Memoirs of the Faculty of Engineering, Nagoya University
Vol. 28, No. 1. (1976) — Research Report —

RIGOROUS TREATMENT OF THE SPECTRAL DISTORTION CAUSED BY THE USE OF TIME-TO-AMPLITUDE CONVERTER

by

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(Received June 8, 1976)

Abstract

A rigorous expression for the spectral distortion involved in the experimental data obtained by the use of the time-to-amplitude converter is derived from statistical treatment of the problem. And a proposal for the use of "over-range" pulses in this instruments is made.

1. Introduction

The time-interval distributions measured by the use of Time-to-Amplitude Converter (TAC) are distorted by various causes,[1] and different from true distributions.

Excepting those distortions attributed to imperfectness of logic and inadequate performances of electronic circuit components, still there exists a inevitable distortion arising from the statistical nature of the detection of events. This effect is well known and called "blocking" or "pile-up" effect, and the method of correction has been given[2], [3] as

$$N_i = NP_i \prod_{j=1}^{i-1} (1-P_j), \tag{1}$$

or in the more convenient form

$$P_i = N_i / (N - \sum_{j=1}^{i-1} N_i), \tag{2}$$

where P_i is defined as the total probability of detecting an event during the time interval defined by channel i, N is the total number of timing cycles performed, and N_i is the number of counts in channel i.

In this paper, a rigorous expression which takes the place of Eq. (2) is derived, and an additional proposal for the use of "over-range" pulses in this instruments is made.

2. Statistical Treatment of the "Blocking" Effect

At first, we introduce a probability density $\varphi(t)$ which is the time interval distribution to be measured with a TAC. In other wards, $\varphi(t)\Delta t$ is the probability of occurrence of an event in time interval Δt between time t and $t+\Delta t$ after a start of conversion ($t=0$).

It is assumed that the phenomenon under observation is periodic, namely, $\varphi(t)$'s are immutable in every cycle, as is made in practical experiments.

For simplicity, in this chapter we confine ourselves in the TAC which accept only one pulse in a particular conversion cycle, namely, sweep. Then $\varphi(t)\Delta t$ is the probability of an interruption of a sweep in a time interval Δt at t.

Now we discuss the result obtained after S times sweeps have performed.

By $P_{N,S}(t)$, let us denote the probability that N sweeps out of S are interrupted in the time interval (0, t). In successive interval (t, $t+\Delta t$), the probability of occurrence of one interruption is

$$\binom{S-N}{1}\varphi(t)\Delta t(1-\varphi(t)\Delta t)^{S-N-1}=(S-N)\varphi(t)\Delta t+O(\Delta t)^2,$$

and that of no occurrence is

$$(1-\varphi(t)\Delta t)^{S-N}=1-(S-N)\varphi(t)\Delta t+O(\Delta t)^2.$$

So, we have

$$P_{N,S}(t+\Delta t)=[1-(S-N)\varphi(t)\Delta t]P_{N,S}(t)$$
$$+(S-N+1)\varphi(t)\Delta tP_{N-1,S}(t)+O(\Delta t)^2.$$

Rearranging and putting $\Delta t\to 0$, we get the system of differential equations

$$\frac{dP_{N,S}(t)}{dt}=-(S-N)\varphi(t)P_{N,S}(t)+(S-N+1)\varphi(t)P_{N-1,S}(t). \quad (3)$$

Equations (3) can be calculated in order of N, beginning with $N=0$, with attention to the fact that $P_{0,S}(0)=1$, $P_{N,S}(0)=0$ for $N\geqq 1$, and $P_{-1,S}(t)=0$.
The solutions are

$$P_{N,S}(t)=\binom{S}{N}(1-e^{-c})^N(e^{-c})^{S-N}, \quad (4)$$

with

$$C=C(t)\equiv\int_0^t\varphi(t)\,dt. \quad (5)$$

Equation (4) reveals an interesting fact that the probability $P_{N,S}(t)$ follows the binomial distribution.*

We have at a glance

$$P_{0,1}(t)=e^{-c} \tag{6}$$

and it is clear from the definition of $P_{N,S}(t)$ that e^{-c} is the probability of no interruption of a particular sweep in $(0, t)$.

Hence the mean $\langle N\rangle$ and variance $\langle (N-\langle N\rangle)^2\rangle$ of the random variable N are

$$\langle N\rangle=\sum_{N=0}^{S} NP_{N,S}(t)=S\alpha \tag{7}$$

and

$$\langle N^2\rangle-\langle N\rangle^2=S\alpha\beta, \tag{8}$$

where

$$\alpha\equiv 1-e^{-c},\quad \alpha+\beta=1.$$

Since the problem now we are concerned is not homogeneous in time, that is $\varphi(t)$ depends on t, an expression for the probability $P_{n,S}(t,\ t')$ of n interruption in any time interval $(t,\ t+t')$ is needed further. This can be written as

$$\begin{aligned} P_{n,S}(t,\ t') &= \sum_{m=0}^{S-n} P_{m,S}(t)\cdot P_{n,S-m}(t') \\ &= \binom{S}{n}(\alpha'\beta)^n(1-\alpha'\beta)^{S-n}, \end{aligned} \tag{9}$$

where

$$\alpha'\equiv 1-e^{-c'} \quad \text{and} \quad \alpha'+\beta'=1,$$

with

$$C'=C'(t,t')\equiv\int_t^{t+t'}\varphi(t)\,dt=C(t+t')-C(t).$$

Equation (9) shows the $P_{n,S}(t,\ t')$ obeys the binomial law, too. Then the mean and variance of n are

$$\langle n\rangle=S\alpha'\beta \tag{10}$$

and

$$\langle (n-\langle n\rangle)^2\rangle=S\alpha'\beta(1-\alpha'\beta). \tag{11}$$

Substituting Eq. (7) into Eq. (10), we get

$$\langle n\rangle=(S-\langle N\rangle)\cdot\alpha'. \tag{12}$$

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* Regarding "sweep" itself as a random variable whose state (value) is "in sweeping" or "already stopped", this result may be expected more directly.

For practical convenience, Eq. (12) is slightly modified as follows. Adding $<N>$ and $<n>$, we observe

$$<N>+<n>=S(1-e^{-c})+Se^{-c}(1-e^{-c'})$$
$$=S(1-e^{-(c+c')})=<N+n>.*$$

This relation is easily extended to

$$<N>=\sum_{j=1}^{i-1}<n_j> \tag{13}$$

with the understandings that n_j is the number of interruptions out of S sweeps in (t_{j-1}, t_j), $t_0=0$, $t_{i-1}=t$, and $t_i=t'$, to reduce Eq. (12) to

$$<n_i>=(S-\sum_{j=1}^{i-1}<n_j>)(1-e^{-c_i}). \tag{14}$$

This is the result which takes the place of Eq. (2).
Here

$$C_i\equiv\int_{t_{i-1}}^{t_i}\varphi(t)\,dt, \tag{15}$$

and this quantity is connected to n_{0i} which is the "true" counts of the "stop" events arrived at the TAC in time inteval (t_{i-1},t_i), that is,

$$n_{0i}=SC_i. \tag{16}$$

Additionally it should be mentioned that if we take a random variable x of

$$x=\frac{n}{S-N}, \tag{17}$$

its mean is

$$<x>=\sum_{N=0}^{S}\sum_{n=0}^{S-N}\frac{n}{S-N}P_{N,\,S}(t)\cdot P_{n,\,S-N}(t,t')$$
$$=\alpha'=1-e^{-c'}. \tag{18}$$

In comparing Eq. (18) with Eq. (12), we have favourably

$$\left\langle\frac{n}{S-N}\right\rangle=\frac{<n>}{S-<N>}, \tag{19}$$

as implicitly assumed in practice.

* This relation is resulted from the recurcive nature of the binomial distribution for the procedure of convolution.

3. Notes on the Converting Procedures of the Pulse Height Spectra to the Time Spectra

3. 1. Proposal for the Use of "Over-range" Pulses

In most cases, a TAC is used in combination with a multi-channel pulse height analyzer (MCA). Thereupon, a care should be taken on the performance of the MCA in use. That is, a few tens of the lower channels of the MCA are usually "masked-off" so as to protect the MCA from being occupied by noise pulses whose height distribution is crushing into lower portion. Accrodingly, the summation of n_j in Ep. (14) — the number of sweeps already terminated before reaching the channel i — can not be obtained.

However, it is easy to remove this difficulty, with the number R of sweeps which have been spent without a stop pulse, namely, the number of the "over-ranged" sweeps. Regardless of the MCA connected to the TAC, there exist a relation,

$$S=\sum_{j=1}^{M} n_j+R$$

or

$$S-\sum_{j=1}^{i-1} n_j=\sum_{j=i}^{M} n_j+R, \tag{20}$$

where M is the maximum channel number in the TAC range.

Thus, a more useful version of Eq. (14) is obtained as

$$1-e^{-c_i}=\frac{<n_i>}{\sum_{j=i}^{M}<n_j>+R}, \tag{21}$$

or

$$n_{0i}=S\cdot ln\frac{\sum_{j=i}^{M}<n_j>+R}{\sum_{j=i+1}^{M}<n_j>+R}. \tag{21$'$}$$

Based on the above discussion, it is proposed that "every time-to-amplitude converter, which accepts only one event in a sweep, should be equipped with an output terminal so as to provide a logic signal indicating the over-ranging of the sweeps".

To realize the above proposal, there is no difficulty at all in the case of start-stop type TAC's emplying a conversion capacitor and a constant current source, since they equip the over-range trigger circuit. On the other hand, overlap-type TAC's may require a few circuit components additionally for this purpose.

3. 2. On the "Multi-Analysis" type TAC[4), 5)]

Let us consider a case where a "multi-analysis" type TAC that accepts one more pulses in a sweep is connected to a MCA possesing dead time T_d.

Since the input gate of the MCA is closed during the time interval T_d after a reception of a pulse, the numver of effective sweeps, out of S, for the channel i is

$$S-\sum_{j=k}^{i-1} n_j. \tag{22}$$

Therefore, the lower limit of the summation in Eq. (14) is shifted to k, and the same expression that has been obtained for the multi-channel time analyzer[6), 7)] is encountered. Using the channel width τ relating to the magnitude of a conversion range, k is given by

$$k=i-T_d/\tau.$$

For usual (Wilkinson type) MCA's, T_d is given as

$$T_d=a+bk,$$

so

$$k=\frac{\tau i-a}{\tau+b}. \tag{23}$$

For the TAC of this type, it may be mentioned further that the consideration stated in section 3-1 is unnecessary, because any of sweeps is not interrupted before the maximum conversion time and such a pulses that does not exceed the input discriminator level of the MCA have no effect on the dead time of MCA.

4. Concluding Remarks

The present approach to this problem is considered to be the most fundamental and straightforward, since it is based on the differential difference equation governing the process.

The results presented here have been adopted successfully in our laboratory.

Acknowledgements

The authors wish to express their sincere gratitude to Dr. K. Nishina for many helpful discussions, and to Mr K. Kawano and Mr M. Katoh for their assistance in numerical work and computer-coding.

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