THE RELATION BETWEEN TURNING ANGLE AND AXIAL VELOCITY RATIO OF SOLID WALL COMPRESSOR CASCADES*

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Summary

The fact, that there exists a relationship between the turning angle and the axial velocity ratio which depends only on the inflow angle, inflow boundary layer thickness and aspect ratio, but is independent of the pitch chord ratio and stagger angle on the solid wall compressor cascade experiment, was clarified, and an experimental equation was derived from experimental data.

1. Introduction

One of the authors has once found that there exists a relationship between the turning angle ε and the axial velocity ratio w_{a2}/w_{a1}, which depends only on the inflow angle γ_1 but is independent of the pitch chord ratio a/c and stagger angle α, on the solid wall compressor cascade experiment.[1] Fig. 1 is the reproduction of the result. A similar result was also published by Inoue[2] which is reproduced in Fig. 2.

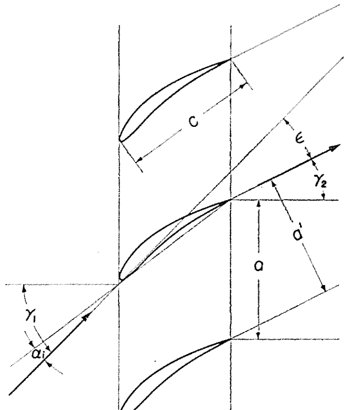

Fig. S-1 Definition of symbols.

These two are different not only in their values of w_{a2}/w_{a1} but also in qualitative sense, curves of the latter are straight and pass through the origin, but curves of the former are not straight and do not pass through the origen. Thinking about experimental errors we can put curves in Fig. 1 to be straight and pass through the origin. and let us, therefore, consider so.

The feature common to the both is that, taking γ_1 as a parameter, w_{a2}/w_{a1} is the function of ε alone and may not be the function of a/c. At the same time, values of w_{a2}/w_{a1} are different at both cases even in the same γ_1 and ε. This is

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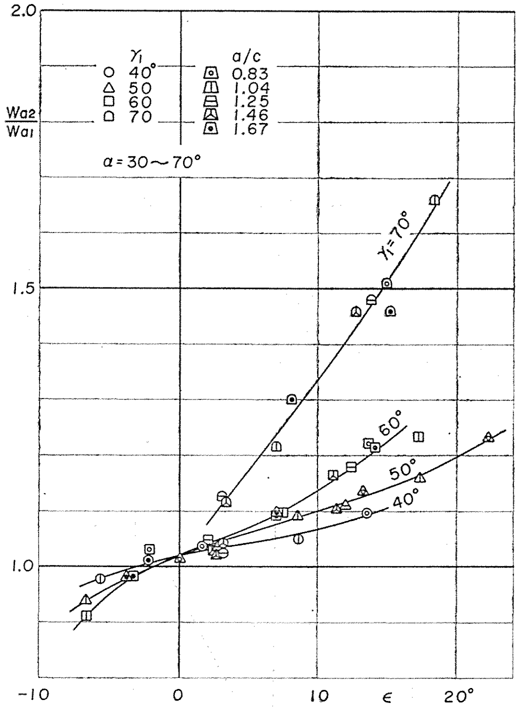


Fig. 1. Axial velocity ratio and turning angle relation by Otsuka(1)

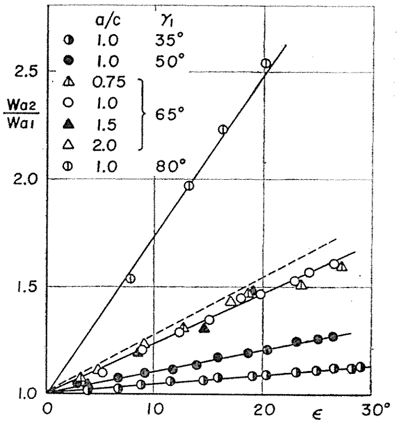


Fig. 2. Axial velocity ratio and turning angle relation by Inoue.(2)

thought to be effects of inflow boundary layer and aspect ratio as explained later.

The aim of this report is to clarify these situations and make an experimental formula. Although the relatioship of turning angle and axial velocity ratio can be anticipated from the secondary flow theory (but not sufficiently because of the imperfectness of the theory), the problem was investigated in this report only from an empirical view point.

2. Symbols

AR	aspect ratio
a	blade pitch
B	a half of blade span
c	blade chord
f_A	[see equation (1) or (1′)]
f_Δ	[see equation (1) or (1′)]
g	[see equation (1) or (1′)]
w_a	axial velocity
α	stagger angle (degree)
α_i	incidence angle [see Fig. S-1] (degree)
γ_1	inlet flow angle [see Fig. S-1] (degree)
δ^*	inlet boundary layer displacement thickness
ε	turning angle (degree)

2. 1 *Subscript*

1 before cascade
2 behind cascade

3. Procedures, Considerations and Results

As mentioned in the introduction, w_{a2}/w_{a1} is affected by not only γ_1 and ε but also inlet boundary layer thickness (which is expressed by displacement thickness δ^*) and aspect ratio. The effect of Reynolds Number is also to be considered, but there exists an opinion that we can consider turning angle and axial velocity ratio being constant above some definite Reynolds Number, and we follow this opinion.[5] All data used in this report belong in this range, and were taken from references (1), (3), (4) and (5).

3. 1 *The Effect of Pitch Chord Ratio*

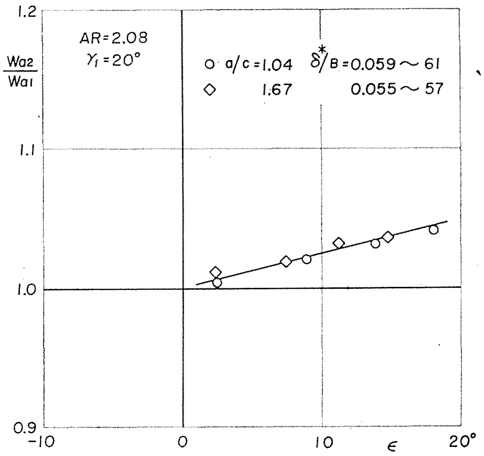


Fig. 3-a. Effect of pitch chord ratio.

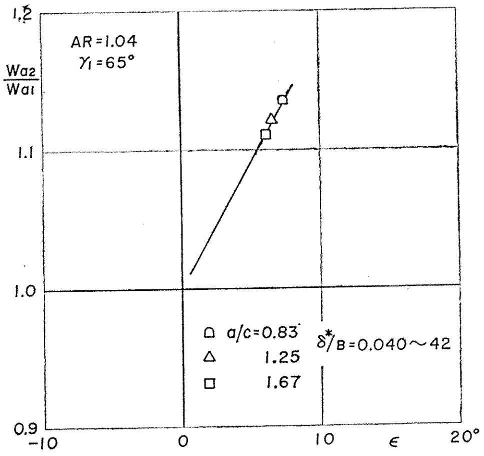
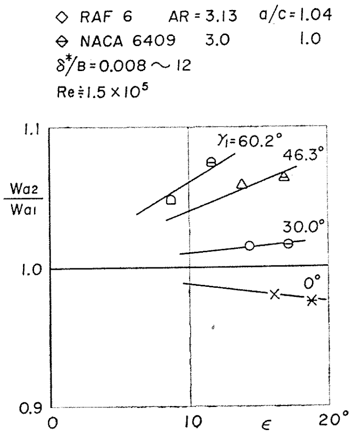


Fig. 3-b. Effect of Pitch chord ratio.

The effect of pitch chord ratio seems to be very small. Examples are shown in Fig. 3-a and b. Although the effect of δ^*/B is not sufficiently removed in Fig. 1 and Fig. 2, the fact that the effect of a/c is small can be recognized pretty clearly.

3. 2 *The Difference of Blade Profiles*

An example of the result of measurements under conditions other than blade profiles (which are RAF 6 and NACA 6409) being practically identical is shown in Fig. 4. There exists no problem on the difference of blade profiles.



Fig, 4. Effect of blade profile.

3. 3 *Making an Experimental Formula*

We have noted in the above that there exists a relation between the turning angle ε and the axial velocity ratio w_{a2}/w_{a1}, which can be put on one straight line despite of the pitch chord ratio a/c and blade profile being different, provided the inflow angle γ_1 is settled. However, the slope of this straight line changes with changes of the inlet boundary layer thickness δ^*/B and the aspect ratio AR. Authors, therefore, tried to deal with these relations by assuming an equation shown below.

$$w_{a2}/w_{a1}=g(\gamma_1,\ \varepsilon)+\varepsilon[f_\Delta(\gamma_1,\ \delta^*/B)+f_A(\gamma_1,\ AR)] \tag{1}$$

Where, $g(\gamma_1,\ \varepsilon)$ is w_{a2}/w_{a1} characteristic of a cascade having definite δ^*/B and AR which will be decided later to be 0.04 and 2.0 respectively.

Since the experiments done in our laboraory contain very many ones with RAF 6 profile and $AR=2.08$, we decided at first to make $g\ (\gamma_1,\ \varepsilon)$ from data having these conditions. However, δ^*/B showed pretty large scatter and could not select a definite value. An example of values of δ^*/B is shown in Fig. 5, and roughly speaking we could think them being the function of γ_1. As a first step to get g, therefore, we selected data contained in the range shown by shadowed portion in the figure or by the next equation,

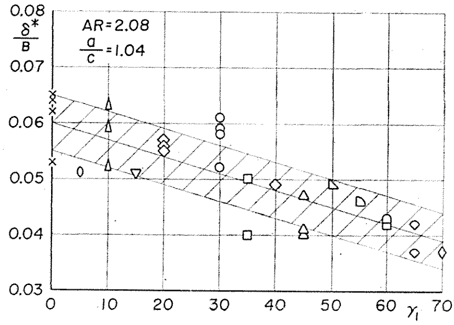


Fig. 5. Condition of boundary layer.

$$\delta^*/B=-0.0003\gamma_1+0.060(\pm 0.005) \tag{2}$$

Then the equation (1) must be modified to

$$w_{a2}/w_{a1}=\bar{g}(\gamma_1,\ \varepsilon,\ \delta^*/B)+\varepsilon[\bar{f}_\Delta(\gamma_1,\ \delta^*/B)+\bar{f}_A(\gamma_1, AR)] \tag{1'}$$

We can easily recover $\bar{g}$ into g, if we can clarify the effect of δ^*/B.

Curves of $\bar{g}$ are shown in Fig. 6. Although there exists scattering of points, straight lines in the figure are expressed by the following equation,

$$\bar{g}(\gamma_1,\ \varepsilon, \delta^*/B)=1+\varepsilon\left[0.007-\frac{45-\gamma_1}{90-\gamma_1}\ \frac{1}{60+0.4(45-\gamma_1)}\right] \tag{3}$$

This equation was assumed to give the regularity to straight lines and has no special meaning.

3. 4 The Value of $\bar{f}_\Delta$

For the effect of boundary layer thickness (displacement thickness), we can get $\bar{f}_\Delta\ (\gamma_1,\ \delta^*/B)$ from the difference of data, which have $AR=2.08$ and have δ^*/B different from equation (2), and $\bar{g}\ (\gamma_1,\ \varepsilon)$ shown in Fig. 6. The plot of results for a few γ_1 is shown in Fig. 7. Straight lines in the figure are relations of $\bar{f}_\Delta \sim \delta^*/B$ being thought to be suitable for each γ_1. These straight lines must take the value 0 at δ^*/B indicated in Fig. 5. Slopes of straight lines are given by the following equation to have regularity.

$$d\bar{f}_\Delta/d(\delta^*/B)=0.0375+5000/(90-\gamma_1)^3 \tag{4}$$

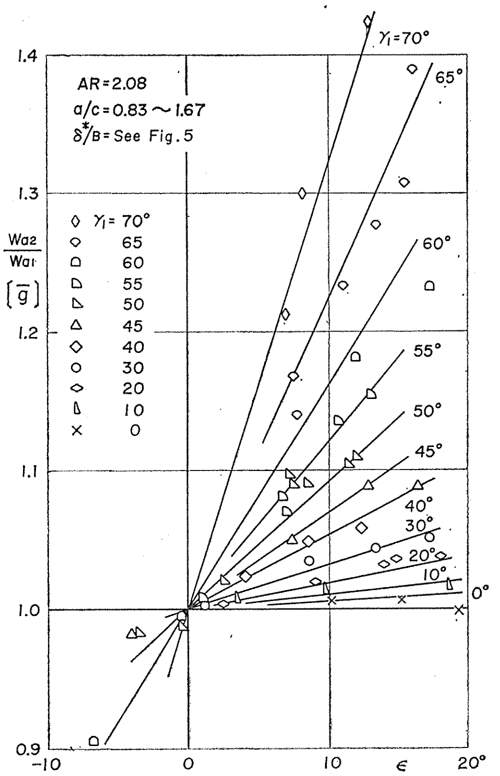


Fig. 6. $\bar{g}$ curve.

As aforesaid we want to take g for a specific δ^*/B, and this is possible if we modify $\bar{g}$ into g by using Fig. 7. we employed 0.04 as the specific value of δ^*/B. The results will be shown later.

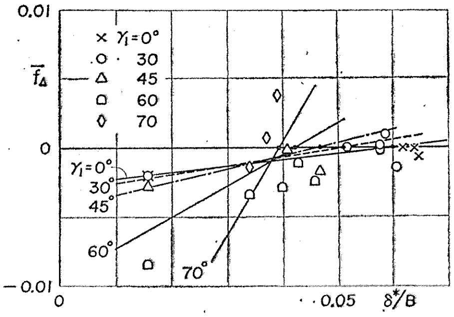


Fig. 7. $\bar{f}_A$ curve.

3. 5 *The value of* $\bar{f}_A$

For the effect of aspect ratio, we can get $\bar{f}_A$ (γ_1, AR) from the difference of data, which satisfy equation (2) among many data having various AR, and $\bar{g}$ (γ_1, ε) shown in Fig. 6. The plot of results for a few γ_1 is shown in Fig. 8. Straight lines in the figure are relations of $\bar{f}_A \sim 1/AR$ being thought to be suitable for each γ_1. These straight lines are given by the following equation to have regularity.

$$\bar{f}_A=\frac{1}{1000}\left(\frac{2.08}{AR}-1\right)(0.07\gamma_1-2) \qquad (5)$$

Althought we used values in the case of AR=2.08 for the curve of $\bar{g}$, the curve with respect to AR=2.0 may be more convenient. Modification will be done by using Fig. 8, and the result will be shown later.

3. 6 *The Decision of Experimental Formula*

The modification of equation (1′) into (1) is easy, provided the

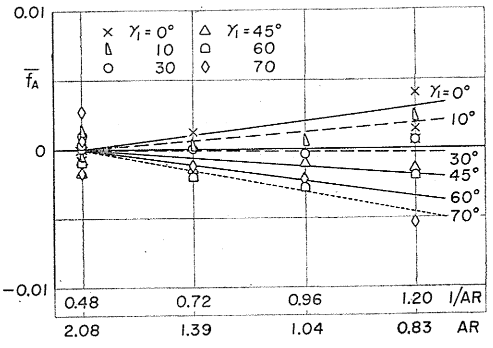


Fig. 8. $\bar{f}_A$ cucrve.

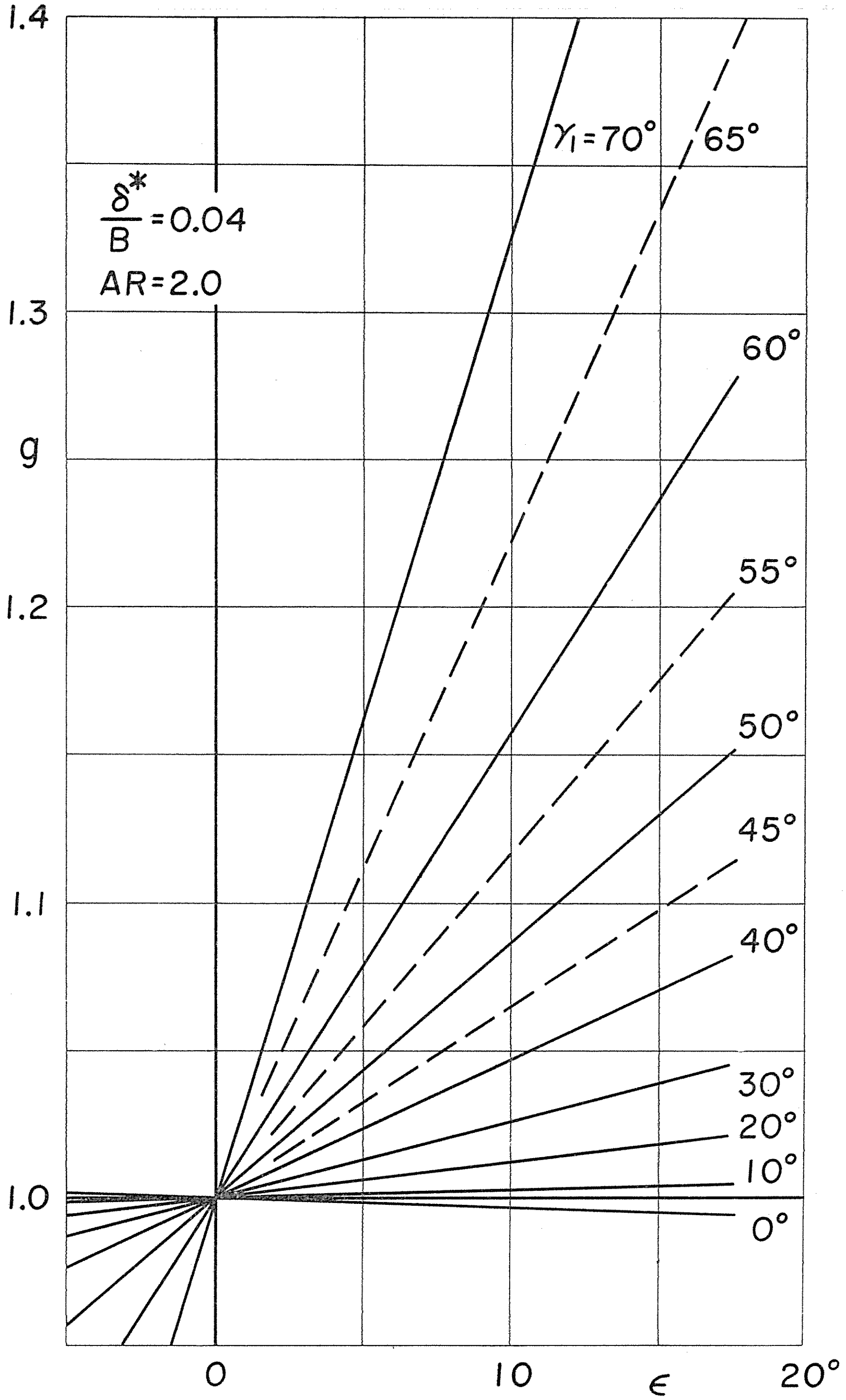

1.4
1.3
g
1.2
1.1
1.0
$\frac{\delta^*}{B}=0.04$
AR=2.0
$\gamma_1=70°$
65°
60°
55°
50°
45°
40°
30°
20°
10°
0°
0
10
ϵ
20°

Fig. 9. *g* curve.

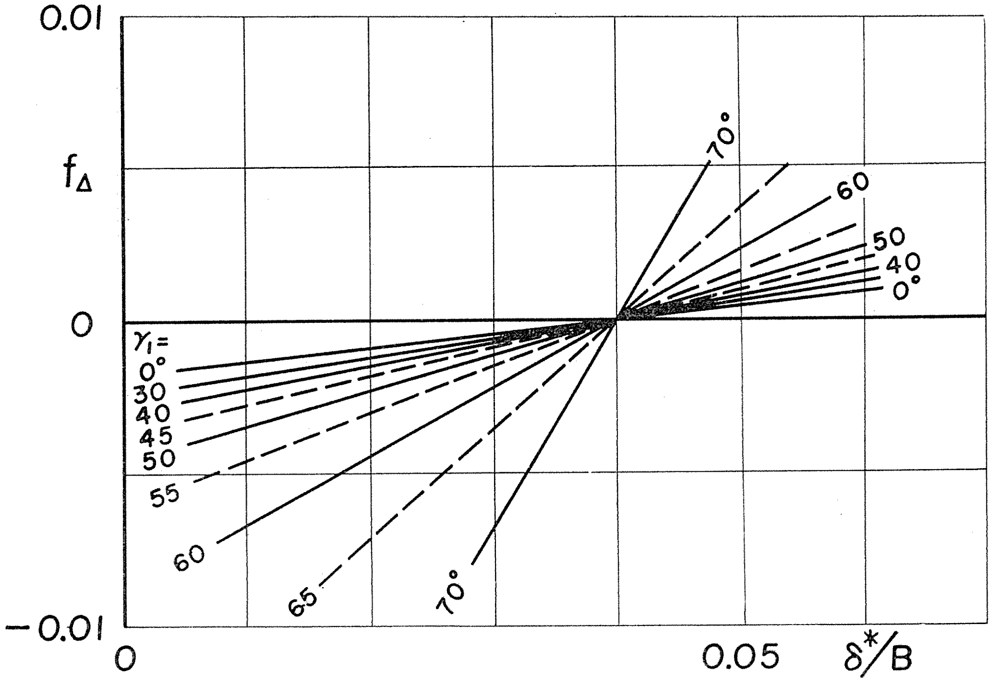


Fig. 10. f_Δ curve.

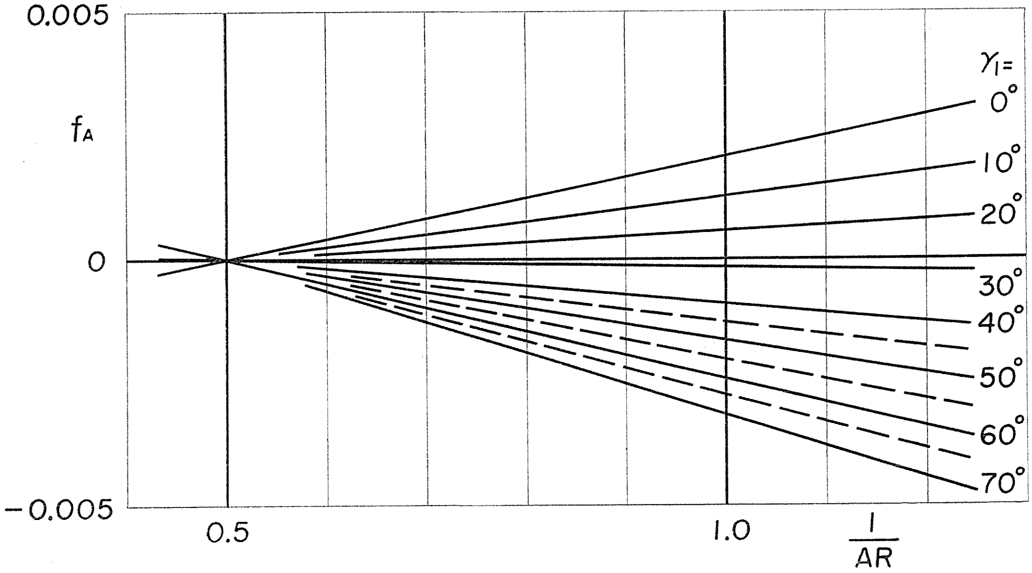


Fig. 11. f_A urve.

effects of each factor are clear. Using Fig. 7 and Fig. 8 $\bar{g}$ was adjusted into g in respect of the case of $\delta^*/B=0.04$ and $AR=2.0$. g is indicated in Fig. 9. f_Δ and f_A are also shown in Fig. 10 and Fig. 11 respectively. It will need no explanation that f_Δ $(\delta^*/B=0.04)=0$ and $f_A(AR=2.0)=0$.

3. 7 *The Examination of Results*

As an examination of results, w_{a2}/w_{a1} which was calculated from our experimental data using the result got in the last paragraph was compared with the experimental datum itself. Table 1 the comparison of the both which were picked up arbitrarily.

Table 1. Examination of results.

Profile	AR	a/c	α	α_i	γ_1	ε	δ^*/B	Calculated w_{a2}/w_{a1}	Experimental w_{a2}/w_{a1}
RAF 6	2.08	1.67	45	5	50	7.1	0.040	1.06	1.10
	〃	1.67	65	0	65	3.3	32	1.07	1.07
	〃	0.83	30	5	35	9.1	36	1.03	1.06
	〃	1.46	50	5	55	6.7	38	1.08	1.08
	〃	1.46	70	0	70	3.3	32	1.09	1.11
	〃	1.04	20	10	30	13.4	58	1.05	1.05
	〃	1.67	55	5	60	7.0	36	1.10	1.09
	〃	1.04	45	5	50	8.5	44	1.08	1.09
	〃	1.04	35	5	40	8.6	42	1.04	1.05
	〃	1.04	65	5	70	7.0	34	1.20	1.21
	1.39	1.04	0	10	10	13.5	61	1.02	1.02
	1.04	1.04	30	5	35	8.0	53	1.03	1.03
	0.83	1.04	15	5	20	8.3	59	1.03	1.02
	〃	1.04	60	5	65	6.7	41	1.13	1.15
	〃	1.04	0	5	5	8.7	68	1.03	1.03
	3.13	1.04	19.3	10.7	30	14.3	12	1.02	1.02
	〃	1.04	35.6	10.7	46.3	13.8	9	1.06	1.06
NACA 6409	1.5	1	35.6	10.7	46.3	16.0	17	1.07	1.08
	〃	1	54.2	6.0	60.2	11.6	24	1.13	1.10
	〃	1	-10.7	10.7	0	18.4	21	0.99	0.98
	3.0	1	19.3	10.7	30	16.0	17	1.07	1.08
	6.0	1	35.6	10.7	46.3	16.4	8	1.08	1.06

If we assume that the difference of w_{a2}/w_{a1} less than ± 0.01 represents a good agreement, the results are thought to show rather better agreement than we expected excepting a few cases. Frankly speaking, the fact, that the results showed such good agreement despite of experimental data being insufficient and considerably bold conjectures we did, surprised us. One reason for this might come from the fact that the data considered had same source as ones used for the determination of experimental equation. But, neverthless, authors could not anticipate such good agreement in the course of the derivation.

4. Concluding Remarks

It was supposed that there exists a relationship between the turning angle and the axial velocity ratio which depends only on the inflow angle but is independent of the pitch chord ratio and stagger angle, and the experimental equation was derived. Besides inflow angle, inflow boundary layer thickness and aspect ratio were clarified to have effects on the relationship. The effect of boundary layer was especially noticeable.

Comparison was made between the axial flow ratio, which was calculated from our experiment by the experimental equation obtained, and experimental datum itself, and showed rather better agreement than we expcted. Authors think, therefore, that this experimental equation is very useful.

It is one probelm to be checked that data used for the derivation of experimental equation are ones obtained only in authors' laboratory. This situation came from the fact that many data published by many researchers lack sufficient descriptions of their experimental conditions (especially inflow boundary layers), and could not be used for this study. Authors wish to make efforts to gather as many data as possible to clarify the usefulness of this experimental formula.

5. References

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