EFFECTS OF ACCOMODATION COEFFICIENTS ON EVAPORATION AND CONDENSATION

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Abstract

A linearized BGK model equation is solved using half-range Hermite polynomials. Effects of evaporation and condensation coefficients on the evaporation and condensation problems are studied. It is found that in the two-surface problem macroscopic jumps of temperature and density are weak functions of the condensation coefficient and the temperature jump is independent of the evaporation coefficient, while mass flux (evaporation rate) is strongly dependent upon the both coefficients.

Introduction

In the previous paper [1], the authors obtained a solution to the BGK model equation [2] for a one-dimensional evaporation and condensation problem. Application of the solution to the half-space problem confirmed the validity of the method used.

In the existing analyses of the problem [3] - [6], the following boundary conditions were utilized: Particles impinging into the condensed phase are hundred-per-cent captured while particles emitting from the interphase surface have a Maxwellian distribution function corresponding to the saturated vapor condition at the surface. These boundary conditions are physically unpractical in the sense, for instance, emitting particles are entirely independent of impinging particles. The purpose of this paper is, then, to release such restrictions on the boundary conditions, by using macroscopic ones instead of being based on a priori assumption on the distribution function itself. In other words, we use conventional accomodation coefficients; an evaporation coefficient α_e and a condensation coefficient α_c [7], [8]. The boundary conditions at the interphase surface are given using these coefficients as follows: The α_c fraction of the particles impinging into the condensed phase is captured and the remaining $(1-\alpha_c)$ reflected from the surface either specularly or diffusely, while the α_e fraction of the particles emitting from

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the surface is able to evaporate and the remaining $(1-\alpha_e)$ of them recaptured by the condensed phase. Of course, no essential problem arises in resorting to any other meaningful boundary conditions if available. The solution obtained in the previous paper will be applied to study the effects of the accomodation coefficients on the evaporation and condensation problem. The results will suggest the necessity of further study of the problems included in evaporation and condensation phenomena.

Formulation of the Problem

Let us consider the following evaporation and condensation problem. Two vapor-liquid (or vapor-solid) interphase surfaces are maintained at $x=0$ and $x=L$. The liquid phases in the spaces $x<0$ and $x>L$ are kept at constant temperatures T_s and T_L; $T_L=T_s(1+\Delta T)$, respectively. Due to a difference of the saturated vapor pressures maintained between the two condensed phase, the vapor gas is evaporating from the interphase surface at $x=L$ and condensating into the other interphase surface at $x=0$: As the results of the evaporation and condensation processes at the interphase surfaces, there are fluxes of mass and energy in the $-x$ direction (See Fig. 1).
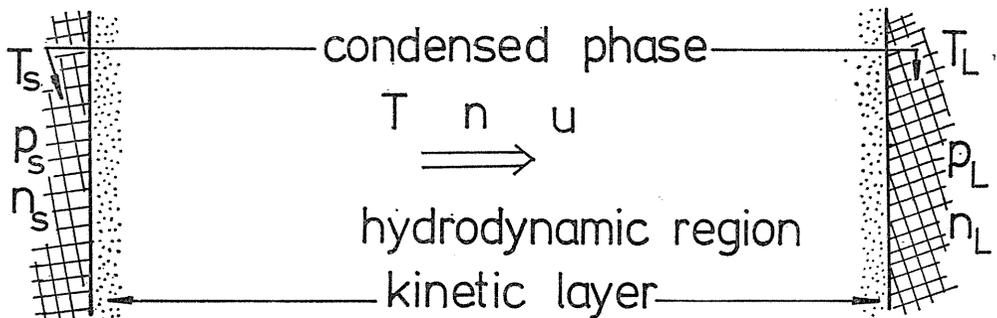

Fig. 1. Schematic drawing of the Two-surface problem.

Let $f(x, V)$ be the distribution function of the vapor gas where V is the molecular velocity. The Boltzmann equation with BGK collision model [2] is written as follows:

$$V_x(\partial f/\partial x)=\nu_c(f_0-f) \tag{1}$$

where ν_c is a collision frequency and f_0 is a Maxwellian,

$$f_0=n(2\pi RT)^{-3/2}exp[-(V-u)^2/(2RT)]$$

where R is the gas constant.

In order to evaluate effects of imperfect accomodation at the interphase surface, we introduce an effective evaporation coefficient α_e and an effective condensation coefficient α_c defined by the following way: The α_c fraction of the particles

impinging into the condensed phase is captured and the remaining $(1-\alpha_c)$ reflected from the surface specularly, while the α_e fraction of the particles emitting from the surface is able to evaporate and the remaining $(1-\alpha_e)$ of them recaptured by the condensed phase. Then, the distribution functions of the particles emitting from the interphase surfaces are give by

$$f(0, \ V_x>0)=\alpha_e f_s+(1-\alpha_c)f(0, \ V_x<0)$$

and

$$f(L, \ V_x<0)=\alpha_e f_L+(1-\alpha_c)f \ (L, \ V_x>0) \tag{2}$$

where f_s and f_L are the Maxwellians corresponding to the saturated vapor conditions at the surfaces at $x=0$ and $x=L$, respectively. A perturbed distribution $\phi*$ may be defined by

$$f=(\alpha_e/\alpha_c)f_s(1+\phi*) \tag{3}$$

Entering (3) into (1), we obtain a linearized version of (1) using a length scale $2\lambda_s/\sqrt{\pi}$; λ_s is the mean free path corresponding to the saturated vapor condition at $x=0$,

$$c_x(\partial\phi*/\partial x)=\Phi*-\phi* \tag{4}$$

where

$$\Phi*=\nu*+2c_x u*+\tau*(c^2-3/2)$$

where $\nu*$, $u*$ and $\tau*$ are the perturbed density, flow velocity and temperature, respectively, and c_x is the x component of the peculiar velocity $\boldsymbol{c}=(c_x, \ c_y, \ c_z)$; $c^2=\boldsymbol{c}\cdot\boldsymbol{c}$.

For convenience, we introduce reduced distribution functions g and h defined by

$$g=\pi^{-1}\iint_{-\infty}^{\infty}\phi*e^{-c_y^2-c_z^2}dc_ydc_z, \quad h=\pi^{-1}\iint_{-\infty}^{\infty}(c_y^2+c_z^2-1)\phi*e^{-c_y^2-c_z^2}dc_ydc_z$$

In terms of g and h, Eq. (4) reduced to

$$c_x(\partial g/\partial x)=\nu*+2cu*+\tau*(c_x^2-1/2)-g$$

$$c_x(\partial h/\partial x)=\tau*-h \tag{5}$$

Perturbed values are given by

$$\nu*=<g, \ 1>, \quad u*=<g, \ c_x>$$

and

$$\tau*=(2/3)\{<g, \ c_x^2-1/2>+<h, \ 1>\} \tag{6}$$

where $<A, \ B>$ implicates

$$<A, \ B>=\pi^{-1/2}\int_{-\infty}^{\infty}A\cdot B\cdot e^{-c_x^2}dc_x$$

An actual density, n, a flow velocity u and a temperature T are easily found from the relation (3) as follows:

$$n=(\alpha_e/\alpha_c)n_s(1+\nu*), \quad u=u*, \quad T=T_s(1+\tau*) \tag{7}$$

The boundary conditions (2) are reduced to

$$g^+(0)=(1-\alpha_c)g^-(0), \quad h^+(0)=(1-\alpha_c)h^-(0) \tag{8, a}$$

and

$$g^-(L)=\alpha_c\{\Delta n+\Delta T(c_x^2-1/2)\}+(1-\alpha_c)g^+(L)$$

$$h^-(L)=\alpha_c\Delta T+(1-\alpha_c)h^+(L) \tag{8, b}$$

where superscript "+" and "−" implicate $c_x>0$ and $c_x<0$, respectively. From Eq. (5), the first four lowest moments on c_x, which are corresponding to the moments of collision invariants and energy flux, become

$$(d/dx)<g, \; c_x>=0 \quad (9,\,a), \qquad (d/dx)<g, \; c_x^2>=0 \tag{9, b}$$

$$(a/dx)\{<g, \; c_x(c^2-1/2)>+<h, \; c_x>\}=0 \tag{9, c}$$

and

$$(d/dx)\{<g, \; c_x^2(c_x^2-1/2)>+<h, \; c_x^2>\}=Q* \tag{9, d}$$

Here, we find from (9, a) and (9, c) that $u*$ and $Q*$, which implicates a heat flux, are constants. Any other higher moments include moments of g and/or h in the righthand-side of the moment equations.

In order to solve (5), we introduce half-range Hermite polynomials $H_n(\eta)$ using the Gram-Schmidt method [1], which have the following relations:

$$\eta_n=T^{(n,\,n)}\cdot H_n, \quad H_n=F^{(n,\,n)}\cdot \eta_n \text{ and } \eta H_{n-1}=M^{(n-1,\,n)}\cdot H_n \tag{10}$$

Here, $H_n=(H_1, H_2, \cdots, H_n)^T$ and $\eta_n=(1, \eta, \eta^2, \cdots, \eta^{n-1})^T$; superscript T implicates the transpose of a vector or a matrix, where matrixes $T^{(n,\,n)}$, $F^{(n,\,n)}$ and $M^{(n,\,n)}$ are shown in Ref. 1. Let us define half-range distribution functions $g^\pm$ and $h^\pm$ by $g^+=g$ $(c_x>0)$, $g^-=g$ $(c_x<0)$, $h^+=h$ $(c_x>0)$ and $h^-=h$ $(c_x<0)$. Entering $g^\pm$ and $h^\pm$ into (5, a) and (5, b), we have

$$\pm\eta(\partial g^\pm/\partial x)=\nu*\pm2\eta u*+\tau*(\eta^2-1/2)-g^\pm$$

$$\pm(\partial h^\pm/\partial x)=\tau*-h \tag{11}$$

where $\eta=|c_x|$. The half-range distribution functions $g^\pm$ and $h^\pm$ may be expanded using the half-range Hermite polynomials,

$$g^\pm=\sum_{i=1}^{\infty}H_i(\eta)a_i^\pm(x), \quad h^\pm=\sum_{i=1}^{\infty}H_i(\eta)b_i^\pm(x) \tag{12}$$

where

$$a_i^\pm=<H_i(\eta), \; g^\pm(x)> \text{ and } b_i^\pm=<H_i(\eta), \; h^\pm(x)>$$

The macroscopic moments are given by

$$\nu^* = \pi^{-1/2}T_{11}(a_1^+ + a_1^-), \quad u^* = \pi^{-1/2}[T_{21}(a_1^+ - a_1^-) + T_{11}(a_2^+ - a_2^-)]$$

$$\tau^* = (2/3)\pi^{-1/2}\{T_{32}(a_2^+ + a_2^-) + T_{33}(a_3^+ + a_3^-) + T_{11}(b_1^+ + b_1^-)\} \tag{13}$$

Substituting (12) into (11), multiplying (11) by $H_k(\eta) \exp(-\eta^2)$ and integrating it from $\eta = 0$ to $\eta = \infty$, we obtain a simultaneous differential equation on the coefficients $a_i^{\pm}$ and $b_i^{\pm}$,

$$(d/dx)X = \Gamma \cdot X \tag{14}$$

where $X = (a^{+T}, b^{+T}, a^{-T}, b^{-T})^T$; $a^{\pm} = (a_1^{\pm}, a_2^{\pm}, \cdots, a_n^{\pm})^T$ and $b^{\pm} = (b_1^{\pm}, b_2^{\pm}, \cdots, b_n^{\pm})^T$. In obtaining (14), the relation (10) and an orthogonal relation $\langle H_k, H_l \rangle = \delta_{kl}$ are utilized. From the concrete form of the matrix Γ, a characteristic equation of Γ; $|\Gamma - \lambda I| = 0$, can be expressed by [1]

$$\lambda^4 \prod_{k=1}^{2n-2} (\lambda^2 - \lambda_k^2) = 0$$

where λ_k is an eigenvalue of the characteristic equation of Γ. Thus, a general solution of Eq. (14) is expressed by

$$X = \sum_{k=1}^{2n-2} p_k \exp[\lambda_k^P x] \cdot u_k^P + \sum_{k=2n-1}^{4n-4} p_k \exp[\lambda_k^N x] \cdot u_k^N + X_F \tag{15}$$

where u_k is an eigenvector corresponding to the eigenvalue λ_k and superscript "P" and "N" implicate $\lambda_k > 0$ and $\lambda_k < 0$, respectively. A fluid dynamic solution X_F, which is directly related to the four-fold degenerated eigenvalue $\lambda^4 = 0$, i. e., relations (9, a) through (9, d), is given by

$$X_F = X_1\nu^0 + X_2 u^* + X_3 \tau^0 + X_4 Q^* + X_5 Q^* \cdot x \tag{16}$$

where $X_2 = (T_{21}, T_{22}, 0, \cdots, 0_{2n}, -T_{21}, -T_{22}, 0, \cdots, 0_{4n})^T$ and other vectors $X_i s$ are given in Ref. 1. ν^0 and τ^0 are called the macroscopic jumps of density and temperature, respectively. The general solution (15) and (16) include $4n$ adjustable parameters, p_k ($k = 1, 2, \cdots, 4n-4$), ν^0, u^*, τ^0 and Q^* so as to be able to satisfy the boundary conditions at $x = 0$ and $x = L$.

Effects of the Accomodation Coefficients

The boundary conditions (8, a) and (8, b) are rewritten in terms of $a_i^{\pm}$ and $b_i^{\pm}$ as follows:

$$\left. \begin{array}{l} a^+(0) - (1-\alpha_c)a^-(0) = 0 \\ b^+(0) - (1-\alpha_c)b(0) = 0 \end{array} \right\} \tag{17, a}$$

and

$$\left. \begin{array}{l} a(L) - (1-\alpha_c)a^+(L) = \alpha_c n \cdot \Delta n + \alpha_c t \Delta T \\ b^-(L) - (1-\alpha_c)b^+(L) = \alpha_c \cdot n \cdot \Delta T \end{array} \right\} \tag{17, b}$$

where $n = (T_{11}, 0, \cdots, 0_n)^T$ and $t = (0, T_{32}, T_{33}, 0, \cdots, 0_n)^T$. Imposing (17, a) and (17, b) on (15) and (16), we obtain

$$\sum_{k=1}^{4n-4} p_k \{ \boldsymbol{u}_k^+ - (1-\alpha_c) \boldsymbol{u}_k^- \} + \alpha_c X_1^+ \nu^0 + (2-\alpha_c) X_2^+ u^* + \alpha_c X_3^+ \tau^0 + (2-\alpha_c) X_4^+ Q^* = 0$$

$$(18, a)$$

$$\sum_{k=1}^{4n-4} p_k e^{\lambda_k L} \{ \boldsymbol{u}_k^- - (1-\alpha_c) \boldsymbol{u}_k^+ \} + \alpha_c X_1^- \nu^0 + (2-\alpha_c) X_2^- u^* + \alpha_c X_3^- \tau^0$$

$$+ (2-\alpha_c) X_4^- Q^* + \alpha_c X_5^- Q^* L = \alpha_c \binom{n}{o} \Delta n + \alpha_c \binom{t}{n} \Delta T \qquad (18, b)$$

where superscript on the vector X_i, "+" and "−", denote the uper half and lower half of the column vector, respectively. When $\alpha_c = 1$, Eq. (18) is reduced to

$$\sum_{k=1}^{2n-2} p_k' e^{-\lambda_k^P L} \boldsymbol{u}_k^+ + \sum_{k=2n-1}^{4n-4} p_k^N \boldsymbol{u}_k^+ + X_1^+ \nu^0 + X_2^+ u^* + X_3^+ \tau^0 + X_4^+ Q^* = 0 \qquad (19, a)$$

$$\sum_{k=1}^{2n-2} p_k' \boldsymbol{u}_k^- + \sum_{k=2n-1}^{4n-4} p_k e^{\lambda_k^N L} \boldsymbol{u}_k^- + X_1^- \nu^0 + X_2^- u^* + X_3^- \tau^0 + X_4^- Q^* + X_5^- Q^* L$$

$$= \binom{n}{o} \Delta n + \binom{t}{n} \Delta T \qquad (19, b)$$

where $p_k' = p_k^P \exp(\lambda_k^P L) \sim O(\Delta T)$ and $q = Q^* L \sim O(\Delta T)$ because X must be finite at $x = L$. Neglecting terms of the orders $O(\exp(-\lambda_k^P L))$ and $O(1/L)$ for $L \gg 1$, (19, a) reduced to the Eq. (35) in Ref. 1. Thus, we have $\nu^0 = c_n \dot{m}$ and $\tau^0 = c_T \dot{m}$ [1] where $\dot{m} = u^*$. In the same manner, (19, b) can be solved and results in

$$\dot{m} = (1/2) \Delta p / (c_n + c_T), \quad q = -\Delta T + \Delta p \cdot c_T / (c_n + c_T)$$

and

$$p_{k+2n-2} = p_k ; \quad \Delta p = \Delta n + \Delta T \qquad (20)$$

where a symmetric relation of the solution $X^{\pm}(-x) = X^{\pm}(x)$, i, e.,

$$\lambda_k = -\lambda_{k+2n-2}, \qquad u_{i, k+2n-2} = u_{i+2n, k} \quad \text{and} \quad u_{i, k} = u_{i+2n, k+2n-2} ;$$

$u_{i, k}$ is a component of vector $\boldsymbol{u}$, is used. Then, we obtain

$$\nu^0 = (1/2) \Delta p \cdot c_n / (c_n + c_T) \quad \text{and} \quad \tau^0 = (1/2) \Delta p \cdot c_T / (c_n + c_T) \qquad (21)$$

Results of finite values of L are listed in Table I and shown in Figs. 2 - 4. A parameter β is defined by $\Delta n = \beta \Delta T$ which is obtained from the Clausius-Clapeyron's equation

$$d(\ln n)/d(\ln T) = L_0/kT^* - 1 = \beta$$

where T^* is a boiling temperature and L_0 is a latent heat.

When $L \gg 1$, (18, a) and (18, b) can be simplified for cases $\alpha_c \neq 1$ as well as the case $\alpha_c = 1$:

$$\sum_{k=2n-1}^{4n-4} p_k \{ \boldsymbol{u}_k^+ - (1-\alpha_c) \boldsymbol{u}_k^- \} + \alpha X_1^+ \nu^0 + (2-\alpha_c) X_2^+ u^* + \alpha_c X_3^+ \tau^0 = 0 \qquad (22, a)$$

Table 1. Macroscopic jumps, evaporation rate and heat flux

L	α_c	ν^0		u		τ^0		Q^* or q^*		β_c
		A_1	B_1	A_2	B_2	A_3	B_3	A_4	B_4	
1	0.1	0.49715	0.10029×10^{-1}	-1.47104×10^{-2}	-0.75958×10^{-2}	0.28458×10^{-2}	0.48907	0.56917×10^{-2}	-0.20058×10^{-1}	3.5240
	0.5	0.48509	0.53997×10^{-1}	-0.89702×10^{-1}	-0.52428×10^{-1}	0.14910×10^{-1}	0.44600	0.29820×10^{-1}	-0.10799	3.6215
	1.0	0.46733	0.12243	-0.25569	-0.17402	0.32671×10^{-1}	0.37757	0.653418×10^{-1}	-0.24485	3.7474
10	0.1	0.47712	0.80759×10^{-1}	-1.43648×10^{-2}	-0.86452×10^{-2}	0.22878×10^{-1}	0.41924	0.45757×10^{-2}	-0.16152×10^{-1}	3.5299
	0.5	0.43489	0.23725	-0.84220×10^{-1}	-0.67942×10^{-1}	0.65112×10^{-1}	0.26275	0.13022×10^{-1}	-0.47450×10^{-1}	3.6437
	1.0	0.41533	0.31953	-0.23895	-0.21778	0.84674×10^{-1}	0.18047	0.16935×10^{-1}	-0.63906×10^{-1}	3.7737
∞	0.1	0.38962	0.38962	-0.13102×10^{-1}	-0.13102×10^{-1}	0.11038	0.11038	0.22075	-0.77925	3.5299
	0.5	0.39232	0.39232	-0.80712×10^{-1}	-0.80712×10^{-1}	0.10768	0.10768	0.21535	-0.78465	3.6435
	1.0	0.39525	0.39525	-0.23450	-0.23450	0.10475	0.10475	0.20949	-0.79051	3.7735

$\nu^0 = (A_1\beta + B_1)\varDelta T$, $u = (A_2\beta + B_2)\varDelta T$, $\tau^0 = (A_3\beta + B_3)T$, Q^* or $q^* = (A_4\beta + B_4)\varDelta T$
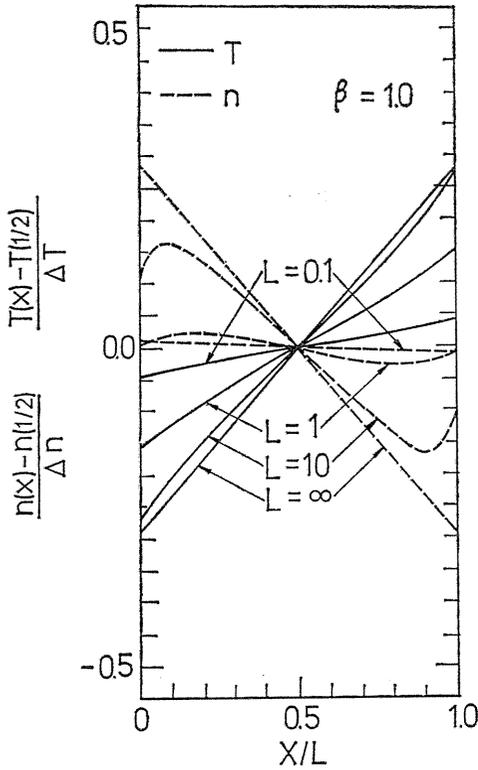
Fig. 2. Density and temperature
distributions ; $\beta=1$.
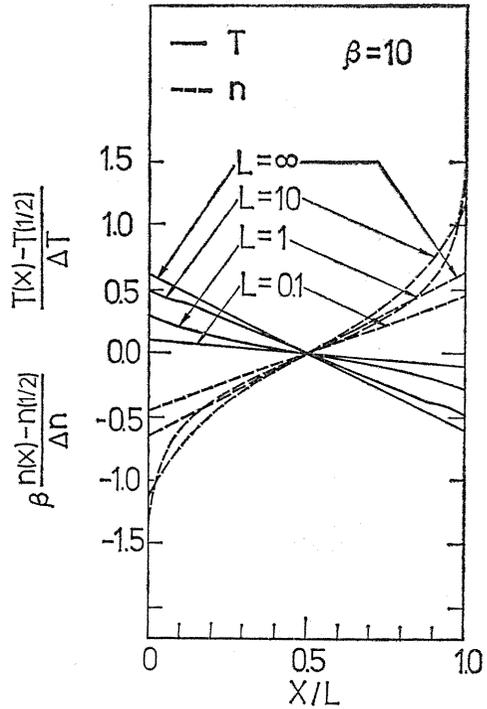

Fig. 3. Density and temperature
distributions ; $\beta=10$.

$$\sum_{k=1}^{2n-2} p'_k \{u_k^- - (1-\alpha_c)u_k^+\} + \alpha_c X_1^- \nu^0 + (2-\alpha_c)X_2^- u^* + \alpha_c X_3^- \tau^0 + \alpha_c X_5^- q^*$$

$$= \alpha_c \binom{n}{o} \varDelta n + \alpha_c \binom{t}{n} \varDelta T \qquad (22, b)$$

Solving (22, a), we find the macroscopic jump coefficients

$$c^\infty = -\nu^0/(2\dot{m}) = -\nu^0/(2\alpha_e u^*/\alpha_c) = -(c_n^*/2)(\alpha_c/\alpha_e)$$

$$d^\infty = -\tau^0/(2\dot{m}) = -\tau^0/(2\alpha_e u^*/\alpha_c) = -(c_T^*/2)(\alpha_c/\alpha_e) \qquad (23)$$

where $c_n^* = \nu^0/u^*$ and $c_T^* = \tau^0/u^*$. Microscopic jump coefficients are also obtained as
follows :

$$\gamma^0 = -(c_n^*(0)/2)(\alpha_c/\alpha_e) \quad \text{and} \quad \partial^0 = -(c_T^*(0)/2)(\alpha_c/\alpha_e)$$

where $c_n^*(0) = \nu^*(0)/u^*$ and $c_T^*(0) = \tau^*(0)/u^*$. As the values with superscript "*"
depend only upon the condensation coefficient α_c, the jump coefficients are inversely
propotional to the evaporation coefficient α_e. When $\alpha_e = \alpha_c$, the superscript "*" can
be removed (See (7)). In Table II, the macroscopic and microscopic jump coeffi-
cients are given for a few values of α_c ($= \alpha_e$). In the same manner as in the case
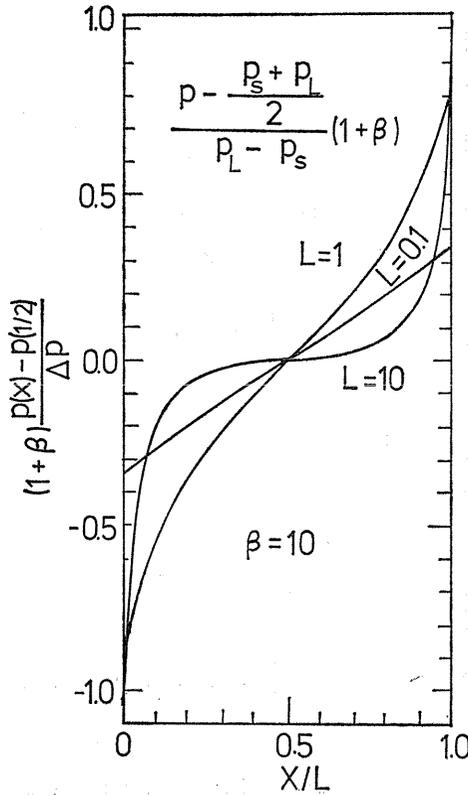
Fig. 4. Pressure distribution.

Table 2. Jump coefficients

α_c	$1 \cdot 10^{-8}$	0.1	0.5	1.0
c^∞	1.55026×10^3	1.48687×10	2.43038	0.84275
d^∞	4.42895×10^2	4.2122	0.66704	0.22334
γ^0	1.54987×10^3	1.44991×10	2.14756	0.66124
δ^0	4.42859×10^2	4.17821	0.63968	0.20480

$\alpha_c = 1$, (22, b) can be solved and results in

$$u^*_\infty = (1/2)\Delta p/(c^*_n + c^*_T), \quad q^* = -\Delta T + \Delta p \cdot c^*_T/(c^*_n + c^*_T)$$

$$\nu^0 = (1/2)\Delta p \cdot c^*_n/(c^*_n + c^*_T), \quad \tau^0 = (1/2)\Delta p \cdot c^*_T/(c^*_n + c^*_T)$$

and

$$p'_k = p_{k+2n-2}$$

In the limit $\alpha_c \to 0$, $c^*_n/c^*_T \to 7/2$, $u^* \to (2/9)/\pi^{1/2}\alpha_c\Delta p$, $q^* \to (2/9)\Delta p - \Delta T$, $\nu^0 \to (7/18)\Delta p$ and $\tau^0 \to (1/9)\Delta p$, while $\dot{m} \to (\alpha_e/\alpha_c)u^*$ and an actual heat flux $Q = (\alpha_e/\alpha_c)q^*/L$.

Solutions to Eq. (21, a) and (21, b) for the finite values of L are listed in Table I and shown in Figs. 5 - 7.
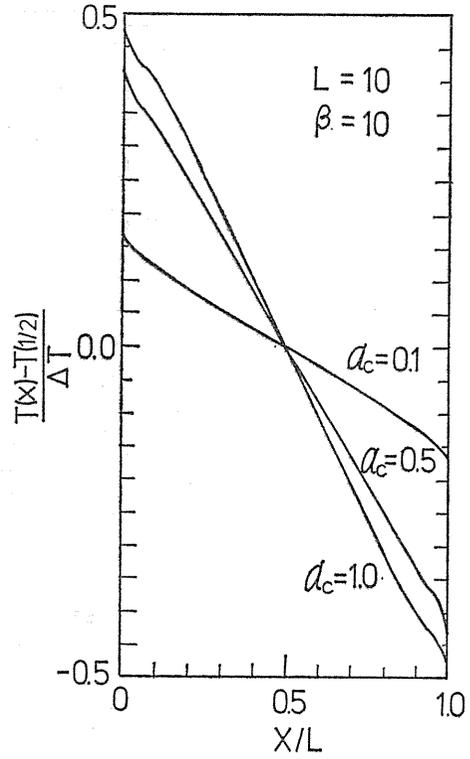
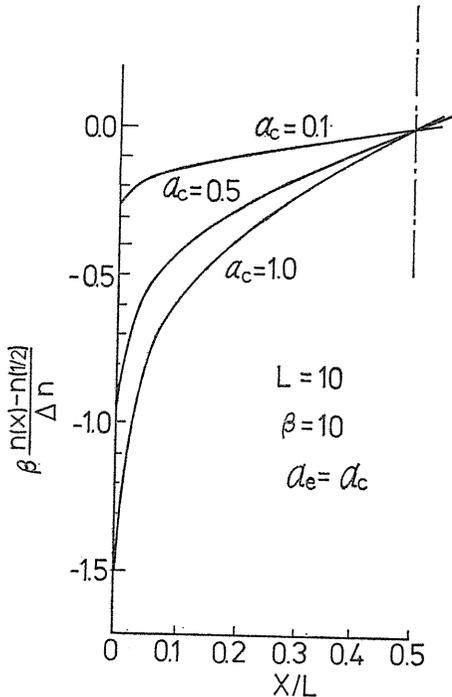Fig. 5. Effects of the accomodation coefficient on temperature distribution.

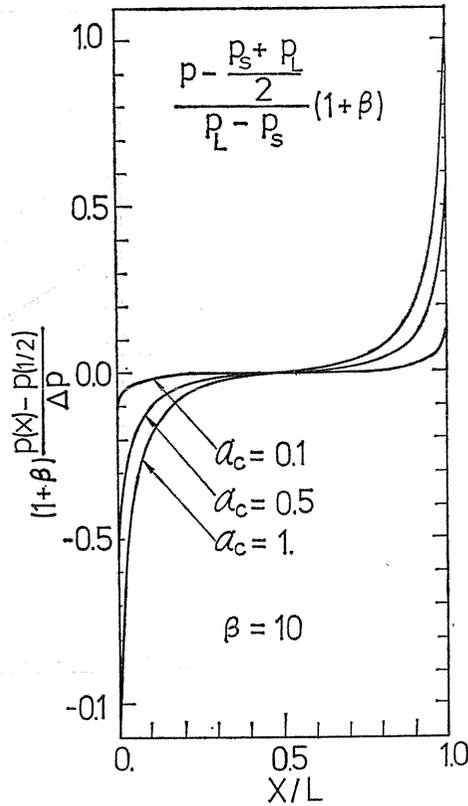Fig. 6. Effects of the accomodation coefficient on density distribution.

Fig. 7. Effects of the accomodation coefficient on pressure distribution.

Results and Discussions

The Case $\alpha_c = 1$.

Equation (19) can be easily solved for arbitrary values of L to yield the evaporation rate, energy flux, flow field etc. of the two-surface problem. The important results are listed in Table I for typical values of L where β_c is the critical value of β for which the heat flux Q becoms zero. The results for $L=\infty$ are obtained neglecting the terms including exp $(-L)$ and $1/L$. It is interesting to note that the macroscopic jumps of the density ν^0 and the temperature τ^0 depend only upon the pressure difference Δp while it is expected that the evaporation rate $\dot{m}$ is propotional to the pressure difference Δp. On the contrary, the heat flux Q^* depends upon ΔT and Δn with opposite sings. Thus, the temperature gradient, which is indicated as Q^* in Eq. (30) in Ref. 1, can be of the sign opposite to the maintaing temperature difference, as has been predicted by Pao [9] using the approximate method, when β exceeds the critical value β_c. For the given temperature difference ΔT, the evaporation rate increases with Δp because the pressure difference is propotional to $(\beta+1)$. The transient flow fields for finite values of L,

from free molecular to continum, are shown in Fig. 2 - 4. The temperature gradient for $\beta < \beta_c$ is positive and the temperature jump decreases as L increases while the density distribution behaves oppositely. When $\beta > \beta_c$, the profiles of the density and the temperature are reversed, as seen from comparison between Fig. 2 and 3. The density profiles for $L=10$ in Figs. 2 and 3 show that Knudsen layer emerges adjacent to the interphase surfaces while the Knudsen layer in the temperature profile seems rather ambiguous. Steep decrease of the density near the interphase surfaces causes rapid change of the pressure in the Knudsen layer, as shown in Fig. 4, while Δp is constant for large value of L outside of the Knudsen layer. In the cases for $L \leq 10$, present results show a good agreement with those of Matsushita [6].

The Case $0 < \alpha_c < 1$

It may be convenient to discuss the results for $\alpha_e = \alpha_c$ because ν^* and τ^* give the actual perturbations of density and temperature in that case. Of couse, we can easily obtain the results for case $\alpha_e \neq \alpha_c$.

In the half-space problem (See Table II), the jump coefficients c_n^* and c_T^* increase as α_c decreases; when α_c decreases, the vapor gas accomodates less to the condensed phase and consequently the Knudsen layer becomes ambiguous. In the limit $\alpha_c \to 0$, where the vapor gas cannot accomodate to the condensed phase at all, the Knudsen layer disappeares. Although this situation seems analogus to the free molecular flow conditions, the vapor gas is in thermal equilibrium at $x \to \infty$, which is essentially different from the free molecular results. In Fig. 8, evaporation rate $\dot{m}$ and the macroscopic jumps of density and temperature are plotted against the
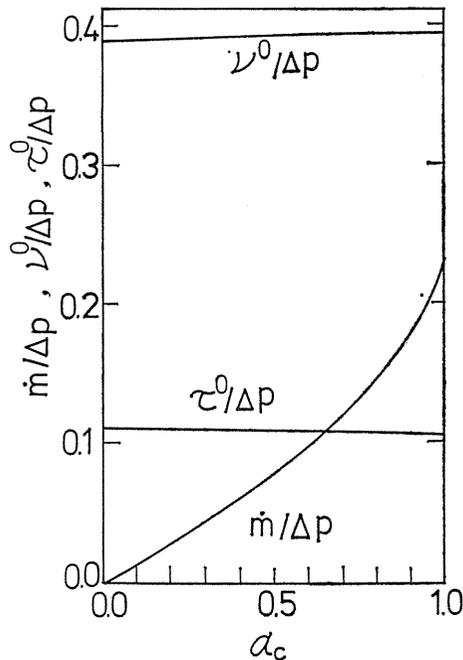

Fig. 8. Macroscopic jumps and evaporation rate vs α_c

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accomodation coefficient α_c ($\alpha_e = \alpha_c$). It is worth noting that the macroscopic jumps of density and temperature are weak functions of α_c, while the evaporation rate is strongly dependent upon the coefficient. Behaviors of the values in the limit $\alpha_c \to 0$ are already mentioned in the previous section, except that the critical value β_c approach 7/2. Effects of the accomodation coefficient for finite values of L on the evaporation rate etc. are found from Table II where A_i implies an effective density difference and B_i implies an effective temperature difference: In the expression of ν^0, the decrease of the accomodation coefficient is accompanied by the decrease of the effective temperature difference B_1, while it is insensitive for the effective density difference A_1. We find a reversed tendency in the expression of τ^0. In the evaporation rate $\dot{m}$ and heat flux Q, both of the effective differences A_i and B_i decrease in the same degree as α_c decreases.

Throughout the analysis, the effective accomodation coefficients α_e and α_c have been utilized. As a matter of fact, these coefficients must be related to some real physical phenomena of evaporation and condensation. Decrease of α_e implies the decrease of saturated vapor pressure at the interphase surface; it may be caused from a temperature gradient at the interphases surface, inside of the condensed phase. In the future analysis, therefore, the surface structure has to be taken into account. On the other hand, decrease of α_c may be caused by contamination due to some noncondensable gases; the condensed phase may contain small amount of other substances the saturated vapor pressure of which is much higher than that of the main substance of the consensed phase, or noncondensable gases exist between two condensed phase. Thus, the study of noncondensable gas effects on evaporation and condensation is required. This problem is now in progress by the authors using the method applied here.

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