EXPERIMENTAL FORMULAE ON THE HYDRAULIC LOSS IN PIPE BENDS *

Mitsukiyo MURAKAMI** and Yukimaru SHIMIZU***

1. Introduction

A pipe line of a fluid machine or other hydraulic plant has many complicated bent portions. The bends which are located closely in a pipe line have a strong interference, and when two or more elbows are installed closely in different planes, the water flowing through them acquires a strong spiral component of velocity. In this case the hydraulic loss cannot be predicted from the results of a single elbow.[1][2][3]

Very few data for the interference effects are available. This paper gives the results for the combined loss due to two or three 45°- or 90°- screw type elbows and also experimental formulae for the hydraulic loss.

2. Nomenclature

d	:	diameter of pipe, $=2R$
g	:	acceleration of gravity
L_d	:	distance of the downstream section from the exit of the last elbow
L_{m1}	:	spacer length between the first and the second elbows
L_{m2}	:	spacer length between the second and the third elbows
L_u	:	distance of upstream section from the entrance of the first elbow
L	:	gauge length, $= L_u + \sum_{i=1}^{n} L_{mi} + L_d$
R	:	radius of pipe, $= d/2$
p_1	:	static pressure at the upstream pressure tap, $L_u=5d$
p_2	:	static pressure at the downstream pressure tap, $L_d=300d$
Re	:	Reynolds number, $= V_m d/\nu$
v_z	:	axial component of velocity
v_z'	:	dimensionless expression of axial velocity, $v_z' = v_z / V_m$

* Received 20th May, 1974

** Professor, Department of Mechanical Engineering, Nagoya University

*** Lecturer, Department of Mechanical Engineering, Mie University

v_θ	:	tangential component of velocity
v'_θ	:	dimensionless expression of tangential velocity, $v'_\theta = v_\theta / V_m$
V_m	:	mean flow velocity
γ	:	specific weight of water
ρ	:	density of water
ζ_n	:	coefficient of total elbow loss as defined by Eq. (1) (subscript n expresses the number of elbows)
ζ_2^*	:	correction factor for total elbow loss (two elbows)
ζ_3^*	:	correction factor for total elbow loss (three elbows)
$\bar{\zeta}'_1$	:	coefficient of an elbow loss obtained by pressure difference between the entrance and the exit sections of an elbow ($L_u = L_d = 1d$)
λ	:	friction factor of straight pipe
ψ	:	turned angle of pipe line made by the first and the second elbows (Fig. 1)
ϕ	:	turned angle of pipe line made by the second and the third elbows (Fig. 1)
δ	:	angle of an elbow ($\delta=90°$ for 90°-elbow, $\delta=45°$ for 45°-elbow)
ν	:	kinematic viscosity
M'_u	:	dimensionless expression of angular momentum flux through the section one pipe diameter upstream from an elbow entrance
M'_d	:	dimensionless expression of angular momentum flux through a section situating arbitrary distance downstream from an elbow

3. Loss and Secondary Flow due to Pipe Bends

1) *Coefficient for Bend Loss*

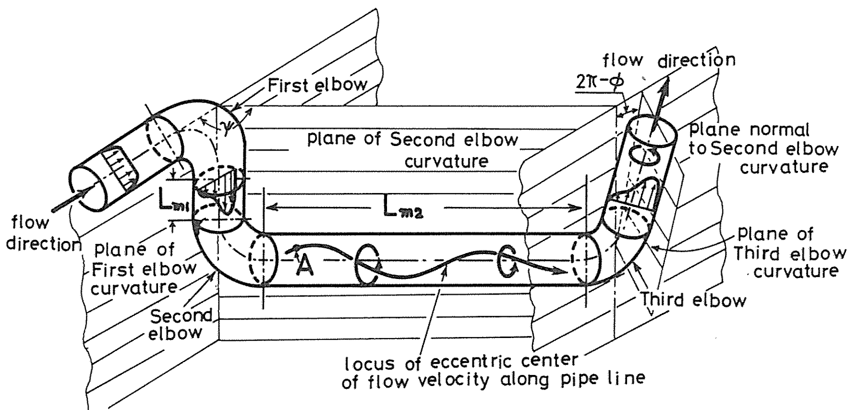


Fig. 1 Schematic diagram of turned pipe line

Elbows employed in this study are 45°- and 90°- screw type commercial ones (inner diameter = 2 in., JIS B 2301). The turned angles of the pipe line (ψ and ϕ) and the spacer length (L_{m1} and L_{m2}) are indicated in Fig. 1. The coefficient of total elbow loss ζ_n can be calculated by the following equation.

$$\frac{P_1 - P_2}{\gamma} = \lambda\left(\frac{L}{d}\right)\left(\frac{V_m^2}{2g}\right) + \zeta_n\left(\frac{V_m^2}{2g}\right) \quad (1)$$

Coupling depth of screw joints between an elbow and its tangent pipes has a considerable effect on the fitting loss of the elbows.

For example, when every joint between an elbow and the pipes are half screwed, the value of $\zeta_n (\zeta_1)$ reduces to 85 percent of that which would be obtained with fully screwed joints. The loss coefficient ζ_n in this report is referred to the case when the pipe ends are fully screwed.

Within the range of Re = $3\times10^4 \sim 2\times10^5$, the change of ζ_n does not exceed ten percent, and hence, the data for Re $\fallingdotseq 10^5$ are only described here.

2) Swirling Flow Component and Loss

As a measure of the strength of secondary swirling flow, the angular momentum of water through a pipe section is considered as (Fig. 3).

$$M = \rho \int_0^{2\pi} \int_0^R V_z V_\theta r^2 dr d\theta \quad (2)$$

Putting $r = R\,r'$, $v_z = V_m v_z'$, and $v_\theta = V_m v_\theta'$, a dimensionless expression of Eq. (2) can be written as

$$M' = M/\rho V_m^2 R^3 = \int_0^{2\pi} \int_0^1 V_z' V_\theta' r'^2 dr' d\theta \quad (3)$$

When two symmetrical vortices with opposite signs are formed in a pipe section as seen in the flow behind a single bend, the vortices cancel each other and $M' = 0$. When the strength of two vortices is not equal, a stronger vortex absorbs a weaker one in course of flow, and a forced vortex type swirling flow is formed behind the section at which the symmetric vortices disappear.

A dimensionless expression for the energy of the swirling motion is given by

$$\zeta_\theta = \int_0^R 2\pi r dr V_z \left(\frac{V_\theta^2}{2g}\right) / \pi R^2 V_m \left(\frac{V_m^2}{2g}\right) \quad (4)$$

This energy will be dissipated in course of flow. As a reference, the flow with an uniform axial velocity and a forced vortex type tangential velocity is assumed; i.e. $v_z = V_m$ and $v_\theta = \Omega r$ where Ω denotes the angular velocity of the forced vortex flow. For this flow Eqs. (3) and (4) yield

$$\zeta_\theta = \frac{1}{2}\left(\frac{\Omega R}{V_m}\right)^2 \quad (4')$$

$$M_d' = \frac{1}{2}\left(\frac{\Omega R}{V_m}\right) \quad (5)$$

From Eqs. (4') and (5)

$$\zeta_\theta = 2 M_d'^2 \quad (6)$$

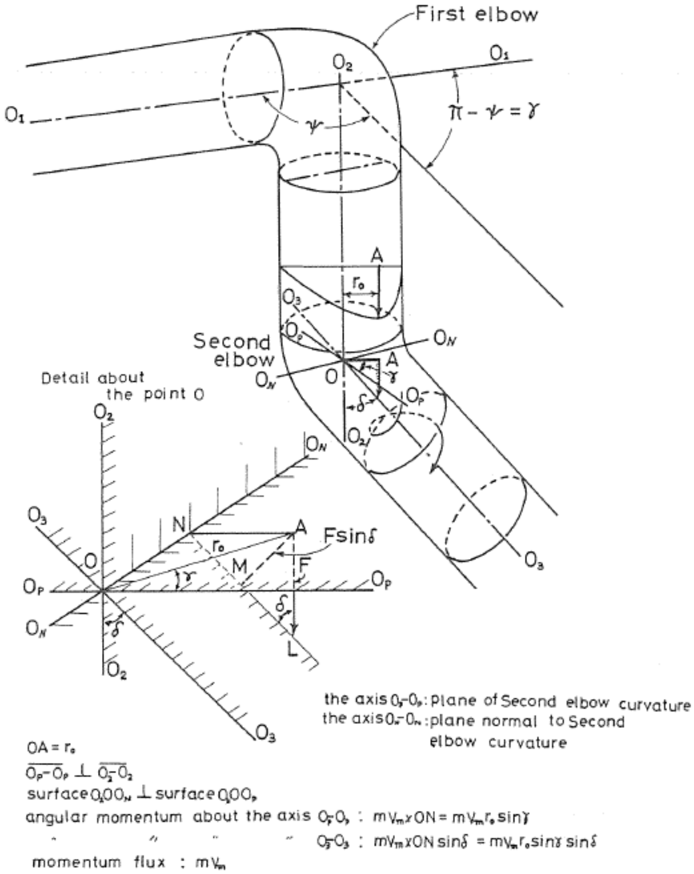


Fig. 2 Explanation of swirling motion produced by uneven axial velocity (r_o/R).

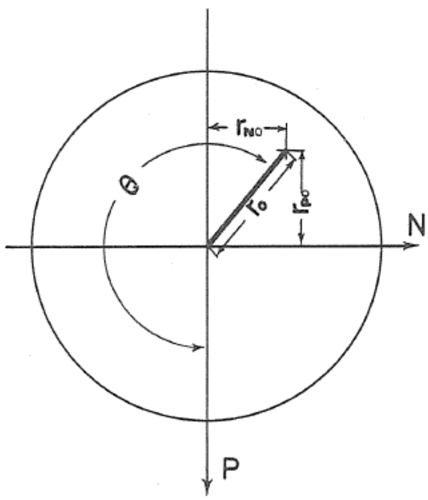


Fig. 3 Notation for velocity center

3) Center of Velocity Distribution

When two elbows are closely located in a pipe line, the flow entering the downstream elbow will be strongly disturbed. In order to define the center of velocity distribution, the following expressions for the moment of momentum of the flowing water are introduced

$$M_N = \rho V_m^2 R^3 \int_0^{2\pi} \int_0^1 V_z'^2 r'^2 \sin\theta dr' d\theta \tag{7}$$

$$M_P = \rho V_m^2 R^3 \int_0^{2\pi} \int_0^1 V_z'^2 r'^2 \cos\theta dr' d\theta \tag{8}$$

where M_N and M_p are the momentum about the axes N-N and P-P, respectively (Fig. 3).

Making use of the relationships

$$r = R\ r', \ r_N = r\cos\theta, \ r_p = r\sin\theta, \text{ and } v_z = V_m v_z' \tag{9}$$

and denoting the distances of the velocity center from the axes of N-N and P-P by r_{op} and r_{oN}, respectively, the following expressions can be derived

$$M_N = \rho\pi R^2 V_m^2 r_{oP}, \ M_P = \rho\pi R^2 V_m^2 r_{oN}$$

These and together with Eqs. (7), (8) and (9) give

$$r_{oP}/R = \frac{1}{\pi}\int_o^{2\pi} \int_o^1 V_z'^2 r'^2 \sin\theta dr' d\theta \tag{10}$$

$$r_{oN}/R = \frac{1}{\pi}\int_0^{2\pi} \int_0^1 V_z'^2 r'^2 \cos\theta dr' d\theta \tag{11}$$

When the radial distance of the velocity center A is denoted by r_o, the following relation is obtained:

$$(r_0/R)^2 = (r_{oN}/R)^2 + (r_{oP}/R)^2 \tag{12}$$

4) Generation of Secondary Flow

A fairly intensive secondary flow is set up in an elbow, and at its exit the maximum axial velocity is displaced from the center of the pipe towards the outer wall, as is shown in Fig. 1. This non-uniform velocity recovers its uniformity at the section where L_d = (6~7)d downstream of the elbow exit. When the deformed flow enters the next (the second) elbow as in Fig. 2, the centrifugal force due to this non-uniform flow produces a secondary circulating flow in the second elbow. Provided that the water at the entrance of the second elbow is concentrated at the velocity center A of which offset distance is r_o, the moment arm of the centrifugal force with respect to the center line of the second elbow is given by

$$(r_0/R)\sin(\pi-\psi) = (r_0/R)\sin\gamma$$

Thus, the angular momentum created by the non-uniform inlet is given approximately by

$$M'' = (r_0/R)\sin\gamma\sin\delta \tag{13}$$

where δ is the angle between the lines $o_2 - o_2$ and $o_3 - o_3$ (δ=90° for 90°-elbow and δ=45° for 45°-elbow). M'' multiplyed by a numerical factor K gives the angular momentum of swirling secondary flow M_d'' in the exit section of the second elbow:

$$M_d'' = KM'' = K(r_0/R)\sin\gamma\sin\delta \tag{14}$$

The value of M_d' becomes zero when ψ = o° and 180°, namely, when the pipe line is turned two dimensionally in U or S shape. When the pipe line is turned three dimensionally by two elbows, the flow downstream of the elbows has an asymmetric axial velocity accompanying a swirling component. The asymmetry disappears at about the section at L_d = 7d downstream of the second elbow. The disturbed axial velocity behind the first elbow recovers its uniformity nearly in the same axial distance.

The flow pattern behind two elbows combined is shown schematically in Fig. 2, where $0° < \gamma < 180°$ and $L_d < 7d$. If the third elbow is installed in the non-uniform flow region, the flow in the third elbow will be affected by the non-uniform axial velocity and the swirling velocity existing just upstream of the third elbow.

The secondary flow in the exit section of the third elbow is classified into the following three cases:

i) The second and the third elbows are closely located in a twisted S shape. Distortion of axial velocity distribution just before the third elbow becomes to a considerable amount and it governs the flow in the third elbow.

ii) The first and the second elbows are closely located in a twisted S shape and the third elbow is installed in a some distance from the second elbow. The axial velocity is nearly uniform before the third elbow an axi-symmetric swirling flow enters the third elbow.

iii) The swirling motion and non-uniformity of the axial velocity distribution have nearly the same effects on the flow in the third elbow.

4. Experimental Formulae for Bend Loss

4.1 Hydraulic Loss in Two Combined Elbows

When the spacer length (L_{m1}) is reduced, an interference between the two elbows is enhanced progressively, and when the elbows are arranged in twisted S shape, a strong swirling flow is created by the elbow ends. The energy of this swirling motion is dissipated ultimately and it constitutes a part of the elbow loss. If the total loss due to the two elbows is assumed to be the sum of the loss due to the swirling motion ζ_θ and the other loss ζ_2^*, the following relation is obtained

$$\zeta_2 = \zeta_\theta + \zeta_2^* \tag{15}$$

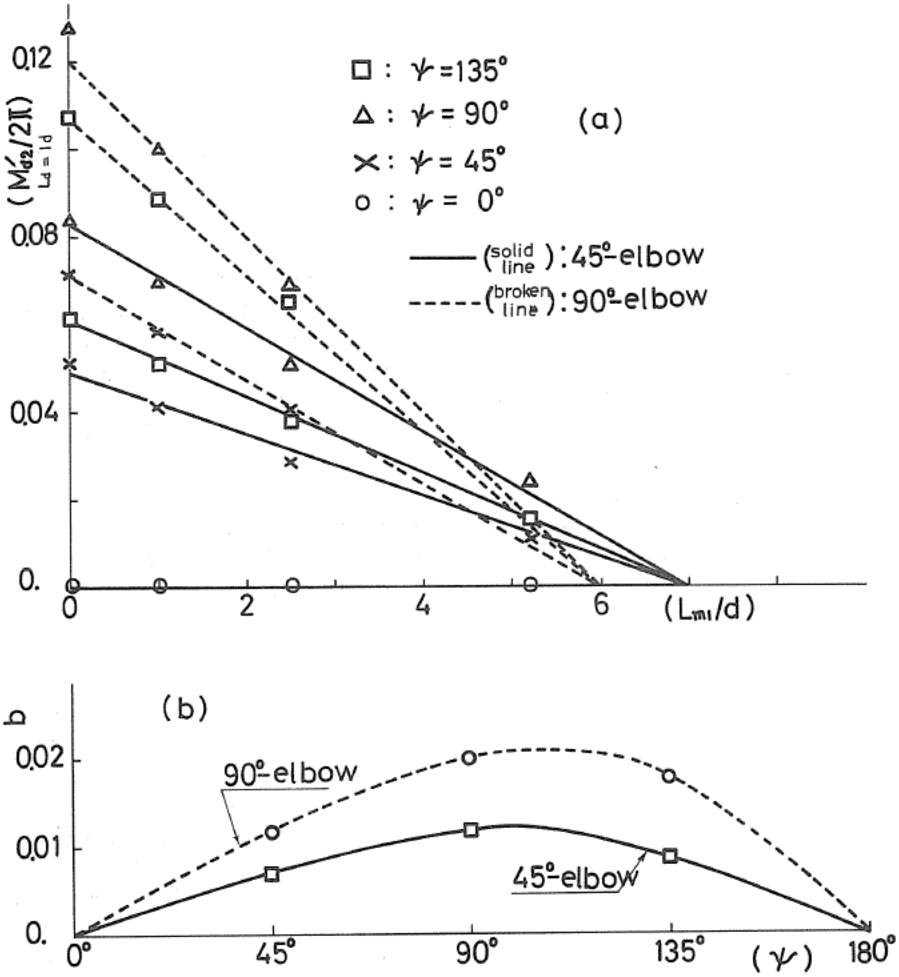


Fig. 4

The swirling loss ζ_θ is found from Eq. (6) when M'_{d2} in the exit section of the second elbow is known. The values of M'_{d2} against the spacer length L_{m1} are shown in Fig. 4 (a). M'_{d2} changes linearly with L_{m1} and tends to zero approximately when $L_{m1} \geqq 6d$ for 90°-elbows and $L_{m1} \geqq 7d$ for 45°-elbows, irrespective of the twisted angle ψ. These relationships are given by

$$\left.\begin{array}{ll} 90^\circ-\text{elbows:} & M'_{d2}=b_{90^\circ}(6-L_{m1}/d)\ (L_{m1}\leqq 6d) \\ 45^\circ-\text{elbows:} & M'_{d2}=b_{45^\circ}(7-L_{m1}/d)\ (L_{m1}\leqq 7d) \end{array}\right\} \quad (16)$$

The inclination of lines of M'_{d2} vs. L_{m1} is obtained from Fig. 4 (a) as

$$\left.\begin{array}{l} b_{90^\circ}=\psi\times10^{-3}(0.305-0.95\times10^{-3}\psi),\quad 0^\circ\leqq\psi\leqq90^\circ \\ b_{90^\circ}=(180^\circ-\psi)\times10^{-3}(0.567-0.38\times10^{-2}(180^\circ-\psi)),\ \ 90^\circ\leqq\psi\leqq180^\circ \end{array}\right\} \quad (17)$$

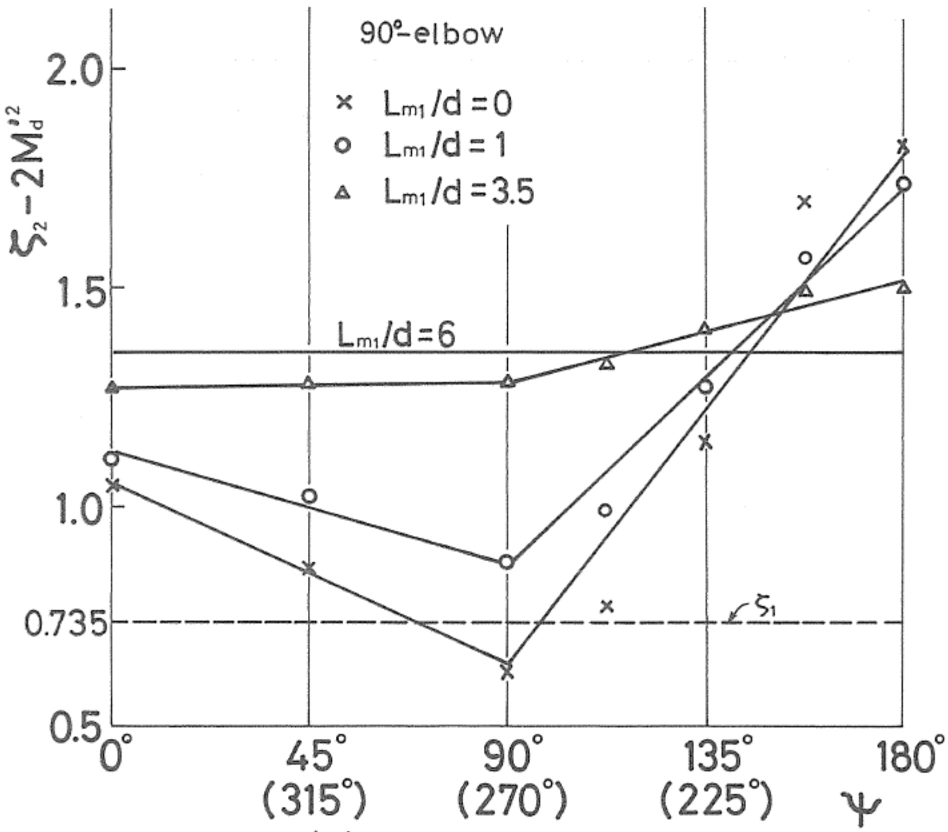

90°-elbow
× $L_{m1}/d=0$
○ $L_{m1}/d=1$
△ $L_{m1}/d=3.5$
$L_{m1}/d=6$
ζ_1
$\zeta_2-2M_d'^2$
2.0
1.5
1.0
0.735
0.5
0°
45° (315°)
90° (270°)
135° (225°)
180°
ψ

Fig. 5 (a)

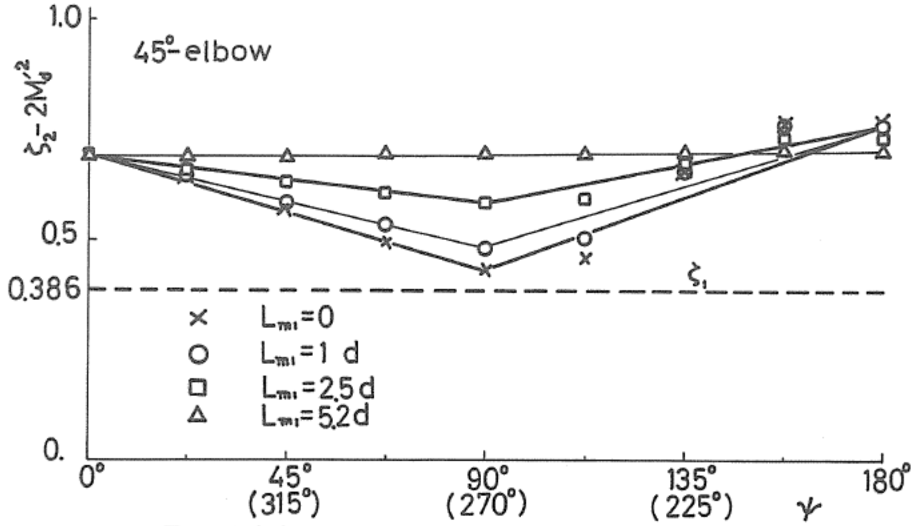

45°-elbow
$\zeta_2-2M_d'^2$
1.0
0.5
0.386
0.
ζ_1
× $L_{m1}=0$
○ $L_{m1}=1\ d$
□ $L_{m1}=2.5\ d$
△ $L_{m1}=5.2\ d$
0°
45° (315°)
90° (270°)
135° (225°)
180°
ψ

Fig. 5 (b)

$$\left.\begin{aligned} b_{45^\circ} &= \psi\times10^{-3}(0.184-0.62\times10^{-3}\psi), \quad 0^\circ\leqq\psi\leqq90^\circ \\ b_{45^\circ} &= (180^\circ-\psi)\times10^{-3}(0.244-0.114\times10^{-2}(180^\circ-\psi)), \quad 90^\circ\leqq\psi\leqq180^\circ \end{aligned}\right\} \tag{18}$$

With these results and the measured values of total elbow loss ζ_2, the value of ζ_2^* may be calculated by Eq. (15).
Fig. 5 (a) and (b) give the results and their empirical relationship is expressed by

$$\zeta_2^* = \zeta_2 - \zeta_\theta = n\,(\psi-90^\circ) + \zeta_E + c \tag{19}$$

where for 90°-elbows

$$c = -0.0272(L_{m1}/d)^2 + 0.278(L_{m1}/d) + 0.64 \tag{20}$$

$$\left.\begin{aligned} &0^\circ\leqq\psi\leqq90^\circ \\ n &= -0.000122\left(\frac{L_{m1}}{d}-6\right)^2 \\ \zeta_E &= 0 \\ &90^\circ\leqq\psi\leqq180^\circ \\ n &= 0.000265\left(\frac{L_{m1}}{d}-7\right)^2 \\ \zeta_E &= -0.00265\left(\frac{L_{m1}}{d}-7\right)^2\sin(4(\psi-90^\circ)) \end{aligned}\right| \tag{21}$$

and for 45°-elbows

$$c = 0.05\left(\frac{L_{m1}}{d}\right)+0.44 \tag{22}$$

$$\left.\begin{aligned} &0^\circ\leqq\psi\leqq90^\circ \\ n &= \left(0.556\frac{L_{m1}}{d}-2.77\right)\times10^{-3} \\ \zeta_E &= 0 \\ &90^\circ\leqq\psi\leqq180^\circ \\ n &= \left(-0.788\frac{L_{m1}}{d}+4.1\right)\times10^{-3} \\ \zeta_E &= -0.00265\left(\frac{L_{m1}}{d}-7\right)^2\sin(4(\psi-90^\circ)) \end{aligned}\right| \tag{23}$$

The foregoing formulae are valid for the range, $L_{m1} < 6d$ for 90°-elbows and $L_{m1} < 7d$ for 45°-elbows.

When the values of L_{m1} exceed these limits, ζ_2 becomes approximately equal to $2\zeta_1$. The above results are compared with experiments in Figs. 6 (a) and (b).

4.2 Hydraulic Loss in Three Combined Elbows

When the third elbow is placed closely behind the two elbows, a complicated flow enters the third elbow and brings a special hydraulic loss. Total hydraulic loss of the three combined elbows ζ_3 can be conveniently expressed as the sum of the hydraulic loss due to the two elbows situating upstream and the

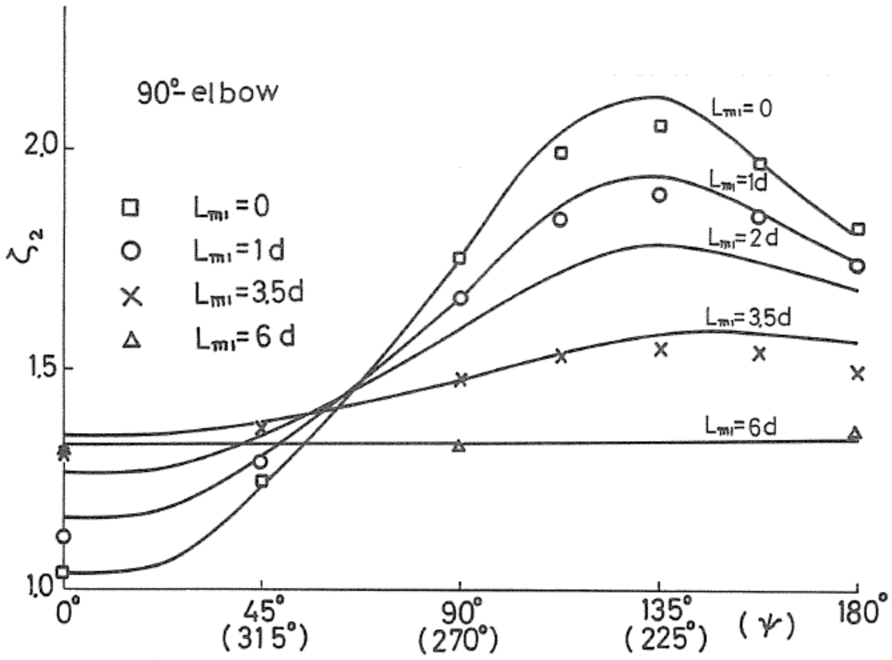


Fig. 6(a)

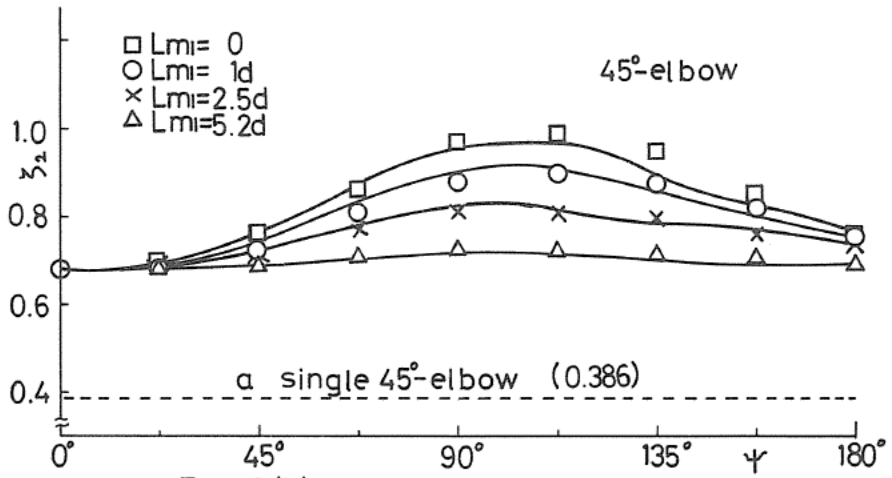


Fig. 6(b)

hydraulic loss due to the third elbow, namely,

$$\zeta_3 = \zeta_2 + \overline{\zeta}'_1 + \zeta_3^* + \zeta_{3\theta} \tag{24}$$

where $\overline{\zeta}'_1$ is a dimensionless expression of the pressure loss across the third elbow when an axi-symmetrical swirling flow enters the elbow. ζ_3^* shows the excess of the hydraulic loss due to the third elbow over the hydraulic loss $\overline{\zeta}'_1$ when an asymmetrical swirling flow enters the third elbow. The value of $\zeta_{3\theta} = 2M''^2_{d3}$

exhibits the loss due to the swirling motion behind the third elbow.

The pipe flow with three elbows combined can be classified into the following two cases:

i) All elbows are located closely and the velocity profile just before the third elbow is extremely distorted. The effects of the distortion is predominant in the flow through the third elbow. In this case:

$$\zeta_3 = \zeta_2 + \zeta_3^* + 2M''^2_{d3} \tag{25}$$

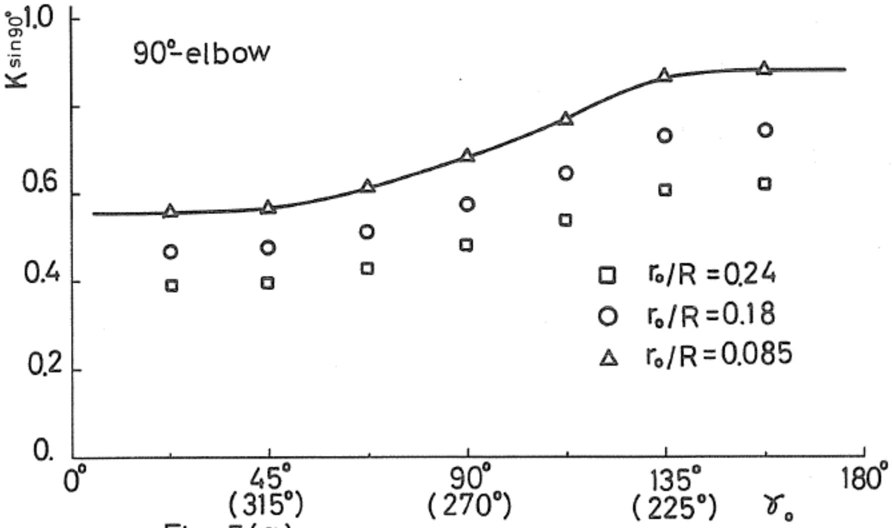


Fig. 7(a)

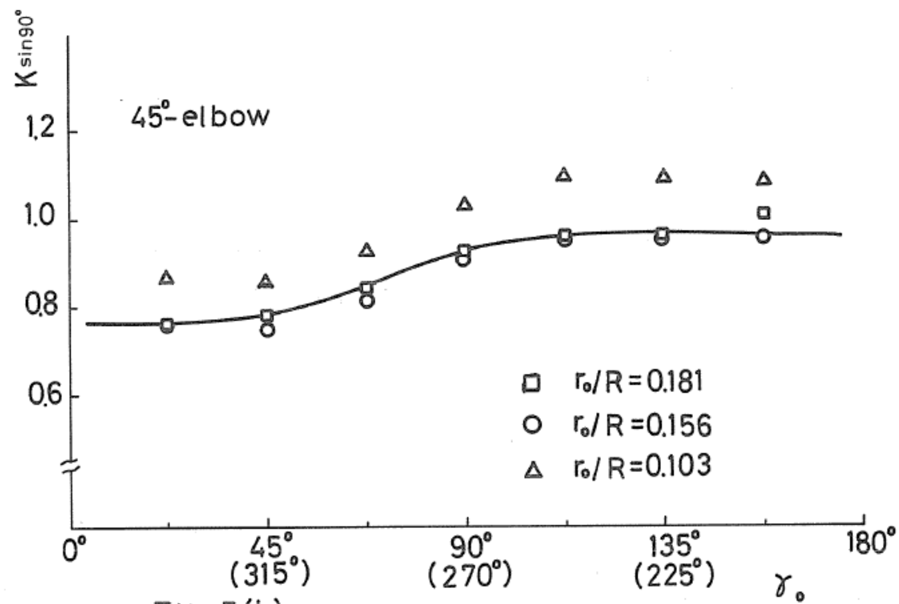


Fig. 7(b)

ii) The first and the second elbows are closely located in a twisted S shape and the third elbow is installed in a some distance from them. The flow entering the third elbow is nearly axi-symmetric, and M''_{d3} due to the swirling flow created by the third elbow is small amount. In this case: $\bar{\zeta}'_1 \gg \zeta^*_3$, and hence,

$$\zeta_3 = \zeta_2 + \bar{\zeta}'_1 + 2M''^2_{d3} \tag{26}$$

By using Eq. (14), M''_{d3} in this equation is expressed as follows:

$$M''_{d3} = K\pi(r_o/R)\sin\gamma_o \tag{14'}$$

where K is a numerical constant, which depends on the angular position of the velocity center A in the section just before the second elbow (L_u = 1d). In Figs. 7 (a) and (b), the value of K against γ_o are given for various values of r_o / R.

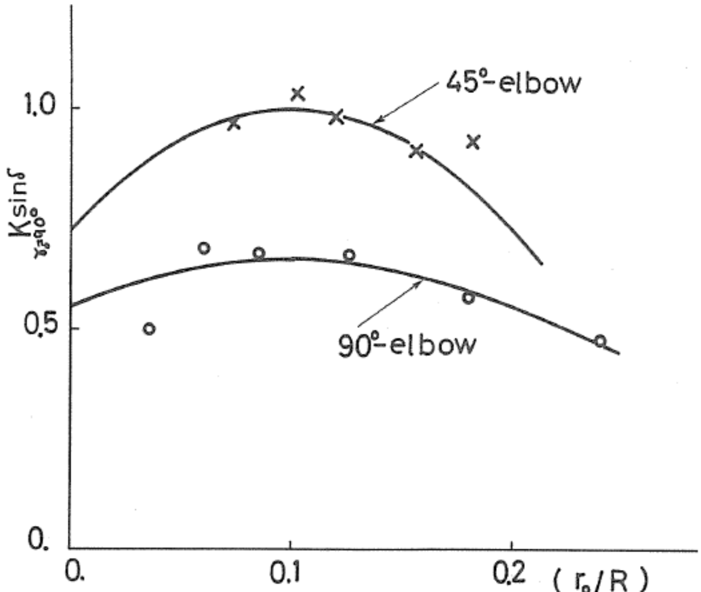


Fig. 8

Fig. 8 shows the value of K against r_o / R, when γ_o = 90°.
The results in Figs. 7 and 8 can be summarized by the following formulae:

For 90°- elbows:

$$\left.\begin{array}{ll} \gamma_0 = 0^\circ,\ 180^\circ, 360^\circ & K = 0 \\ 0^\circ < \gamma_0 < 180^\circ,\ \gamma_0 > 360^\circ & \\ K \sin 90^\circ = -11\left(\frac{r_0}{R} - 0.1\right)^2 + 0.66 + \frac{0.0044(\gamma_0 - 100^\circ)}{\sqrt{4.788\times10^{-4}(\gamma_0-100^\circ)^2+1}} & \\ 180^\circ < \gamma_0 < 360^\circ & \\ K \sin 90^\circ = -11\left(\frac{r_0}{R} - 0.1\right)^2 + 0.66 - \frac{0.0044(\gamma_0 - 260^\circ)}{\sqrt{4.788\times10^{-4}(\gamma_0-260^\circ)+1}} & \end{array}\right\} \tag{27}$$

For 45°-elbows:

$$\left.\begin{array}{ll} \gamma_0 = 0^\circ,\ 180^\circ,\ 360^\circ & K = 0 \\ 0^\circ < \gamma_0 < 180^\circ,\ \gamma_0 > 360^\circ & \\ K \sin 45^\circ = -27.7\left(\frac{r_0}{R} - 0.1\right)^2 + 1 + \frac{0.0044(\gamma_0 - 80^\circ)}{\sqrt{1.004 \times 10^{-3}(\gamma_0 - 80^\circ)^2 + 1}} & \\ 180^\circ < \gamma_0 < 360^\circ & \\ K \sin 45^\circ = -27.7\left(\frac{r_0}{R} - 0.1\right)^2 + 1 - \frac{0.0044(\gamma_0 - 280^\circ)}{\sqrt{1.004 \times 10^{-3}(\gamma_0 - 280^\circ)^2 + 1}} & \end{array}\right\} \quad (28)$$

In order to find the values of ζ_3^*, $\bar{\zeta}_1'$ and M_{d3}'', the flow pattern downstream of the second elbow should be determined. In Fig. 9, the decay of the swirling flow behind the second elbow is shown for various bent configurations.

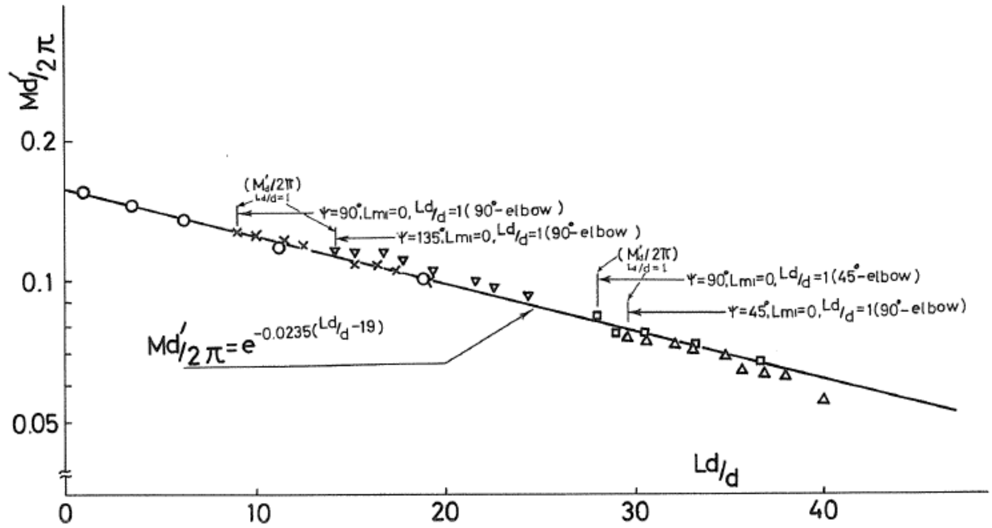


Fig. 9

$\left(M_d'/2\pi\right)_{Ld=1d}$ denotes the swirling intensity at the second elbow exit (L_d = 1d). The circular mark o denotes the results for the three combined elbows with $\psi = 0°$, ϕ = 112.5°, and L_{m1} = L_{m2} = 0. The decay of the swirling flow can be expressed by a single line, which is given by

$$M_d'/2\pi = \exp(-0.0235(L_d/d - 19)) \qquad (29)$$

The velocity center A behind the two elbows follows a spiral path and approaches the pipe center at about seven pipe diameters from the second elbow exit.

The offset amount of the center A, namely, r_o / R decreases exponentially downstream and is expressed by

$$r_0/R = (r_0/R)_{Ld=1d} \exp(-m(L_d/d - 1)) \qquad (30)$$

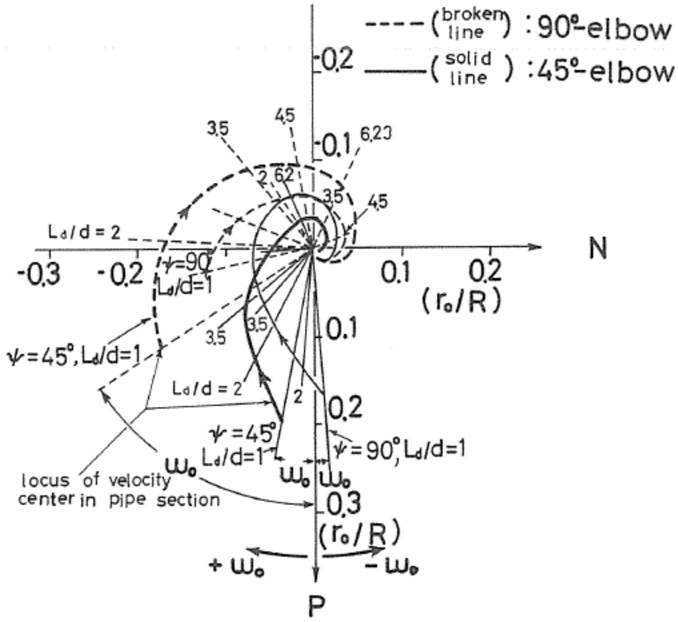


Fig. 10 Change of velocity center along pipe line.
(for two elbows, $L_{m1}=0$)
Clockwise swirling motion from P-axis
(plus) : $\omega_0 > 0$.

where m is a constant to be determined by the experiments. It may be chosen 0.36 for 90°-elbows and 0.17 for 45°-elbows, within the range of this study.

The values of $(r_0/R)_{Ld=ld}$ at the exit of the second elbow are given by the following formulae.

For 90°-elbows:

$$
\left.
\begin{array}{l}
0^\circ \leqq \psi \leqq 45^\circ \\
\quad (r_0/R)_{Ld=1d} = 0.21 \\
45^\circ \leqq \psi \leqq 180^\circ \\
\quad (r_0/R)_{Ld=1d} = \left[0.00486\left(\frac{L_{m1}}{d}\right)^2 - 0.0641\left(\frac{L_{m1}}{d}\right) + 0.21\right] \times 10^{-4} (\psi - 112.5^\circ)^2 + R_0 \\
\qquad R_0 = 0.04(L_{m1}/d) + 0.11, \quad L_{m1}/d \leqq 1 \\
\qquad R_0 = 0.012(L_{m1}/d) + 0.138, \; 1 \leqq L_{m1}/d \leqq 6
\end{array}
\right\} \quad (31)
$$

For 45°-elbows:

$$
\left.
\begin{array}{l}
L_{m1}/d \leqq 4 \\
(r_0/R)_{Ld=1d} = 0.4325 \times 10^{-3}\left(\frac{L_{m1}}{d} - 4\right)(\psi - 90^\circ) - 0.0156\frac{L_{m1}}{d} + 0.184 \\
\quad L_{m1}/d > 4 \\
(r_0/R)_{Ld=1d} = -0.0156(L_{m1}/d) + 0.184
\end{array}
\right\} \quad (32)
$$

Rate of change of the angular position of the point A along the pipe axis is affected by the swirling intensity M'_{d2} and is generally expressed by

$$q = 361.5^{\circ}(M'_{d2}/2\pi)_{Ld=1d} \tag{33}$$

The initial angular position of A at the second elbow exit is a function of the twisted angle of the pipe line ψ and of the spacer length L_{m1}. The relationship is given approximately by the following equation.

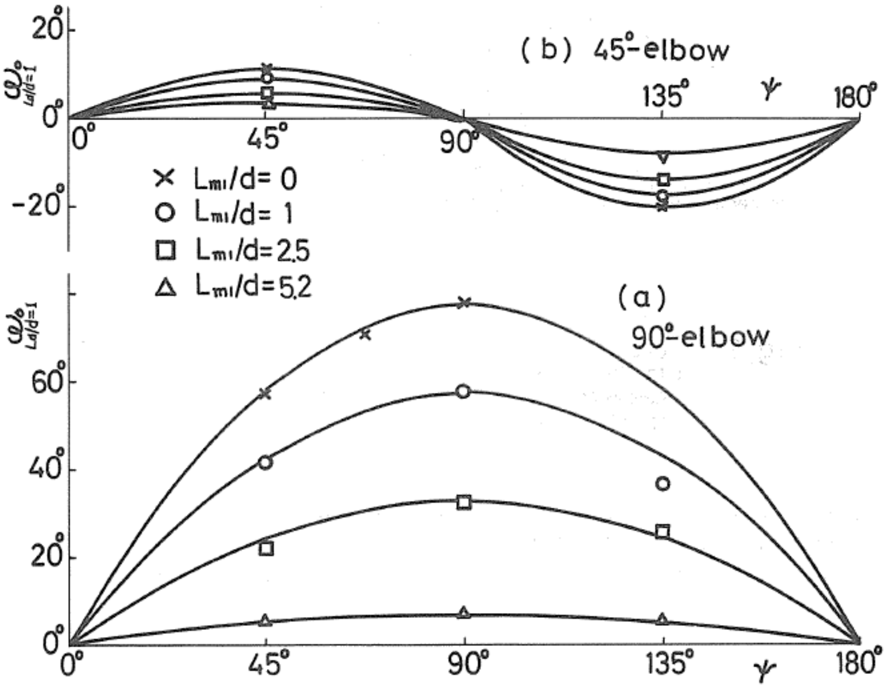
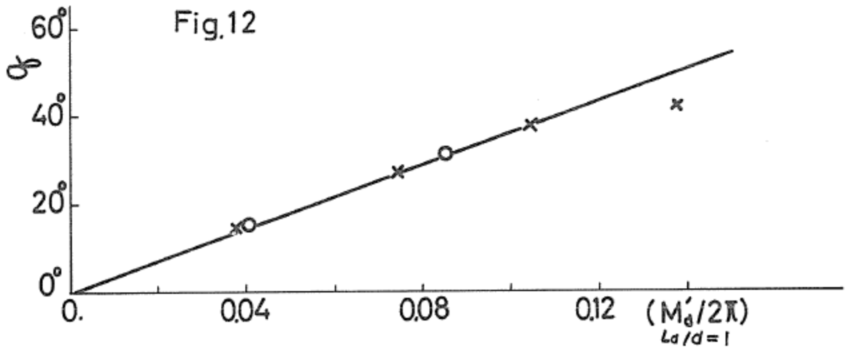


Fig.11



For 90°-elbows:

$$\omega_0 = -\left[0.197\left(\frac{L_{m1}}{d}\right)^2 - 2.716\left(\frac{L_{m1}}{d}\right) + 9.629\right] \times 10^{-3}(\psi - 90°)^2 + 1.6\left(\frac{L_{m1}}{d}\right)^2 - 22\frac{L_{m1}}{d} + 78 \tag{34}$$

For 45°-elbows:

$$\begin{array}{ll} 0° \leqq \psi \leqq 90°, & \omega_0 = (-1.83°(L_{m1}/d) + 11.16°)\sin(2\psi) \\ 90° \leqq \psi \leqq 180°, & \omega_0 = (2.52°(L_{m1}/d) - 20.3°)\sin(2\psi) \end{array} \tag{35}$$

With these results the angular position of the velocity center A at any section (L_d) measured from the second elbow exit is given by

$$\gamma' = \omega_0 + q(L_d/d - 1) \tag{36}$$

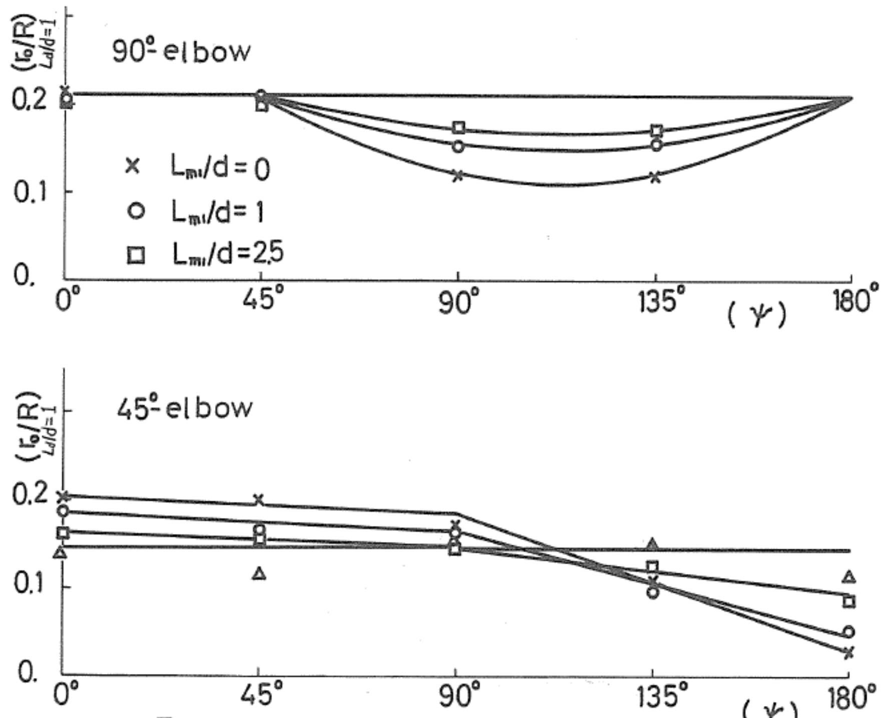
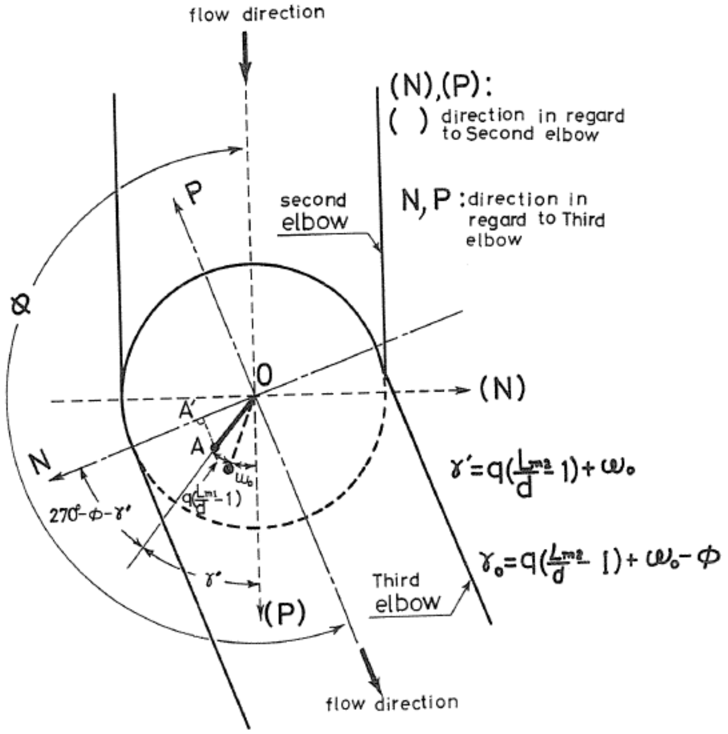


Fig. 13

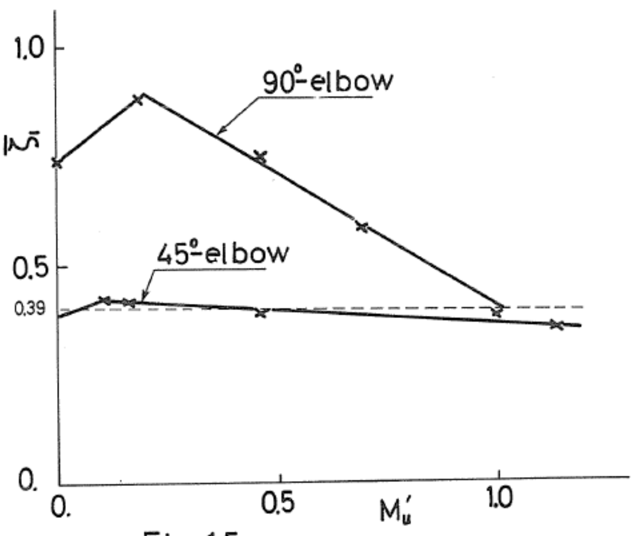
If the third elbow is installed with an angle ϕ against the plane of the second elbow, the angular position of the velocity center A referred to the second elbow exit is given by

$$\gamma_0 = \omega_0 + q(L_d/d - 1) + \phi \tag{37}$$



flow direction
(N),(P):
( ) direction in regard to Second elbow
N, P : direction in regard to Third elbow
second elbow
O
(N)
P
N
A'
A
Φ
270°-Φ-γ'
γ'
(P)
γ'=q(Lm2/d - 1)+ω₀
γ₀=q(Lm2/d - 1)+ω₀-Φ
Third elbow
flow direction

Fig. 14



1.0
0.5
0.39
0.
90°-elbow
45°-elbow
0.
0.5
1.0
M'u

Fig. 15

(a) Formulae for Hydraulic Loss in the Case (i)

In Eq. (25), the value of M''_{d3} can be estimated by Eq. (20), in which the values of K, r_o/R and γ_o for the third elbow can be found from Eqs. (28), (30), and (37) together with the results in Figs. 11, 12 and 13. By using the measured values of ζ_3, ζ_2 and also the values of M''_{d3} described just above, the value of ζ_3^* in Eq. (25) can be found from the relation $\zeta_3^* = \zeta_3 - (2M''^2_{d3} + \zeta_2)$, which can be expressed in the empirical formulae:

For 90°-elbows:

$$\left.\begin{aligned} \zeta_3^* &= 0.2 && 0^\circ \leqq \gamma_0 \leqq 90^\circ, \quad 270^\circ \leqq \gamma_0 \leqq 450^\circ \\ \zeta_3^* &= 0.014\exp\left[-0.3\frac{L_{m1}}{d}(\gamma_0 - 112.5^\circ)\right] + 0.2, && 90^\circ < \gamma_0 \leqq 180^\circ \\ \zeta_3^* &= -0.014\exp\left[-0.3\frac{L_{m1}}{d}(\gamma_0 - 247.5^\circ)\right] + 0.2, && 180^\circ \leqq \gamma_0 < 270^\circ \end{aligned}\right\} \tag{38}$$

For 45°-elbows:

$0^\circ \leqq \phi \leqq 180^\circ, \quad L_{m1},\ L_{m2} \geqq 0$

$$\zeta_3^* = \left|\left[-0.617\times10^{-6}(\phi - 90^\circ)^2 + 0.5\times10^{-2}\right]\times(\phi - 90^\circ)\right| + 0.235\times10^{-4}(\phi - 90^\circ)^2 - 0.19 \tag{39-1}$$

$180^\circ \leqq \phi \leqq 360^\circ$

$$L_{m2} = 0, \qquad \zeta_3^* = -0.042\left(\frac{L_{m1}}{d} - 6\right) \tag{39-2}$$

$$L_{m2} \neq 0, \qquad \zeta_3^* = -0.048\left(\frac{L_{m2}}{d} - 5\right) \tag{39-3}$$

(b) Formulae for Hydraulic Loss in the Case (ii)

The value of M''_{d3} in Eq. (26) can be found from Eq. (2). The value of $\overline{\zeta}_1$, exhibiting the pressure loss across the third elbows, can be found from experiments performed in axi-symmetric flow. The results can be expressed as:

For 90°-elbows:

$$\left.\begin{aligned} \overline{\zeta}_1 &= 0.775M'_u + 0.735, && M'_u \leqq 0.2 \\ \overline{\zeta}_1 &= -0.6125M'_u + 1.0125, && M'_u \geqq 0.2 \end{aligned}\right\} \tag{40}$$

$$\overline{\zeta}'_1 = \overline{\zeta}_1 - 0.2 \tag{41}$$

In 45°-elbows, the value of $\overline{\zeta}_1$ changes with M'_u, but little as is seen in Fig. 15, and it may be taken approximately to be a constant:

$$\overline{\zeta}'_1 = 0.39 = \overline{\zeta}_1 \tag{42}$$

The value of M'_u in Eq. (38) is the dimensionless expression of angular momentum of water through the section at $L_d = (L_{m2}/d) - 1$, behind the second elbow exit. The section situates also one pipe diameter upstream from the third elbow entrance. The value of M'_u can be obtained from Eq. (29) as

$$\left.\begin{array}{l} M_u' = 0.628\exp(\log(10(M_{d2}'/2\pi)_{Ld=1d}) - 0.0235(L_{m2}/d-2)) \\ \text{when } (M_{d2}'/2\pi)_{Ld=1d} \neq 0 \quad \text{and} \\ M_u' = 0 \qquad \text{when } (M_{d2}'/2\pi)_{Ld=1d} = 0 \end{array}\right\} \tag{43}$$

In the case of 45°-elbows, the swirling component behind the third elbow M_{d3}'' is always very small, and $\zeta_3 \gg 2M_{d3}''^2$.
In this case

$$\zeta_3 = \zeta_2 + \bar{\zeta}_1' \tag{44}$$

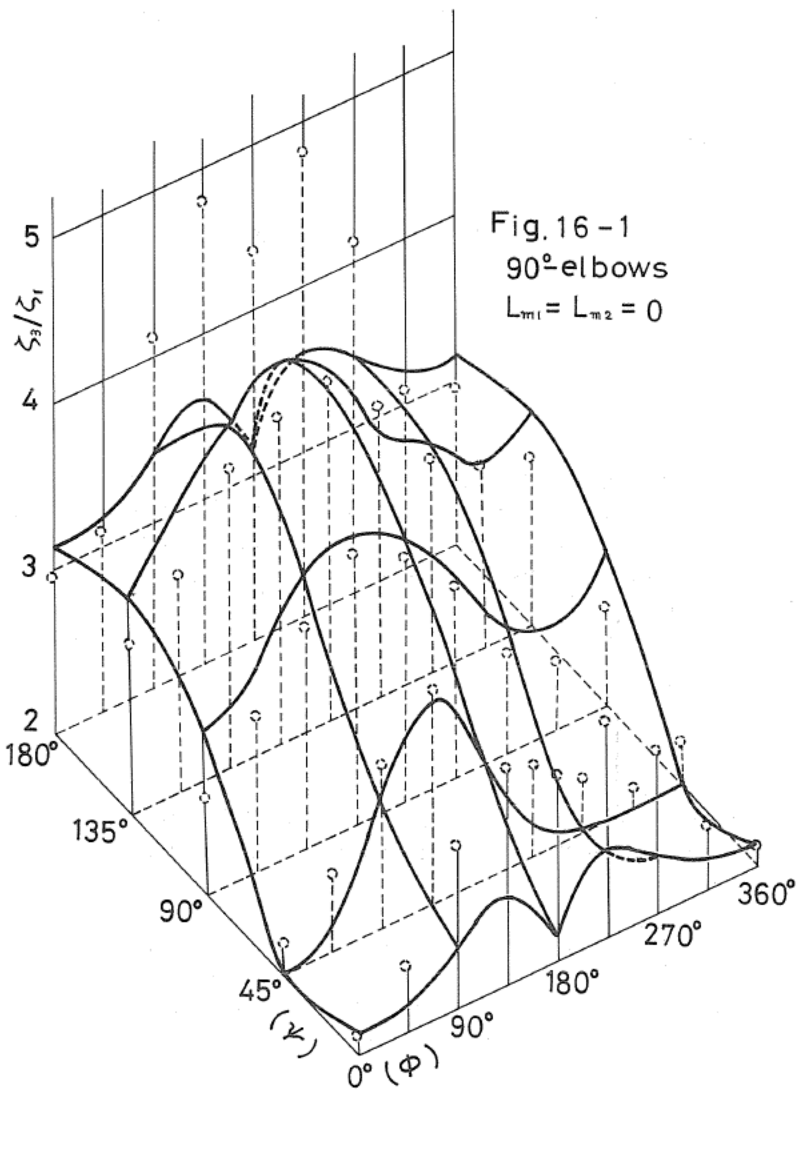


Fig. 16-1
90°-elbows
$L_{m1} = L_{m2} = 0$

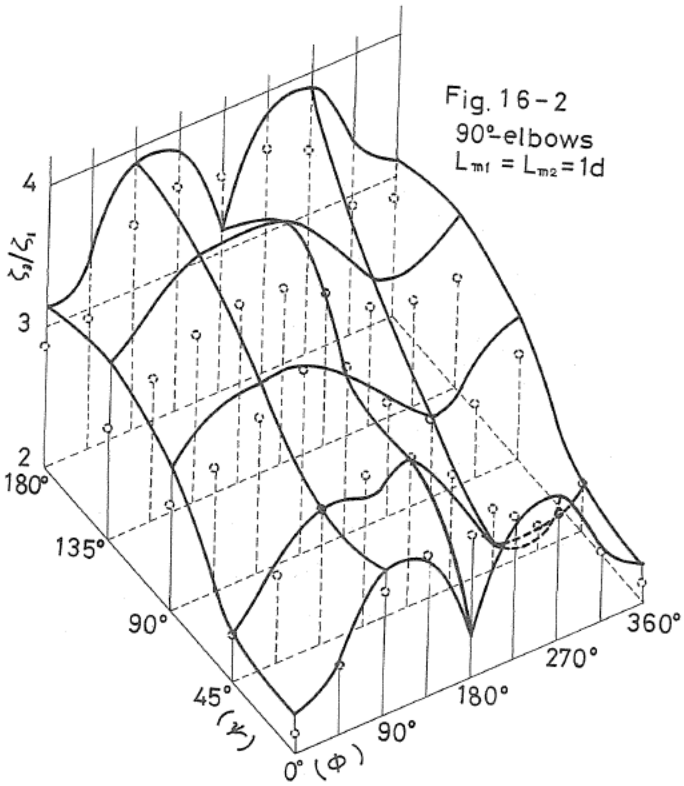


Fig. 16-2
90°-elbows
$L_{m1} = L_{m2} = 1d$

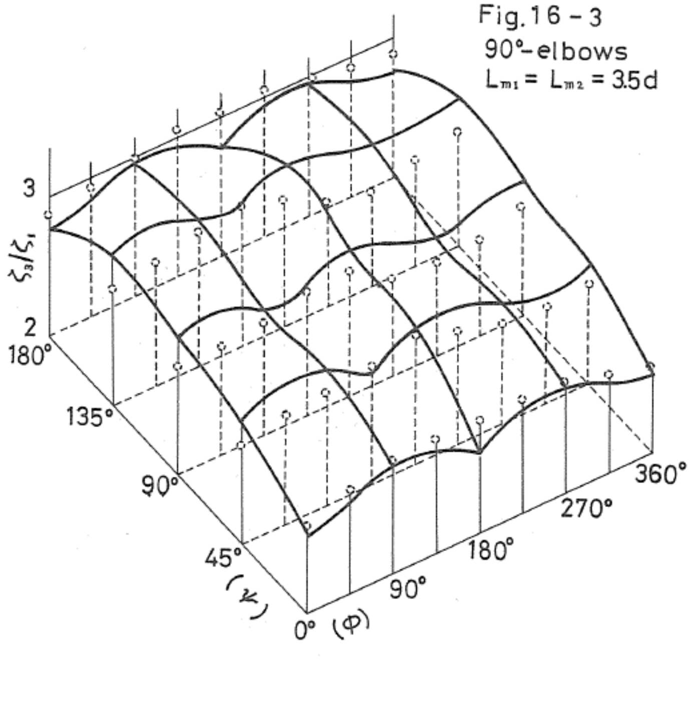


Fig. 16-3
90°-elbows
$L_{m1} = L_{m2} = 3.5d$

Fig. 16-4

45° elbows

$L_{m1} = L_{m2} = 0$

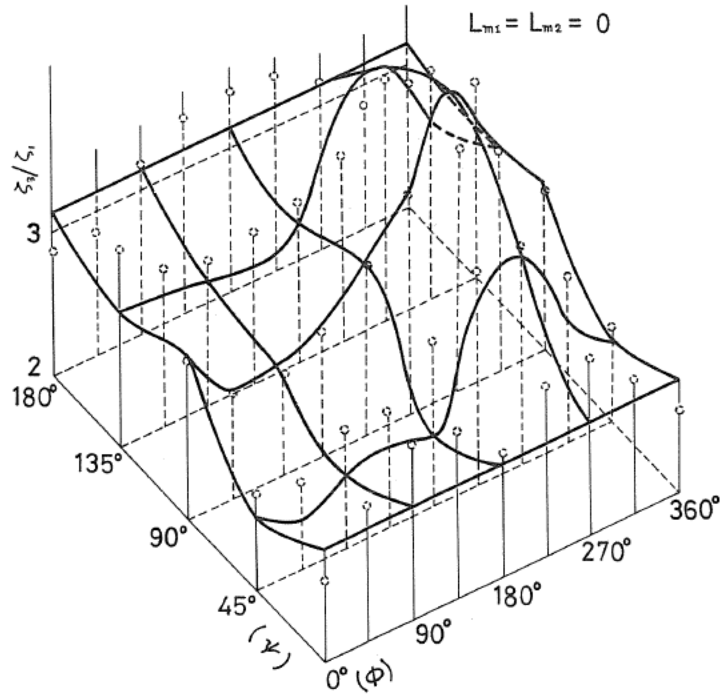
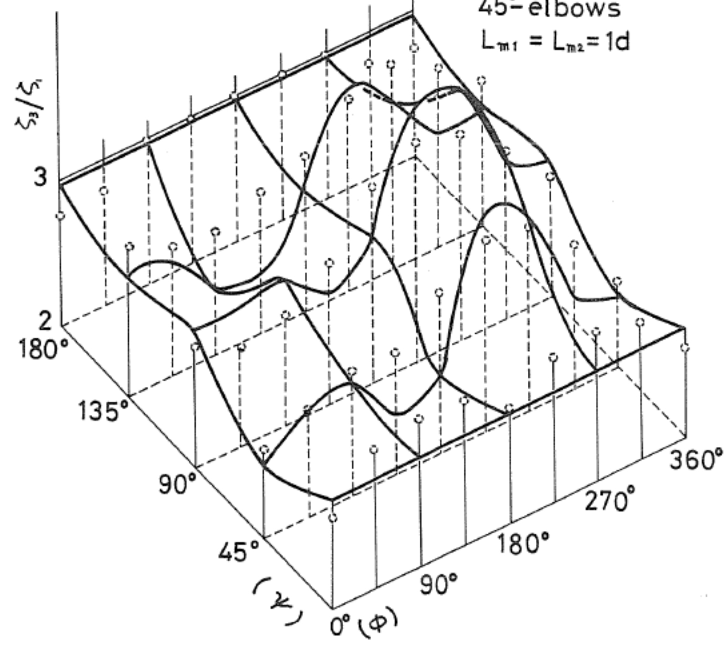
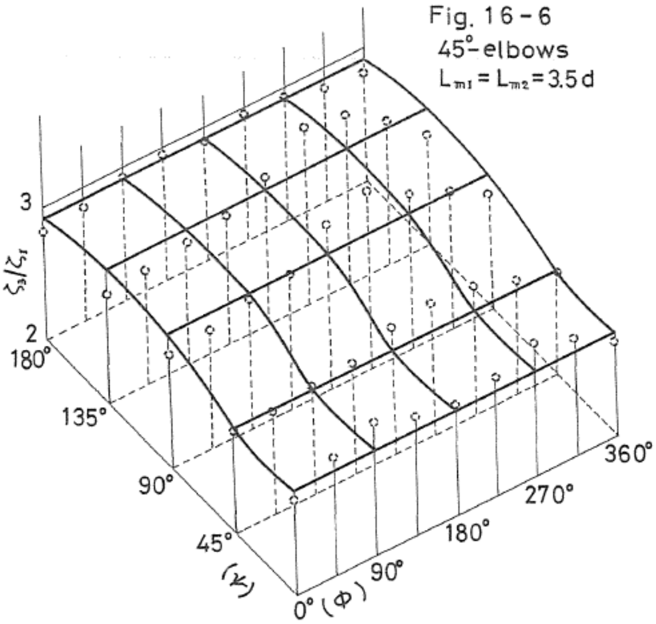


Fig. 16-5

45° elbows

$L_{m1} = L_{m2} = 1d$





	Two 90°-elbows	Two 45°-elbows
ζ_2	$\zeta_2^* - 2(M_{\theta 1}^{*})^2_{L_{m1}=1d}$	$\zeta_2^* - 2(M_{\theta 1}^{*})^2_{L_{m1}=1d}$
$(M_{\theta 1}^{*})_{L_{m1}=1d}$	$-6.28b(L_{m1}/d-6)$	$-6.28b(L_{m1}/d-7)$
b	Refer to Fig. 4(b)	Refer to Fig. 4(b)
ζ_2^*	$0°\leq\psi\leq 90°$ $-0.122\times10^{-3}(L_{m1}/d-6)^2(\psi-90°)-0.02724(L_{m1}/d)^2+0.2782(L_{m1}/d)+0.64$	$0°\leq\psi\leq 90°$ $\psi(0.556(L_{m1}/d)-2.77)\times10^{-3}+0.69$
	$90°\leq\psi\leq 180°$ $0.265\times10^{-3}(L_{m1}/d-7)^2(\psi-90°)-10\sin(4(\psi-90°)))-0.02724(L_{m1}/d)^2+0.2782(L_{m1}/d)+0.64$	$90°\leq\psi\leq 180°$ $(0.788(L_{m1}/d)-4.1)\times10^{-3}(\psi-90°)+0.44+0.05(L_{m1}/d)+(0.0123(L_{m1}/d)-0.064)\sin(4(\psi-90°))$

	Three 90°-elbows	Three 45°-elbows
ζ_3	$\zeta_2+\zeta_3^*+2M_{\theta 3}^{*2}$ ······(1) $\zeta_2+\bar{\zeta}_1'+2M_{\theta 3}^{*2}$ ······(2)	$\zeta_2+\zeta_3^*+2M_{\theta 3}^{*2}$ ······(1) $\zeta_2+\bar{\zeta}_1'$ ············(2)
ζ_3^*	$0°\leq\gamma_e\leq 90°$, $270°\leq\gamma_e\leq 450°$, 0.2	$0°\leq\varphi\leq 90°$, $L_{m1},L_{m2}\geq0$: $(-0.617\times10^{-7}(\psi-90°)^2+0.5\times10^{-3})(\varphi-90°)+0.235\times10^{-7}(\psi-90°)^2-0.19$
	$90°\leq\gamma_e\leq 180°$ $0.014\exp(-0.3\frac{L_{m}}{d})\sin(\gamma_e-112.5°))+0.2$	$90°\leq\varphi\leq 180°$, $L_{m1},L_{m2}\geq0$: $-(-0.617\times10^{-7}(\psi-90°)^2+0.5\times10^{-3})(\varphi-90°)+0.235\times10^{-7}(\psi-90°)^2-0.19$
	$180°<\gamma_e<270°$ $-0.014\exp(-0.3\frac{L_{m}}{d})\sin(\gamma_e-247.5°))+0.2$	$180°<\varphi<360°$, $L_{m1}\neq0$; $L_{m2}=0$: $-0.0424(L_{m1}/d-6)$ $180°<\varphi<360°$, $L_{m1}=0$; $5d\geq L_{m2}>0$: $-0.048(L_{m2}/d-5)$; $5d<L_{m2}$: 0.0
$\bar{\zeta}_1'$	$M_\theta'\leq0.2$, $0.775M_\theta'+0.735$ $M_\theta'>0.2$, $-0.613M_\theta'+1.013$	0.39
M_θ'	$M_{\theta 3}^{*}=0$, 0.0 $M_{\theta 3}^{*}\neq0$, $0.628\exp(\mathrm{Log}00(\frac{M_{\theta 3}^{*}}{2\pi})_{L_{m}=1d})-0.0235(L_{m1}/d-2))$	
$M_{\theta 3}^{*}$	$K\pi(r_e/R)\sin\gamma_e\sin90°$	$K\pi(r_e/R)\sin\gamma_e\sin45°$
$K\sin\Gamma$	$\gamma_e=0°,180°,360°$: 0.0	$\gamma_e=0°,180°,360°$: 0.0
	$0°<\gamma_e<180°$, $\gamma_e>360°$ $-11(r_e/R-0.1)^2+0.66+\frac{0.0044(\gamma_e-100°)}{\sqrt{4.788\times10^{-4}(\gamma_e-100°)^2+1}}$	$0°<\gamma_e<180°$ $-27.7(r_e/R-0.1)^2+1+\frac{0.0044(\gamma_e-80°)}{\sqrt{1.004\times10^{-3}(\gamma_e-80°)^2+1}}$
	$180°<\gamma_e<360°$ $-11(r_e/R-0.1)^2+0.66-\frac{0.0044(\gamma_e-260°)}{\sqrt{4.788\times10^{-4}(\gamma_e-260°)^2+1}}$	$180°<\gamma_e<360°$ $-27.7(r_e/R-0.1)^2+1-\frac{0.0044(\gamma_e-280°)}{\sqrt{1.004\times10^{-3}(\gamma_e-280°)^2+1}}$
γ_e	$\omega_e+q(L_{m2}/d-1)+\varphi$	$\omega_e+q(L_{m2}/d-1)+\varphi$
r_e/R	$(r_e/R)_{L_{m}=1d}\exp(-0.36(L_{m1}/d-1))$	$(r_e/R)_{L_{m}=1d}\exp(-0.17(L_{m1}/d-1))$
	q, ω_e and $(r_e/R)_{L_{m}=1d}$: Refer to Fig. 11 and 12, 13	q, ω_e and $(r_e/R)_{L_{m}=1d}$: Refer to Fig. 11 and 12, 13

When $L_{m1}, L_{m2} > 6d$,

For 45°- or 90°-elbows $\zeta_2 = 2\zeta_1$

$\zeta_3 = 3\zeta_1$

Table 1

Tables 2(a) and (b) show the suitable range of applications of the cases (i) and (ii). The experimental formulae described above are compared with measurements in Fig. 16. Except the region of $L_{m1} = L_{m2} = 0$ and $\psi = 0°$ or $180°$, the agreement is satisfactory for engineering purpose.

L_{m2}/d \ L_{m1}/d	0	1	2.5	3.5	5.2
0	eq. 1	eq. 1	eq. 1	eq. 1	eq. 1
1	〃 1	〃 1	〃 1	〃 1	〃 1
2.5	〃 1	〃 1	〃 2	〃 2	〃 2
3.5	〃 2	〃 2	〃 2	〃 2	〃 2
5.2	〃 2	〃 2	〃 2	〃 2	〃 2

(b) For 45°-elbows

L_{m2}/d \ L_{m1}/d	0	1	2.5	3.5	5.2
0	eq. 1	eq. 1	eq. 1	eq. 1	eq. 1
1	〃 1	〃 1	〃 1	〃 1	〃 1
2.5	〃 2	〃 2	〃 1	〃 1	〃 1
3.5	〃 2	〃 2	〃 2	〃 2	〃 2
5.2	〃 2	〃 2	〃 2	〃 2	〃 2

(a) For 90°-elbows

Table 2 Applied examples of eq.(1) and (2) in Table 1

5. Conclusion

Experimental formulae for the hydraulic loss due to elbows combined in series in a pipe line are obtained and the results is given in Tables 1 and 2. Fortran program for the formulae is supplemented.

References

1) M. Murakami, Y. Shimizu and H. Shiragami, Bull. JSME, Vol. 12, No. 54 (1969-12), p. 1369.
2) M. Murakami and Y. Shimizu, Bull. JSME, Vol. 16, No. 96 (1973-7), p. 981
3) M. Murakami and Y. Shimizu, Trans. JSME, Vol. 40, No. 336 (1974-6). p. 1739.