

1 Primitive chain network simulations for the interrupted shear response of entangled polymeric liquids

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9  
10 **ABSTRACT**

11 The non-linear viscoelastic response under interrupted shear flows is one of the interesting  
12 characteristics of entangled polymers. In particular, the stress overshoot in the resumed shear has been  
13 discussed concerning the recovery of the entanglement network in some studies. In this study, we  
14 performed multi-chain slip-link simulations to observe the molecular structure of an entangled  
15 polymer melt. After confirming the reasonable reproducibility of our simulation with the literature  
16 data, we analyzed the molecular characteristics following the decoupling approximation. We  
17 reasonably found that the segment orientation dominates the stress overshoot even under the resumed  
18 shear with minor contributions from the segment stretch and entanglement density. We defined the  
19 mitigation function for the recovery of stress overshoot as a function of the rest time and compared it  
20 with the relaxation of the molecular quantities after the initial shear. As a result, we have found that  
21 the mitigation of stress overshoot coincides with the relaxation of entanglement density.

22  
23 **Keywords**

24 viscoelasticity, polymers, molecular simulation

25  
26 **INTRODUCTION**

27 Stress growth under interrupted shear flows is one of the interesting non-linear viscoelastic behaviors  
28 of entangled polymers<sup>1,2</sup>. In this rheological test, an equilibrated polymeric liquid is subjected to fast,  
29 steady shear. After a certain amount of applied shear, the flow is interrupted to relax the material.  
30 During the relaxation, before reaching equilibrium, the material is exposed to the second shear. The  
31 material response under the second shear depends on the interval between initial and resumed flows-  
32 the so-called rest time. With a sufficiently long rest time, the material equilibrates, and the stress  
33 growth under resumed flow is identical to that under the initial flow. More specifically, the stress  
34 increases up to a strain of approximately 2 to 3 and shows an overshoot before reaching a steady value.  
35 The maximum stress at the overshoot monotonically decreases with decreasing rest time. Because no  
36 overshoot occurs in the absence of a rest, the decline of the overshoot is intuitive in a qualitative sense.

37 However, no quantitative explanation has been established.

38

39 In some studies, the mitigation of stress overshoot has been attributed to a structural change of the  
40 entanglement network. To our knowledge, network modification under deformation was firstly  
41 mentioned by Tobolsky et al.<sup>3,4</sup> according to the transient network idea. Graessley<sup>5</sup> described the shear  
42 thinning of polymeric liquids by considering the rates of creation and destruction of entanglements.  
43 Doi and Edwards<sup>6</sup> derived the universal damping function under large step shear deformations from  
44 the loss of entanglements induced by chain contraction. Similar discussions have been made for  
45 interrupted shear<sup>7-11</sup>.

46

47 The non-linear response under interrupted shear is not necessarily solely due to modification of the  
48 entanglement network. For example, Santangelo and Roland<sup>12</sup> showed that an unentangled  
49 polystyrene melt exhibits similar rest time dependence of stress overshoot. They mentioned that the  
50 suppression of overshoot is due to the unrelaxed segmental orientation. Even for entangled polymers,  
51 orientation contributes to stress, as discussed for the stress overshoot during the initial shear<sup>13-16</sup> on  
52 the basis of the stress-optical law. Indeed, Ianniruberto and Marrucci<sup>17</sup> demonstrated that the integral  
53 constitutive equation proposed by Doi and Edwards<sup>18</sup> qualitatively reproduces the interrupted shear  
54 results reported by Wang et al.<sup>11</sup>, even though the model incorporates neither chain stretch nor loss of  
55 entanglement. In addition, Graham et al.<sup>19</sup> semi-quantitatively reproduced the data of Wang et al.<sup>11</sup>  
56 using a tube model, in which chain stretch is incorporated but the entanglement density is unchanged.  
57 Nevertheless, as noted by Ianniruberto and Marrucci<sup>17</sup>, further studies are required to discriminate the  
58 contributions from orientation, stretch and entanglement density in the stress response under  
59 interrupted shear.

60

61 In this study, through the multi-chain slip-link simulation (the so-called primitive chain network (PCN)  
62 simulation<sup>20-23</sup>), we investigated the stress response of an entangled polymer melt under interrupted  
63 shear flows. The observed stress response was in reasonable agreement with the experimental results  
64 of Roy and Roland<sup>24</sup>. We obtained the time development of segmental orientation, stretch, and  
65 entanglement density separately. The results demonstrate that, for the stress overshoot, the contribution  
66 from the orientation is dominant, whereas the contribution from the entanglement density is minor.  
67 Meanwhile, the mitigation of overshoot coincides with the relaxation of entanglement density.

68

## 69 **MODEL AND SIMULATIONS**

70 Because the model and code used in this study are the same as those used for previous studies<sup>25-29</sup>,  
71 only a brief description is given here. A melt of entangled polymer was replaced by a network  
72 consisting of network nodes, strands and dangling ends. Each polymer chain was represented by a

73 path connecting two dangling ends through the nodes and strands. At each node, a slip-link was  
74 positioned to bundle two chains according to the binary assumption of entanglement. The slip-link  
75 allows sliding of the bundled chains, and it is destroyed if one of the chains slides off. By contrast,  
76 when a chain end protrudes from the connected slip-link by a critical amount, a new slip-link is created  
77 on the dangling segment by hooking another strand within a cut-off distance equal to the average  
78 strand length at equilibrium. The position of the slip-link obeys a Langevin-type equation of motion,  
79 in which the force balance is considered among the drag force, osmotic force, tension acting on  
80 diverging strands, and the random force. The chain sliding is described by a rate equation for the  
81 number of Kuhn segments on each strand. The transport rate of the Kuhn segments between  
82 consecutive strands is calculated along the chain according to the force balance considered for the  
83 slip-link position.

84  
85 As summarized earlier<sup>21-23</sup>, the PCN model is located in a niche between the tube<sup>30-32</sup> and the bead-  
86 spring<sup>33</sup> models; namely, the unit of calculation is the entanglement segment, which is also used in  
87 single-chain slip-link models<sup>34-37</sup>. Meanwhile, the multi-chain dynamics is calculated as performed  
88 for the bead-spring simulations and the other coarse-grained models that prohibits chain crossing<sup>33,38-</sup>  
89 <sup>40</sup>. The multi-chain nature of the model excludes the mean-field type assumptions for thermal<sup>41-43</sup> and  
90 convective<sup>44</sup> constraint release. The disturbance from the affine deformation owing to the force  
91 balance around the entanglement<sup>45</sup> is also considered. The multi-chain construction was followed by  
92 multi-chain slip-spring models<sup>46-49</sup>, in which many Rouse chains dispersed in a simulation box are  
93 connected by virtual springs. Except for the level of coarse-graining, these multi-chain models exhibit  
94 similar features to reproduce entangled polymer dynamics<sup>50</sup>.

95  
96 In this study, we examined the experiment reported by Roy and Roland<sup>24</sup> for a polyisobutylene (PIB)  
97 melt ( $M_w = 46,000$  and  $M_w/M_n = 3.2$ ). From this value of  $M_w$ , we determined the number of  
98 strands per chain was  $Z_0 = 12$ . This value of  $Z_0$  means that the molecular weight carried by a single  
99 network strand is  $M_0 = M_w/Z_0 \sim 3800$ . We note that  $M_0$  is much smaller than the entanglement  
100 molecular weight  $M_e = 9400$ <sup>24</sup>. This difference is due to the additional fluctuations implemented in  
101 the PCN model<sup>51,52</sup>. We also note that the value of  $M_0$  determines the unit of modulus,  $G_0$ , by the  
102 dimensional analysis as  $G_0 = \rho RT/M_0$ , where  $\rho$  is the density. Although this relation works well for  
103 monodisperse systems, in this specific study,  $G_0$  was determined independently of  $M_0$  to  
104 accommodate for the effect of the molecular weight distribution (MWD) in the experiment<sup>24</sup>, as shown  
105 later. Nevertheless, for simplicity in the molecular analysis, we simulated the monodisperse system.  
106 The simulations were performed with non-dimensional units, where the units of length, energy, and  
107 time were chosen as the average strand length,  $a$ , thermal energy,  $k_B T$ , and the diffusion time for a  
108 node,  $\tau_0 = \zeta a^2 / 6 k_B T$ . Here,  $\zeta$  is the friction of the single node. The simulation box dimension was

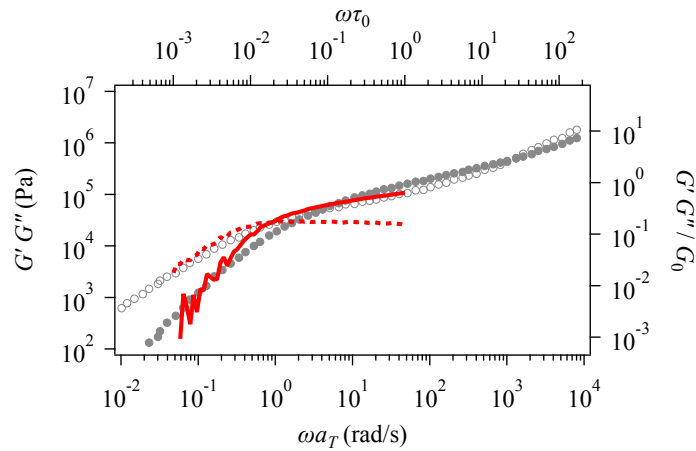
109  $16^3$ , which is sufficiently larger than the chain dimension even under fast shear. The end-to-end  
 110 distance  $\sqrt{\langle \mathbf{R}^2 \rangle}$  was 3.7 at equilibrium, and 8.2 under the fastest shear among all examined shear  
 111 rates. The strand number density under equilibrium was 10. For this system, the Rouse time is given  
 112 as  $\tau_R = Z_0^2 / 2\pi^2 = 7.3\tau_0$  according to a previous study<sup>53</sup>. The longest relaxation time for the  
 113 viscoelastic relaxation was obtained from the equilibrium simulation<sup>50,54</sup> as  $\tau_d = 77\tau_0$ . To resolve  
 114 the slight changes in stress, we performed 24 independent simulation runs for each condition for  
 115 different initial equilibrated configurations.

116

## 117 RESULTS AND DISCUSSION

### 118 Comparison with the experimental results

119 Figure 1 shows the linear viscoelasticity of the sample. Comparison between the experimental data  
 120 (symbols) and the simulation results (curves) gives the model parameters as  $G_0 = 0.174$  MPa, and  
 121  $\tau_0 = 0.021$  s. As we mentioned in the previous section,  $G_0$  must generally be determined from  $M_0$ .  
 122 However, in this specific study, we optimized  $G_0$  separately from  $M_0$  to accommodate for the effects  
 123 of the MWD on the compliance (i.e., the crossover between  $G'$  and  $G''$ ). The value of  $\tau_0$  was  
 124 determined to reproduce the longest relaxation mode.



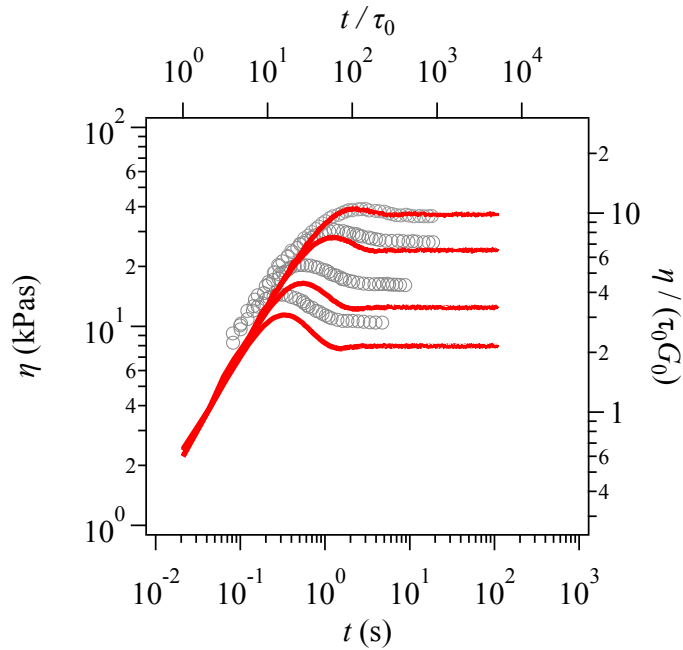
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126 **Figure 1** Linear viscoelasticity. The filled and unfilled circles are the experimental data taken from  
 127 the literature<sup>24</sup>. The solid and broken curves are the simulation results.

128

129 Figure 2 shows the viscosity growth under start-up shear from equilibrium. The experimental data<sup>24</sup>  
 130 are shown for comparison. The simulation reproduces the viscosity overshoot and shear thinning only  
 131 qualitatively. In particular, the steady state viscosity is overestimated in the simulation. Similar  
 132 discrepancies have been reported even for monodisperse systems<sup>25-29</sup>. The tube model also has a  
 133 similar feature, and the mechanism of this discrepancy is unknown. Nevertheless, hereafter, we discuss  
 134 the response under  $1 \text{ sec}^{-1}$  in detail.

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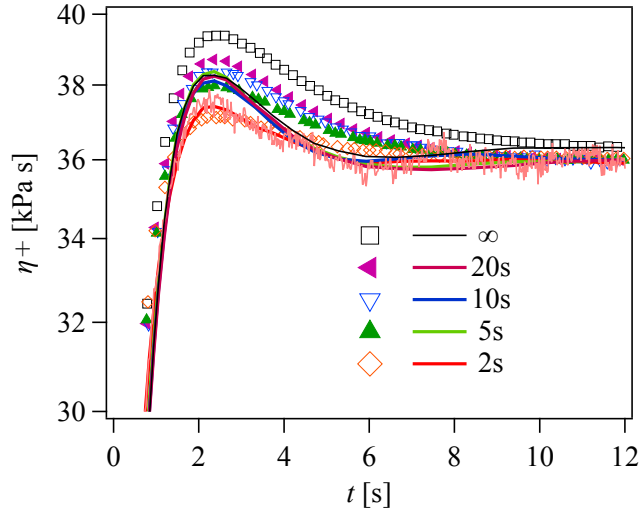
136

137 **Figure 2** Viscosity growth under start-up shear at shear rates of 1, 2, 5 and 10  $\text{sec}^{-1}$  from top to bottom.

138 The solid curves and symbols represent the simulation and experimental results<sup>24</sup>, respectively.

139

140 Figure 3 shows the viscosity growth under resumed shear at a shear rate of  $1\text{s}^{-1}$ . The prediction for the  
 141 first start-up run (black curve) exhibits a discrepancy from the experimental data (black symbols) in  
 142 this comparison. Indeed, the stress fluctuations in the simulation are concealed in the double-log plot  
 143 in Figure 2, whereas they are visible in Figure 3 even after ensemble averaging for 24 independent  
 144 simulation runs, as shown by the red thin curve for a rest time of 2 s. To extract the peak, we smoothed  
 145 the simulation results using the second-order Savitzky-Golay method<sup>55</sup>. For the smoothed curves, the  
 146 simulation qualitatively captures the experiment; namely, the magnitude of the viscosity overshoot  
 147 increases with increasing rest time to recover the behavior exhibited under initial shear. Possibly owing  
 148 to the MWD, the simulated viscosity decreases to a steady value faster than the experimental one.



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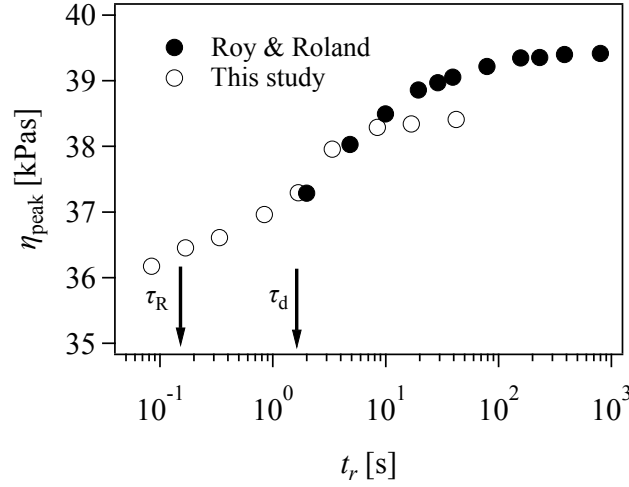
150 **Figure 3** Viscosity growth under resumed shear at a shear rate of  $1\text{s}^{-1}$  for various rest times after the  
 151 initial shear. The data for the initial startup run are shown as an infinite rest time. The bold curves are  
 152 the smoothed simulation results. The thin curve is the result for a rest time of 2s without smoothing.  
 153 The symbols indicate the experimental results from the literature<sup>24</sup>.

154

155 Figure 4 shows the peak value of the viscosity plotted as a function of the rest time,  $t_r$ . The  
 156 viscoelastic relaxation time ( $\tau_d = 77\tau_0 = 1.6\text{ s}$ ) and the Rouse time ( $\tau_R = 7.3\tau_0 = 0.15\text{ s}$ ) for the  
 157 simulation are indicated for comparison. The experimental data are not available for short  $t_r$  possibly  
 158 due to experimental limitations. Consequently, direct comparison can only be made within a limited  
 159 range of  $t_r$ . Nevertheless, the simulation reproduces the experiment reasonably well when  $t_r \sim \tau_R$ ,  
 160 where the peak value increases with increasing  $t_r$  to reach the same value as under the initial shear.  
 161 However, for  $t_r > \tau_d$ , the viscosity overshoot is insensitive to  $t_r$  in the simulation, whereas it  
 162 increases with  $t_r$  in the experiment. This difference is probably due to the MWD, which is neglected  
 163 in the simulation.

164

165



166

167 **Figure 4** Peak value of the viscosity overshoot under resumed shear as a function of rest time at a  
 168 shear rate of  $1\text{sec}^{-1}$ . The filled and unfilled symbols are the experimental<sup>24</sup> and simulation results,  
 169 respectively. The viscoelastic relaxation time and the Rouse time for the simulation are indicated for  
 170 comparison.

171

#### 172 **Decoupling analysis**

173 We carried out molecular analysis based on the decoupling approximation<sup>13</sup>, which is widely used for  
 174 molecular constitutive equations;

$$175 \quad \sigma = \frac{3}{V} \sum r_x r_y \approx 3\nu\lambda^2 S \quad (1)$$

176 Here,  $\sigma$  is the shear stress, and the sum is taken for all the strands in the simulation box with volume  
 177  $V$ .  $\mathbf{r} \equiv (r_x, r_y, r_z)$  is the strand vector,  $\nu$  is the strand number density,  $\lambda^2 (\equiv \langle \mathbf{r}^2 \rangle)$  is the average  
 178 strand stretch, and  $S (\equiv \langle (r_x r_y) / \mathbf{r}^2 \rangle)$  is the average strand orientation.

179

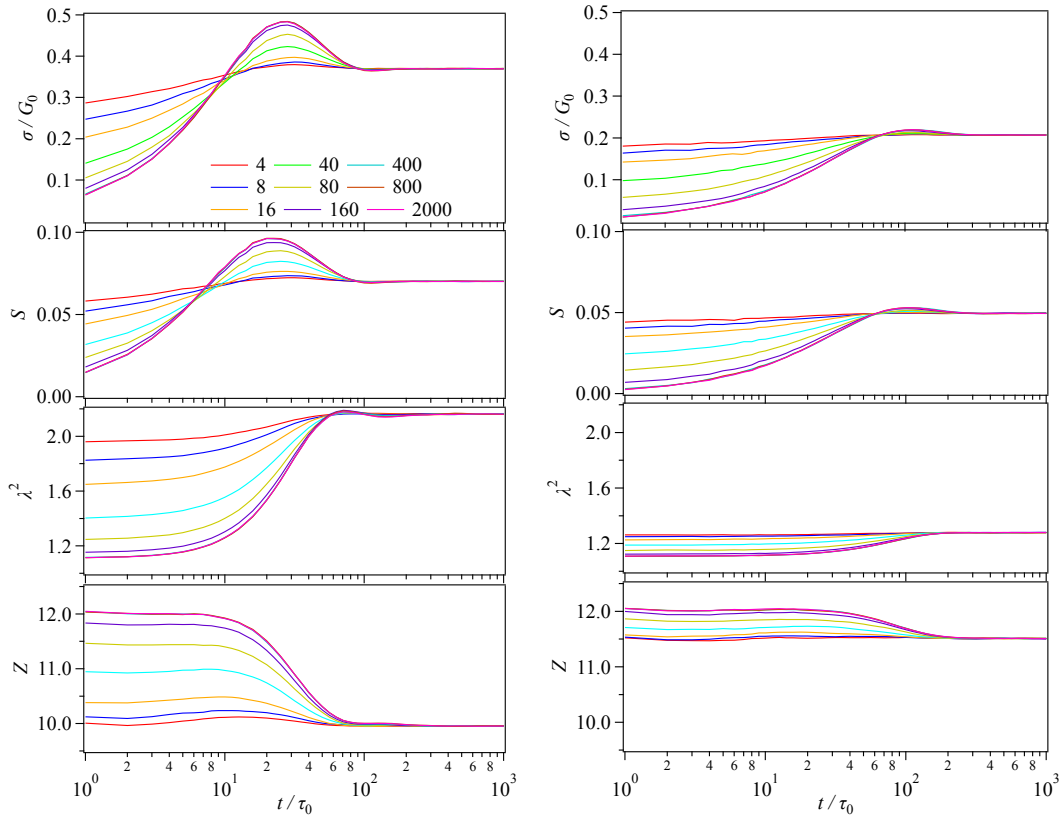
180 Figure 5 shows the transient behavior of the aforementioned molecular quantities under the resumed  
 181 shear for various  $t_r/\tau_0$  values at shear rates of  $10\text{sec}^{-1}$  and  $1\text{sec}^{-1}$ . Note that we hereafter discuss the  
 182 segment number per chain,  $Z$ , instead of  $\nu$  hereafter. As shown in Fig 3, the magnitude of stress  
 183 overshoot decreases with an increase of  $t_r$ . Being consistent with the stress-optical rule, the  
 184 orientation exhibits a similar behavior. For a shear rate of  $10\text{ s}^{-1}$  ( $Wi_d = 16$ ) (Figure 5; left panels),  
 185 the stretch contributes to the stress to delay the peak position from that of the orientation. The segment  
 186 number develops similarly to the stretch, but the magnitude of change is much smaller than that for  
 187 the stretch. Specifically,  $\lambda^2$  increased by about 100% (Fig 5, purple line in the second panel from the  
 188 bottom on the left), whereas the change for  $Z$  is less than 20%. The situation is similar for lower  
 189 strain rates. We note that  $\lambda^2$  does not start from unity even when the rest time is sufficiently long.  
 190 Indeed, in our code,  $\lambda^2$  is slightly ( $\sim 10\%$ ) larger than unity at equilibrium because of the artificial

191 stiffness induced by a numerical cut-off for the monomer number on each strand.

192

193 Figure 5 (right panels) shows the case of  $\dot{\gamma}=1\text{ s}^{-1}$  ( $Wi_d = 1.6$ ), where the stress is dominated by the  
 194 orientation, and the changes for  $\lambda^2$  and  $Z$  are less pronounced. Consequently, the stress overshoot  
 195 is dominated by the orientation, and the strand density plays a secondary role. This result is consistent  
 196 with that of Ianniruberto and Marrucci<sup>17</sup>, who qualitatively reproduced the interrupted shear response  
 197 using a tube model without chain stretch and variable entanglement density.

198



199

200 **Figure 5** Time development of stress, orientation, stretch, and segment number per chain (from top to  
 201 bottom) under resumed shear. The shear rates are  $10\text{ s}^{-1}$  (left panel) and  $1\text{ s}^{-1}$  (right panel). The  
 202 normalized rest time ( $t_r/\tau_0$ ) is indicated in the figure. Note that the curves for rest times longer than  
 203 400 ( $\gg \tau_d$ ) overlap.

204

### 205 Structural relaxation during the rest

206 We now discuss whether the recovery of stress overshoot under resumed shear reflects structural  
 207 relaxation during the rest after the initial shear. Although the stress overshoot is dominated by the  
 208 orientation under resumed shear, as seen in Figure 5, the magnitude of overshoot may correlate with  
 209 the relaxation of other molecular characteristics.

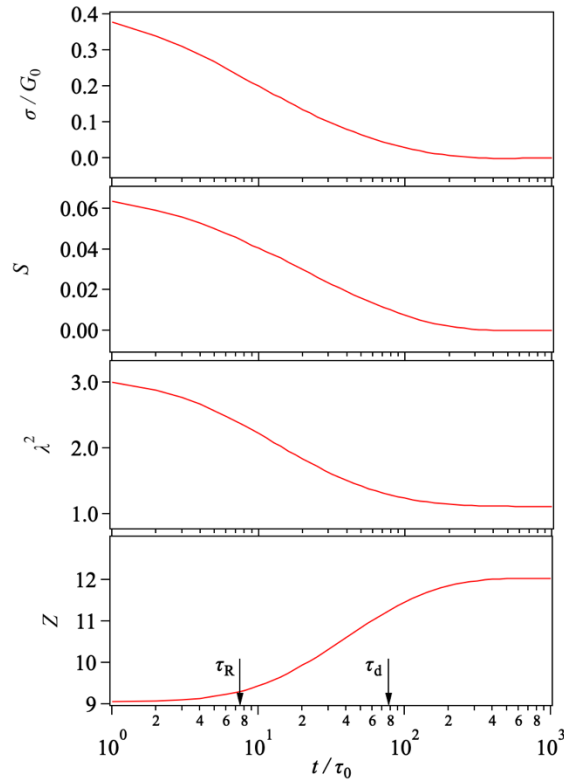


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211 Figure 6 shows the relaxation of stress and molecular characteristics after the initial shear. The  
 212 orientational relaxation is similar to the stress relaxation, which decays to zero during  $\tau_d$ . The stretch  
 213 decreases to an equilibrium value reflecting the chain contraction, and the longest relaxation time is  
 214 comparable to  $\tau_d$  rather than  $\tau_R$ . The number of entanglements per chain recovers from a reduced  
 215 value, and the recovery time is close to  $\tau_d$ . We note that in the original CCR theory<sup>44,56</sup>, the  
 216 entanglement density is assumed to be constant, whereas the entanglement renewal is accelerated  
 217 under shear. However, molecular simulations<sup>27,57-59</sup> have revealed that the entanglement density  
 218 decreases under fast flows as a consequence of CCR.

219

220 One may argue that the relaxation time of stretch would be  $\tau_R$  rather than  $\tau_d$ . However, we note that  
 221 the contraction with  $\tau_R$  is considered for the contour length in the tube theory<sup>30</sup>, and not for each  
 222 segment. Indeed, even for our model, the relaxation time of the contour length is  $\tau_R$ , as reported  
 223 previously<sup>18</sup>. Meanwhile, for the segment stretch, a retarded contraction has been reported<sup>60,61</sup>. This  
 224 retardation is due to the reduced number of entanglements as a result of CCR, as mentioned above.  
 225 When a fast and large deformation results in disentanglements, the number of Kuhn steps on each  
 226 strand becomes larger than the equilibrium value. In such a configuration, the tension acting on each  
 227 strand becomes weaker, thereby slowing contraction. Because the entanglement density recovers its  
 228 equilibrium state via reptation, the relaxation time of segment stretch is close to  $\tau_d$  rather than  $\tau_R$ <sup>60,61</sup>.



229

230 **Figure 6** Relaxation from the steady state under a shear rate of  $10 \text{ s}^{-1}$  for stress, segment orientation,  
 231 segment stretch, and segment number per chain from top to bottom.

232

233 To compare the relaxation behaviors in Figure 6 with each other, we define the relaxation functions as  
 234 follows:

$$235 \quad \varphi_\sigma(t) = A_\sigma \sigma(t) \quad (2)$$

$$236 \quad \varphi_S(t) = A_S S(t) \quad (3)$$

$$237 \quad \varphi_\lambda(t) = A_\lambda (\lambda^2(t) - 1) \quad (4)$$

$$238 \quad \varphi_Z(t) = A_Z (1 - Z(t)/Z_0) \quad (5)$$

239 Figure 7 shows the relaxation functions plotted with the normalized linear relaxation modulus  
 240  $G(t)/G_0$ . The parameters in Eqs 2-5,  $A_\sigma$ ,  $A_S$ ,  $A_\lambda$  and  $A_Z$ , were chosen to match the functions with  
 241  $G(t)/G_0$  for  $t \geq \tau_d$ ; these parameters depend on the shear rate. In this regime, all relaxation  
 242 functions overlap with  $G(t)/G_0$  (broken curve) showing the identical relaxation time of  $\tau_d$ . In the  
 243 short-time range ( $t < \tau_d$ ), most of the relaxation functions depend on the shear rate. Because of the  
 244 non-linearity imposed by the shear, the stress relaxation,  $\varphi_\sigma(t)$  (red curve), does not coincide with  
 245  $G(t)$ , and the intensity of the fast relaxation modes increases with an increase of shear rate. Similar  
 246 behavior is seen for the orientational relaxation,  $\varphi_S(t)$  (yellow curve), which overlaps with  $\varphi_\sigma(t)$   
 247 for low shear rates; this similarity is consistent with the stress-optical rule. Meanwhile, for high shear  
 248 rates, the orientational relaxation becomes much lower than  $\varphi_\sigma(t)$  owing to the contribution of  
 249 stretch. The stretch relaxation  $\varphi_\lambda(t)$  (green curve) shows a growth of the intensity for fast relaxation  
 250 modes for fast shear rates due to the chain stretch.  $\varphi_Z(t)$  (blue curve) is almost insensitive to the  
 251 shear rate. Interestingly, only for this relaxation function, fast modes do not appear, even for high shear  
 252 rates. This behavior can be rationalized if the recovery of  $Z$  is dominated by reptation motion as  
 253 assumed in the tube theory<sup>30</sup>. Furuichi et al.<sup>60</sup> have reported such behavior for PCN simulations under  
 254 large step shear deformations.

255

256 We now compare the relaxation function and the recovery of stress overshoot under resumed shear.  
 257 From the recovery behavior shown in Figure 4, we define the mitigation function,  $M_\eta$ , for the recovery  
 258 of viscosity overshoot as a function of  $t_r$  as

$$259 \quad M_\eta(t_r) = B_\eta (1 - \eta_{\text{peak}}(t_r)/\eta_{\text{peak}}(\infty)) \quad (6)$$

260 Here,  $B_\eta$  is a numerical constant that depends on the shear rate. Assuming that  $t_r$  and the relaxation  
 261 period in Figure 5 are compatible, we plot  $M_\eta$  (unfilled circles in Figure 7) with a value of  $B_\eta$  that  
 262 attains reasonable comparison between  $M_\eta$  and the relaxation functions. Note that the value of  $M_\eta$   
 263 is not available for the lowest shear rate, because we did not observe any overshoot for this case. For  
 264 the other shear rates,  $M_\eta$  is close to  $\varphi_Z$ , and these two functions are almost insensitive to the shear  
 265 rate, exhibiting no growth of the intensity for fast relaxation modes. To be fair, we note that there is

266 considerable uncertainty for  $M_\eta$  under low shear rates.

267

268 One may argue that similar mitigation functions could be defined for the molecular quantities as well.

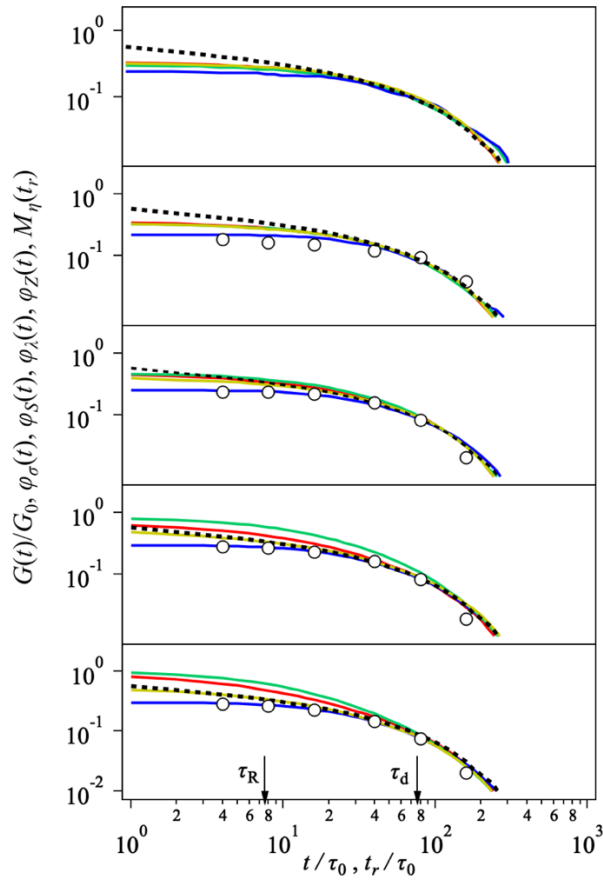
269 However, such an analysis is not straightforward. For example, as seen in Figure 4, there is no peak

270 for  $Z$ . For  $\lambda^2$ , although a faint peak is observed, it is not located at the same position as that for  $\eta$ . As

271 mentioned above,  $S$  behaves similarly to  $\eta$ . However, the peak positions do not coincide at high

272 shear rates owing to the contributions of  $Z$  and  $\lambda^2$  to the stress.

273



274

275 **Figure 7** Relaxation functions for stress (red), orientation (yellow), stretch (green), and entanglement

276 density (blue), as defined in Eqs. 2-5. The linear relaxation modulus is shown by the broken curve.

277 The mitigation function for the viscosity overshoot defined by Eq. 6 is marked with open circles. The

278 shear rates are 0.5, 1, 2, 5, and 10  $\text{s}^{-1}$  from top to bottom. The Rouse time and the longest relaxation

279 time are indicated by the arrows.

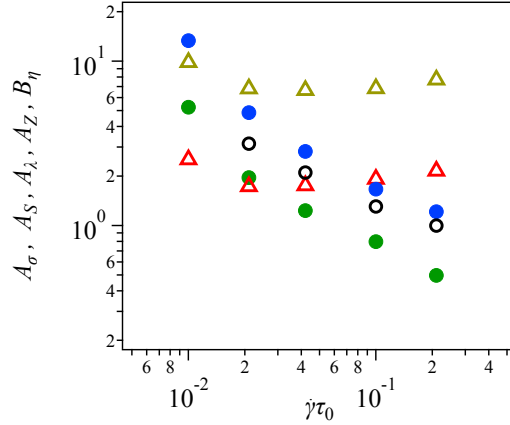
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281 Figure 8 shows the numerical constants  $A$  and  $B$  in Eqs. 2-6 plotted as functions of the shear rate.  $A_\sigma$

282 (red) and  $A_S$  (yellow) are almost insensitive to the shear rate, whereas  $A_\lambda$  (green) and  $A_Z$  (blue)

283 decrease with an increase of the shear rate in a power-law manner, for which the exponent is  
 284 approximately -1. For the mitigation function,  $M_\eta$ ,  $B_\eta$  (black) exhibits a similar decay to  $A_\lambda$  and  
 285  $A_Z$ . These shear rate dependence of  $A_\lambda$ ,  $A_Z$  and  $B_\eta$  suggest a linear response for the longest  
 286 relaxation mode of  $\varphi_\lambda(t)$ ,  $\varphi_Z(t)$  and  $M_\eta(t_r)$ .

287  
 288



289

290 **Figure 8** Numerical constants defined for Eqs. 2-6 plotted as functions of the normalized shear rate.  
 291  $A_\sigma$  and  $A_S$  are represented by unfilled triangles in red and yellow, and  $A_\lambda$  and  $A_Z$  are indicated  
 292 by filled circles in green and blue, respectively.  $B_\eta$  is represented by black unfilled circles.

293

294

295 The similarities between  $\varphi_Z$  and  $M_\eta$  seen in Figures 7 and 8 are unexpected, and we are unable to  
 296 provide a reasonable explanation for this behavior. We emphasize that the magnitude of the overshoot  
 297 is not measured for the system during the relaxation. Rather, the overshoot reflects a transient state  
 298 between a partly relaxed state and a non-equilibrium state under steady shear. Because of this  
 299 fundamental difference between the relaxation functions and the mitigation function, the results in  
 300 Figures 7 and 8 cannot be explained by conventional molecular pictures.

301

302 We note that the presented results may be related to the model settings and the algorithm used in the  
 303 conducted simulation, and that other models may give different results. Indeed, because the definition  
 304 of entanglement is model dependent, the entanglement density and its response to deformations are  
 305 also model dependent. For example, as mentioned in the Introduction, Ianniruberto and Marrucci<sup>17</sup>  
 306 argued that the response to interrupted shear can be qualitatively described using a the tube model,  
 307 whereby the entanglement density is insensitive to deformations. Even among models that assume the  
 308 entanglement density is reduced under fast and large deformations, the magnitude of entanglement  
 309 reduction is not universal<sup>59,61</sup>. Meanwhile, we note the network statistics assumed in the PCN model

310 is consistent with the topological network extracted from atomistic molecular simulations<sup>62,63</sup>. In this  
311 respect, the presented results would reflect the dynamics of the topological network in some extent.

312

313 We also note that some experimental studies suggest flaws in the conventional modeling of  
314 entanglement. For example, for the dielectric measurement of polyisoprene under steady shear,  
315 Watanabe et al.<sup>64,65</sup> have reported that the relaxation time and the relaxation intensity measured in the  
316 shear gradient direction do not depend on the shear rate. This result means that the end-to-end  
317 dimension and its fluctuations along the shear gradient direction do not change, even under fast shear,  
318 even when the viscosity shows the shear thinning. To the best of our knowledge, conventional  
319 molecular theories cannot reproduce this behavior<sup>25,27</sup>. Teixeira et al.<sup>66</sup> have conducted a direct  
320 observation of single DNA molecules under start-up shear. They compared the observed  
321 conformational change with that predicted by the tube theory, and found that the observed DNA  
322 response is much slower than the theoretical one. PCN simulations failed to reproduce this result as  
323 well, even though the viscosity growth was in quantitative agreement<sup>67</sup>. These flaws mean that the  
324 conventional molecular theories may not be compatible with the conformational dynamics of polymers,  
325 even though they have achieved remarkable success for describing the rheological response of  
326 polymers. In this respect, molecular modeling of entanglement is still a challenge, and further  
327 improvement is necessary for the conventional models, including PCN. The commonly applied  
328 assumptions, such as the homogeneous deformation and the binary contact at the entanglement point,  
329 might be problems that should be addressed in future investigations.

330

## 331 **CONCLUSIONS**

332 We performed multi-chain slip-link simulations for an entangled polymer under interrupted shear. The  
333 simulation reproduced the experimental data for the recovery of viscosity overshoot as a function of  
334 rest time. Owing to this agreement, we analyzed the molecular behavior following the decoupling  
335 approximation. From the results, we reasonably confirmed that the overshoot is mainly due to segment  
336 orientation. To determine if the recovery of the overshoot is related to structural relaxation during the  
337 rest after the initial shear, we observed the relaxation of segment orientation, segment stretch, and  
338 entanglement density. We compared these relaxation functions with the mitigation of the viscosity  
339 overshoot, assuming that the waiting time before the resumed flow is compatible to the relaxation  
340 period. The comparison revealed that the mitigation of viscosity overshoot is similar to the relaxation  
341 of entanglement density. The similarity between the mitigation of overshoot and the relaxation of  
342 entanglement is nontrivial, and might be model-dependent. The effects of the molecular weight  
343 distribution would be worth investigating as well. We are currently conducting researches in these  
344 directions and the results will be reported elsewhere.

345

346 **ACKNOWLEDGEMENT**

347 This study is supported in part by Grant-in-Aid for Scientific Research (A) (17H01152) and for  
348 Scientific Research on Innovative Areas (18H04483) from JSPS, Ogasawara foundation, and JST-  
349 CREST (JPMJCR1992).

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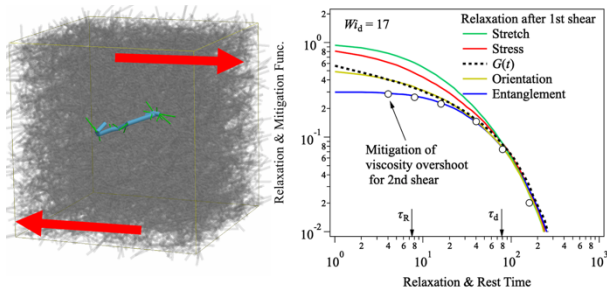
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444 TOC graphic:

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446 In slip-link simulations, recovery of viscosity overshoot in interrupted shear reflects relaxation of  
 447 entanglement density during interval

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