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KINETIC THEORY OF ONE-DIMENSIONAL EVAPORATION AND CONDENSATION PROBLEM

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Abstract

A linearized Boltzmann equation with BGK collision model is solved using half-range Hermite polynomials. The solution is applied to the study of half-space evaporation and condensation problem. The validity of the analysis is confirmed by the comparison with other existing results. From the present result, it is more convincing to interpret that the mass flux rate is caused by the microscosic pressure jump at the interphase surface rather than the macroscopic one maintained between the saturated vapor and the vapor gas outside of the knudsen layer.

Introduction

In the past several years, one-dimensional problems of vapor motion in contact with interphase boundaries were intensively studied by many authors [1]-[6] as the fundamental model of general evaporation and condensation problems. When a thermostatic equilibrium is not established between a condensed phase and an associated vapor gas, evaporation or condensation will take place while mass and energy fluxes are closely related to the vapor motion in the Knudsen layer adjacent to the surface of the condensed phase. Consequently, the phenomenon must be considered from the viewpoint of the Boltzmann equation. Existing analyses other than by Matsusita [6] include essentially only one reference length, the mean free path of the vapor gas; the analyses are only valid in the limit of the Knudsen number $K_n \to 0$. Since practical problems have variety of length scales (clyogenic pumps, heat pipes, etc.), the study of the phenomena in the transient regime from free molecular ($K_n = \infty$) to continum flow ($K_n = 0$) is necessary: Matsusita studied such a regime using a finite element method.

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Thus, the purpose of this paper is to obtain an analytical solution to the evaporation and condensation problem which is valid for the entire range of the Knudsen number. For the sake of the simplicity, the present analysis uses a version of the linearized Boltzmann equation with the single relaxation collision model suggested by Bhatnager et al. [7]. In order to solve the above-mentioned model equation, the distribution function will be expanded using half-range Hermite polynomials which were applied to the studies of rarefied gas dynamics originally by Gross et al. [8] and later by Huang et al. [9] for the discrete ordinate method. A more generalized solution will be obtained for a wide range of applications in the present analysis. An example of the applications of the general solution will be presented in order to show its validity: We treat the half-space evaporation (condensation) problem, a basic problem in the theory of evaporation (condensation), where there is only one interphase surface from which the gas vaporizes eventually to infinity.

More extensive applications will be made in the following papers.

Formulation of the Problem

We consider the following problem. A vapor-liquid (or vapor-solid) interphase surface is maintained at $x=0$ and the liquid phase in the space $x<0$ is kept at a constant temperature T_s. The vapor gas occupied the half-space $x\geq 0$ (Fig. 1). Let $f\ (x,\ V)$ be the distribution function of the vapor gas where V is the molecular velocity. The Boltzmann equation with the BGK collision model is written as folllows:

$$V_x(\partial f/\partial x)=\nu_c(f_0-f) \tag{1}$$

Here ν_c is the collision frequency and f_o is the Maxwellian,

$$f_0=n(2\pi RT)^{-3/2}\exp[-(V-u)^2/(2RT)]$$

where R is the gas constant. Following assumptions are introduced in the boundary conditions: The particles emitting from the interphase surface have the Maxwellian distribution corresponding to the saturated vapor conditions at the interphase surface, while the particles impinging into the condensed phase are captured completely by the condensed phase. Thus, the boundary conditions at $x=0$ and $x=L$ become

$$f(0,\ V)=n_s(2\pi RT_s)^{-3/2}\exp[-V^2/(2RT_s)],\quad (V_x>0) \tag{2—a}$$

and

$$f(L,\ V)=n_L(2\pi RT_L)^{-3/2}\exp[-V^2/(2RT_L)],\quad (V_x<0) \tag{2—b}$$

respectively. Here n_s and n_L are the number densities of the saturated vapors corresponding to the temperatures T_s and $T_L=T_s\ (1+\Delta T)$, respectively. The boundary condition at $x=L$ is taken into account so that the general solution might be obtained. For the half-space problem, the boundary condition at $x=L$ must be substituted with that of at $x\to\infty$.

Let us assume that the perturbation of the temperature is small; $\Delta T/T\ll 1$ and then $\Delta n = (n_L - n_s)/n_s \ll 1$. We may define the perturbed distribution function $\phi(x, c)$ by $f = f_s\ (1 + \phi)$ where c is the peculiar velocity scaled by $c_m = (2RT_s)^{1/2}$. A linearized version of the BGK model equation can be written by

$$c_x(\partial\phi/\partial x) = \nu + 2uc_x + \tau(\boldsymbol{c}^2 - 3/2) - \phi \tag{3}$$

Here the distance x is normalized with the mean free path λ_s pertinent to the saturated vapor condition at $x = 0$ where perturbed number density ν, temperature τ and flow velocity u are given by

$$\begin{pmatrix} \nu \\ u \\ \tau \end{pmatrix} = \pi^{-3/2} \iiint_{-\infty}^{\infty} \begin{pmatrix} 1 \\ c_x \\ (2/3)(\boldsymbol{c}^2 - 3/2) \end{pmatrix} \phi e^{-c^2} d\boldsymbol{c} \tag{4}$$

Linearization of the boundary conditions (2) gives

$$\phi(0, \boldsymbol{c}) = 0, \quad (c_x > 0) \tag{5—a}$$

$$\phi(L, \boldsymbol{c}) = \Delta n + \Delta T(\boldsymbol{c}^2 - 3/2), \quad (c_x < 0) \tag{5—b}$$

Three conservation equation for mass, momentum and energy are obtained from Eq. (3) by multiplying the collision invariants m, mc_x and $mc^2/2$ and integrating in the velocity space as follows:

$$\iiint_{-\infty}^{\infty} \begin{pmatrix} c_x \\ c_x^2 \\ (1/2)c_x c^2 \end{pmatrix} \cdot \phi e^{-c^2} d\boldsymbol{c} = \pi^{3/2} \begin{pmatrix} u \\ \Delta p^{\infty} \\ \dot{E} \end{pmatrix} \begin{matrix} (6—a) \\ (6—b) \\ (6—c) \end{matrix}$$

where u, Δp_∞ and $\dot{E}$ are constants.

Moreover, the moment equation on the energy flux $mc_x c^2$ becomes

$$(d/dx)\iiint_{-\infty}^{\infty} (1/2)c_x^2 \boldsymbol{c}^2 \phi e^{-c^2} d\boldsymbol{c} = (5/4)u - \dot{E} = Q \tag{6—d}$$

where Q denotes heat flux, while it is constant from Eqs. (6—a) and (6—c).

For combenience, we introduce the reduced distribution functions g and h defined by

$$\begin{pmatrix} g(x, c_x) \\ h(x, c_x) \end{pmatrix} = \pi^{-1} \iint_{-\infty}^{\infty} \begin{pmatrix} 1 \\ (c_y^2 + c_z^2 - 1) \end{pmatrix} \phi e^{-c_y^2 - c_z^2} dc_y dc_z$$

Equation (3) is rewritten in terms of g and h as follows:

$$c_x(\partial g/\partial x) = \nu + 2uc_x + \tau(c_x^2 - 1/2) - g \tag{7—a}$$

$$c_x(\partial h/\partial x) = \tau - h \tag{7—b}$$

Perturbed properties ν, τ and u are given by

$$\begin{pmatrix} \nu \\ u \\ \tau \end{pmatrix} = \pi^{-1/2} \int_{-\infty}^{\infty} \left[\begin{pmatrix} 1 \\ c_x \\ (2/3)(c_x^2 - 1/2) \end{pmatrix} \cdot g + \begin{pmatrix} 0 \\ 0 \\ 2/3 \end{pmatrix} \cdot h \right] e^{-c_x^2} dc_x \tag{8}$$

Boundary conditions (5—a) and (5—b) become

$$q(0,\ c_x) = 0, \qquad h(0,\ c_x) = 0, \quad (c_x > 0) \tag{9—a}$$

$$q(L,\ c_x) = \Delta n + \Delta T(c_x^2 - 1/2), \qquad h(L,\ c_x) = \Delta T, \quad (c_x < 0) \tag{9—b}$$

In the limit $1/L \to 0$, we may deduce the half-space evaporation or condensation problem adopting the boundary conditions, $(d\tau/dx)_\infty = 0$ and $(d\nu/dx)_\infty = 0$, or

$$g(\infty,\ c_x) = \nu_\infty + 2u_\infty c_x + \tau_\infty (c_x^2 - 1/2), \quad h(\infty,\ c_x) = \tau_\infty \tag{10}$$

where ν_∞, τ_∞ and $u_\infty (=u)$ are not known a priori and should be obtained from the results.

Analysis

In order to solve the basic equation (7), we introduce the half-range distribution functions $g^\pm$ and $h^\pm$ defined by $g^+ = g\ (c_x > 0)$, $g^- = g\ (c_x < 0)$, $h^+ = h\ (c_x > 0)$, and $h^- = h\ (c_x < 0)$. In the Knudsen layer adjacent to the interphase surface, vapor gas is far from the thermal equlibrium due to the collisions of particles emitting from the interphase surface with those of impinging into the condensed phase, so that above-mentioned half-range distribution functions are relevant to the study of the present phenomena.

In the similar manner as Gross et al. [8], we introduce the half-range Hermite polynomials $H_n(\eta)$ using the Gram-Schmidt method as follows:

$$H_n(\eta) = \frac{\eta^n - \sum_{i=1}^{n-1} \langle H_i | \eta^n \rangle H_i(\eta)}{\left\| \eta^n - \sum_{i=1}^{n-1} \langle H_i | \eta^n \rangle H_i(\eta) \right\|} \tag{11}$$

where

$$\langle H_n | \eta^k \rangle = \int_0^\infty H_n \eta^n e^{-\eta^2} d\eta$$

and $\|A\| = \langle A | A \rangle$. Here, $\boldsymbol{H}_n(\eta) = (H_1,\ H_2,\ H_3,\ \cdots,\ H_n)^T$ is expressed by $\eta_n = (1,\ \eta^1,\ \eta^2,\ \eta^3,\ \cdots,\ \eta^{n-1})^T$ as follows:

$$\boldsymbol{H}_n = \boldsymbol{F}^{(n,\ n)} \cdot \eta_n \tag{12}$$

where $\boldsymbol{B}^T$ denotes the transpose vector of $\boldsymbol{B}$ and $\boldsymbol{F}^{(n,\ n)}$ is a square matrix of order n whose elements $\boldsymbol{F}_{ij}$ is obtained from Eq. (11). The inverse relation is

$$\eta_n = \boldsymbol{T}^{(n,\ n)} \cdot \boldsymbol{H}_n \tag{13}$$

where $\boldsymbol{T}^{(n,\ n)}$ is the inverse matrix of $\boldsymbol{F}^{(n,\ n)}$ and its components are shown in

Appendix. Following recursive relation is available:

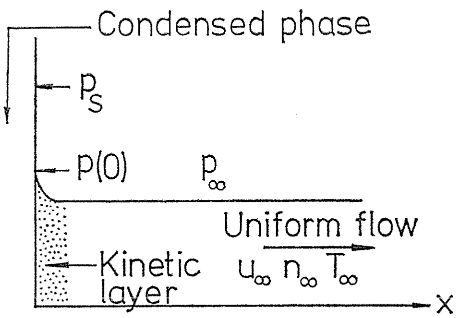

Fig. 1. Schematic drawing of the half-space problem

$$\eta \boldsymbol{H}_{n-1}=\boldsymbol{F}^{(n-1,\,n-1)}\cdot\eta\eta_{n-1}=\boldsymbol{F}^{(n-1,\,n-1)}\cdot\begin{pmatrix} 0\,1\,0\,0\cdots 0\\ 0\,0\,1\,0\cdots 0\\ 0\,0\,0\,1\cdots 0\\ \cdots\cdots\cdots\cdots\\ \cdots\cdots\cdots\cdots\\ \cdots\cdots\cdots\;1\,0\\ 0\cdots\cdots\cdots 0\,1\end{pmatrix}\cdot\eta_n=\boldsymbol{M}^{(n-1,\,n)}\cdot\boldsymbol{H}_n \tag{14}$$

where

$$\boldsymbol{M}^{(n-1,\,n)}=\boldsymbol{F}^{(n-1,\,n-1)}\cdot\begin{pmatrix} T_{21} & T_{22} & 0 & \cdots\cdots & 0\\ T_{31} & T_{32} & T_{33} & \cdots & 0\\ \cdots & \cdots & \cdots & \cdots & \cdots\\ \cdots & \cdots & \cdots & \cdots & \cdots\\ T_{n1} & T_{n2} & T_{n3} & \cdots & T_{nn}\end{pmatrix}$$

where T_{ij} is the element of the matrix $T^{(n,\,n)}$.

Entering $g^{\pm}$ and $h^{\pm}$ into Eq. (7), we get

$$\pm\eta(\partial g^{\pm}/\partial x)=\nu\pm 2\eta u+\tau(\eta^2-1/2)-g^{\pm} \tag{15—a}$$

$$\pm\eta(\partial h^{\pm}/\partial x)=\tau-h^{\pm} \tag{15—b}$$

where $\eta=|c_x|$. Half-range distribution functions $g^{\pm}$ and $h^{\pm}$ may be expanded using half-range Hermite polynomials;

$$g^{\pm}(x,\eta)=\sum_{i=1}^{\infty}H_i(\eta)a_i^{\pm}(x),\quad h^{\pm}(x,\ \eta)=\sum_{i=1}^{\infty}H_i(\eta)b_1^{\pm}(x) \tag{16}$$

where

$$a_i^{\pm}(x)=\langle H_i(\eta)|g^{\pm}(x)\rangle \quad\text{and}\quad b_i^{\pm}(x)=\langle H_i(\eta)|h^{\pm}(x)\rangle. \tag{17}$$

Entering Eq. (16) into Eq. (15) and taking into account the relation (14), we obtain

$$\sum_i \sum_j M_{ij} H_j \dot{a}_i^{\pm} = \pm \sum_i H_i (\langle H_i | G^{\pm} \rangle - \langle H_i | g^{\pm} \rangle) \qquad (18\text{—a})$$

and

$$\sum_i \sum_j M_{ij} H_j \dot{b}_i^{\pm} = \pm \sum_j H_i (\langle H_i | H^{\pm} \rangle - \langle H_i | h^{\pm} \rangle) \qquad (18\text{—d})$$

where "$\cdot$" denotes the derivative on x and M_{ij} is the component of the matrix $M^{(n,n)}$.

Multiplying Eq. (18) by $H_k e^{-\eta^2}$ and integrating over the velocity space, we obtain a set of simultaneous differential equation of the coefficients $a_i^{\pm}$ and $b_i^{\pm}$:

$$\sum_j M_{jk} \dot{a}_j^{\pm} = \pm (\langle H_k | G^{\pm} \rangle - a_k^{\pm}) \qquad (19\text{—a})$$

$$\sum_j M_{jk} \dot{b}_j^{\pm} = \pm (\langle H_k | H^{\pm} \rangle - b_k^{\pm}). \qquad (19\text{—b})$$

Here, the orthogonal relation $\langle H_i | H_j \rangle = \delta_{ij}$ is applied where δ_{ij} is the Kronecker's delta. Constructive form of $\langle H_k | G_{\pm} \rangle$ and $\langle H_k | H^{\pm} \rangle$ are as follows:

$$\langle H_1 | G^{\pm} \rangle = T_{11}\nu \pm 2T_{21}u, \quad \langle H_2 | G^{\pm} \rangle = \pm 2T_{22}u + T_{32}\tau,$$

$$\langle H_3 | G^{\pm} \rangle = T_{33}\tau, \quad \langle H_1 | H^{\pm} \rangle = T_{11}\tau, \quad \langle H_k | G^{\pm} \rangle = 0 \ (k \geq 4),$$

$$\langle H_k | H^{\pm} \rangle = 0 \ (k \geq 2).$$

In terms of $a_i^{\pm}$ and $b_i^{\pm}$, perturbed values ν, τ, and u become

$$\nu = \pi^{-1/2} T_{11} (a_1^+ + a_1^-), \quad u = \pi^{-1/2} [T_{21}(a_1^+ - a_1^-) + T_{22}(a_2^+ - a_2^-)],$$

$$\tau = (2/3)\pi^{-1/2} [T_{32}(a_2^+ + a_2^-) + T_{33}(a_3^+ + a_3^+) + T_{11}(b_1^+ + b_1^-)]. \qquad (20)$$

The four relations (6—a) $\sim$ (6—d) are reduced to

$$T_{21}(a_1^+ - a_1^-) + T_{22}(a_2^+ - a_2^-) = \pi^{1/2} u \qquad (21\text{—a})$$

$$T_{31}(a_1^+ + a_1^-) + T_{32}(a_2^+ + a_2^-) + T_{33}(a_3^+ + a_3^-) = \pi^{1/2} \Delta p_\infty / 2 \qquad (21\text{—b})$$

$$(T_{41} + T_{21})(a_1^+ - a_1^-) + (T_{42} + T_{22})(a_2^+ - a_2^-) + T_{43}(a_3^+ - a_3^-)$$
$$+ T_{44}(a_4^+ - a_4^-) + T_{21}(b_1^+ - b_1^-) + T_{22}(b_2^+ - b_2^-) = \pi^{1/2} \dot{E} \qquad (21\text{—c})$$

$$(T_{51} + T_{31})(a_1^+ + a_1^-) + (T_{52} + T_{32})(a_2^+ + a_2^-) + (T_{53} + T_{33})(a_3^+ +$$
$$a_3^-) + T_{54}(a_4^+ + a_4^-) + T_{55}(a_5^+ + a_5^-) + T_{31}(b_1^+ + b_1^-) + T_{32}(b_2^+ + b_2^-)$$
$$+ T_{33}(b_3^+ + b_3^-) = -(5/4)Qx + Q_0 \quad (Q_0 : \text{constant}) \qquad (21\text{—d})$$

For convenience, we introduce new variables $\hat{a}_1^{\pm}$ and $\hat{a}_{2\,2}^{\pm}$ defined by

$$\hat{a}_1^{\pm} = a_1^{\pm} \mp 2T_{21}u \quad \text{and} \quad \hat{a}_2^{\pm} = a_2^{\pm} \mp 2T_{22}u \qquad (22)$$

Without confusion we omit the " ∧ " hereafter. Then, the Eqs (19—a, b) are reduced to

$$\sum_i M_{i1}\dot{a}_i^{\pm}=\mp(a_1^{\pm}-a_1^{\mp})/2 \tag{23—a}$$

$$\sum_i M_{i1}\dot{b}_i^{\pm}=\pm[-b_1^{\pm}+(2/3)\pi^{-1/2}T_{11}\{T_{32}(a_2^{+}+a_2^{-})+T_{33}(a_3^{+}+a_3^{-}) +T_{11}(b_1^{+}+b_1^{-})\}] \tag{23—b}$$

$$\sum_i M_{i2}\dot{a}_i^{\pm}=\pm[-a_2^{\pm}+(2/3)\pi^{-1/2}T_{32}\{T_{32}(a_2^{+}+a_2^{-})+T_{33}(a_3^{+}+a_3^{-}) +T_{11}(b_1^{+}+b_1^{-})\}] \tag{23—c}$$

$$\sum_i M_{i3}\dot{a}_i^{\pm}=\pm[-a_3^{\pm}+(2/3)\pi^{-1/2}T_{33}\{T_{32}(a_2^{+}+a_2^{-})+T_{33}(a_3^{+}+a_3^{-}) +T_{11}(b_1^{+}+b_1^{-})\}] \tag{23—d}$$

$$\sum_i M_{ik}\dot{a}_i=\mp a_k^{\pm}\ (k\geq 4)\quad (23—e),\qquad \sum_i M_{ik}\dot{b}_i=\mp b_k^{\pm}\ (k\geq 2)\quad (23—f)$$

The Eqs. (23—a) ~ (23—f) are reduced to $4n$ linear simultaneous ordinary differential equations when the first n terms of the expansion (16) are retained. Let us $\boldsymbol{X}$ be the column vector composed of coefficients $a_i^{\pm}$ and $b_i^{\pm}$, $\boldsymbol{X}=(a_1^{+}, a_2^{+}, \cdots, a_n^{+}, b_1^{+}, b_2^{+}, \cdots, b_n^{+}, a_1^{-}, a_2^{-}, \cdots, a_n^{-}, b_1^{-}, b_2^{-}, \cdots, b_n^{-})^T$, and $\boldsymbol{M}$ be the square matrix of order $4n$ defined by

$$\boldsymbol{M}=\begin{pmatrix} \boldsymbol{M}^T & & & \boldsymbol{O} \\ & \boldsymbol{M}^T & & \\ & & \boldsymbol{M}^T & \\ \boldsymbol{O} & & & \boldsymbol{M}^T \end{pmatrix}$$

where $\boldsymbol{M}^T$ is the transpose matrix of $\boldsymbol{M}^{(n,n)}$. Equations (23—a) ~ (23—f) are, then, expressed in the simple form;

$$\boldsymbol{M}\dot{\boldsymbol{X}}=\boldsymbol{A}\boldsymbol{X} \tag{24}$$

where $\boldsymbol{A}$ is a square matrix of order $4n$, whose components are easily found from the right members of the Eqs. (23—a) ~ (23—f) while $\boldsymbol{A}$ has the following structure:

$$\boldsymbol{A}=\begin{pmatrix} \alpha & \beta \\ -\beta & -\alpha \end{pmatrix}$$

where α and β are the block matrix of order $2n$. Multipling M^{-1}, inverse matrix of $\boldsymbol{M}$, from the left to the both sides of Eq. (24), we have

$$\dot{\boldsymbol{X}}=\Gamma\boldsymbol{X}. \tag{25}$$

Here,

$$\Gamma=\boldsymbol{M}^{-1}\boldsymbol{A}=\begin{pmatrix}\Gamma_1 & \Gamma_2\\ -\Gamma_2 & -\Gamma_1\end{pmatrix}$$

where Γ_1 and Γ_2 are as follows:

$$\Gamma_1=\begin{pmatrix}\boldsymbol{M}^{-T} & \boldsymbol{O}\\ \boldsymbol{O} & \boldsymbol{M}^{-T}\end{pmatrix}\cdot\alpha,\quad \text{and}\quad \Gamma_2=\begin{pmatrix}\boldsymbol{M}^{-T} & \boldsymbol{O}\\ \boldsymbol{O} & \boldsymbol{M}^{-T}\end{pmatrix}\cdot\beta$$

where $\boldsymbol{M}^{-T}$ is the inverse matrix of $\boldsymbol{M}^{T}$.

Entering an ansatz of solution,

$$\boldsymbol{X}=\exp\ (\lambda x)\cdot\boldsymbol{u},$$

into Eq. (25), we obtain

$$\Gamma\cdot\boldsymbol{u}=\lambda\boldsymbol{u}.$$

Thus, λ must be the eigenvalue of the matrix Γ;

$$|\Gamma-\lambda\boldsymbol{I}|=0 \tag{26}$$

where $\boldsymbol{I}$ is the unit matrix. As the coefficients $a_{i\pm}$ and $b_{i\pm}$ satisfy the four linear dependent relations (21—a) ~ (21—d), the characteristic equation (26) has the four-fold degenerated eigenvalues $\lambda^4=0$. Moreover, we can find the following relation for matrix Γ;

$$\begin{pmatrix}\boldsymbol{O} & \boldsymbol{I}_1^{-1}\\ \boldsymbol{I}_1 & \boldsymbol{O}\end{pmatrix}\cdot\Gamma\cdot\begin{pmatrix}\boldsymbol{O} & \boldsymbol{I}_1\\ \boldsymbol{I}_1 & \boldsymbol{O}\end{pmatrix}=-\Gamma \tag{27}$$

where $\boldsymbol{I}_1$ is the unit matrix of order $2n$. If λ is a root of Eq. (26), it also satisfy

$$|-\Gamma-\lambda\boldsymbol{I}|=0$$

through Eq. (27). Thus, Eq. (26) may be expressed in the following form:

$$\lambda^4\prod_{i=1}^{2n-2}(\lambda^2-\lambda_i^2)=0 \tag{28}$$

where λ_i is a eigenvalue of Eq. (26).

The four-fold degenerated eigenvalues $\lambda^4=0$ are the direct reflection of the four relations (21—a) ~ (21—d) and they allow a fluid dynamic solution to the Eq. (25). Kriese et al. [10] also show the existence of the four-fold degenerated eigenvalues of a integral equation reduced from the BGK model equation. A fluid dynamic solution must be the Chapman-Enskog solution to the Eq. (15) and may be expressed, including four unknowns τ_∞, u_∞, ν_∞ and Q, as follows:

$$g_F^{\pm}=\nu_\infty\pm2u_\infty\eta+\tau_\infty(\eta^2-1/2)+Q\eta(\eta^2-3/2)$$

$$h_F^{\pm}=\tau_\infty\pm Q\eta \tag{29}$$

Substituting (29) into (15), we get

$$\dot{\nu}_\infty+\dot{\tau}_\infty(\eta^2-1/2)=-Q(\eta^2-3/2), \quad \dot{\tau}_\infty=-Q.$$

Thus, we have

$$\tau_\infty=\tau^0-Qx, \quad \nu_\infty=\nu^0+Qx, \quad \text{and} \quad \Delta p_\infty=\tau_\infty+\nu_\infty=\tau^0+\nu^0$$

where ν^0 and τ^0 are the macroscopic jumps of number density and temperature at the interphase surface. The fluid dynamic solution is, then, given by

$$\begin{aligned} g_F^\pm &=\nu^0\pm 2u_\infty\eta+\tau^0(\eta^2-1/2)+Q(\pm\eta-x)(\eta^2-3/2) \\ h_F^\pm &=\tau^0+Q(\pm\eta-x) \end{aligned} \tag{30}$$

This solution is expressed in terms of vector X as follows:

$$\boldsymbol{X}_F=\boldsymbol{X}_1\nu^0+\boldsymbol{X}_3\tau^0+\boldsymbol{X}_4Q+\boldsymbol{X}_5Qx \tag{31}$$

where nonzero components of X_i 's are listed in Table I; relations (22) are taken into account.

Table I. Nonzero components of vector $\boldsymbol{X}_i$

k	X_{1k}	X_{3k}	X_{4k}	X_{5k}
1	T_{11}	O	$T_{41}-\ (3/2)T_{21}$	T_{11}
2		T_{32}	$T_{42}-\ (3/2)T_{22}$	$-T_{32}$
3		T_{33}	T_{43}	$-T_{33}$
4			T_{44}	
$n+1$		T_{11}	T_{21}	$-T_{11}$
$n+2$			T_{22}	
$2n+1$	T_{11}	O	$-T_{41}+\ (3/2)T_{21}$	T_{11}
$2n+2$		T_{32}	$-T_{42}+\ (3/2)T_{22}$	$-T_{32}$
$2n+3$		T_{33}	$-T_{43}$	$-T_{33}$
$2n+4$			$-T_{44}$	
$3n+1$		T_{11}	$-T_{21}$	$-T_{11}$
$3n+2$			$-T_{22}$	

The general solution to the Eq. (25) is, then, given by

$$\boldsymbol{X}=\sum_{k=1}^{2n-2}p_k\exp(\lambda_k^P x)\cdot\boldsymbol{u}_k^P+\sum_{k=2n-1}^{4n-4}p_k\exp(\lambda_k^N x)\cdot\boldsymbol{u}_k^N+\boldsymbol{X}_F \tag{32}$$

where $\boldsymbol{u}_k$ is the eigenvector corresponding to the eigen value λ_k and "P" and "N" denote plus eigenvalue and minus one, respectively. Arbitrary parameters $p_k(k=1, 2, \cdots, 4n-4)$ as well as ν^0, τ^0, u_∞, and Q will be determined from bondary conditions.

Half-space Proalem

In the half-space problem, vapor gas at $x \to \infty$ is in thermal equilibrium with constant mass and energy fluxes, $\dot{m}(=u_\infty)$ and $\dot{E}$, where heat flux $Q=0$. The boundary conditions for $g^\pm$ and $h^\pm$ are, then, given by

$$x=0: \quad g^+=h^+=0 \tag{33—a}$$

$$x\to\infty: \quad g^\pm=\nu_\infty \pm 2u_\infty\eta+\tau_\infty(\eta^2-1/2) \quad \text{and} \quad h^\pm=\tau_\infty. \tag{33—b}$$

In terms of $a_i^\pm$ and $b_i^\pm$, (33—a) and (33—b) become

$$a_1^+(0)=-2T_{21}u_\infty,\ a_2^+(0)=-2T_{22}u_\infty,\ a_k^+(0)=0\ (k\geq 3),\ \text{and}$$

$$b_k^+(0)=0 \ (k\geq 1) \tag{34—a}$$

and

$$a_1^\pm(\infty)=T_{11}\nu_\infty,\quad a_2^\pm(\infty)=T_{32}\tau_\infty,\quad a_3^\pm(\infty)=T_{33}\tau_\infty,$$

$$a_k^\pm(\infty)=0\ (k\geq 4),\quad b_1^\pm(\infty)=T_{11}\tau_\infty,\ \text{and}\ b_k^\pm(\infty)=0\ (k\geq 2). \tag{34—b}$$

The parametrs p_k of the term with positive eigenvalue in (32) should be zero because the solution X must be finite at $x\to\infty$. The other $2n-2$ coefficients p_k, ν_∞, u_∞ and τ_∞ can be determined imposing the boundary conditions (34—a) and (34—b) on (32);

$$\begin{pmatrix} -2T_{21} \\ -2T_{22} \\ 0 \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ 0 \end{pmatrix} u_\infty = \sum_{k=2n-1}^{4n-4} \begin{pmatrix} u_{2n-1,k} \\ u_{2n,k} \\ u_{2n+1,k} \\ \cdots \\ u_{i,k} \\ \cdots \\ u_{3n-2,k} \\ \cdots \\ \cdots \\ u_{4n-4,k} \end{pmatrix} \cdot p_k + \begin{pmatrix} T_{11} \\ 0 \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ 0 \end{pmatrix} \cdot \nu_\infty + \begin{pmatrix} 0 \\ T_{32} \\ T_{33} \\ 0 \\ \cdots \\ \cdots \\ T_{11} \\ 0 \\ \cdots \\ 0 \end{pmatrix} \cdot \tau_\infty \tag{35}$$

Then, we have

$$\begin{pmatrix} u_{1,\,2n-1} & u_{1,\,2n} & \cdots\cdots & u_{1,\,4n-4} & T_{11} & 0 \\ u_{2,\,2n-1} & u_{2,\,2n} & \cdots\cdots & u_{2,\,4n-4} & 0 & T_{32} \\ u_{3,\,2n-1} & u_{3,\,2n} & \cdots\cdots & u_{3,\,4n-4} & 0 & T_{33} \\ \cdots\cdots & & & & & 0 \\ \cdots\cdots & & & & & \cdots \\ u_{n+1,\,2n-1} & \cdots\cdots & & u_{n+1,\,4n-4} & 0 & T_{11} \\ \cdots\cdots & & & & & 0 \\ \cdots\cdots & & & & & \\ u_{2n,\,2n-1} & \cdots\cdots & & u_{2n,\,4n-4} & 0 & 0 \end{pmatrix} \begin{pmatrix} p_{2n-1} \\ p_{2n} \\ p_{2n+1} \\ \cdots \\ \cdots \\ \cdots \\ p_{4n-4} \\ \nu_{\infty} \\ \tau^{\infty} \end{pmatrix} = \begin{pmatrix} -2T_{21} \\ -2T_{22} \\ 0 \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ 0 \end{pmatrix} \cdot u_{\infty} \quad (36)$$

From Eq. (36), we can find a definite relationship between the condition of the vapor gas far away from the interphase surface and the mass and energy fluxes while from (6—a) and (29) $\dot{E}=(5/4)\dot{m}$. The results vs the retained term number n are listed in Table II. Values of the perturbed number density and temperature, as the ratio to mass flux; $\nu/\dot{m}$ and $\tau/\dot{m}$, at arbitrary x are easily obtained from (20) once the p_k/u_{∞} are obtained from (36). The results are shown in Fig. 1.

Table II. Macroscopic jumps of density and temperature vs expansion term number n

	$n=5$	$n=6$	$n=7$	Siewert et al. [4]
ν_{∞}/u_{∞}	−1.68549	−1.68538	−1.68534	−1.68529
τ_{∞}/u_{∞}	−0.44667	−0.44671	−0.44673	−0.44675

Results and Discussions

On Convergence of the Expansion Method

As the expansion method using an orthogonal polynomial is a sort of moment method, the results obtained may be dependent upon where to truncate the expansion term. The present results of the half-space problem shown in Table II indicate the rapid approach of the macroscopic moments to the results of Siewert et al. [4] when the retained terms increase. In order for the present results to agree with those of Siewert et al. up to the first 4 digits, one needs to retain only five or six terms in the expansion. It is obvious that the present solution can become accurate to the required extent when the sufficient terms are retained.

Results of Half-Space Problem

From Eq. (36), we can consistently evaluate $p_k/\dot{m}$, $\nu_{\infty}/\dot{m}$ and $\tau_{\infty}/\dot{m}$, where ν_{∞}, $u_{\infty}(=\dot{m})$ and τ_{∞} specify the respective equilibrium flow conditions at $x\to\infty$. This implicates that for any value of $u_{\infty}(u_{\infty}\ll 1)$ stationary evaporation or condensation causing no heat flux is possible, although the density and the temperarure in the flow field are related to the evaporation or condensation rate $\dot{m}$. We define the following four coefficients originally introduced by Pao [3]:

$$c_\infty = -(n_\infty/n_s - 1)/(2\dot{m}), \quad d_\infty = -(T_\infty/T_s - 1)/(2\dot{m}),$$
$$\gamma^0 = -(n(0)/n_s - 1)/(2\dot{m}), \quad \text{and} \quad \delta^0 = -(T(0)/T_s - 1)/(2\dot{m})$$

where we designate $c_\infty = -c_n/2$ and $d_\infty = -c_T/2$. Here, c_∞ and d_∞ represent the macroscopic density jump and the macroscopic temperature jump whereas γ^0 and δ^0 imply the microscopic slips of density and temperature at the interphase surface. In Table III, the slip coefficients obtained using the present half-range Hermite expansion with $n=5$ are evaluated for the purpose of comparison with other results. It should be mentioned that evaporation (condensation) rate is closely related to the microscopic pressure jump (slip) rather than the macroscopic pressure jump: If the gas expands isentropically through the pressure difference Δp, velocity increment Δu is give by

Table III. Values of slip coefficients

	Siewert et al.	Sone et al	Matsusita	present results
c^∞	0.842645	0.842645	0.842694	0.842745
d_∞	0.223375	0.223375	0.223073	0.223335
γ^0	0.661130	0.661130	0.661035	0.661124
σ^0	0.204789	0.204789	0.205550	0.204800

$$\Delta u / c_m = (2\gamma)^{-1/2} \cdot \Delta p / p_s,$$

where γ is the specific heat ratio. Thus. we have

$$\dot{m}_{\text{isen}} = \Delta u / c_m = (3/10)^{1/2} \times 2(c_\infty + d_\infty)\,\dot{m} = 1.17\dot{m}$$

$$\dot{m}_{\text{isen}} = (3/10)^{1/2} \times 2(\gamma^0 + \delta^0)\,\dot{m} = 0.95\dot{m}$$

The latter relation suggests us to interpret that the evaporation or condensation is caused primarily through the isentropic expansion or compression due to the pressure difference (See Fig. 1) between the saturated vapor and the vapor gas just outside of the interphase surface as long as heat flux plays no important role. The remaining about 5 % of the evaporation (condensation) rate may be attributed to the adopted boundary condition; an assumption is made with respect to the boundary condition on emitting particles, in other words, emitting particles are independent of the effects of impinging particles, *i. e.*, the emitting particles are pre-accelerated.

The Structure of the Knudsen layer in the half-space evaporation or condensation problem is shown in Fig. 2. Temperature relaxation seems somewhat slower than that of density even though the heat flux is zero elsewhere in the flow field.

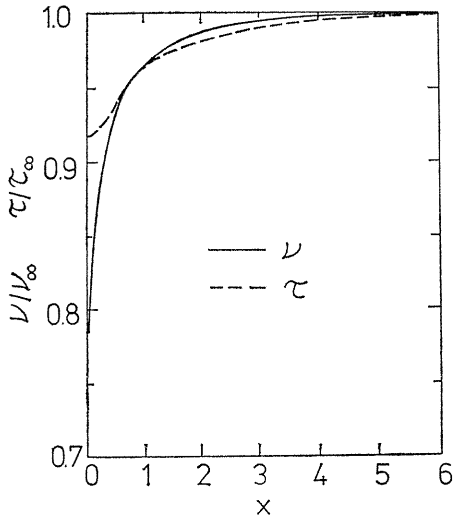

Fig. 2. Density and temperature profile in the Knudsen layer

In conclusion, the applicability of the general solutions (32) and (31) to the half-space problem is confirmed. Other applications will appear in the future.

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Appendix

Matrix $\boldsymbol{T}^{(10,10)}$

0.94140									
0.53113	0.40127								
0.47070	0.62302	0.23443							
0.53113	0.95341	0.66546	0.16659						
0.70605	1.55755	1.53115	0.72689	0.13639					
1.06225	2.73502	3.39551	2.30801	0.83108	0.12472				
1.76512	5.13991	7.60449	6.61235	3.44153	0.99860	0.12486			
3.18676	10.2756	17.4808	18.3033	12.2811	5.18284	1.25956	0.13498		
6.17791	21.7278	41.4895	50.3840	40.9730	22.4086	7.95552	1.66337	0.15597	
12.7470	48.3491	101.851	139.849	132.631	88.3103	40.7855	12.4985	2.29312	0.19112