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ELECTNOHYDRODYRAMCIS IN THE WIDE SENSE

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Abstract

To describe the phenomena obserbed in the ion drag pump, the ion transport generator, the corona wind etc., we have proposed the two-fluid model which is consisted with the charged particle fluid and the uncharged particle fluid. A set of fundamental equations is the equation of continuity and the momentum equation for both fluids and the Maxwell epuations. With those equations, the electrohydrostatic equation has been solved only for a one-dimensional parallel plane case. Furthermore, a one-dimensional steady flow has been treated from which we have got the modified Poiseuille flow and the mobified Cobine space charge relation. Finally a numerical example has been tabulated for an oil.

1. Introduction

We shall call, "the electrohydrodynamics in the narrow sense", with which we can discuss the behaviors of a single charged particle fluid in a vacuum or in a static fluid. In that dynamics, main problem is to find or to calculate the space charge relation; for example, Langmuir 3/2 powers law in a vacuum diode tube and Cobines law in a gas-filled diodc tube. In the other words, we can explain the current-voltage characteristic curve in a vacuum diode tube or in a gas-filled diode tube without any ionization[1)~13)].

"The electrohydrodynamics in the wide sense", which we shall here develope, can describe the various phenomena of the system consisted with the single charged particle fluid and the unchrged fluid. Those various phenomena, for example, are the ion drag pump[19)~21)], the ion transport generator[22)], the corona wind[15)~18)], the electrical osmostics and so on. The electrical osmostics[14)], however, is concearned with an electrical double layer so that we shall not diseuss here.

2. The fundamental equations

Comparing the electrohydrodynamics (EHD) and the magnetohydrodynamics (MHD), O. M. Stuetzer formulated a set of ebuations governing the electrohydrodynamics[23], but it seems to authors too formal. So we shall here propose a new set of equations which is consisted with the equation of continuity and the hydrodynamic momentum equation for both fluids and then the Maxwell equations.

They may be written as follows,

$$\frac{\partial n}{\partial t}+\nabla\cdot n\boldsymbol{v}=0 \tag{1}$$

$$mn\left\{\frac{\partial \boldsymbol{v}}{\partial t}+(\boldsymbol{v}\cdot\nabla)\boldsymbol{v}\right\}=-mn-\nabla\cdot\boldsymbol{p}-mm_c nn_c\alpha(\boldsymbol{v}-\boldsymbol{v}_c) \tag{2}$$

$$\frac{\partial n_c}{\partial t}+\nabla\cdot n_c\boldsymbol{v}_c=0 \tag{3}$$

$$m_c n_c\left\{\frac{\partial \boldsymbol{v}_c}{\partial t}+(\boldsymbol{v}_c\nabla)\boldsymbol{v}_c\right\}=-m_c n_c\boldsymbol{g}+en_c(\boldsymbol{E}+\boldsymbol{v}_c\times\boldsymbol{B})$$
$$-\nabla\boldsymbol{p}_c-nm_c nn_c\alpha(\boldsymbol{v}_c-\boldsymbol{v}) \tag{4}$$

$$\nabla\times\boldsymbol{H}=\boldsymbol{j}+\frac{\partial\varepsilon\boldsymbol{E}}{\partial t}\qquad \nabla\cdot\mu\boldsymbol{H}=0 \tag{5}$$

$$\nabla\times\boldsymbol{E}=-\frac{\partial\mu\boldsymbol{H}}{\partial t}\qquad \nabla\cdot\varepsilon\boldsymbol{E}=en_c \tag{6}$$

$$\boldsymbol{j}=en_c\boldsymbol{v}_c \tag{7}$$

Here the subscript c implies the quantities for the charged particle fluid, m is the mass, n the number density, $\boldsymbol{v}$ the flow speed $\boldsymbol{g}$ the gravitational constant, α the coefficient[24], $\boldsymbol{E}$ the intensity of electric field, $\boldsymbol{H}$ the intensity of magnetic field, ε the dielectric constant, μ the magnetic permiablity and $\boldsymbol{p}$ the pressure tensor. The pressure tensor $\boldsymbol{p}$ is approximately written as

$$(\boldsymbol{p})_{ik}=p\delta_{ik}+\frac{2}{3}(\nabla\cdot\boldsymbol{v})\delta_{ik}-\eta\left(\frac{\partial v_k}{\partial x_i}+\frac{\partial v_i}{\partial x_k}\right) \tag{8}$$

where p is the scalar pressure, η the coefficient of viscousity, δ_{ik} the Cronecker symbole. The equation (8) is valid when the Newtonian flow and storke's approximation is assumed. The pressure tensor of the charged particle, p_c, may express as same as Eq. (8), althongh there is a problem arised from Rutherford's scattering. The problem will be solved by selecting $n_c^{-1/3}$ as the maximum impact paramerter.

Thus, all quantities can be determined by Eqs. (1)~(8) as a function of the time t and the position r. But there is an another problem how is produced the charged particles. The ansower is that two ways are possible. One way is to supply into the uncharged particle fluid by an exteral source; for instance, by the discharge corona. The other way is to utilize the charged particle in the so-called diffuse layer produced by a liquid contact to a solid. It is believed that if a liquid contact to a solid then the profile of electrical potential is as shown in the case (1) or the case (2) in Fig. 1. In that figure, the charged particles in Helmholtz's

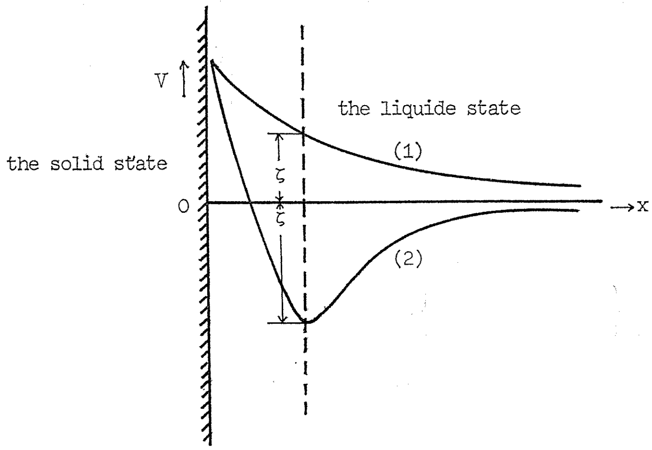

Fig. 1. The profile of electrical potential of the liquide state contact to the solid state.

layer do not move, because the charged particles attache to the solid. The charged particles in the biffuse layer can be transported with the moving fluid. It can also be accelarated by an externally applied electric potential and as the result the uncharged particle fluid is derived, which is called "the electrical osmostics" or "the ion drag pump". Thus if the profile of electric potential in the diffuse layer can be measured by a methode then n_c will be known by making use of the Poisson equation. In the age of Helmholtz, the surface charge density q is as

$$q \approx \varepsilon\zeta/\delta, \tag{9}$$

where δ is the thickness of electical duble layer and ζ is of the so-called ζ-potential. However, ζ and δ themselves should be determined theoretically which itself is a theoretical problem of physics. So, here, we shall assume that charged particles are supplied by an external source.

3. The electrohydrostatic equilibrium

We shall call "the electrohydrostatic equilibrium" when $v=0$, $v_c=0$ and $g=0$. Then, a set of equations becomes

$$\nabla p=0, \tag{10}$$

$$\nabla p_c=en_c\boldsymbol{E}, \tag{11}$$

$$\nabla \boldsymbol{E}=\frac{en_c}{\varepsilon}, \tag{12}$$

$$\boldsymbol{E}=-\nabla v. \tag{13}$$

Immediately, we get

$$p = \text{constant}. \tag{14}$$

If the ideal gas law, $p_c \simeq n_c k T_c$, is valid and if T_c is constant because the static equilibrium is in the thermal equilibrium, then it will be hold

$$n_c = n_{c0} exp(-eV/kT_c) \tag{15}$$

where $n_c = n_{co}$ at $V=0$. Combining Eqs, (12), (13) and (15), we have the dependency of V on x

$$\frac{eV}{kT_c} = 2\log\left(1+\frac{x}{2x_0}\right)$$
$$x_0^2 = (\varepsilon k T_c / 2e^2 n_{c0}) \tag{16}$$

for a one-dimensional parallel plane case, where $n_c = n_{co}$ at $x=0$ and, by the refernce (1),

$$E_0 = (2n_{c0} k T_c / \varepsilon)^{1/2} \tag{17}$$

at $x=0$.

4. The ion drag pump and the ion transport generator

In this section we shall treat with the one-dimensional steady flow neglecting the magnetic field.
Seeing the figure 2, we set

$$v = (v_1, 0, 0), \quad n = \text{constant},$$
$$v_c = (v_{c1}, 0, 0), \quad E = (E_1, 0, 0), \tag{18}$$
$$j = (j_1, 0, 0)$$

Then, from $\nabla \cdot v = 0$, we have

$$v_1 = v_1(x_3). \tag{19}$$

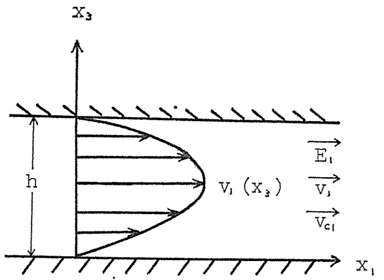

Fig. 2. The profile of the flow velocity v_1 (x_3), which implies the modified Poiseuille flow.

From Eq. (2), we get, assuming collisions dominant,

$$0 \gtrsim -\frac{\partial p}{\partial x_1} + \eta \frac{\partial^2 v_1}{\partial x_3^2} - m m_c n n_c (v_1 - v_{c1}). \tag{20}$$

The equation (3) becomes

$$n_c v_{c1} = j(x_3)/e. \tag{21}$$

Assuming collisions dominant in Eq. (4) we have

$$0 \simeq en_c E_1 - \frac{\partial p_c}{\partial x_1} + \eta_c \frac{\partial^2 x_{c1}}{\partial x_3^2} - mm_c nn_c (v_{c1} - v_1), \tag{22}$$

in which if the third term in the right hand side is negligible small compared with the other term, $p_c \simeq n_c k T_c$ and T_c constant, then we have

$$v_{c1} = v_1 + b\left(E_1 - \frac{kT_c}{en_c} \frac{\partial n_c}{\partial x_1}\right) \tag{23}$$

where

$$b = \frac{b_0}{n} = \frac{e}{mm_c n\alpha} \tag{24}$$

is the mobility and its value are $10^{-4} \sim 10^{-7}$ cm^2/sec.V for oil.[25),26)] Combining Eq. (20) and Eq. (23), we have

$$en_c E_1 - \frac{\partial p}{\partial x_1} - \frac{\partial p_c}{\partial x_1} + \eta \frac{\partial^2 v_1}{\partial x_3^2} \simeq 0, \tag{25}$$

which, using Poisson's equation, is arranged as

$$\eta \frac{\partial^2 v_1}{\partial x_3^2} = \frac{\partial}{\partial x}\left(p + p_c - \frac{\varepsilon E_1^2}{2}\right) = \text{constant} = -C, \tag{26}$$

That equation is as same as the poiseuille flow by putting $p + p_c - \varepsilon E^2/2$ into p.[27)] The solution of Eq. (26) is

$$v_1 = \frac{C}{\eta}(h - x_3) x_3, \tag{27}$$

under the conditions that $v_1 = 0$ at $x_3 = 0$ and $x_3 = h$. And

$$Cx_1 = \left(p_0 + p_{c0} - \frac{\varepsilon E_{10}^2}{2}\right) - \left(p + p_c - \frac{\varepsilon E_1^2}{2}\right), \tag{28}$$

where the subscript 0 means the quantities at $x_1 = 0$.

Now the current density is given by the following equations,

$$j_1(x_3) = en_c v_{c1} = \varepsilon \frac{\partial E_1}{\partial x_1} \left\{ v_1 + b\left(E_1 - \frac{kT_c}{en_c} \frac{\partial n_c}{\partial x_1}\right)\right\}. \tag{29}$$

That equation can be easily integrated if we neglect the diffusion term, which leads to the second order algebraic equation on E_1. Thus, we have

$$E_1 = -\frac{v_1}{b} + \left\{\left(E_{10} + \frac{v^1}{b}\right)^2 + \frac{2j_1 x_1}{b\varepsilon}\right\}^{1/2}. \tag{30}$$

In that equation, $j_1 \geqq 0$ and $x_1 \geqq 0$ so that if $E_{10} \geqq 0$ then $E_1 \geqq 0$ for all x_1 as it was shown in Fig. 3(a). Under that condition, the ion drag pump will be much likely derived. If, however, $E_{10} < 0$ then E_1 becomes partly positive and partly negative as shown in Fig. 3(b). When E_{10} is furthrmore negative, then E_1 is negative for all x_1 as it was shown in Fig. 3(c). In that case, the ion transport generator will be, under better condition, operated. The case of Fig. 3(b) is not in a good condi-

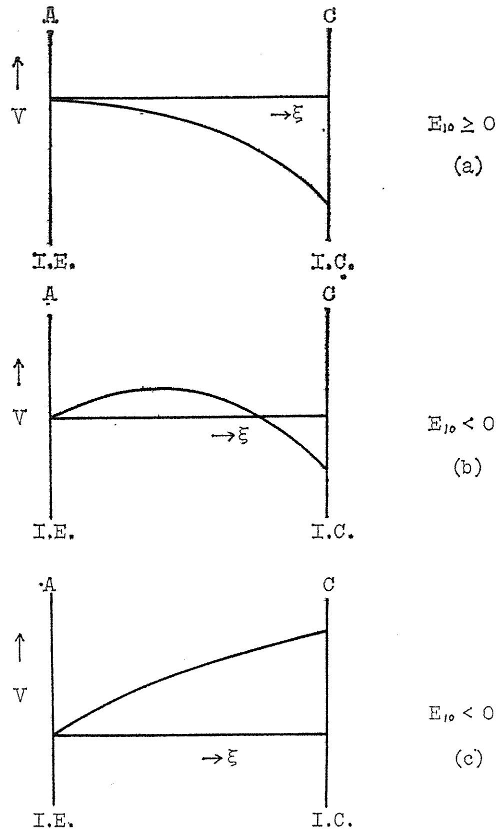

Fig. 3. The profe of electric potential, where A is the anode, C the cathode, I. E. the ion emitter and I. C. the ion collector.

tion for the ion drag pump and also for the ion transport generator. As it was stated above, wheather E_{10} is negative and positive and wheather E_{10} is large or small are play an important role upon $E_1(x_1)$ or upon the profile of electric potential. But it is a quiet bad that E_{10} can not be determined theoretically.

We should, however, rememedr that in the above disussion we have neglected the diffusion term in Eq. (30). Indeed, if we set $v_1=0$ and if we calculate the space charge relation including the diffusion term then E_{10} can be determined[3]. In the next section, we shall develope the calculation of space charge relation including the diffusion term. It is noted that electrical potential is easily obtained by an integration, but that calculation was already carried out[19)~21)], so that we are no more any discussion in this section.

5. The effect of the diffusion term upon the space charge relation

Combining Eq. (29) and the Poisson equation, the electric field E_1 and the electric potential V is easily obtained as follows: we introduce the iollowing new notations,

$$\zeta=\frac{ex_0}{kT_c}E_1\quad \zeta_0=\frac{ex_0}{kT_c}E_{10}\quad \zeta=\frac{x}{x_0},\quad \beta=\frac{x_0ev_1}{bkT_c}$$

$$i=\frac{j}{j_0},\quad j_0=\frac{en_{c0}bkT_c}{ex_0},\quad \gamma=\frac{1-(\zeta_0+\beta)^2}{i},\quad y=\left(\frac{i}{4}\right)^{1/3}(\gamma-\zeta),\tag{31}$$

$$\zeta=-2\frac{d}{d\xi}ln\phi-\beta=-\frac{dW}{d\xi}$$

then we have

$$\frac{d^2\phi}{dy^2}+y\phi=0\tag{32}$$

whose solution is

$$\phi(z)=A_1z^{1/3}J_{1/3}(z)+B_1z^{1/3}J_{-1/3}(z),\ \text{for}\ y>0,$$

and

$$\phi(z^*)=A_2z^{*1/3}I_{1/3}(z^*)+B_2z^{*1/3}I_{-1/3}(z^*),\ \text{for}\ y<0\tag{33}$$

where

$$z=(2/3)y^{3/2}$$

$$z^*=(2/3)(-y)^{3/2}\tag{34}$$

From Eq. (33), $\zeta+\beta$ is

$$\zeta(z)+\beta=\frac{(3i)^{1/3}}{\phi(z)}\{A_1z^{2/3}J_{-2/3}(z)-B^1z^{2/3}J_{2/3}(z)\},\ \text{for}\ y>0,$$

and

$$\zeta(z^*)+\beta=\frac{(3i)^{1/3}}{\phi(z^*)}\{A_2z^{*2/3}I_{-2/3}+B_2z^{*2/3}I_{2/3}(z^*)\},\ \text{for}\ y<0.\tag{35}$$

The quantities A_1, A_2, B_1, and B_2 are determined by the following conditions;

(1) ϕ and $\zeta+\beta$ are continuous at $z=z^*=0$,

(2) when $z^*\to\infty$, then the solution terminate to the solution neglecting the diffusion term, namely

$$\zeta(z^*)+\beta\ \to\ (3i)^{1/3}z^{*1/3}=\{i\xi+(\zeta_0+\beta)^2-1\}^{1/2}$$

(3) $\phi=1$ $(W=0)$, $\zeta+\beta=\zeta_0+\beta$ at $\xi=0$.

The conditions (1) and (2) lead to

$$A_1=B_1,\ B_1=B_2\ \text{and}\ A_1=-A_2.$$

Thus,

for $y>0$,

$$\phi(z)=A_1R(z),$$

$$\zeta(z)+\beta=\frac{(3i)^{1/3}}{\phi(z)}A_1s(z),\tag{36}$$

and for $y<0$

$$\phi(z^*)=A_1R(z^*),$$
$$\zeta(z^*)+\beta=\frac{(3i)^{1/3}}{\phi(z^*)}A_1S(z^*). \tag{37}$$

Here

$$R(z)=z^{1/3}(J_{1/3}(z)+J_{-1/3}(z)),$$
$$R(z^*)=z^{*1/3}(I_{-1/3}(z^*)-I_{1/3}(z^*)), \tag{38}$$

$$S(z)=z^{2/3}(J_{-2/3}(z)-J_{2/3}(z)),$$
$$S(z^*)=z^{*2/3}(I_{-2/3}(z^*)-I_{2/3}(z^*)). \tag{39}$$

To determine A_1 and $\zeta_0+\beta$, we put

$$z_0=\frac{1}{3}i^{1/2}\gamma^{3/2},$$
$$z_0^*=\frac{1}{3}=i^{1/2}(-r)^{3/2}, \tag{40}$$

and we apply to the third condition. Then we can determine A_1 and $\zeta_0+\beta$ as a funetion of z_0 or z_0^*; in the other words, we can determine A_1 and $\zeta_0+\beta$ as a function of i, as it will be described in Appendix.

The function $R(z)$ or $R(z^*)$ is a scale of electric potential and is shown in Fig. 4. That R is zero at $z=2.381$, $R=1.404$ at $z=0.68555$ and approches to zero with the increasing z^*. Fig. 5 shows the function S and S is a scale of electric field. The function S is zero at $z=0.6855$, is in maximum (the value 0.5925) and terminate to zero with the increasing z^*. Both Fig. 4 and 5 can use for z_0 or z_0^*.

From Appendix, the relation between A_1 and i in Fig.6. The value of A_1 has a minimam ($A_1=0.7123$) at $i=0.4863$ and, for $i\lesssim 10^{-2}$, it is hold

$$A_1=0.4320\, i^{-1/3}, \tag{41}$$

whose relation shows the straight line AA' in Fig.6. Now, the relation between $\zeta_0+\beta$ and i is shown in Fig. 7. If $i=0$ then $\zeta_0+\beta=-1$, if $i=0.4863$ then $\zeta_0+\beta=0$ and for the large i (say $i\lesssim 4$) it is valid

$$\zeta_0+\beta\simeq i. \tag{42}$$

Furthermore, $|\gamma|$ is shwn in Fig. 8 as a function of i. If $i<1.291$ then γ ispositive, if $i=1.291$ then $\gamma=0$ and $i>1.291$ then γ is negative. Especially $i\leqq 10^{-2}$ it gives

$$\gamma=3.709\, i^{-1/3} \tag{43}$$

whose relation is shown the straight line AA' in Fig. 8. If the approximate relation (42) is valid then the following equation can be used,

$$-\gamma\simeq i. \tag{44}$$

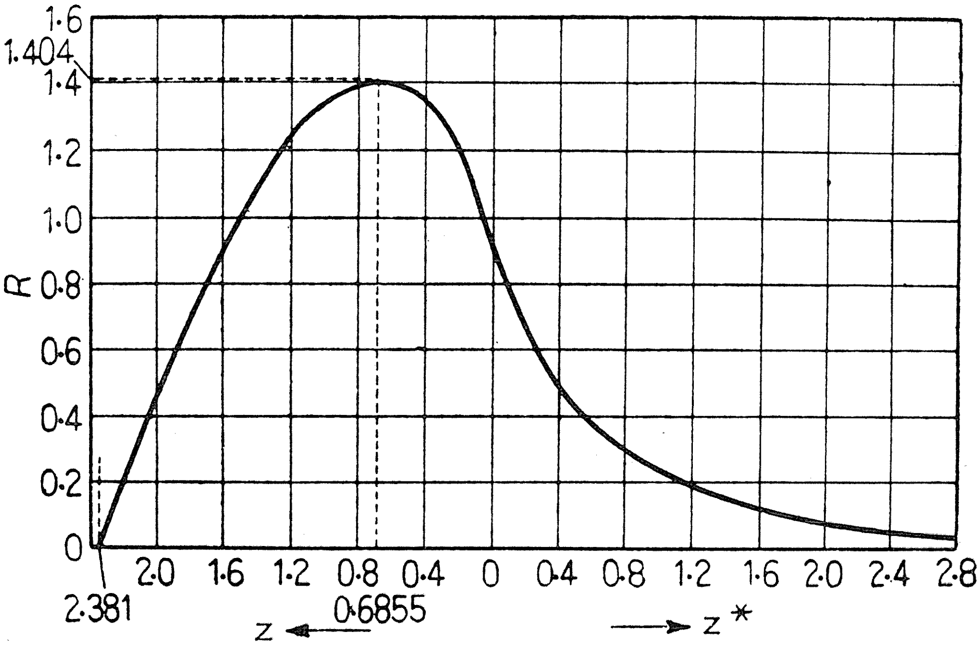

Fig. 4. The curve of $R(z)$ or $R(z^*)$.

The quantities R, S, i, $\zeta_0+\beta$ and γ are tabulated in Table 1. (a) or (b) as a function of z_0 or z_0^*. While, if z_0 and z_0^* are equal to or larger than 4 then the effect diffusion term will be able to neglect: namely $R(z^*)$ and $R(z_0^*)$ are given by the following equations,

$$R(z^*)\simeq\left(\frac{3}{2\pi z^*}\right)^{1/2}\exp(-z^*),$$
$$R(z_0^*)\simeq\left(\frac{3}{2\pi z_0^*}\right)^{1/2}\exp(-z_0^*),\tag{45}$$

approximately. Thus, we have

$$W=2\,ln\phi+\beta\simeq 2ln\frac{R(z^*)}{R(z_0^*)}+\beta\xi$$
$$=\frac{2}{3}i^{1/2}\{(-\gamma)^{3/2}-(\xi-\gamma)^{3/2}\}+\frac{3}{2}ln\frac{(-\gamma)}{(\xi-\gamma)}+\beta\xi$$

which is, using $-\gamma\simeq i$ for $i\gtrsim 4$, rewritten as

$$W=\frac{2}{3}i^{1/2}\{i^{3/2}-(\xi+i)^{3/2}\}+\frac{3}{2}ln\frac{i}{(\xi+i)}+\beta,\xi\tag{46}$$

That equation becomes, for $\xi\gg i$,

$$W\simeq-\frac{2}{3}i^{1/2}\xi^{3/2}+\beta\xi,\tag{47}$$

approximataly.

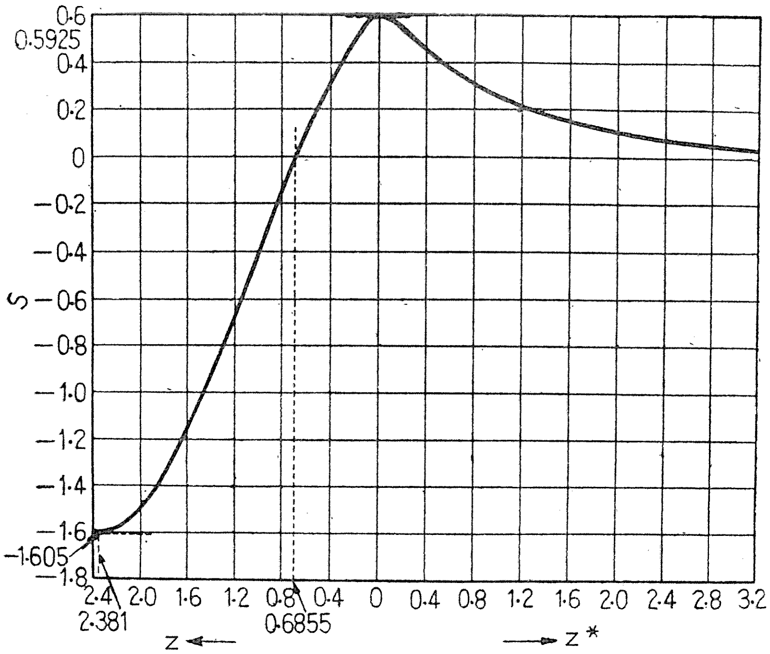

Fig. 5. The curve of $S(z)$ or $S(z^*)$.

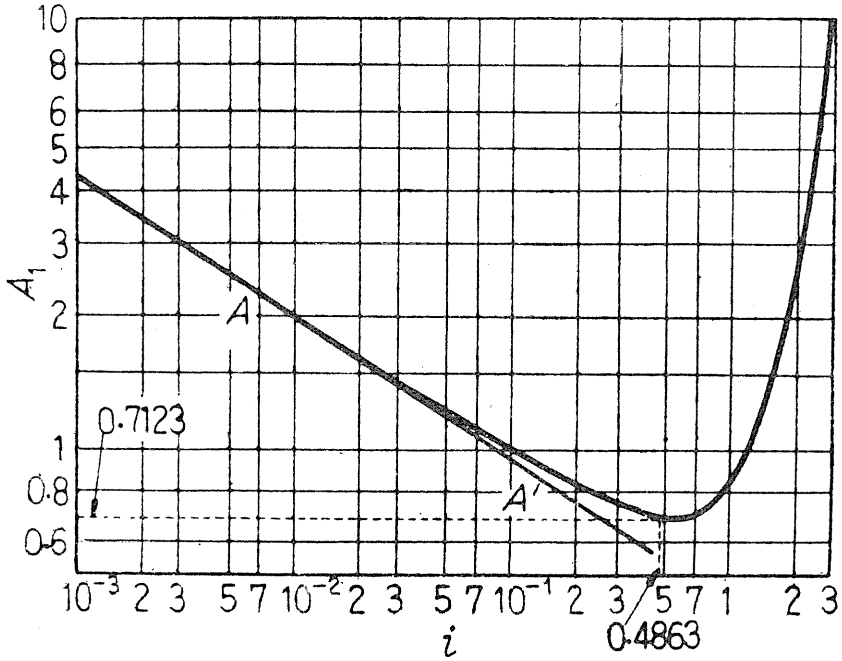

Fig. 6. The relation between A_1 and i, where the straight line AA' shows $A_1=0.4320i$

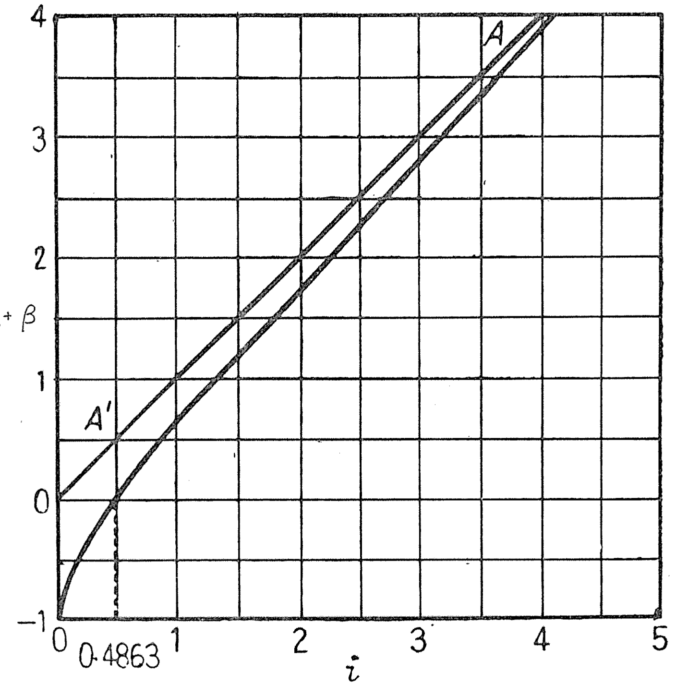

Fig. 7. The relation between $\zeta_0+\beta$ and i, where the straight line AA' shows $\zeta_0+\beta=i$

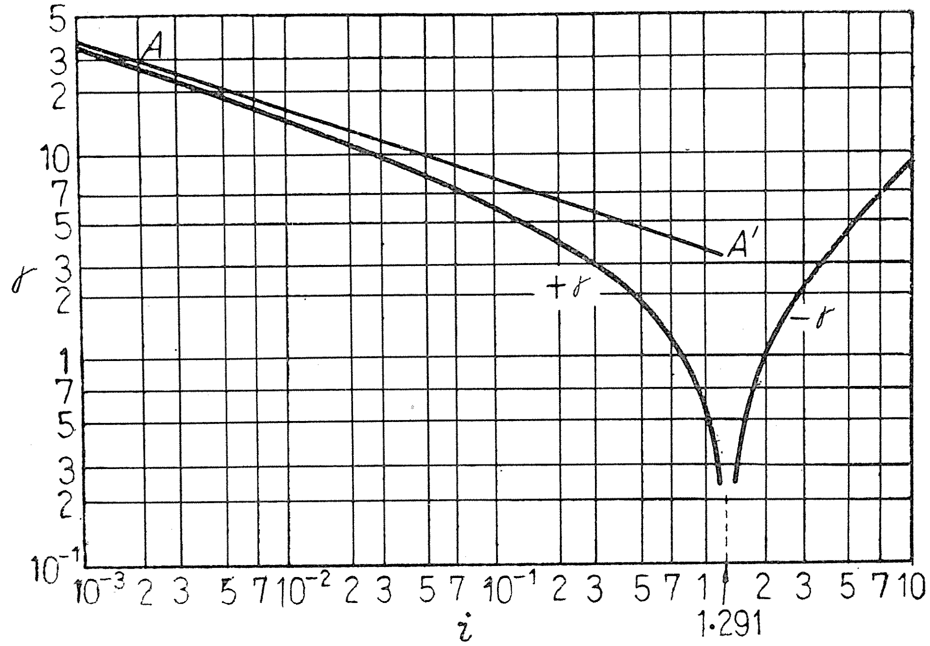

Fig. 8. The relation bet ween γ and i. The straight line AA' shown $\gamma=3.709i^{-1/3}$ and one should take $-\gamma$ for $i>1.291$ and $+\gamma$ for $i<1.291$.

6. A left problem

As was mentioned aboye, the uncharged particle fluid follows the Poiseuille flow by putting $p+p_c-\varepsilon E/2$ in p, and $v_1=0$ at the wall; *i. e.*, on the one hand, v_1 is a function of the coordinate x_3 perpendicular to the flow. On the other hand, the electrcal potential V and the intensity of electric E_1 are independent on x_3. So, a pionering reseach coworker. O. M. Stuetzer, takes the average value for V_1 and thereby the calculated equation is not exact.

If one does not take the diffusion term, then Eq. (30) changes as

$$\left\{\left(E_1+\frac{v_1}{b}\right)^2-\left(E_{10}+\frac{v_1}{b}\right)^2\right\}=\frac{2j_1x_1}{b\varepsilon}, \tag{48}$$

so that, by the integration from $x_3=0$ to $x_3=h$, we get

$$(E_1-E_{10})\left\{(E_1+E_{10})h+\frac{Ch^3}{3\eta b}\right\}=\frac{Ix_1}{b\varepsilon},$$

where $I=\int_0^h j_1(x_3)dx_3$ is the total current. That equation is rewritten as

$$E_1=-\frac{Ch^2}{6b\eta}+\left\{\left(E_{10}+\frac{Ch^2}{6b\eta}\right)^2+\frac{2Ix_1}{\varepsilon bh}\right\}^{1/2}, \tag{49}$$

from which the electrical potential become

$$V=\frac{Ch^2}{6b\eta}x-\frac{2}{3}\cdot\frac{\varepsilon bh}{2I}\left\{\left(E_{10}+\frac{Ch^2}{6b\eta}\right)^2+\frac{2Ix_1}{\varepsilon bh}\right\}^{3/2}$$
$$+\frac{2}{3}\cdot\frac{\varepsilon bh}{2I}\left(E_{10}+\frac{Ch^2}{6b^3}\right)^3.$$

If one takes the diffusion effect into acount then the results will be much complicated, so that the problem will be left as it will be solved in the future.

7. A numerical example

We have treated the space charge charge relation by taking into account the diffusion effect in the section 5. As a numerical example of the present theory, we put

$$n_{c0}\simeq 10^{16}/\mathrm{m}^3,\quad T_c\simeq 500°\mathrm{K},\quad b\simeq 10^{-8}\mathrm{m}^2/\mathrm{sec}\cdot\mathrm{V},$$
$$v_1\simeq 10^{-1}\mathrm{m/sec.},\quad d\simeq 1\mathrm{m},$$

where d is the anode-cathode distance. Then

$$x_0\simeq 10^{-5},\quad \beta\simeq 2\times 10^3,\quad j_0\simeq 7\times 10^{-8}A/\mathrm{m}^2$$
$$\xi_d=d/x_0\simeq 10^5,\quad \beta\xi_d\simeq 2\times 10^8,$$

since $+\beta \simeq i$ if $i \gtrsim 4$, thereby if $i \gtrsim 2\times10^3$ then $\zeta_0 > 0$ and $\zeta > 0$ for all x_1. In the other words, when $i \gtrsim 2\times10^3$ then the ion drag pump is likely derived when, however, $i < 2\times10^3$ then at the near anode (orat the ion emitter) $\zeta < 0$ and thus the electrical potential begins to becomes positive with the decreasing i. In Table 1, we have shown a numerical example with the above data.

Table 1. An numerical example of γ, $\zeta_0+\beta$ and V where $\beta=2\times10^3$

i	γ	$\zeta_0+\beta$	V (Volt)
10^6	-10^6	10^6	-4.52×10^9
10^5	-10^5	10^5	-5.42×10^8
10^4	-10^4	10^4	-9.81×10^7
10^3	-10^3	10^3	-2.07×10^7
10^2	-10^2	10^2	-7.23×10^5
10	-10	10	5.01×10^6
1	0.60	0.73	8.09×10^6
10^{-1}	5.7	-0.64	8.5×10^6
10^{-2}	17.2	-0.91	8.5×10^6
10^{-3}	37.1	-0.97	8.5×10^6

8. Conclusion

We have here developed the electrohydrodynamics to describe the ion drag pump and the ion transport generator. That dynamics is consisted with the equation of continuity and the momentum equation for an uncharged particle and for a charged particle fluid and furthermore tԛe Maxwell equations.

With these fundamental equation, we have given a solution of electrohydrostatic equilibrium for one-dimensional parallel plane geometry. Next, we have treated the one-dimensional flow when the uncharged particle fluid is incompressible taking into accound the diffusion effect for the charged particle fluid. Because if we neglect the diffusion effect, one can not theoretically determine the strength of electric field at the ion emitter. The results of the present theory shows that the ion drag pump is likely operated for the higher electric current whereas for the smaller current the higher electric potential is generated leading to the ion transport generator. It is noted that the Poiseuille flow will be kept unchanged if we put $p-\varepsilon E^2/2$ into p (where $\varepsilon E^2/2$ expresees the electrical stress).

The auths wish to appreciate to Prof. Minoru Ueda for his interest in this paper.

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Appendix

Two quantities A_1 and $\zeta_0+\beta$ cav be determined as a function of i by making use of $z=1$ and $\zeta+\beta=\zeta+\beta$ at $z=z_0$ or $z^*=z_0^*$. Namely

$$R(z_0)=A_1^{-1},\qquad R(z_0^*)=A_1^{-1}, \tag{A1}$$

$$A_1S(z_0)=(\zeta_0+\beta)/(3i)^{1/3},\qquad A_1S(z_0^*)=(\zeta_0+\beta)/(3i)^{1/3}. \tag{A2}$$

We define

$$f=\frac{\zeta_0+\beta}{\{1-(\zeta_0+\beta)^2\}^{1/2}}=\frac{A_1S(z_0)}{z},\qquad g=\frac{\zeta_0+\beta}{\{(\zeta_0+\beta)^2-1\}^{1/2}}=\frac{A_1S(z_0^*)}{z} \tag{A3}$$

from which we have

$$\zeta_0+\beta=f/(1+f^2)^{1/2},$$
$$\zeta_0+\beta=g/(g^2-1)^{1/2}, \tag{A4}$$

Then, γ and i are also expressed as a function of z_0 or z_0^* by

$$i=1/3z_0(1+f^2)^{3/2}$$
$$=1/3z_0^*(g^2-1)^{3/2} \tag{A5}$$

$$\gamma=3z_0(1+f^2)^{1/2}$$
$$=-3z_0^*(g^2-1)^{1/2} \tag{A6}$$

Thus A_1, $\zeta_0+\beta$, i and γ can be calculated as a function of z_0 or z_0^* from (A1), (A4), (A5) and (A6) respectively. The calculated results have been tabulated in Table 2.

Table 2(a) The values of R, S, i, $\zeta_0+\beta$ and γ as a function of Z

Z	R	S	i	$\zeta_0+\beta$	γ
2.5	−0.1459	−1.593	—	—	—
2.4	−0.0236	−1.605	—	—	—
2.3	+0.0881	−1.600	5.523×10^{-5}	−1.000	9.518×10
2.2	0.2210	−1.580	8.163×10^{-4}	−0.9845	$3.765\times$ ·0
2.1	0.3435	−1.543	3.286×10^{-3}	−0.9633	2.295×10
2.0	0.4624	−1.492	8.206×10^{-3}	−0.9315	1.612×10
1.9	0.5812	−1.427	1.602×10^{-2}	−0.8929	1.266×10
1.8	0.6924	−1.346	2.761×10^{-2}	−0.8474	1.018×10
1.7	0.8001	−1.257	4.342×10^{-2}	−0.7962	8.430
1.6	0.9037	−1.158	6.389×10^{-2}	−0.7389	7.387
1.5	0.9983	−1.048	9.095×10^{-2}	−0.6811	6.061
1.4	1.086	−0.9298	1.193×10^{-1}	−0.6076	5.288
1.3	1.164	−0.8033	1.546×10^{-1}	−0.5346	4.619
1.2	1.233	−0.6749	1.952×10^{-1}	−0.4577	4.049
1.1	1.291	−0.5422	2.408×10^{-1}	−0.3771	3.563
1.0	1.338	−0.4094	2.914×10^{-1}	−0.2927	3.138
0.9	1.373	−0.2762	3.475×10^{-1}	−0.2040	2.758
0.8	1.395	−0.1451	4.099×10^{-1}	−0.1114	2.410
0.7	1.404	−0.0179	4.762×10^{-1}	−0.001436	2.100
0.6	1.399	0.1036	5.499×10^{-1}	0.08750	1.806
0.5	1.379	0.2196	6.284×10^{-1}	0.1968	1.530
0.4	1.344	0.3218	7.168×10^{-1}	0.3092	1.261
0.3	1.291	0.4147	8.142×10^{-1}	0.4327	0.9983
0.2	1.219	0.4937	9.258×10^{-1}	0.5694	0.7299
0.1	1.118	0.5558	1.061	0.7306	0.3855
0	0.9306	0.5925	1.2914	1.000	0.000

Table 2(b). The values of R, S, i, $\zeta_0+\beta$ and γ as a function of Z^*.

Z^*	R	S	i	$\zeta_0+\beta$	γ
0	0.9306	0.5925	1.2914	1.000	0.000
0.1	0.7424	—	1.522	—	0.3896
0.2	0.6387	0.5283	1.667	1.414	0.6000
0.3	0.5571	—	—	—	—
0.4	0.4900	0.4541	1.877	1.649	0.9155
0.5	0.4331	—	1.868	—	—
0.6	0.3839	0.3855	2.074	1.846	1.160
0.8	0.3038	0.3250	2.210	2.010	1.376
1.0	0.2422	0.2728	2.388	2.172	1.556
1.2	0.1937	0.2283	2.507	2.304	1.729
1.4	0.1556	0.1906	2.677	2.453	1.875
1.6	0.1250	0.1587	2.764	2.570	2.028
1.8	0.1009	0.1322	2.887	2.691	2.162
2.0	0.08227	0.1100	—	—	—
2.2	0.06495	0.09154	—	—	—
2.4	0.05048	0.07611	—	—	—
2.6	0.03601	0.06347	—	—	—
2.8	—	0.05289	—	—	—
3.0	—	0.04530	—	—	—