1)$.

(b) Several notices relative to the dead-beat response time $(s_0 T)$ must be introduced as in *3. 4. 1.* That is, in order to realize the dead-beat response, it is necessary to estimate s_0 to derive a simultameous equation being analogous to Eq. (3.52).

3. 4. 2. 4. Computer Flow Diagram

Fig. 3. 9 shows the flow diagram of computer algorithm for automatic design for unknown time-variable linear control system.

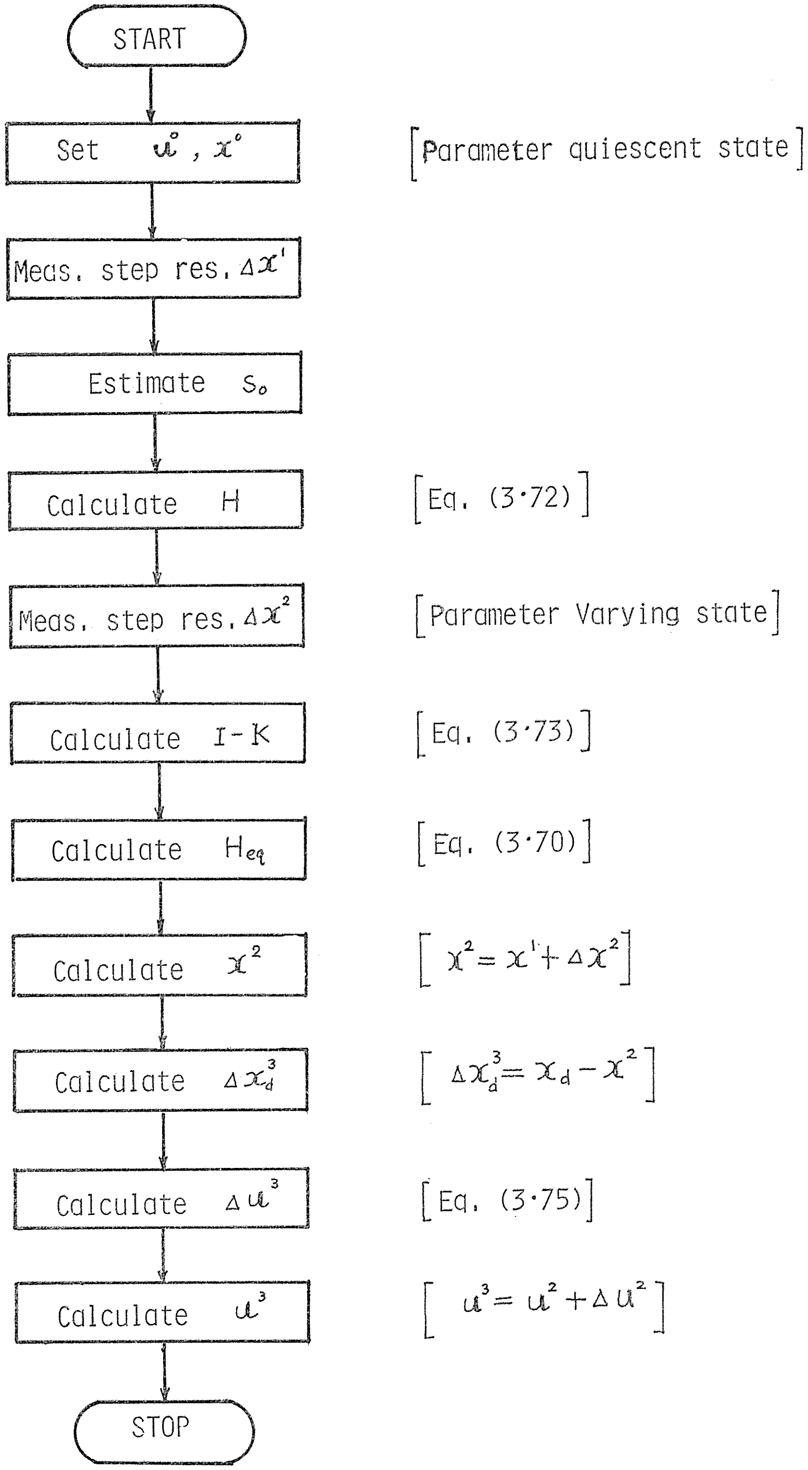


Fig. 3. 9. Computer flow diagram of automatic control design for time-variable linear system

3. 5. Real-Time Design of Control by Convergent Recursive Method for Unknown Non-Linear Objects

3. 5. 1. Proposition of LCDS

In this section, we will investigate the real-time design of control for unknown non-linear objects.

Now, consider a non-linear control system operating at a control situation ($\boldsymbol{u}^0$, $\boldsymbol{x}^0$) and let the initial control situation be disturbed by an external stimulus. It will be desirable for the system to shift into the vicinity of a new desirable state corresponding to that disturbance. In this condition, the real-time measurement of moved system characteristics and the real-time redesign of optimum control are both very difficult. The author believes that the most poweful procedure for this case will be composed of learning processes, and now proposes a new leaning system of LCDS (Learning Control Dynamics System), which learns the control-dynamics of non-linear control system by the convergent recursive redesigning process.

This procedure is based on the following two essential principles:

(a) Behavior of continuous non-linear system for a small excitation (perturbation) can be generally regarded as linear.

(b) Linear control system (involving time-variable system) can be designed, in general, by the unique method through S. T. M..

According to the above principles, it is clear that the non-linear system, in each stage of repetitive procedure, can be considered as a linear, time-variable system, and that the achievement of the subgoal in each stage can be realized by the design procedure for time-variable system explained in *3. 4. 2.*. Thus, the approach to the final desired state will be established by the repetition of (1) measurement of locally linearized equivalent S. T. M. and (2) determination of incremental control in each stage. Several important subjects about this procedure will be discussed in the following sections.

Here, we must pay attention to the next important features. That is, the system which has learned optimal policy relative to the external disturbances can quickly adapt to the repeated similar disturbances. From the above view-point, this learning procedure is an application of the "*Learntrol*" principle to the improvement of control-dynamics. The recurrent behaviors and the quick adaptation in this learning procedure correspond to the step-by-step search and the jumping search in Leantrol respectively. (As to the detail of Learntrol, see *4. 3. 2.*).

3. 5. 2. Computation of Linearized Equivalent S. T. M.

This section is assigned to the explanation of the equivalent S. T. M. which is linearized in each stage of the recursive approach.

Now, we will consider, for simplicity, the second order system described by Eq. (3.77).

$$\ddot{x}(t)+a(x,\ \dot{x})\dot{x}(t)+b(x,\ \dot{x})x(t)=u(t) \tag{3.77}$$

For this system, the system parformance for small incremental input can be considered to be linear and time-variable, and the measurement principle of equivalent S. T. M. is the same as that explained in *3. 4. 2.*.

Let the small incremental control in the i-th stage be Δu^i, and the corresponding output and its derivatives be Δx^i, $\Delta\dot{x}^i$, and $\Delta\ddot{x}^i$. The time-variable system in the i-th stage is expressed by Eq. (3.78).

$$\Delta\ddot{x}^i(t)+a^i(t)\Delta\dot{x}^i(t)+b^i(t)\Delta x^i(t)=\Delta u^i(t)^* \tag{3.78}$$

The initial control situation of the recursive procedure can be considered as a stationary mode**. Then the system in the first stage is regarded as a time-invariant system with constant coefficients of a^1 and b^1. We can thus rewrite Eq. (3.78) in the form of Eq. (3.79).

$$\Delta\ddot{x}^i(t)+a^1\Delta\dot{x}^i(t)+b^1\Delta x^i(t)$$
$$=\Delta u^i(t)-[a^i(t)-a^1]\Delta\dot{x}^i(t)-[b^i(t)-b^1]\Delta x^i(t) \tag{3.79}$$

Eq. (3.79) has the same form as Eq. (3.57), then the equivalent S. T. M. in i-th stage can be measured by the same method as in *3. 4. 2. 1.*.

Let the equivalent S. T. M. in i-th and 1st stages be H^i_{eq} and H^1 respectuvely. In accordance with the notation in *3. 4. 2. 1.*, we can get the following vector equation.

$$\Delta\boldsymbol{x}^i=H^1[I-K^i]\Delta\boldsymbol{u}^i=H^i_{eq}\Delta\boldsymbol{u}^i \tag{3.80}$$

where

$$H_{eq}{}^i=H^1[I-K^i] \tag{3.81}$$

All the components of H^1 can be secured by the data of incremental response $\Delta\boldsymbol{x}^1$ measured in the first stage. That is, the computing equation for H^1 of (3.82) can be easily derived from the relation of Eq. (3.71).

$$h^1_j=\frac{\Delta x^1_j-\sum_{l=0}^{j-1}h_l\Delta u^1_{j-l}}{\Delta u^1_0}, \quad j=0,\ 1,\ \cdots,\ n \tag{3.82}$$

And all the diagonal elements ef $[I-K^i]$ can be also determined by Eq. (3.83) using the measured value of incremental response $\Delta\boldsymbol{x}^i$.

$$1-k^i_j=\frac{1}{h^i_0}\cdot\frac{\Delta x^i_j-\sum_{l=1}^{j}h^i_l(1-k^i_{j-l})\Delta u^i_{j-l}}{\Delta u^i_j} \tag{3.83}$$

If $h^i_0=0$ but $h^i_1\neq0$, Eq. (3.84) should be used instead of Eq. (3.83).

$$1-k^i_j=\frac{1}{h^i_1}\cdot\frac{\Delta x^i_{j+1}-\sum_{l=2}^{j+1}h^i_l(1-k^i_{j-l+1})\Delta u^i_{j-l+1}}{\Delta u^i_j} \tag{3.84}$$

---

*) As far as the small response in the i-th stage is concerned, coefficients a^i and b^i which are the functions of x^i and $\dot{x}^i$ can be regarded as an explicit function of t because the x^i and $\dot{x}^i$ are invariable in this stage.

**) The appearance period of the step-wise disturbance is selected so long that the system can arrive at a stationary state at the end of each stage. Then the initial control state preceding the appearance of disturbance can be also regarded as a stationary mode.

At last, each element of H^i_{eq} is computed by Eq. (3.85).

$$(h_{eq})^i_{jl} = \begin{cases} h^i_{j-l}(1-k^i_l), & j \geq l \\ 0, & j < l \end{cases} \tag{3.85}$$

3. 5. 3. Determination of Control in Each Stage

The incremental control $\Delta \boldsymbol{u}^i$ corresponding to the desired increments $\Delta \boldsymbol{x}^i_d$ in the i-th stage can be calculated through Eq. (3.86) in which H^{i-1}_{eq} has been secured in the $(i-1)$-th stage.

$$\Delta \boldsymbol{x}^i_d = H^{i-1}_{eq} \Delta \boldsymbol{u}^i \tag{3.86}$$

The recursive form of Eq. (3.86) is given as Eq. (3.87), referring to Eq. (3.75) and Eq. (3.76).

$$\left.\begin{aligned} \Delta u^i_j &= \frac{\Delta x^i_{d,j} - \sum_{l=0}^{j-1} (h_{eq})^{i-1}_{jl} \Delta u^i_l}{(h_{eq})_{jj}}, \quad (h_{eq})_{jj} \neq 0 \\ \Delta u^i_j &= \frac{\Delta x^i_{d,j} - \sum_{l=0}^{j-2} (h_{eq})^{i-1}_{jl} \Delta u^i_l}{(h_{eq})_{j,j-1}}, \quad (h_{eq})_{jj} = 0 \\ & \qquad\qquad\qquad\qquad\qquad\qquad\quad (h_{eq})_{j,j-1} \neq 0 \end{aligned}\right\} \tag{3.87}$$

In the actual evaluation of Eq. (3.87), we must take account of the dead-beat condition in the same way as in *3. 4. 2.*.

3. 5. 4. Computer Algorithm

The algorithm for LCDS which is shown in Fig. 3. 10 has been made by referring to Fig. 3. 9.

3. 6. Proposition of Learndynatrol

"*Learntrol-* II" is a learning control system which conducts itself to hold the static optimum condition by learning search strategy under the recursive disturbances of step-wise load changes. And the remarkable features of the system is the improvement of its hill-climbing behavior from the step-by-step search without any experiences to the jumping search with the same experience.

The author intends to propose a new learning control system which acts to improve not only the searching behavior to attain the statical optimum, but also the dynamical performance which is the transient process shifting from starting state to the new optimum state. We call this system as "*Learndynatrol*" (Learning Dynamics Control System) which is a kind of advanced Learntrol in the direction of the improvement of dynamical behavior.

3. 6. 1. Setting up of the Problem

Fig. 3. 11 shows the control loop of Learndynatrol, which is the same diagram as that of Learntrol in Fig. 4. 3. The controlled object is an unknown system whose output is the performance index (P. I.) of the lower order dynamical system, and whose input is the corresponding control which is to be secured by learning

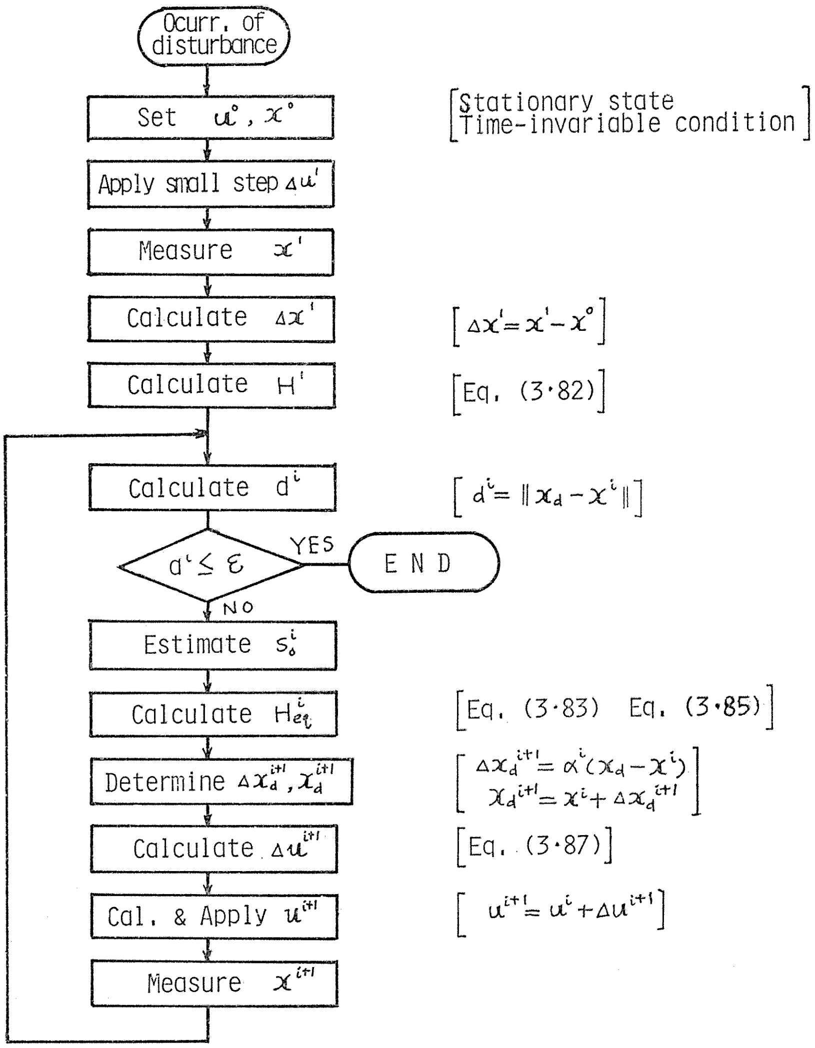


Fig. 3. 10. Computer algorithm of real-time design for unknown non-linear objects

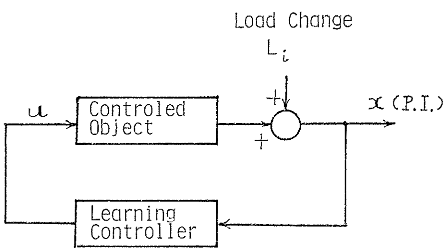


Fig. 3. 11. Control loop of Learndynatrol

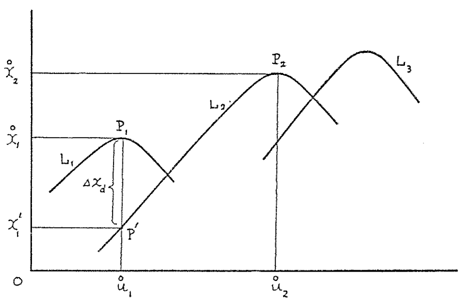


Fig. 3. 12. Shifting of static character of controlled object due to load changes

controller. Furthermore, the object has a set of unimodel static curves, each of which corresponds to each different load L_i, $i=1, 2, 3, \cdots$ (c.f. Fig. 3. 12).

Now, we will assume that a step-wise change of load ($L_1 \rightarrow L_2$) suddenly occurs when the system has been operating at an optimum control situation ($\mathring{u}_1$, $\mathring{x}_1$), as shown in Fig. 3. 12. The learning aim of Learntrol in such circumstance is to make an association between the summarized disturbance (Δx_{L_1}, $\mathring{u}_1$) and the new optimum control $\mathring{u}_2$, by recursive learning procedure. As the learning progresses, the external behavior of Learntrol is gradually improved from the step-by-step search to the jumping search (c.f. Fig. 3. 13).

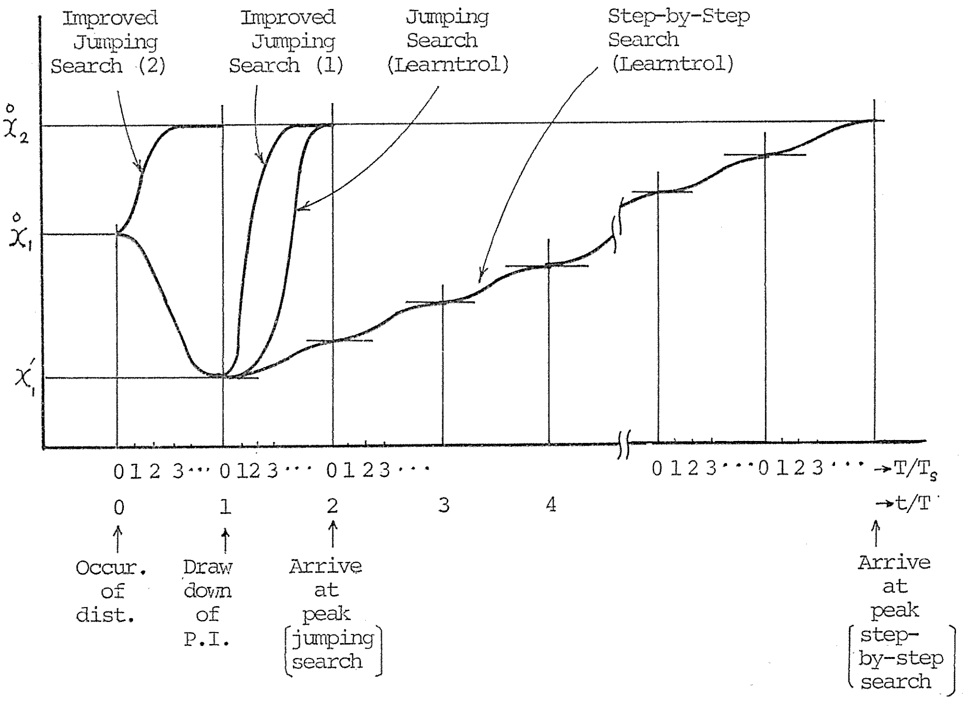


Fig. 3. 13. Explanation of searching behaviors of step-by-step, jumping, improved jumping searches in Learndynatrol. T_s is sampling period; T is length of a transient response, hense is the period of recursive learning process.

The new learning control scheme of Learndynatrol which is investigated in this chapter is Learntrol with additional ability to improve the transient behavior of the jumping search. And we have now the following two interesting cases.

Case-1 To improve the jumping behavior from the drawn down point P′ to the new peak P_2. (c. f. Fig. 3. 12 and curve (1) in Fig. 3. 13).

Case-2 To improve the jumping process from the occurrence poinţ P_1 to the new peak P_2. (c. f. Fig. 3. 12 and curve (2) in Fig. 3. 13).

The former is the case when the occurrence and the nature of disturbance can be detected indirectly only by the drawing down of output, while the latter is the case when the disturbance itself can be directly observed. For both cases, the main subject reduces to the problem how to self-improve the dynmaic character of control for the unknown object. Thus, if we know only that the object is a non-linear system with single peak performance function, the problem can be generalized in the form of "Real-time redesign of control for unknown non-linear objects".

3. 6. 2. Planning of Learndynatrol

As stated in *3. 6. 1.*, the problem comes to the general approach to "Real-time self-design of control for unknown general object". Therefore, the planning of this problem can be also made by the principle of LCDS proposed in *3. 5.* However, we will here adopt another procedure in which Leantrol Data (the data obtained in the step-by-step search in the early period of Learntrol II) are used effectively, and hereafter we will name it as *Learndynatrol.*

The fundamental form of real-time design in Learndynatrol is the convergent recursive mode, whose process is composed of measurement of equivalent S. T. M. and determination of incremental control in each stage. The noticeable features of Learndynatrol can be stated as follows:

In order to know the system character in each stage, we must perform a small step-input test in that stage. Since the object should be regarded as non-linear, this small step response is that of a time variable system whose parameter varies as a time-function dependent upon the output variation in that stage, and it is exemplified as $\Delta x^i(t)$ in Fig. (3. 14). By the way, to know all the element of the equivalent S. T. M. of each stage, we must try many step tests corresponding to various output levels starting at each sampling time. However, this procedure is too troublesome to be adopted. In Learndynatrol, the procedure is much simplified by using the step response data obtained in the step-by-step search in the early period of Learntrol-II, because the step-by-step search data is considered as a set of small step responses at various output level existing in the searching range of output. The repeating period of step-by-step search in Learntrol-II is selected so

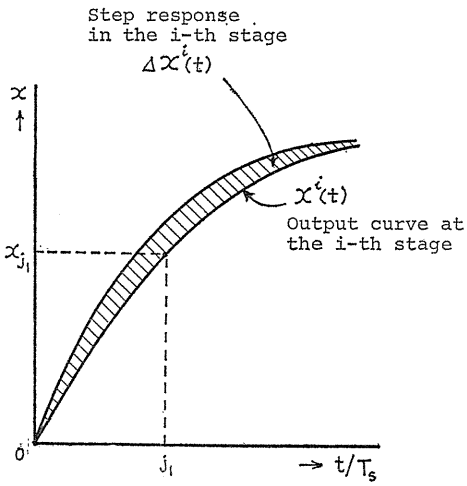


Fig. 3. 14. Step-input test in each stage of recursive approach

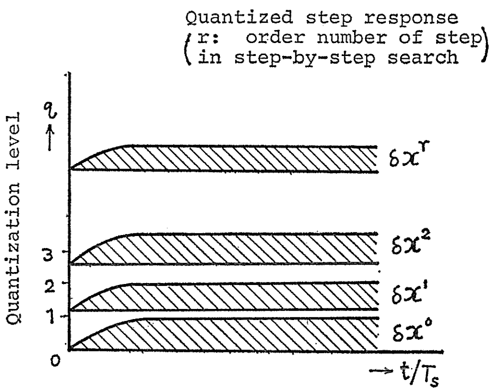


Fig. 3. 15. Learntrol data i. e. quantized step responses in step-by-step search of Learntrol

long that the system state at the end of each period can be considered stationary (constant output). So that the transient behavior in the step-by-step search is the step response of a time-invariant system whose parameter takes a constant value corresponding to the output level in each step. Each of δx^r, $r=0, 1, 2, \cdots$ shown in Fig. 3. 15 is a small step response of the time-invariable system, assigned to the quantized level of output x. The group of these responses becomes naturally Learntrol data. The important aim of Learndynatrol's procedure is to relate the Learntrol data to the computation of equibalent S. T. M. of the time-variable system in each stage. The detailed discussion on this problem will be shown mathematically in the following sections.

The essential process of the real-time design is composed of the following few terms:

(1) Strage of the Learntrol Data.

(2) Computation of system parameters of the time-invariant system corresponding to each step of step-by-step search in Learntrol by using the data obtained in Term (1).

(3) Computation of equivalent S. T. M. in each stage of the recursive redesigning procedure.

(4) Determination of the subgoal and the incremental control to get to the subgoal.

(5) Repeating the process of Terms (3) $\sim$ (4), the system approaches to the optimum control situation.

The explanation of this process will be developed in the succeeding section, in the order of (3) $\rightarrow$ (1) $\rightarrow$ (2) $\rightarrow$ (4).

3. 6. 3. Computation of Linearized Equivalent S. T. M.

Now we consider the object of second order non-linear system given by Eq. (3. 88).

$$\ddot{x}(t)+a(x)\dot{x}(t)+b(x)x(t)=u(t) \tag{3.88}$$

where, the coefficients of a and b are both a function of only x. The system equation for small input in the i-th stage of repetitive redesign process is expressed by Eq. (3. 89).

$$\Delta\ddot{x}^i(t)+a(x^i)\Delta\dot{x}^i(t)+b(x^i)\Delta x^i(t)=\Delta u^i(t) \tag{3.89}$$

The association $(\Delta u^i, \Delta x^i)$ is a small control variation around $x^i(t)$, therefore, Eq. (3. 89) can be transformed to a time-variable equation of Eq. (3. 90).

$$\Delta\ddot{x}^i(t)+a^i(t)\Delta\dot{x}^i(t)+b^i(t)\Delta x^i(t)=\Delta u^i(t) \tag{3.90}$$

Now, rewritting Eq. (3. 90) in the form of difference equation, Eq. (3. 91) can be derived.

$$\frac{1}{T^2}(\Delta x_j^i-2\Delta x_{j-1}^i+\Delta x_{j-2}^i)+\frac{1}{T}a_j^i(\Delta x_j^i-\Delta x_{j-1}^i+b_j^i\Delta x_j^i=\Delta u_j^i \tag{3.91}$$

Thus the relation of Eq. (3. 92) can be obtained.

$$\Delta x_j^i=\frac{\Delta u_j^i+(2T^{-2}+T^{-1}a_j^i)\Delta x_{j-1}^i-T^{-2}\Delta x_{j-2}^i}{T^{-2}+T^{-1}a_j^i+b_j^i}, \quad j=0, 1, 2, \cdots \tag{3.92}$$

Here, putting as Eq. (3.93)

$$\left.\begin{aligned} \frac{1}{T^{-2}+T^{-1}a_j^i+b_j^i}&=\alpha_j^i \\ \frac{2T^{-2}+T^{-1}a_j^i}{T^{-2}+T^{-1}a_j^i+b_j^i}&=\beta_j^i \\ \frac{-T^{-2}}{T^{-2}+T^{-1}a_j^i+b_j^i}&=\gamma_j^i \end{aligned}\right\} \qquad (3.93)$$

Eq. (3.92) can be expressed as Eq. (3.94)

$$\left.\begin{aligned} \Delta x_0^i&=\alpha_0^i\Delta u_0^i \\ \Delta x_1^i&=\alpha_1^i\Delta u_1^i+\beta_1^i\Delta x_0^i \\ \Delta x_j^i&=\alpha_j^i\Delta u_j^i+\beta_j^i\Delta x_{j-1}^i+\gamma_j^i\Delta x_{j-2}^i, \quad j=2,\ 3,\ 4,\ \cdots \end{aligned}\right\} \qquad (3.94)$$

From Eq. (3.94), Eq. (3.95) can be derived.

$$\left.\begin{aligned} \Delta x_0^i&=\alpha_0^i\Delta u_0^i \\ \Delta x_1^i&=\alpha_1^i\Delta u_1^i+\beta_1^i\alpha_0^i\Delta u_0^i \\ \Delta x_j^i&=\alpha_j^i\Delta u_j^i+\beta_j^i\alpha_{j-1}^i\Delta u_{j-1}^i+(\beta_j^i\beta_{j-1}^i+\gamma_j^i)\alpha_{j-2}^i\Delta u_{j-2}^i+,\ \cdots,\ j=2,\ 3,\ 4,\ \cdots \end{aligned}\right\} \qquad (3.95)$$

Thus, the equivalent S. T. M. of the i-th stage can be given by Eq. (3.96).

$$H_{eq}^i=\begin{pmatrix} \alpha_0^i & 0 & 0 & \cdot & \cdot & \cdot & 0 & 0 \\ \beta_1^i\alpha_0^i & \alpha_1^i & 0 & & & & \cdot & \cdot \\ (\beta_2^i\beta_1^i+\gamma_2^i)\alpha_0^i & \beta_2^i\alpha_1^i & \alpha_2^i & & & & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & & & \cdot & \cdot \\ \cdot & \cdot & \cdot & & \cdot & & \cdot & \cdot \\ \cdot & \cdot & \cdot & & & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \beta_n^i\alpha_{n-1}^i & \alpha_n \end{pmatrix} \qquad (3.96)$$

If the coefficients of α_j^i, β_j^i, γ_j^i, $j=0,\ 1,\ 2,\ \cdots,\ n$ can be evaluated by an adequate procedure, we can secure all the element elements of H_{eq}^i. The author nowproposes an interesting method using the Table 3. 1 which is shown in *3. 6. 4.*.

As is shown in Eq. (3.93), all the coefficients of α_j^i, β_j^i, γ_j^i are the functions of j or x_j^i. Consequently, we can secure the value of these coefficients by reading out the value of α, β and γ from the column in Table 3. 1 corresponding to the quantization level of x_j^i.

3. 6. 4. Computation and Storage of Coefficients α, β, γ

We will investigate here how to cnmpute the coefficients of α, β and γ by using the Learntrol data which is obtained in the step-by-step search in early period of Learntrol.

Now, we will use vectoral expression of the transient behavior by dividing one period of stage iteration (T) into n-sections (c. f. Fig. 3. 13). Let the magnitude of step-wise control applied in the r-th step of step-by-step search of Learntrol be δu^r, and the output variation caused by δu^r be δx^r. The normalized step response of the r-th step can be given by Eq. (3. 97)

$$\delta\tilde{\boldsymbol{x}}^r = \frac{\delta \boldsymbol{x}^r}{\delta u^r} \tag{3.97}$$

Now, we will consider to secure the coefficient of α^r, β^r and γ^r by applying the data of normalized step response to Eq. (3. 94) Although we can get a set of (α, β, γ) by solving the recursive equation composed of the first three equations for $j=0$, 1, 2 in Eq. (3. 95), the solution is not correct. We must use the simultaneous equations for $j=2$, 3, 4 in Eq. (3. 95). As far as the small-size step response in the step-by-step search of Learntrol is concerned, all the coefficients of α_j, β_j, and γ_j are independent on j. Then, by applying Eq. (3. 98) to Eq. (3. 94), the simultaneous equation of Eq. (3. 99) can be obtained.

$$\alpha_j^r = \alpha^r, \quad \beta_j^r = \beta^r, \quad \gamma_j^r = \gamma^r, \quad j=2,\ 3,\ 4,\ \cdots \tag{3.98}$$

$$\left.\begin{aligned} \delta x_2^r &= \alpha^r \delta u^r + \beta^r \delta x_1^r + \gamma^r \delta x_0^r \\ \delta x_3^r &= \alpha^r \delta u^r + \beta^r \delta_2^r + \gamma^r \delta x_1^r \\ \delta x_4^r &= \alpha^r \delta u^r + \beta^r \delta x_3^r + \gamma \delta x_2^r \end{aligned}\right\} \tag{3.99}$$

Dividing the both sides of Eq. (3. 99) by δu^r, the normalized equation of Eq. (3. 100) are obtained.

$$\left.\begin{aligned} \delta\tilde{x}_2^r &= \alpha^r + \beta^r \delta\tilde{x}_1^r + \gamma^r \delta\tilde{x}_0^r \\ \delta\tilde{x}_3^r &= \alpha^r + \beta^r \delta\tilde{x}_2^r + \gamma^r \delta\tilde{x}_1^r \\ \delta\tilde{x}_4^r &= \alpha^r + \beta^r \delta\tilde{x}_3^r + \gamma_r \delta\tilde{x}_2^r \end{aligned}\right\} \tag{3.100}$$

where $\delta\tilde{x}_j^r \triangleq \delta x_j^r / \delta u^r$, $j=2$, 3, 4

Since δx_j^r, $j=2$, 3, 4 are given from Learntrol data, the coefficients α^r β^r and γ^r can be secured by solving Eq. (3. 100).

The value of α, β, γ evaluated by the above procedure, are stored in the cor-

Table 3. 1. Table of coefficients α, β, γ

q	1	2	…………	Q
α				
β				
γ				

responding column in Table 3. 1 where q is the quantization level of x^r. It will be better, in practice, to store an average of many evaluated values which have been obtained from many step response data in Learntrol.

The above theory on equivalent S. T. M. can be directly extended to the higher order system. However, it must be remembered for this general case that the making of the simultaneous equation of Eq. (3. 100) must be preceded by the pre-estimation of system lag s_0. When the system lag is estimated to be s_0, number of coefficients come to (s_0+1) and the (s_0+1) simultaneous equations must be solved.

3. 6. 5. Determination of Control

The desired incremental control Δu_j^i in i-th stage of the iterative Learndynatrol process can be derived from the relation of Eq. (3. 101) ~ Eq. (3. 103).

$$\Delta \boldsymbol{x}_d^i = H_{eq}^i \Delta \boldsymbol{u}^i \tag{3.101}$$

The component equation of Eq. (3. 101) comes to Eq. (3. 102) or Eq. (3. 103).

$$\left.\begin{aligned} &\Delta x_{d,0}^i = \alpha_0^i \Delta u_0^i \\ &\Delta x_{d,1}^i = \alpha_1^i \Delta u_1^i + \beta_1^i \alpha_0^i \Delta u_0^i \\ &\Delta x_{d,j}^i = \alpha_j^i \Delta u_j^i + \beta_j^i \alpha_{j-1}^i \Delta u_{j-1}^i + \cdots, \quad j=2,\ 3,\ \cdots \end{aligned}\right\} \tag{3.102}$$

$$\left.\begin{aligned} &\Delta x_{d,0}^i = \alpha_0^i \Delta u_0^i \\ &\Delta x_{d,1}^i = \alpha_1^i \Delta u_1^i + \beta_1^i \Delta x_0^i \\ &\Delta x_{d,j}^i = \alpha_j^i \Delta u_j^i + \beta_j^i \Delta x_{d,j-1}^i + \gamma_j^i \Delta x_{d,j-2}^i, \quad j=2,\ 3,\ \cdots \end{aligned}\right\} \tag{3.103}$$

The explicit equation relative to Δu_j^i comes to Eq. (3. 104)

$$\left.\begin{aligned} &\Delta u_0^i = (\alpha_0^i)^{-1} \Delta x_{d,0}^i \\ &\Delta u_1^i = (\alpha_1^i)^{-1} (\Delta x_{d,1}^i - \beta_1^i \Delta x_{d,0}) \\ &\Delta u_j^i = (\alpha_j^i)^{-1} (\Delta x_{d,j}^i - \beta_j^i \Delta x_{d,j-1}^i - \gamma_j^i \Delta x_{d,j-2}^i), \quad j=2,\ 3,\ \cdots \end{aligned}\right\} \tag{3.104}$$

In the above equations, Δx_{dj}^i is the desired incremental output in the i-th stage and α_j^i, β_j^i, γ_j^i are the values read out from the column in Table 3. 1, corresponding to the quantization level of x_j^i.

3. 6. 6. Computer Algorithm

Fig. 3. 16 shows the algorithm for computer simulation of Learndynatrol.

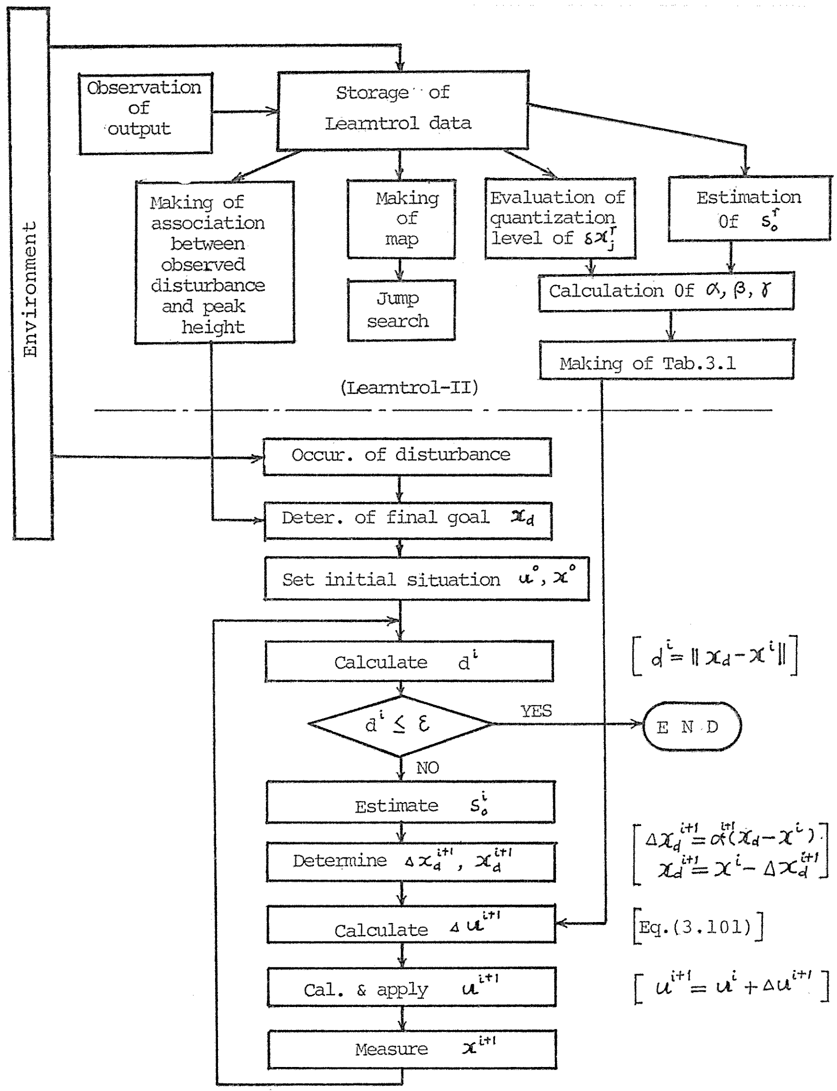


Fig. 3. 16. Computer algorithm of Learndynatrol.

4. Heuristic Approach to the Intelligent Control for Unknown Objects

4. 1. Positioning of Heuristic Approach

The concept of artificial intelligence includes various psychological functions such as learning, self-organizing, inference, association, heuristics, creation, evolution and so on, and plays serious roles in many information and control problems. Of these functions, the two of learning and heuristics are considered, at present time, to be most contributive to the control of unknown objects. Research on learning control which started about twenty years ago has grown up as an interesting control scheme succeeding to introduce widely the human intelligence into the control engineering field. Learning is the psychological function which is developed by human when he wants to remove unknowns and uncertainties about the objects, and to improve their performance and to discover new control policies on the objects, etc. "*Heuristics*" is a sophisticated intelligence which human presents in the selection of a decision from various complex possibilities. That is, heuristics is the characteristic human idea for complex decision making problems. Thus, heuristics seems to be most befitting to express behavior features of human or human-like machines to decide control policy for unknown objects. The significanece adopting the term of "*Heuristic Approach*" as one of basic classification of intelligent control schemes lies in this essential human sense of "*Heuristics*".

The planning of intelligent machine must be preceded by exhaustive analysis of human intelligence. From this view point, the study of author's group on artificial intelligence was started by experimental analysis of human intelligences. From the phsycho-engineering experiments on human subjects, some essetial properties of specified intelligence are abstracted and summarized. Basing on the experimental results, have been developed various simulation principles, and further have been proposed several unique plans of intelligent control machines. In this Chapter, are surrveyed some interesting research results on learning- and heuristic controls, and further are presented some new researches on control- and search technologies.

4. 2. Definition of Learning, Heuristics and their Related Concepts

4. 2. 1. Definition of Learning

There are various definitions and interpretations on learning in psychological sense. However, the essence of learning is the "alternation of behavior possibility by experience". Thus, we have defined it in engineering sense that learning (control) system is a (control) system which purposively alters the "possibility of (control) behavior by experience". By the term "purposively" is meant here that learning has its own distinct purpose. Besides, by the "alternation of behavior possibility" is meant here the modification of internal structure and / or internal state which may take place in the system preceding the appearance of overt behavior. Moreover, "experience" in the above definition includes both "self-experience" which is attained by the human itself and "external-experience" attained through the experience of another human. The other important point is that learning is a bilateral behavior caused as the mutual communications and actions between human and their environment.

According to tsypkin's definition, learning process is not a mere recursive process like the apparent behavior of ordinary closed (autonomous) discrete system, but is the behavior of open-towards-environment system which behaves in recursive manner and is self-improved by utilizing system experiences, in other words, by modifying the system character (parameter or structure) through the feedback reaction from environment corresponding to the system's actions in the preceding stages. According to a series of researches on "Learntrol", developed by M. Oda and K. Nakamura, the same or similar experiences are used positively in step-by-step or jumping behavior of Learntrol.

We can introduce the artificial learning to various engineering functions such as estimation, indentification, prediction, search, control, decision, etc. They are called respectively as leaning estimation, learning identification, learning prediction, learning search, learning control, leaning decision, etc.

4. 2. 2. Definition of Heuristics

The psychological definition of heuristics is expressed in various forms. Although the engineering sense definition of heuristics is not fixed so clearly, the authors' research group has proposed a definition that heuristics is a method which solves a specified problem through an algorithm (heuristic algorithm) which utilizes a series of clues (heuristic elements) having a special order of priority. We consider now a thinking or searching system whose behavior progresses step-by-step wise from the starting state to the optimal final state. There are so many possible branches on any reachable path that it is unfeasible to check all the composed paths sequentially. In such problems, heuristic element means the branches which have higher possibility of success to reach at the end point among many possible branches in each steps. On the other hand, heuristic method means a method making a more successful path to reach at the optimal end state by connecting branches of higher possibility of success according to their priority. Thus, the essence of heuristics is to choose a plausible way from many possible way to get the optimal goal (c. f. Fig. 4. 1).

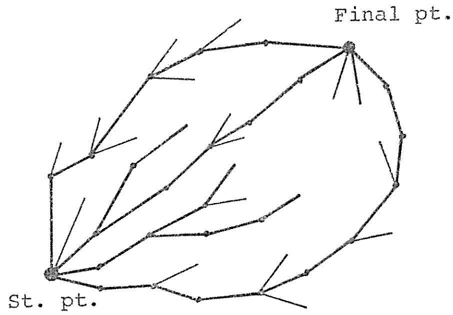


Fig. 4. 1. Tree structure of heuristic system; Heuristic element and heuristic algorithm.

4. 2. 3. Relation among Learning, Heuristics and Intuition

Learning, Heuristics and Intuition are three sophisticated native functions of human. The investigation of actual mechanisms of the above human intelligences concerns closely with the problem how human uses the gathered informations and / or the accumulated experiences. The engineering interpretation of the three intelligences may be as follows: The feature of learning is the active usage of past experiences to improve system behavior. Then, learning is a function relating to time actions like accummulation and utilization of experiences. On the other hand, heuristics is a useful procedure (heuristic algorithm) to extract several relevant factors (heuristic elements) from vast external informations and to make a best path (solution) by connecting these extracted heuristic elements in each step of the sequential decision process. In other words, heuristics is a function concerning spatial factors like the processing of enormous informations and the derivation of spatial solution. Repeating the heuristic processing, man becomes to be able to quicken the heuristic processing and to make a decision quickly, that is, heuristics could be quickened by learning. Moreover, experiences and knowledges obtained by learning will supply more available informations and lead to better probable solutions. Thus, learning can improve the heuristic processing both in speed and accuracy. Heuristics improved by learning may be called *Learned Heuristics*. The extremely quickened heuristics by learning may be able to say *Intuition*. On the other hand, learning is also improved by applying heuristics. Namely, the accumulation and arrangement of informations and experiences can be rationalized by heuristic processing. So that the learning led by heuristic decision presents a considerable improvement in the speed of learning and precision of learned results. Learning improved by heuristics may be called *Heuristic Learning*. The few functions mentioned above, leaning, heuristics, learned heuristics, heuristic learning, and intuition are, in all, important intelligences of human, and they seem to be arranged as above in order of complexity and sophistication. Even if the actual mechanisms of human intelligences, especially of such as intuition, are different from the above description, the above concepts of heuristics and intuition seem to be much convenient from the engineering view point of artificial intelligence. In a decision process, human will probably use these functions in proper way. We can easily guess that human will make proper use of these functions in a peak-searching process. In game-playing, both heuristics and intuition should be in use to decide a move. A poor chess-player or beginner will be always in use of slow heuristics to decide a moving policy; on the other hand, a good, player will use intuition or learned heuristics more often than the average player. So, a good-player can decide every moves in game-playing very quickly or instaneously.

4. 3. Development of Learning Control Machines

4. 3. 1. Analysis of Human Learning

As is pointed out in *4. 2. 1.*, the key functions of learning are the memorization of experiences and their practical utilization for the behavior improvement. While, experience is classified, in general, into the following two categories (1) same experience and similar experience, (2) self-experience and external experience. An available means of memorization by which becomes possible to realize the efficient utilization of experiences is to arrage and store the trial data in a table or ordered boxes systematically.

Now, assume that s_i is a stimulus (input, cause, etc.) to a system (animal or machine), r_i is a response (output, effect, etc.) of the system, and S or R is the *set* of s_i or r_i respectively (($s_i \in S$, $r_i \in R$). When a connection between a certain stimulus s_i and its corresponding response r_i is established, it means that there is generated a *formation of association* between s_i and r_i. In addition, the modification of association means the change of previously established association (cf. Fig. 4. 2). If a system can make such a formation (or modification) of association without or with the aid of human teacher (or external system), it can be interpreted that the system performs a learning by itself (*self-learning*) or a learning by a teacher (*superrised learning*) respectively.

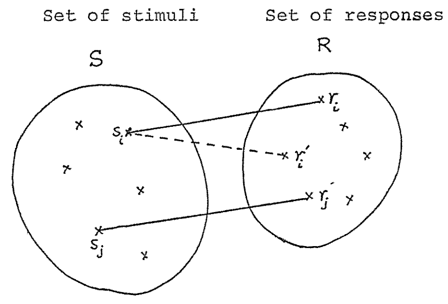


Fig. 4. 2. Formation and modification of association
(s_i, r_i), (s_j, r_j) : Formation
$(s_i, r_i) \rightarrow (s_i, r_i')$: modification.

Thus, an experience means an association between a stimulus s_i and its corresponding response r_i. Such a stimulus s_i

might be a vector (combination) $\boldsymbol{s}_i$ of multi-dimensional stimuli $(s_{i1}, s_{i2}, \cdots)$; similarly a response r_i be a vector $\boldsymbol{r}_i$ of multi-dimensional responses $(r_{i1}, r_{i2}, \cdots)$. Hence, the alteration of behavior possibility which is the essence of learning, is interpreted here as the formation or modification of association between stimulus vector $\boldsymbol{s}_i$ and its corresponding response vector $\boldsymbol{r}_i$.

4. 3. 2. Development of Learntrol

Learntrol is the name of learning control System developed by author's research group about fifteen years ago. The systems considered in Learntrols (Learntrol I ~ V) are restricted to the simple hill-climbing system with single-input, single-output, and unimodal performance function (static characteristics). However, Learntrol is a very interesting scheme which is designed to have enough important functions abstracted from human behaviors not to lose the essence of learning.

4. 3. 2. 1. Problem Statement

The key problems in the design of criterion function are how to learn the location of the optimal peak of criterion function and how to follow the movement of the optimum peak.

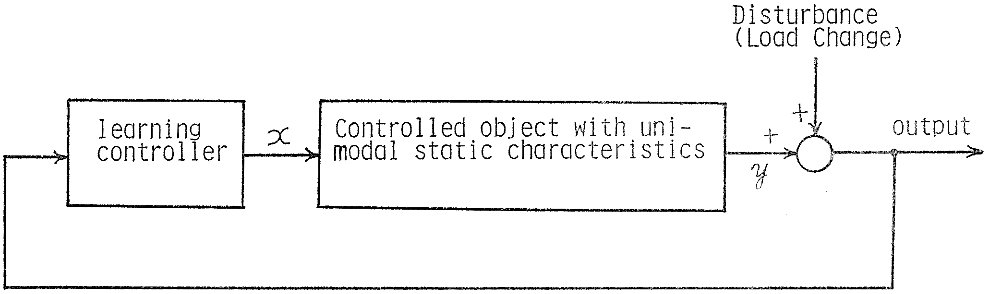


Fig. 4. 3. Block diagram of Learntrols.

The block diagram of Learntrols is shown in Fig. 4. 3. Here, input x is a control setting and output y is a performance index (criterion function) of the system. It is also assume that the static characteristics of the object between input (stimulus) and output (response) is unimodal, and furthermore that the optimal point is abruptly changed by an instantaneous change of load, but it is required to be relatively constant between those changes, occurrence interval of which is made longer than the adaptation time of the system (the time from the moment when the load change has occurred until the system attains the peak). In addition, the measurement noise is neglected.

4. 3. 2. 2. Classification of Learntrols

As summarily listed in Table 4. 1, Learntrols have been developed along two different lines of means to introduce learning functions. One is the increase of the kinds of memory tables which are Map (the chart memorizing the total variation of control setting x or the total magnitude of the learned jump of x), Frequency Table (the chart momorizing the occurrence frequency of experiences) and Order Table (the chart memorizing the occurrence order number of experiences). They are all the fundamenral memory elements which are contrived from human's

Table 4. 1. Classification and grading of Learntols

Structure and function \ Learntrol number		I	II	III	IV	V
Memory element of experiences	Map	○	○	○	○	○
	Frequency table		○	○	○	○
	Order table			○	○	○
Step-by Step search without any experience		○	○	○	○	○
Jumping search with same experience		○	○	○	○	○
Semi-jumping search with similar experience				○	○	○
Self-self-selection of best control strategy by higher-level learning loop					○	○
Acception of external experiences by instruction accepting mechanism						○

learning experience attained in the time and space domains. The other line is the expansion of search methods and / or means to utilize previous experiences. That is, as learning progresses, the search method is altered stepwise from step-by-step search to jumping search (Learntrol I, II) or semi-jumping search (Learntrol III ~ V). The jumping search is a jump behavior to the peak learned through the same or identical experiences. And the semi-jumping search is a learned jump to the neighborhood point of the peak through similar or analoguous experiences. Moreover, the self-selection of the best strategy is performed through the command from the heigher level system of the hierachically structure learning system (Learntrol IV). The sophistication of learning grade is also established through the introduction of external experiences by other systems (Learntrol V).

4. 3. 2. 3. Experiments, Results and Discussions on Learntrols

Experimental simulation of Learntrols have been performed on digital computer. The static character of the object has been simulated by a paraboric function.

$$y(x) = -(x-a_i)^2 \tag{4.1}$$

where a_i is changed instantly and randomly to ocurr the stepwise load disturbances. In Leantrol I ~ III, the step-by-step search is performed in the early stages of learning process, and the number of jumping searches increases gradually as the learning progresses. Thus, the average number of trials carried out until the attainment of the peak decreases gradually and finally approaches to the lower limit of two trials, which is the state carrying out only the jumping searches. The effect of the utilization of similar experimences in Learntrol III appears as "*Plateau*" on the learning curve (average trial number of trials *vs* number of experiences curve) as shown in Fig. 4. 4. By virtue of the increase of semi-jumping searches which appear as the utilization effect of similar experiences, the learning curve begins to descend from the comparatively early period of learning process. Afterword, the learning curve becomes flat and keeps the state until the curve begins again to descend by the effect of inereasing of jumping searches. This flat part of the learning curve is named by *Plateau*, which is the phenomenon caused by

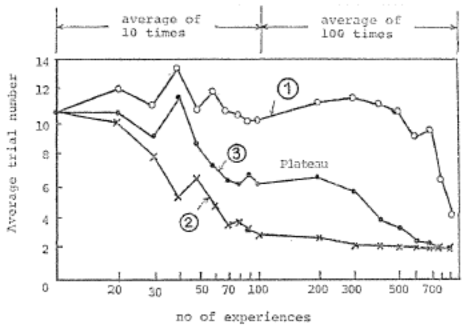


Fig. 4. 4. Representative learning curves by Learntrols Ⅰ ~ Ⅲ, (Experimental results)
①: step-by-step search only
②: step-by-step search → jumping search
③: step-by-step search → semi-jumping search → jumping search.

the utilization of similar experiences and is the most remarkable feature of Learntrol Ⅲ.

The fine discription about Learntrols Ⅳ, V is omitted here because of page limit of this paper. The detail of Learntrols Ⅳ, Ⅴ is presented Ref. (11), (25), (26).

4. 4. Development of Heuristic Searching Machine

The author's group have tried to clarify the heuristics evolved by human searcher in searching process of the highest point (optimum vertex) in multimodal test hills on a two dimensional sheet.

4. 4. 1. Outline of Experiment by Human Subject

In this research, it is planned to extract some heuristics of human though the analysis of experimental search by human subjects performed in the scene shown in Fig. 4. 5. The human subject (searcher) and the experimenter sit facing each other

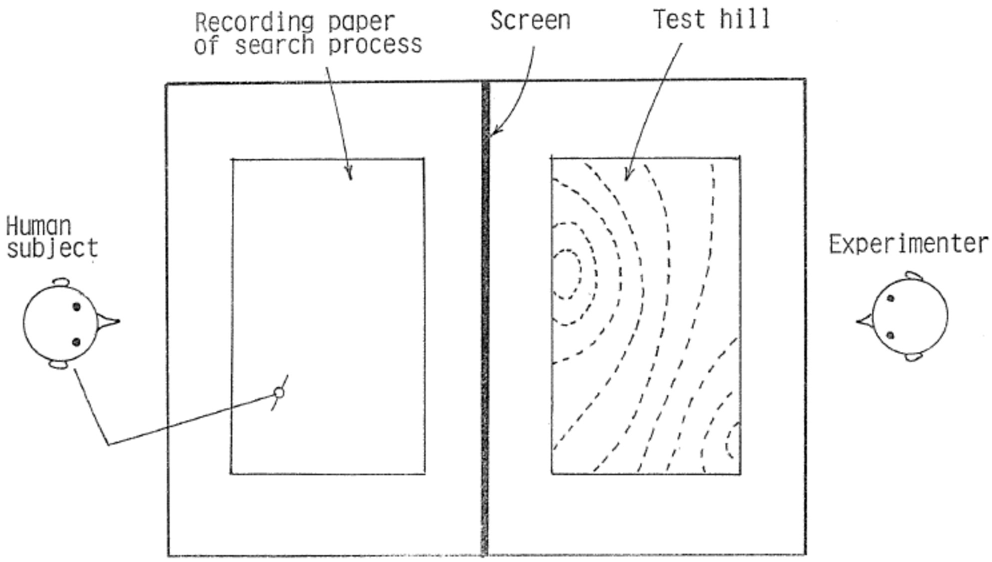


Fig. 4. 5. The scene of experiment.

on the opposite sides of a masking screen, which prevents the subject from seeing the test hill in front of the experimenter. When the subject selects a trial point $\boldsymbol{x}=(x_1, x_2)$, the experimenter tells him the height of the point on the test hill. The subject writes the height on a recording paper, and selects a next triaal point. Repeating the above procedure, the subject finds out an optimum point of the test hill which is hidden to the searcher by the screen, but not to the experimenter, by using the experience of search trial which are recoreded successively on the searching paper. The main goal is to attain an optimum point $\boldsymbol{x}^m$. An incidental goal is to keep the value of $f(\boldsymbol{x}_i)$, $i=1, 2, \cdots$ as large as possible. There is no restriction on the selection of trial points $\boldsymbol{x}_i$, trial step width $\Delta\boldsymbol{x}_i$, and trial number N. All a priori informations told to the subject about a test hill are that $f(\boldsymbol{x})$ is a step-wisely continuous and one-valued function of $\boldsymbol{x}$. The number, location and height of peaks on the test hill are, of course, not informed to the subject.

There are prepared twenty kinds of test hills in which are hidden some regularities (rules) about the number, arrangement, and shape (sharpness) of peaks. Then, the key point for the subject to get the highest peek quickly is to discover the hidden rules as quickly as possible.

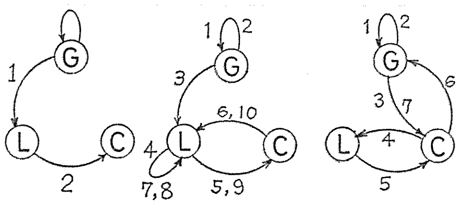
4. 4. 2. Heuristics Abstracted in Human Experiment

In the searching experiment explained in *4. 4. 1.*, heuristics by human subject seems to appear in the conjecture process of figure (gradient of hills, location and height of peaks, etc.), in the selection process of next trial point and in the other various decision processes. So that, it may be possible to abstract various human heuristics through the insight of experimental data. And the abstracted heuristics is much suggestive to plan the artificial heuristic machine.

4. 4. 2. 1. Modes of Heuristic Search by Human

Search modes which appear in the peak searching experiment by human subject are generally devided into the following three modes; (1) global search (G-mode search), (2) local search (L-mode search) and (3) convergent search (C-mode search). The global search is to guess the whole figure of test hill, the local search is to seek the local figure around the spurious or true highest point guessed by the global search, and the convergent search is to comfirm the true peak point by small step trials. In general, the seacher tries G-mode search in the early stage of the searching process, afterward he changes to L-mode in the middle stage and further shifts to C-mode search in the final stage. Thus, the search process evolved is expressed by the transition pattern (c. f. Fig. 4. 6) among these three modes. However, the order, frequency and succession number of mode transitions are not always equal to those of the primitive simple pattern mentioned above. Some examples of mode transition attained in the experiment are expressed as Eq. (4. 2) and Eq. (4. 3).

Here, pattern notatation (e.g. YF-16 on the left side of Eq. (4.2)) means a subject and a test hill. As to the notation $G^{m_i}_i$, $L^{n_j}_j$, $C^{l_k}_k$, upper suffixes show the number of trials and low suffixes show the group number of search trials. Fig. 4. 6 shows some mode transition diagrams corresponding to Eq. (4.2), Eq. (4.3) and other respresentative modes.



a) YF-16 b) KO-13 c) MS-9

Fig. 4. 6. Example of mode transition diagram

4. 4. 2. 2. Heuristics in Global Search by Human

The human subject have no a priori information about number, location and height of peaks in the test hill, so that he usually starts to select randomly few trial points which distribute uniformly on the test hill. This intial search which is developed by human to catch the global information about the figure in the early stage of hill-climbing process is called as *Global search.* What is the number and location of trial points which human subject selects heuristically in grobal search? The experimental results show that the number of trials in global search is 5 for the case in which total number of trials of the whole search process is limitted 20, and is 8 for the case of limitted 30. The propriety of the selected numbers of 5 and 8 is reasonably understood through the fact that the number of circles filled in on the rectangular test sheet under the following four conditions is 5 and 8: (1) radius of all circles are equal, (2) every circles don't have common set, (3) domain area covered by circles is maximum in the test sheet, (4) every circles distribute symmetrically both in right and left and in up and down.

Refering to the hight of few selected points in global search, human intends to presume the whole pattern of test-hill figure. Thus, some higher points of the global search points can be selected as candidates of base points for the succeeding local search. In another type of global search which is transfered backword from local search, (c.f. *4. 4. 2. 1.* and *4. 4. 2. 4.*) human selects, in heuristic way, another few points refering to the figure pattern presumed basing on the searched data in the preceding local search. The heuristically selected few points as above are qualified as candidates of base points for the succeeding local search.

4. 4. 2. 3. Heuristics in the Shifting of Base Point

The interesting behavior in local search by human is the heuristic shifting of base points. Around a base point selected from the global search trials, are selected few trial points. The highest point of those points is qualified as base point for the following local search. Such local search may be repeated until the highest peak is found out. The above approach is named *Shifting Base Point Method.* Around the highest point found out by local search, the convergent search is succeeded to confirm the apparent or spurious peak is highest in truth. After a extremum point is found out by a series of local searches, a new base-point is selected heuristically from the other base points which are selected in the previous global search, and a series of local searchs to seek another peaks is performed repeatedly.

4. 4. 2. 4. Heuristics in Mode Transition

The most interesting heuristics appearing in human search process, is the heuristics in mode transition. As mentioned in *4. 4. 2. 1.*, human's search process repeats the shifting or jumping among various search modes of G-mode, L-mode and C-mode. If we think that heuristics is a conscious jump of logical thought, then the transition of search mode corresponds just to the conscious jump. Heuristics appeared in the selection of a base point may be considered as heuristics in a kind of transition from current point to base point. The transition to a local search area is especially worthy of being called a *heuristic jump*, two kinds of which extracted as follows: One is a jump (transition) to the vicinity of the base point which is selected from the G-mode search in the expectation of giving a higher value of the hill. The other is a jump to a point in an unsearched open local area with a comparable extent including the most plausible highest peak (a candidate of the optimum point) obtained in the preceding searches.

In a practical process of search, the appearances of the transition and the ordering of the search mode are deformed variously according to the searching trials (trial experiences), in other words, by learning of a subject. This deformation by learning is just corresponding to the learned heuristics defined in *4. 2. 3.*. The study of such a deforming mechanism from a microscopic point of view is now under ingestigation at our laboratory.

4. 4. 2. 5. Various Other Heuristics in Local Search

In the local search by human, there are observed various kinds of contrived heuristics excepting the heuristics explained in the preceding sections. Here, we will enumerate only some interesting heuristics.

a) Heuristics on the conjecture of slope at trial point; Acclivity or declivity in what direction? What is the gradient?
b) Heuristics on the cognition of surrounding scene at base point; hill or valey, peak or ridge, sharply or smoothly, etc..
c) Heuristics on the determination of searching policy; the selection order in mode transition, the proper use of learning, heuristics and their combined intelligence etc..

There seems possible to be abstracted many other type of heuristics in human behavior.

4. 4. 3. Simulation Algorithm of Heuristic Peak-searching

We have investigated various kinds of heuristics appearing in various stages of human's search process. Thus, by introducing such interesting human's heuristics, and by planning various contrivances of heuristics for artificial peak searching, there are expected to design ingenious artificial searching machines. Here, we will explain briefly some searching algorithms developed or in studing in our laboratory.

4. 4. 3. 1. Improved Simplex Method with Figure Ground Search Policy

An interesting example of heuristic searching machine is the method applying an improved simplex method, which is developed by K. Fukui et. al.

This work is an engineering-sense simulation of heuristics in multimodal peak-searching process by human, which is a combinative utilization of improved-simplex method and figure-ground search.

The conventional simplex method is a searching method of a peak point by utilizing convergent properties of shifting or midifying (reflection, expansion, contraction and shrinkage) process of a simplex, starting from the highest side of an appropriately selected initial simplex. (Simplex is a (n+1) dimensional figure in n-dimensional space). The improved-simplex method is composed of a series of convergent searches starting to search from each of (n+1) sides of an initial simplex. By one time of searching around an initial simplex, (n+1) or less than (n+1) peakes may be found out. The region which has been searched by the method is called as *Figure*.

After the confirmation of peaks, another initial simplex is selected through predecided rules in the rigion called *Ground* which has not been searched by the above searching process, and the same searching processes as the above are repeated. By the repetition of these processes, almost all region can be searched. This artificial process can be considered as an simulation of human heuristics.

It is noticeable that the *Figure-Ground search principle* mentioned above is very interesting heuristic algorithm contrived by hints from the experimental data of multi-modal searching by human subjects.

4. 4. 3. 2. Shifting Base-points Method

In the determination process of searching policy by human, can be abstracted some interesting heuristics, one of which is the heuristic selection and shifting of base-boint in local search. Firstly, human selects one point from several (5 ~ 8) trial points of a global search and develops a local search around the point, that is, the selected point has the role of base-point in the succeeding local search. After a local search the highest point in the local search is choosed as the base-boint of next local search. These processes are repeated. The above policy for a series of local searches is named as *Shifting base-point method* by T. Nagaoka, a member of our research group. By combining the shifting base-point method and Figure-ground search policy, there have been discovered some interesting searching algorithms for single-and multi-peaks searching problem.

4. 4. 3. 3. Various Contrivances on Heuristics in Local Search

The shifting base-point method and figure-ground search technology are both the powerful basic policies which are great useful for global or rough searching regardless of figure of hill pattern. However, the searching policy in local search should be changed, in general, corresponding to the number of peaks — single-peak or multi-peaks, to the shape or figure of hill-pattern — smooth hill, sharp acclivous top-flat hill, ascending or descending ridge, meandering ridge, crater etc.. We have developed or are in developing various ingeneous searching schemes fitting to various cases. For example, (a) an algorithm to conjecture the location or the direction of peak or ridge by virtue of few auxiliary trial points selected around the base-point (e.g. in the four quadrants), (b) a method to use jointly the two approximation curves (functions) of cordinates trajectory and of peak values trajectory on trial points in local searches, and (c) a method to mix the above proposed schemes (a) and (b). The details of these researches will be presented at proper chances in future.

5. Conclusion

In this paper, the author have surveyed some works on various approaches to the control of unknown systems which has been developed for the past over ten years. Of these descriptions, the research of convergent recursive (learning) scheme in Chapter 3 is new interesting ideas and is expected more development and applications in future. Various contrivances of heuristics in the development of artificial peak-searching machine have been explained briefly only about the philosophical essence in Chapter 4. The simulation experiments on heuristic hill-climbing schemes by our research group supply many interesting results, which give useful suggestions on new applications on mathematical programming and new development of optimization machines. But the presentation of this experimental research remained in near future.

The works presented in this paper are only a part of author's main subject which is the introduction of human's natural intelligence into information and control science. There are remained such interesting problems as new development of hybrid intelligent machines which include the heuristic-learnining machine or the learned-heuristic machine etc., the realization of practical intelligent control machines aided by microcomputer, the multi-purpose robot with the decision ability, and so on.

Although the subject to introduce human's intelligence into engineering fields includes many problems to be developed hurriedly, it is also only a part of interesting researches on general artificial intelligence, which are vastly related to such sciences and engineerings as psychology, language, semantics, biology, medicine, ecology, cybernetics, informatics, computer and so on. There looks extending a hopeful way to bright and boundless fields.

Acknowledgement

The author gives, at first, great thanks to Associate Professor M. Oda, who has been the most active and important promotor in our long years research on artificial intelligence. Many researches surveyed in this paper is greately indebted to him. Furthermore, the author is much grateful to Associate Professor Y. Funahashi and Dr. T. Matsuo for their continuous co-operation through useful advices and suggestions from mathematical and theoretical side, and also to Research Assistant Dr. M. Inagaki for his research effort in system theory. The autor also appreciate Mr. T. Nagaoka's excellent opinions on the research of hill-climbing methodology. Finally, the author is much obliged to Miss M. Ohashi who has kindly prepared the typescript of this article.

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