Memoirs of the Faculty of Engineering, Nagoya University
Vol. 31, No. 2 (1979) — Research Report —

REACTOR PHYSICS GROUP CONSTANTS FOR THE C-CORE OF THE KYOTO UNIVERSITY CRITICAL ASSEMBLY

SUSUMU WAKAMATSU, KOJIRO NISHINA
KENGO HASHIMOTO and TAKAHIRO FUJISHIRO

Department of Nuclear Engineering

(Received November 1, 1979)

Abstract

To facilitate theoretical analyses of Kyoto University Critical Assembly (KUCA), reactor physics group constants were prepared. In this report we describe the procedures of the preparation, and verify the feasibility of the resulting constants through criticality calculations. For this purpose we first use computer code "UGMG-42" for fast-group constant production, and "THERMOS" for thermal group constant production. With these constants as inputs, criticality calculation is carried out by a two-dimensional code "EQUIPOISE-3". The difference between the criticality calculation and experiment proves to be in the range 1.0~1.5%.

In addition, comparison is made between the theoretical prediction and measured values of temperature coefficient, which is one of the fundamental characteristics of the Assembly. The calculation, which is performed with a one-dimensional code "EXPANDA-25-IMPORT," reproduces successfully the experimental lattice-pitch dependence of this quantity.

I. Introduction

Group constants are a set of parameters prepared for the analysis of a nuclear reactor or critical assembly. To facilitate the following presentation of the constants for Kyoto University Critical Assembly (KUCA), we first give a brief introduction of critical assemblies in general, and of the role played by group constants.

A critical assembly is a neutron-multiplying apparatus which is constructed to perform low-power experiments, so that the neutron-economy characteristics of a newly proposed reactor can be determined. The typical characteristics determined for a given arrangement of core materials are: (a) critical size, (b) effectiveness of control rods, (c) temperature coefficient, namely the variation of multiplication factor with temperature rise, (d) void coefficient, namely the variation of multipli-

cation factor upon introduction of voids into the core, and (e) neutron flux distribution.

In attempting to support such experimental characteristics by theoretical estimation, one expresses the system with a one-dimensional (1-D), two-dimensional (2-D) or three-dimensional (3-D) model, and solves diffusion equations with proper boundary conditions. From the results one can evaluate the characteristics mentioned above. The diffusion equations in such practice are set up respectively for many intervals of neutron energies involved, and the constants (neutron diffusion coefficients and neutron cross sections) of the equations must be prepared for all these neutron energies. These constants are called reactor physics constants, or group constants, where the group means energy group. Thus the group constants are indispensable for the analysis of reactor characteristics. While the need for the effort of solving diffusion equations can easily be understood, it is no less important and painstaking to produce proper group constants with reasonable method and make them available to reactor analysts.

In the last five years, a reactor physics group at Nuclear Engineering Department have been participating in experiments to determine the parameters of the KUCA.[1] It is a part of joint research activities involving several universities in Japan. At the same time the theoretical verifications of the typical parameters already mentioned have been underway, which can also be viewed as the test of the group constants proposed. As the result we have reached the following tentative set of constants, which leaves several areas to be investigated and refined. Another set of group constants are being prepared also by the staffs at the Research Reactor Institute of Kyoto University.[2] Judging from the importance of the matter, we feel the investigations by several independent research groups with their own methods are necessary and rewarding.

Ⅱ. The Specifications and the Criticality Data of the Kyoto University Critical Assembly[3,4]

In the building of KUCA there are three cores which are called A-core, B-core (both of them are solid-moderated), and C-core (light water-moderated). Ever since the C-core was brought to its first criticality on August 6, 1974 (B-core on Nov. 16; A-core on Dec. 3 of 1974), the three cores have been used as the apparatus of joint research activities by universities over the whole country. In addition to our participation in its test experiments, we took part in preparing the group constants of the C-core. As shown in Fig. 1, C-core has a base lattice plate of stainless steel which supports the fuel assembly frames (Fig. 2) within a water tank of 1.8 m depth. The core is constructed by placing the fuel frames on the lattice plate (Fig. 1) in a desired core configuration.

The fuel frames can be loaded with fuel plates[5] (Fig. 3; Table 1) with three different pitches, namely 2.96 mm, 3.49 mm and 4.54 mm, corresponding to the $H/^{235}U$ values (atom density ratio of H to ^{235}U) of 159.1, 212.0, and 315.2, respectively, within the unit region illustrated in Fig. 5 (see Sec. Ⅳ). In the following we call these loadings C30, C35, and C45 loading, respetively. In terms of number of plates loaded per frame, these loadings hold 47, 40 and 31 plates, respectively. Fig. 4 illustrates by a plane figure how the arrangement of the fuel frames form a

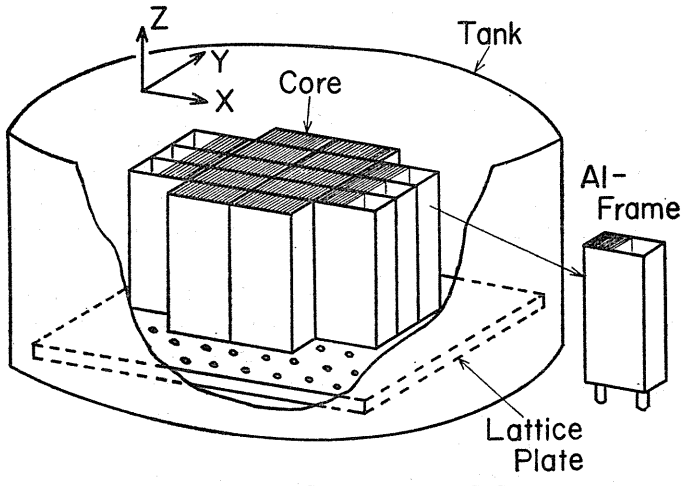


Fig. 1. The Sketch of the C-Core.

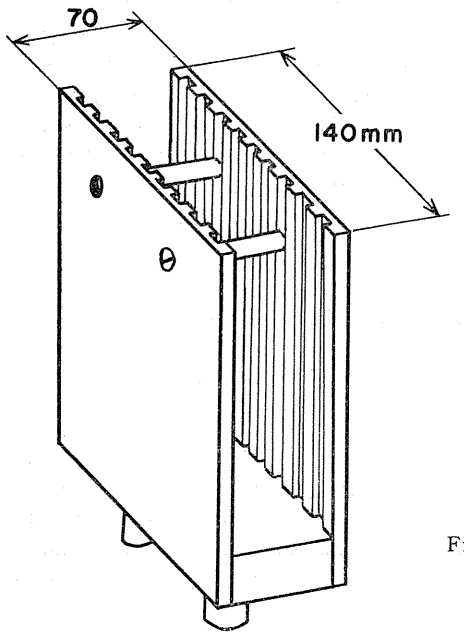


Table 1. Fuel Plate Specifications.[5]

Plate dimension	$62\times600\times1.5\ mm^3$
Meat layer dimension	$52.8\times569.2\times0.5\ mm^3$ (Average value from X-ray photo)
Weight of ^{235}U	8.893 g/plate
Density of meat layer	$3.22\ g/cm^3$
Uranium content in the meat layer of U-Al alloy	19.741 %
Uranium enrichment	93.1 %

Fig. 2. The Fuel Frame Which Holds Fuel Plates. It can hold 47, 40, and 31 plates in the cases of C30, C35, and C45 loading, respectively.

core. It is a rectangle core with four corners cut away.

As for the criticality core dimensions, the height (in z-direction) is determined by the height of the meat content within the fuel plate, while the length in y-direction by the number of the frame arrays (which is fixed at five during routine experiments). The dimensions in x-direction have been determined by criticality experiments of individual loadings, in which fuel plates were added until the core became critical. In such experiments core water was kept at the height of 21~31 cm above the top of fuel plates. Six control rods were pulled off the core com-

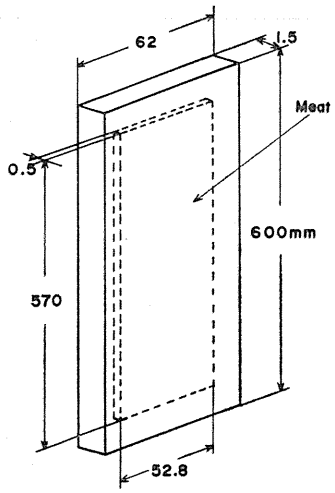


Fig. 3. The Fuel Plate.

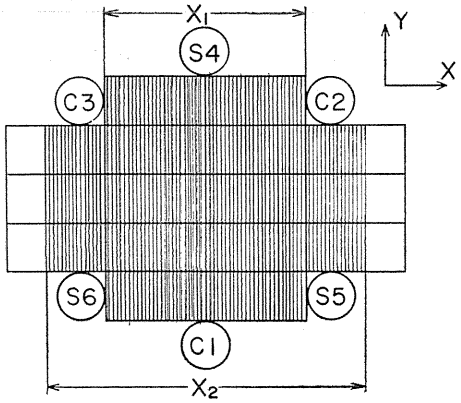


Fig. 4. A Plane Figure Illustrating an Arrangement of Frames in Constructing a Core.
C1～C3: Control Rods
S4～S6: Safety Rods

Table 2. Criticality Data for the Three Ways of Loading.[3,4]

	C30 loading	C35 loading	C45 loading
X_1 of Fig. 4	28.4 cm	21.3 cm	28.4 cm
X_2	32.4 cm	34.6 cm	30.2 cm
Y	35.5 cm	35.5 cm	35.5 cm
Z	57.0 cm	57.0 cm	57.0 cm
Critical number of plate	510	412	322
Critical mass of ^{235}U	4,535 g	3,664 g	2,863 g

pletely. In Table 2[3,4] are given the critical dimensions of three loadings obtained by such experiments.

Ⅲ. Group Constants

To lay out more specific definitions of group constants in connection to the present analysis of the Assembly, we first sort the neutrons into two groups, namely into thermal neutrons (0～0.625 eV), and fast neutrons (0.625 eV ～ 9.99 MeV), and set up the following equations for respective groups.

Core region (the region including fuel plates)

The first group:

$$D_1^c \nabla^2 \phi_1^c - (\Sigma_{1a}^c + \Sigma_{R,1\to 2}^c)\phi_1^c + \nu\Sigma_{1f}^c \phi_1^c + \nu\Sigma_{2f}^c \phi_2^c = 0. \qquad (1)$$

The second group:

$$D_2^c \nabla^2 \phi_2^c - \sum_{2a}^c \phi_2^c + \sum_{R,1\to 2}^c \phi_1^c = 0. \tag{2}$$

Reflector region (light-water region around the core)

The first group:

$$D_1^r \nabla^2 \phi_1^r - (\sum_{1a}^r + \sum_{R,1\to 2}^r) \phi_1^r = 0. \tag{3}$$

The second group:

$$D_2^r \nabla^2 \phi_2^r - \sum_{2a}^r \phi_2^r + \sum_{R,1\to 2}^r \phi_1^r = 0. \tag{4}$$

This system of equations are called two-group equations, and describe the following physical processes:

The reactor is in a stationary state (it is critical in this case). There are high energy neutrons (2 MeV on the average) produced by fissions. They either leak out of the system after colliding with the nuclei of reactor materials ($D_1\nabla^2\phi_1$), or get absorbed by the nuclei ($\sum_{1a}\phi_1$), or trigger fissions ($\nu\sum_{1f}\phi_1$) of ^{235}U and ^{238}U at this energy, or get slowed down ($\sum_{R,1\to 2}\phi_1$) to lower energy (thermal energy: $E<0.625$ eV) by scattering. The neutrons which have been slowed down can be absorbed ($\sum_{2a}\phi_2$), can leak out ($D_2\nabla^2\phi_2$) or can produce high energy neutrons by fission reaction ($\nu\sum_{2f}\phi_2$) with fissile material (^{235}U). The decrease and increase of neutrons by these processes are in balance, and reactor keeps a constant power by repetition of above cycles. Superscripts "c" and "r" indicate core region and reflector region, respectively.

In the diffusion equations Eqs. (1)—(4), neutron fluxes are unknown quantities, and the constants needed in calculation procedure are the diffusion coefficients D, macroscopic absorption cross sections $\sum_a$, macroscopic removal cross sections $\sum_{R,1\to 2}$, and macroscopic fission cross sections $\sum_f$. These constants are obtained as averaged values over the respective energy ranges. The absorption cross section $\sum_{1a}^c$, for example, is evaluated by the following formula:

$$\sum_{1a}^c \equiv \int_{E_1}^{E_0} \sum_a^c (E)\phi^c(E)\,dE / \phi_1^c, \tag{5}$$

where

$$\phi_1^c \equiv \int_{E_1}^{E_0} \phi^c(E)\,dE, \tag{6}$$

and where E_0 and E_1 specify the energy range involved. As $\sum_a^c(E)$ is known as nuclear constants,[6] $\sum_{1a}^c$ can be determined if we could obtain flux $\phi^c(E)$ for the core as a function of energy. The main objective of this report is to describe the computer calculation of $\phi^c(E)$ and $\phi^r(E)$ over the two energy regions mentioned, and to determine the constants appearing in Eqs. (1)—(4) by Eq. (5). We call the resultant constants "two-group constants". If the formulations are carried out with four energy groups, the constants needed are called "four-group constants." In this report, we provide four-group as well as two-group constants (in Secs. Ⅳ and Ⅴ), then calculate critical masses and temperature coefficients of the assembly (in Sec. Ⅵ), and compare them with the experimental values.

The procedures of constant production are described next for the three core loadings, and for the reflector (light water) region. The neutron spectrum $\phi(E)$

within the two groups mentioned previously are governed by entirely different physical mechanisms. In the fast group (0.625 eV $\sim$ 9.99 MeV) the motions and chemical bindings of the medium nuclei can be neglected whereas in the thermal group (0 $\sim$ 0.625 eV) they must fully be considered. The fast group is further divided into fifty-four energy groups in the present calculation, while the thermal group into twenty-nine. These group structures are called multi-energy groups, and are differentiated from few energy-groups. The latter is a general term attached to the final group structure for which the constants are produced (two groups or four groups in our example). For the i-th nuclide from which the core is composed (H, O, Al, ^{235}U and ^{238}U) microscopic cross section σ_j^i for the j-th reaction are available for each of the multi-groups in a form of library tapes prepared for computation codes. The macroscopic cross section $\sum_j$ of the j-th reaction is given by the sum of $\sigma_j^i N_i$ with respect to i, where N_i is the number density of the i-th nuclide. The density N_i is calculated from the specifications of the assembly given in Sec. Ⅱ, and is supplied to the codes as input data. Computer codes solve the transport equations for each of the multi-groups, with such $\sum_j$'s as constants in equations.

The fast enegry spectrum is obtained by the code "UGMG-42"[7,8], which solves transport equations with a continuous slowing-down model for a homogenized core region and the reflector region. For thermal energy group, integral transport equations are solved by "THERMOS" code[9] over a minimum unit (called a "cell") of the periodic structure consisting of fuel plates and water (Fig. 5). Because of the different models included in the two codes, the number densities are evaluated and provided differently. In the following the input data for each case will be described.

Ⅳ. Procedure for Fast Group Constants

Number densities: The UGMG code can perform only homogeneous medium calculation in the case of plane geometry. The number density N_i's for the core region is evaluated as follows. In the allocation of fuel frames, a water region of 2 mm thickness lies between the consecutive frames in x-direction, and a water region of 1 mm thickness in y-direction. Including these regions we regard one unit of a modelled frame as forming a 142mm×71mm rectangle, as seen in Fig. 5 (cf. Fig. 2). Such a modelled frame consists of four kinds of regions: meat layer (which contains U-Al alloy), aluminum cladding, moderator regions between fuel plates, and frame structure. All the nuclides included in these regions are mixed uniformly and the resultant number densities N_i's are used as those for the core, which are given in

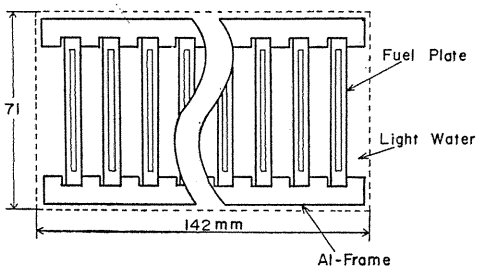


Fig. 5. A Modelled Frame for UGMG. The water channel attached around the frame (indicated by dotted line) is included in the treatment.

Table 3. Number Densities Used for Fast Constants at 20.44℃. Based on this table the densities at 35.12℃ were evaluated with thermal expansion taken into account. ($\times 10^{24}$ n/cm^3)

	Nuclide	C30 loading	C35 loading	C45 loading
Core region	^{235}U	1.8666×10^{-4}	1.5886×10^{-4}	1.2311×10^{-4}
	^{238}U	1.3659×10^{-5}	1.1625×10^{-5}	9.0094×10^{-6}
	Al	3.3096×10^{-2}	2.9537×10^{-2}	2.4962×10^{-2}
	H_2O	1.4845×10^{-2}	1.6840×10^{-2}	1.9405×10^{-2}
Reflector region	H_2O	3.3330×10^{-2}		

Table 3 together with N_i's for the reflector (light water).

Options: By input parameters[7,8] to the UGMG code, the following options were specified:

1. Use of B-1 approximation in solving transport equation.[10]
2. To deal with degradation integral, Selengut-Goertzel approximation[8,11] is employed for H, and consistent-age approximation[8] is used for heavy nuclides, O, Al, ^{235}U, and ^{238}U.
3. Doppler effect is included in resonance integral evaluation.
4. In editing few-group cross sections, neutrons are included which skip an energy group or groups during moderation. It is called "inelastic scattering" in the code manual for convenience, in spite of kinetic energy conservation. The reason for such consideration is the light water content in the system, which slows down neutrons effectively. In our case it means $\sum_{R,1\to3}\neq0$ in the three-group fast constants.
5. Resonance integral is treated by intermediate resonance approximation.[8,12]

Energies and dimensions: Beside the above options, few-group energy widths, and the dimension of the system (dimension of the whole core in this case) have to be specified in the input. As for energy width, UGMG calculates $\phi(E)$ (to be more precise ϕ_j, $j=1, 2, 3, \cdots 54$) for fifty-four energy groups. The intervals of the fifty-four energy groups are fixed in UGMG, whose values are given in Ref. (8) and are not given here. In reducing the resultant fifty-four group cross sections with weights ϕ_j's, we specified the following intervals: (the first group 9.99 MeV$\sim$ 0.8197 MeV; the second group 0.8197 MeV$\sim$5.52 keV; the third group 5.52 keV$\sim$ 0.625 eV). Such three-group fast constants and one-group thermal constants (of Sec. V) form a set of four-group constants. Later, the fast cross sections are further reduced to one-group cross sections with the weights of three group neutron flux $\phi_l(l=1, 2, 3)$.

As to the size specification of the system, UGMG is based on a 1-D model. Thus the thickness of this 1-D slab has to be specified in such a way that the effect of leakage on the spectrum is equivalent to that in the real 3-D system. It is done by providing as input data a constant B^2 (called "buckling"), which are defined by the relation

$$B^2=B_x^2+B_y^2+B_z^2$$

where

$$B_x^2=(\pi/a)^2,\quad B_y^2=(\pi/b)^2,\quad B_z^2=(\pi/c)^2, \tag{7}$$

if we are to carry out the calculation on a bare parallelepiped core with the dimensions a, b and c cm in x-, y-, and z-directions, respectively. Then UGMG solves a bare slab (1-D) problem with a thickness of $a^*=\pi/B$, and obtains a spectrum equivalent to that of the given 3-D system.[10]

To include the reflector effect in such a bare core calculation, we form a fictitious bare core model as follows: In Fig. 6 is given a rough sketch of the neutron flux distribution in x-direction of the system. It was obtained by a preliminary 2-D calculation by the diffusion equation code "EQUIPOISE-3."[13] Deep inside the core region, the flux $\phi_1(x)$ in the figure can be approximated with a function $\phi_1^*(x)=A\cos B_x^* x$, where $B_x^*=\pi/(a+2\delta_x)$. The flux $\phi_1^*(x)$ vanishes at the distance δ_x from the core-reflector boundary. We determine δ_x by fitting on the flux $\phi_1(x)$ of Fig. 6, and call it a "reflector saving" in x-direction. We use B_x^{*2} as the buckling B_x^2 in Eq. (7). It is equivalent to replacing the real reflected system with a fictitious bare core of thickness $a+2\delta_x$ (see Fig. 6). We determined B_y^2 and B_z^2 similarly. Then B^2 given by Eq. (7) is supplied as input data to UGMG. We give those reflector savings (δ_x, δ_y, δ_z) and bucklings (B_x^2, B_y^2, B_z^2) in Table 4.

As to the reflector dimension needed in reflector constant evaluation, this much thickness of water in x-direction can be regarded as infinite[14], and only a single attenuation constant needs to be specified. First, the reflector flux cosine distributions in the y- and z-directions similar to those of the core are confirmed by the preliminary calculation mentioned above, Thus we adopt the same B_y^2 and B_z^2 for the reflector region. In such a problem, the energy spectrum of the 3-D system with spatial attenuation $\exp(-\kappa x)$ in x-direction can be reproduced by a 1-D system spectrum with $\exp(-\mu x)$ attenuation, provided μ is taken as $\mu^2=\kappa^2-(B_y^2+B_z^2)$.[15, 16] Thus we read κ from the figures of neutron flux in Fig. 6, and

Table 4. Reflector Savings and Bucklings for the Three Loadings.

	C30 loading	C35 loading	C45 loading
δ_x (cm)	9.4	8.2	7.8
δ_y (cm)	9.4	8.2	7.8
δ_z (cm)	9.4	8.2	7.8
B_x^2 (cm^{-2})	3.7605×10^{-3}	3.8014×10^{-3}	4.6981×10^{-3}
B_y^2 (cm^{-2})	3.3473×10^{-3}	3.6641×10^{-3}	3.7797×10^{-3}
B_z^2 (cm^{-2})	1.7178×10^{-3}	1.8319×10^{-3}	1.8725×10^{-3}
B (cm^{-1})	9.3945×10^{-2}	9.6423×10^{-2}	1.0174×10^{-1}

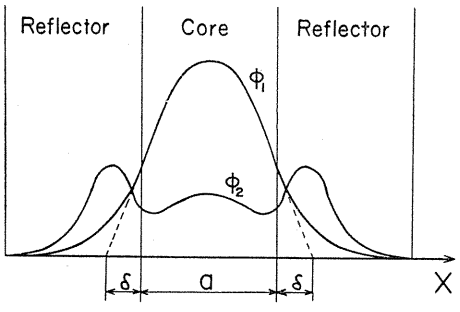


Fig. 6. Neutron Flux Distribution in x-Direction over the Core and Reflector (Output of Two-Dimensional Preliminary Calculation).
ϕ_1: Fast flux
ϕ_2: Thermal flux
a : Core thickness in x-direction
δ : Reflector saving

provide the values of $B^2=-\mu^2$ (for which $\exp(iBx)=\exp(-\mu x)$) as input data ($\mu=0.1565$ cm^{-1} for the reflector attached to the C35). Unfortunately, however, negative flux appeared for such input as the result of UGMG calculation, probably due to the strong attenuation of flux in x-direction. We regarded it as a non-physical solution emerging from numerical procedures, and selected a μ_0 smaller than μ, for which the flux becomes positive. At the same time μ_0 was made as close to μ as possible. The result is $\mu_0=0.1189$ cm^{-1}, which is used for all three loadings hereafter.

From the specifications above as inputs, the three-group fast constants at 20.44℃ and their edited one-group version were obtained, and given in Tables 5 and 6. We also give those at 35.12℃ in Tables 7 and 8, which we will use later

Table 5. Three-Group Fast Constants at 20.44℃.

	Core			Reflector
	C30 loading	C35 loading	C45 loading	H_2O
D_1 (cm)	2.5281	2.5287	2.5190	2.7571
D_2	1.2309	1.1908	1.1453	0.97904
D_3	1.0817	0.99171	0.89545	0.58424
Σ_{1a} (cm^{-1})	5.5931×10^{-4}	5.3599×10^{-4}	5.0471×10^{-4}	7.3403×10^{-4}
Σ_{2a}	5.2740×10^{-4}	4.5288×10^{-4}	3.5674×10^{-4}	0.0
Σ_{3a}	8.6582×10^{-3}	7.6339×10^{-3}	6.2275×10^{-3}	9.6865×10^{-4}
Σ_{1f} (cm^{-1})	2.4446×10^{-4}	2.0812×10^{-4}	1.6133×10^{-4}	
Σ_{2f}	3.3615×10^{-4}	2.8615×10^{-4}	2.2174×10^{-4}	
Σ_{3f}	4.4355×10^{-3}	3.8326×10^{-3}	3.0155×10^{-3}	
$\nu\Sigma_{1f}$ (cm^{-1})	6.5688×10^{-4}	5.5984×10^{-4}	4.3452×10^{-4}	
$\nu\Sigma_{2f}$	8.2618×10^{-4}	7.0329×10^{-4}	5.4499×10^{-4}	
$\nu\Sigma_{3f}$	1.0798×10^{-2}	9.3301×10^{-3}	7.3408×10^{-3}	
Σ_{R1} (cm^{-1})	8.8689×10^{-2}	9.1782×10^{-2}	9.5527×10^{-2}	9.5902×10^{-2}
Σ_{R2}	6.5677×10^{-2}	7.4552×10^{-2}	8.5853×10^{-2}	1.5289×10^{-1}
Σ_{R3}	5.9441×10^{-2}	6.8933×10^{-2}	8.1045×10^{-2}	1.5268×10^{-1}
B (cm^{-1})	9.3946×10^{-2}	9.6423×10^{-2}	1.0174×10^{-1}	1.1890×10^{-1}

Table 6. One-Group Fast Constants at 20.44℃.

	Core			Reflector
	C30 loading	C35 loading	C45 loading	H_2O
D (cm)	1.5941	1.5554	1.5111	1.5161
Σ_a (cm^{-1})	3.0838×10^{-3}	2.7500×10^{-3}	2.2716×10^{-3}	5.8518×10^{-4}
Σ_f (cm^{-1})	1.5910×10^{-3}	1.3826×10^{-3}	1.0906×10^{-3}	—
$\nu\Sigma_f$ (cm^{-1})	3.8955×10^{-3}	3.3852×10^{-3}	2.6703×10^{-3}	—
Σ_R (cm^{-1})	1.8615×10^{-2}	2.1796×10^{-2}	2.5773×10^{-2}	4.9042×10^{-2}
B (cm^{-1})	9.3946×10^{-2}	9.6423×10^{-2}	1.0174×10^{-1}	1.1890×10^{-1}

Table 7. Three-Group Fast Constants at 35.12℃.

	Core			Reflector
	C30 loading	C35 loading	C45 loading	H_2O
D_1 (cm)	2.5329	2.5338	2.5244	2.7668
D_2	1.2342	1.1941	1.1487	0.98277
D_3	1.0854	0.99520	0.89868	0.58656
Σ_{1a} (cm^{-1})	5.5815×10^{-4}	5.3477×10^{-4}	5.0343×10^{-4}	7.3451×10^{-4}
Σ_{2a}	5.2684×10^{-4}	4.5240×10^{-4}	3.5637×10^{-4}	0.0
Σ_{3a}	8.6347×10^{-3}	7.6125×10^{-3}	6.2095×10^{-3}	9.6491×10^{-4}
Σ_{1f} (cm^{-1})	2.4421×10^{-4}	2.0791×10^{-4}	1.6117×10^{-4}	
Σ_{2f}	3.3579×10^{-4}	2.8585×10^{-4}	2.2151×10^{-4}	
Σ_{3f}	4.4249×10^{-3}	3.8233×10^{-3}	3.0080×10^{-3}	
$\nu\Sigma_{1f}$ (cm^{-1})	6.5620×10^{-4}	5.5927×10^{-4}	4.3408×10^{-4}	
$\nu\Sigma_{2f}$	8.2530×10^{-4}	7.0256×10^{-4}	5.4444×10^{-4}	
$\nu\Sigma_{3f}$	1.0772×10^{-2}	9.3073×10^{-3}	7.3226×10^{-3}	
Σ_{R1} (cm^{-1})	8.8443×10^{-2}	9.1516×10^{-2}	9.5235×10^{-2}	9.5381×10^{-2}
Σ_{R2}	6.5404×10^{-2}	7.4243×10^{-2}	8.5499×10^{-2}	1.5231×10^{-1}
Σ_{R3}	5.9179×10^{-2}	6.8634×10^{-2}	8.0699×10^{-2}	1.5210×10^{-1}
B (cm^{-1})	9.3946×10^{-2}	9.6423×10^{-2}	1.0174×10^{-1}	1.1890×10^{-1}

Table 8. One-Group Fast Constats at 35.12℃.

	Core			Reflector
	C30 loading	C35 loading	C45 loading	H_2O
D (cm)	1.5981	1.5594	1.5152	1.5223
Σ_a (cm^{-1})	3.0747×10^{-3}	2.7417×10^{-3}	2.2647×10^{-3}	5.8433×10^{-4}
Σ_f (cm^{-1})	1.5868×10^{-3}	1.3789×10^{-3}	1.0878×10^{-3}	——
$\nu\Sigma_f$ (cm^{-1})	3.8854×10^{-3}	3.3763×10^{-3}	2.6634×10^{-3}	——
Σ_R (cm^{-1})	1.8524×10^{-2}	2.1692×10^{-2}	2.5653×10^{-2}	4.8837×10^{-2}
B (cm^{-})[1]	9.3946×10^{-2}	9.6423×10^{-2}	1.0174×10^{-1}	1.1890×10^{-1}

for temperature coefficient calculation.

V. Procedures for Thermal Group Constants

In order to carry out 1-D calculations by THERMOS, a proper unit cell has to be modelled after the real system. The periodic structure in Fig. 7(A) is deformed to that in Fig. 7(B). In Fig. 7(A), five regions can be identified:

(a) - - meat,

(b) - - aluminum cladding excluding both edges of the plate,

(c) - - side plates of the frame,
(d) - - both edges of the fuel plate,
(e) - - moderator.

As far as dimensions are concerned, the regions (a), (b) and (e) are stretched in y-direction, retaining their original thickness in x-direction. To evaluate N_i's, on the other hand, atoms in

(a) are diluted over the stretched region (a'),
(b) are diluted over the stretched region (b'),
(c), (d) and (e) are all crammed into region (c').

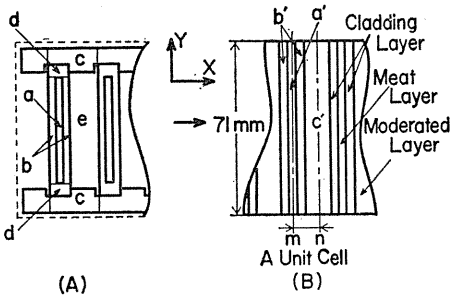


Fig. 7. Modelling a Unit Cell for THERMOS from the Real Frame.

The resulting N_i's are given in Table 9. The THERMOS code solves an infinite medium problem in which a set of regions (a'), (b'), (c'), and their symmetry images repeat themselves indefinitely in x-direction, extending at the same time to infinity in y-direction. The input data specify the axes of symmetry (m) and (n), and impose the law of reflection on neutrons incident on (m) and (n). This law is the particle-based description equivalent of periodic condition. The code solves on the unit cell between these axes.

Table 9. Number Densities Used for Thermal Group Constants at 20.44℃. Based on this table the densities at 35.12℃ were evaluated with thermal expansion taken into account. ($\times 10^{24}$ n/cm^3)

		Nuclide	C30 loading	C35 loading	C45 loading
Unit cell for the Cores	Meat region *	^{235}U	1.1279×10^{-3}	1.1279×10^{-3}	1.1279×10^{-3}
		^{238}U	8.2537×10^{-5}	8.2537×10^{-5}	8.2537×10^{-5}
		Al	4.2898×10^{-2}	4.2898×10^{-2}	4.2898×10^{-2}
	Cladding region **	Al	4.4801×10^{-2}	4.4801×10^{-2}	4.4801×10^{-2}
	Moderator region ***	Al	2.2180×10^{-2}	1.8833×10^{-2}	1.5611×10^{-2}
		H_2O	2.9483×10^{-2}	2.9162×10^{-2}	2.8853×10^{-2}
Reflector medium		H_2O	3.3330×10^{-2}		

* Region (a') in Fig. 7 (B)
** Region (b') in Fig. 7 (B)
*** Region (c') in Fig. 7 (B)

For thermal neutron scattering, the energy of hydrogen-oxygen bonding in a water molecule cannot be neglected. Including this effect by Nelkin model the code "GAKER" prepares for THERMOS the neutron scattering kernel of light water.[9] Then THERMOS obtains the twenty-nine group flux, and calculates macroscopic cross sections by Eq. (5). An exception is diffusion coefficient D, which is calculated manually with a use of the following equation, in contrast to the fast diffusion coefficient given directly by UGMG:

$$D=\sum_s/(3\sum_t^2)$$

$$\sum_t=\sum_s+\sum_a \tag{8}$$

Here, $\sum_s$ is macroscopic thermal scattering cross section, and $\sum_t$ is macroscopic total cross section, both being given by THERMOS as output. Contrary to the case of fast group calculation, twenty-nine thermal groups were collapsed directly to one group. We give the obtained thermal constants for 20.44℃ and 35.12℃ in Tables 10 and 11.

Table 10. Thermal Constants at 20.44℃.

		Core			Reflector
		C30 loading	C35 loading	C45 loading	H_2O
D	(cm)	0.24082	0.21492	0.18843	0.11240
$\sum_a$	(cm^{-1})	9.1123×10^{-2}	8.3429×10^{-2}	7.1475×10^{-2}	1.9100×10^{-2}
$\sum_f$	(cm^{-1})	6.7309×10^{-2}	6.0016×10^{-2}	4.8868×10^{-2}	——
$\nu\sum_f$	(cm^{-1})	1.6356×10^{-1}	1.4584×10^{-1}	1.1875×10^{-1}	——

Table 11. Thermal Group Constants at 35.12℃.

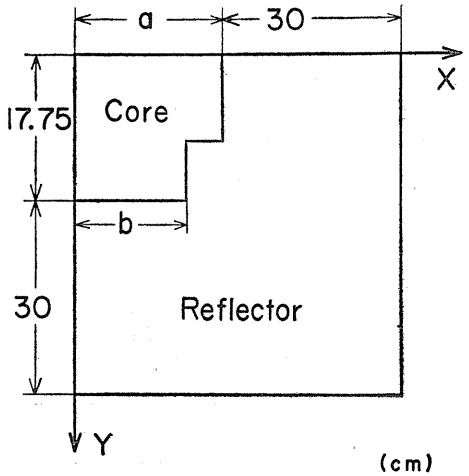
		Core			Reflector
		C30 loading	C35 loading	C45 loading	H_2O
D	(cm)	0.24664	0.21971	0.19228	0.11399
$\sum_a$	(cm^{-1})	8.8521×10^{-2}	8.1043×10^{-2}	6.9459×10^{-2}	1.8558×10^{-2}
$\sum_f$	(cm^{-1})	6.5362×10^{-2}	5.8276×10^{-2}	4.7486×10^{-2}	——
$\nu\sum_f$	(cm^{-1})	1.5883×10^{-1}	1.4161×10^{-1}	1.1539×10^{-1}	——

Ⅵ. Criticality Calculation and Temperature Coefficient

In this section we verify the validity of the constants prepared in Secs. Ⅳ and Ⅴ by carrying out criticality calculation and temperature coefficient evaluation. Both results are compared with measured values.

Criticality Calculation: A 2-D, two-group diffusion equation code "EQUIPOISE-3"[13] is used. Thus the real 3-D Critical Assembly had to be reduced to a 2-D (x-y) model, the core being replaced by a fictitious one which is bare in the z-direction (for x-z geometry calculation, the y-direction is modified to be bare). For this modification we again replace the real flux by cosine distribution in z-direction, with reflector saving δ_z as in Sec. Ⅳ. The value of effective multiplication factor k_{eff} which is obtained by EQUIPOISE-3 as an eigenvalue is to be unity as it is in criticality experiments.

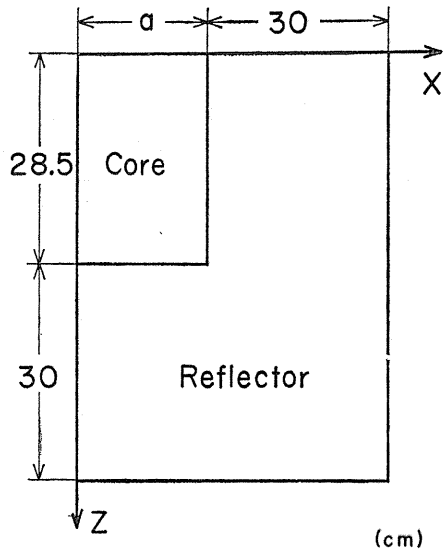
We carried out calculations for two systems: for the x-y geometry of Fig. 8 and the x-z geometry of Fig. 9. The motivation for the x-z geometry calculation is to verify the validity of δ_z assumed for x-y geometry calculation (it disclosed



(cm)

	C30 loading	C 35 loading	C 45 loading
a	16.215	17.277	15.117
b	14.200	10.650	14.200

Fig. 8. The System Analyzed by Two-Dimensional Calculation (x-y Geometry).



(cm)

	C 30 loading	C 35 loading	C 45 loading
a	15.409	14.626	14.750

Fig. 9. The System Analyzed by Two-Dimensional Calculation (x-z Geometry).

a room for improvement as discussed in Sec. Ⅶ). In both cases a quadrant of the cores are treated with 30 cm thick light water attached as the equivalent of infinite reflector. In Fig. 9, the x-y cross sectional shape is artificially made a rectangle

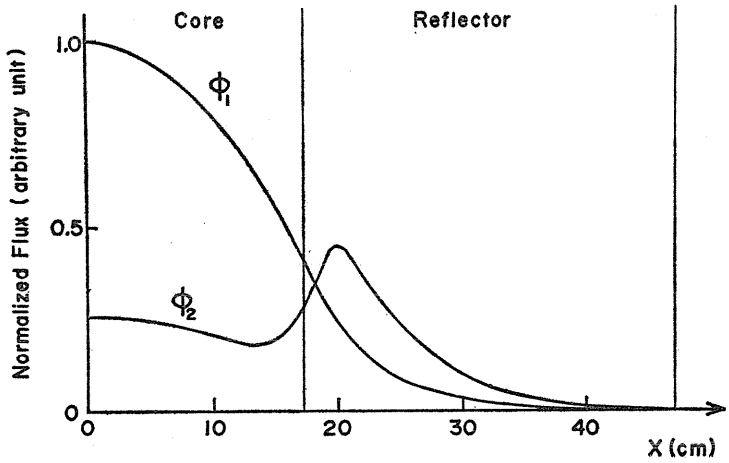


Fig. 10. Neutron Flux Distribution (in x-Direction) of C35 Loading.
ϕ_1: Fast flux ϕ_2: Thermal flux

in such a way that the number of fuel plates included is equal to that in Fig. 8. Thus the dimension a's in Fig. 9 are different from the a's in Fig. 8. The calculated effective multiplication factor k_{eff} for the systems of Figs. 8 and 9 are given in Table 12. The buckling (B_z^2 and B_y^2) used to simultate the leakage in the directions omitted in the calculation are given in Table 4. We see that the calculation predicts slight subcriticality ($k_{eff}<1$), contrary to the criticality in the experiment. In Fig. 10 we illustrate the neutron flux distribution produced by EQUIPOISE-3, which has been normalized to the fast neutron flux at the point close to the center ($x=1.065$cm).

Table 12. Effective Multiplication Factor for Each Loading Analyzed.

	C30 loading	C35 loading	C45 loading
x-y geometry (in Fig. 8)	0.98693	0.99857	0.99343
x-z geometry (in Fig. 9)	0.99139	0.98739	0.99630

Temperature Coefficient: A 1-D, multi-group diffusion code EXPANDA-25-IMPORT is used, which is a modified version of the original EXPANDA-25[17] calculating adjoint flux. Define $k_{eff}(T)$ as the effective multiplication factor at temperature T. Because $\rho(T)\equiv[k_{eff}(T)-1]/k_{eff}(T)$ and $k_{eff}(T)=1$, the change of reactivity is given by

$$\Delta\rho\cong\{k_{eff}(T+\Delta T)-k_{eff}(T)\}/k_{eff}(T+\Delta T), \tag{9}$$

and temperature coefficient α_T as

$$\alpha_T=\Delta\rho/\Delta T. \tag{10}$$

Here we give only the results for a temperature rise from 20.44℃ to 35.12℃.

The system dimensions in y- and z-directions are treated with bucklings (B_y^2, B_z^2) in the same manner as criticality calculation, so that the system is reduced to a 1-D system. By use of the constants in Tables 6 and 10, criticality search at $T=20.44$℃ is carried out to find the critical dimension in x-direction for which $k_{eff}(T)=1$. With this dimension fixed, the calculation at the 35.12℃ is carried ont, with the change of reflector saving $\Delta\delta_y$, $\Delta\delta_z$ upon temperature rise being included through B_y^2 and B_z^2. The relative change $\Delta\delta_y/\delta_y$, for example, is calculated according to the relation

$$\Delta\delta_y/\delta_y=\Delta D_1^c/D_1^c-\Delta D_1^r/D_1^r+\Delta L_T^r/L_T^r,$$
$$L_T^r=\sqrt{D_1^r/\Sigma_{1a}^r}, \tag{11}$$

where we intentionally relied on the fast group constants. These relative change are given in Table 13, together with bucklings for two-directions ($B_\perp^2\equiv B_y^2+B_z^2$) at both temperatures. With such consideration, and with constants of Tables 8 and 11, we obtained $k_{eff}(T+\Delta T)$ and α_T tabululated in Table 14. The calculated temperature coefficient α_T and the measured coefficients[18, 19, 20] are compared in Fig. 11.

Table 13. Fractional Change $\Delta\delta/\delta$ of Reflector Savings Due to Temperature Rise, and Buckling B_1^2's at Two Temperatures.

	C30 loading	C35 loading	C45 loading
$\Delta\delta/\delta$	1.1342%	1.1153%	1.1294%
B_1^2 (cm^{-2}) at 20.44℃	5.0651×10^{-3}	5.4960×10^{-3}	5.6522×10^{-3}
B_1^2 (cm^{-2}) at 35.12℃	5.0293×10^{-3}	5.4612×10^{-3}	5.6172×10^{-3}

Table 14. Calculated Temperature Coefficients and Multiplication Factors.

		C30 loading	C35 loading	C45 loading
α_T	($\Delta k/k$/℃)	-9.22×10^{-5}	-1.60×10^{-4}	-1.70×10^{-4}
k_{eff}	(20.44℃)	1.0000	1.0000	1.0000
k_{eff}	(35.12℃)	0.99865	0.99766	0.99752

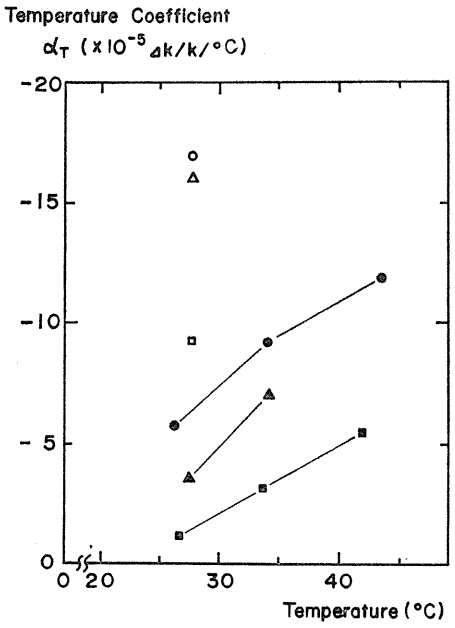


Fig. 11. Calculated and Measured Temperature Coefficient.

Calculation	Experiment
□ ····· C30 loading	■ ····· C30 loading
△ ····· C35 loading	▲ ····· C35 loading
○ ····· C45 loading	● ····· C45 loading

Ⅶ. Discussions on the Results and Plans for Future Investigations

First, we compare two sets of reflector savings: the first is the ones assumed for each loading in Sec. Ⅳ in preparation of fast constants, which we call δ_{x0}, δ_{y0}, and δ_{z0}. The others are the ones which are determined from output neutron flux distribution of criticality calculation, and we call them δ_{x1}, δ_{y1}, and δ_{z1}. The values of δ_{x1}, δ_{y1}, and δ_{z1}, although not given here, are seen to vary with loading. For a given loading, however, it is found that $\delta_{x1}\approx\delta_{y1}\approx\delta_{z1}\equiv\delta_1$, with the differences

within 0.6 cm. (This justifies our approximation of $\delta_{x0} \approx \delta_{y0} \approx \delta_{z0} \equiv \delta_0$ in UGMG inputs.) On the other hand, the value of δ_1 varies from 6.0 cm in C45 loading to 7.0 cm in C30 loading, and is smaller than δ_0 in each loading. For C30 loading, it is found that $\delta_0 - 2.4\,\text{cm} = \delta_1$. From the 2-D preliminary criticality calculation, we can infer that this much difference only in δ_z (and not in δ_x and δ_y) would cause a difference of 0.01 in k_{eff}. Although we should improve the values of δ_0's in future so that $\delta_0 \cong \delta_1$, we judged that this much error in k_{eff} at the present stage can be tolerated. As far as temperature coefficient calculation is concerned, this amount is added to both $k_{\text{eff}}(T+\Delta T)$ and $k_{\text{eff}}(T)$, and the result is not affected.

Next we discuss the scheme of giving μ as a UGMG input for light water calculation in Sec. Ⅳ. We determined μ by subtracting B_y^2 and B_z^2 from $\kappa^2 \equiv (-\phi)^{-1}\partial^2\phi/\partial x^2$ and used a single value $\mu_0 < \mu$ for all loadings. As B_y^2, B_z^2, and $(-\phi)^{-1}\partial^2\phi/\partial x^2$ vary with loading pattern, different μ's must be taken for each loading in principle. The variation of δ_y and δ_z with loadings are, however, small in comparison with the fixed dimension of the core (17.75cm for y-direction in Fig. 8, and 28.5cm for z-direction in Fig. 9). Moreover, the values of $(-\phi)^{-1}\partial^2\phi/\partial x^2$ in the reflector region do not vary much with loadings. Therefore a fixed value of μ for three loadings may be adequate. The more serious problem is the fact that the value of $(-\phi)^{-1}\partial^2\phi/\partial x^2$ obtained by criticality calculation of Sec. Ⅵ is 0.01 cm^{-2} larger than the initial guess UGMG input 0.03 cm^{-2}, which had been determined from the preliminary 2-D calculation (C35 loading). Another problem is the fact that we artificially adopted μ_0 to avoid negative flux.

These debatable points, namely

(a) the use of $\mu_0 (< \mu)$ as input data of UGMG, and

(b) the fact that reflector savings for core regions have not converged in iteration scheme between group constant preparation and criticality calculation,

have to be investigated in future. The constants we prepared here, however, can be used without serious error for ordinary use, if any particular accuracy is not required, because the difference in k_{eff} value between the criticality calculation and experiments ($k_{\text{eff}}=1$) was 1.0~1.5%. Because of this reason, and because the publication of the constants of the Assembly has deadly been needed, we have decided to present them here. The readers are encouraged to utilize these constants, but with reservation regarding the above problems. We also add in passing that 2-D, four-group diffusion calculation (which was performed by KAK[21] code) with similar group constants as those produced here has shown that $(-\phi)^{-1}\partial^2\phi/\partial x^2$ of fast flux in the reflector region did not depend upon the group number, namely on energy.

Next we examine the temperature coefficients. Three years ago, the temperature coefficients were calculated by our group with the use of a different set of group constants. The resulting coefficients showed just the reverse dependence on lattice pitch as compared with the experiments (the measured coefficients of C35 loading are more negative than those of C30 loading). Furthermore, to our disappointment, the calculated coefficients of C45 loading turned out to be positive, contrary to the measurement.

The present calculation succeeded in reproducing the lattice pitch dependence consistent with the experiments as seen in Fig. 11. We consider that we owe the success to the new set of group constant. To summarize the difference between previous scheme and the present scheme in the constant preparation, they are

(1) the use of S-G approximations in dealing with hydrogen degradation integral

(Goertzel-Greuling approximation was used previously),
(2) we determined the bucklings needed for UGMG more systematically,
(3) we included neutrons jumping over few-group energy intervals during moderation.

We regard the present constants to be adequate, judging from the above success of reproducing the pitch dependence, although at a single temperature 27.78℃.

The possible improvement in future in group constants and in the method of temperature coefficient calculations are the followings;

(1) The way to determine the reflector region buckling μ has to be investigated further. We found that the temperature coefficients were sensitive to the way μ is selected, and therefore to reflector cross sections. This is the indication of the fact that the shift in space-energy distribution of reflector flux near the reflector-core boundary plays a key role in the temperature coefficients.
(2) We need to confirm the above consistency with experiment for higher temperatures, covering a broader range.
(3) Although we have managed to evaluate the temperature coefficients with 1-D code EXPANDA-25-IMPORT, the 2-D calculation by EQUIPOISE-3 is expected to be better in treating the effect of reflector on the coefficient.
(4) In evaluating the thermal diffusion coefficient D, anisotropy in scattering in laboratory system has to be included, rather than relying on Eq. (8).

Ⅷ. Acknowledgement

We express our gratitude to Prof. T. Shibata, Associate Prof. K. Kanda and Mr. M. Hayashi of Reactor Institute, Kyoto University, who provided specifications of the Assembly. Mr. Hayashi also gave valuable suggestions in the scheme of constant preparations. Messrs. S. Matsuura, Iwao Kobayashi and A. Ohno of Japan Atomic Energy Research Institute gave enlightening advice and generous help during the course of this work. The UGMG code was made available to the authors by Dr. S. Katsuragi and Mr. H. Takano of Japan Atomic Energy Research Institute. The THERMOS code was adapted by Mr. K. Kawaguchi for the computer system at Nagoya University.

The numerical calculations reported in this report were carried out with FACOM 230/75 Computer at Nagoya University Computation Center. Helps offered in preparing the figures by Mr. H. Yoshii, a graduate student, and reliable typing provided by Miss A. Suzuki are greatly appreciated.

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