PASSIVE EARTH PRESSURE COEFFICIENT DURING EARTHQUAKE

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1. Introduction

The calculation of the passive earth pressure is very important to know the stability of the retaining walls and the ultimate bearing capacity of foundations. This calculation of the passive earth pressure is more difficult than that of the active earth pressure, because the sliding surface of passive earth pressure is clearly curved in difference with the case of active earth pressure.

Therefore, the authors calculated the coefficients of passive earth pressure and obtained the detailed tables of them by assuming the curved sliding surface which is more realistic than the straight line of sliding.

The studies on this problem in the static state have been made since earlier time. Kötter (1888)[1], Müller-Breslau (1906)[2], Reissner (1909)[2], Anzo (1949)[3] *et al.* studied this problem considering plastic equilibrium. It was reported that Schwelder (1887)[2] studied firstly the passive earth pressure by assuming the sliding surface as the logarithmic spiral. Kârmân (1926)[4] obtained the shapes of sliding surfaces and the coefficients of active earth pressure by substituting the Mohr-Coulomb criterion into the equilibrium equations, and expanding these results into infinite serieses. His work was remarkable on the assumption that the wall friction was equal to the internal friction of soil. His method can also be used for the calculation of the passive earth pressure. Fellenius (1927)[1][2] considered firstly that the passive earth pressure was the minimum value of assumed lateral pressure of the circular sliding surface. His method has been utilized in the case of cohesive soil. Krey (1936)[5] and Ohde (1938)[2][6] analyzed the problem of the passive earth pressure by assuming that the sliding surface consisted of a logarithmic spiral and a straight line. Caquot and Kerisel (1949)[7] studied the stress components in the soil mass with the finite difference method and published tables of the coefficients of passive earth pressure.

Later, Sokolovski (1960)[8] studied the problem of the earth pressure and the bearing capacity of foundations and gave the tables of coefficients of earth pressure, solving Karman's Equation by use of the finite difference method. Recently, Mayer-Vorfelder (1970)[2] obtained the detailed tables of coefficients of passive earth pressure in the case of internal friction $\phi=40°$, assuming that the sliding surface consisted of a logarithmic spiral and a straight line.

Although many studies were performed, the coefficients of passive earth pressure during earthquake have not been studied. Therefore, the authors obtained here them with assuming that the sliding surface consisted of a logarithmic spiral and a straight line, and also with extending the Sokolovski method. The Sokolovski method is superior to the logarithmic spiral method from the point of view of the equilibrium of soil mass above the sliding surface and the boundary condition at the point of contact between the wall and the sliding surface. In this paper, the authors assumed many cases of the wall friction and the shape of wall, and compared the coefficients K_{PE} of passive earth pressure

during earthquake between the logarithmic spiral method, the Sokolovski method and the Mononobe method (see Appendix 1). It has become clear that the difference in the results of these methods increased with increasing in the angle of wall friction, with increasing in the inclination of the back face of wall and with increasing in the inclination of ground surface.

2. Calculation of Passive Earth Pressure during Earthquake by Means of Logarithmic Spiral Method

2.1. Method of Calculation

The passive earth pressure indicates the resistance of soil mass against the wall which is displaced toward the soil mass. This pressure is utilized to know the support of soil mass for structures such as retaining walls or sheet pile walls. The following computation is based on the assumption that the wall is moved toward the backfill soil during earthquake and that the shape of sliding surface BD in Fig. 1 is given by a logarithmic spiral, and DC is given by a straight line. The seismic force is treated as positive when it acts toward the backfill from the wall.

Within the soil mass of △ADC, the state of stresses is the same as the passive Rankine pressure acting on the vertical section DF under earthquake. The straight line AD is $+m$ sliding surface and DC is $-m$ sliding surface as shown in Fig. 1. If the center of the logarithmic spiral O is located on the upper extension line of AD, the sliding surface BC is convex surface. On the other hand, BD is a concave surface when the center O is located on the lower extension line of AD as shown in Fig. 2.

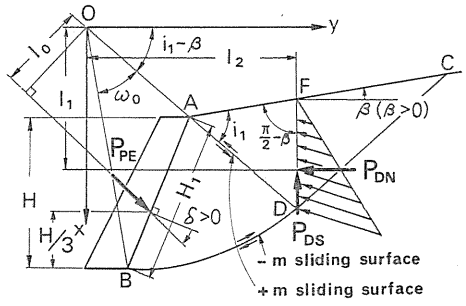

FIG. 1. Sliding surface of convex logarithmic spiral curve.

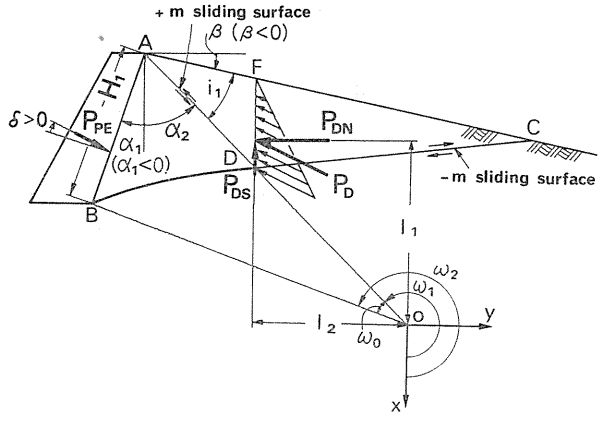

FIG. 2. Sliding surface of concave logarithmic spiral curve.

In the both figures, the sliding surface DC is tangent to the curved surface BD. Since the resultant reaction R along the curved section BD passes through the center O, the moment M of passive earth pressure P_{PE} during earthquake about the center O must be equal to the total moment of the gravity force of soil ABDF, the seismic force acting on the soil mass ABDF and the force acting on the vertical section FD during earthquake.

And the next equations are obtained for Fig. 1 and Fig. 2 respectively;

$$P_{PE} \cdot l_0 = -M_1 + M_2 + M_3 + M_4, \tag{1}$$

$$P_{PE} \cdot l_0 = M_1 - M_2 + M_3 + M_4. \tag{2}$$

where l_0: the arm length of the resulant force of pressure P_{PE} from the center of spiral,

M_1: the moment about the center O of spiral by the body force including seismic effect of assumed soil mass of △OBA,

M_2: the moment about the center O by the body force including seismic effect of assumed soil mass of sector OBD,

M_3: the moment about the center O by the body force including seismic effect of soil mass of △ADF, and

M_4: the moment about the center O by the passive Rankine earth pressure including seismic effect acting upon the vertical section FD;

$$M_4 = l_1 \cdot P_{DN} + l_2 \cdot P_{DS}. \tag{3}$$

In Eq. 3, P_{DN} is the normal resultant force acting on the verticl section FD, and P_{DS} is the shear force acting along the section FD. The force P_{DN} is positive. In Fig. 1 and Fig. 2, the force P_{DS} is considered negative when it acts upward. The lengths of l_1 and l_2 are the arms of two forces of P_{DN} and P_{DS} from the spiral center O respectively.

In the following investigation, it will be assumed that the origin of the X-Y coordinates is situated on the spiral center O as shown in Fig. 1. The passive earth pressure during earthquake can be determined by the minimum value P_{PE} which is calculated with Eq. 1 or Eq. 2. Hence, the passive earth pressure during earthquake is determined with the following equation;

$$P_{PE} = \frac{1}{2} \gamma H_1^2 \cos \alpha_1 \frac{K_{PE}}{\cos \delta}, \tag{4}$$

where H_1: the length of the back face of wall,

γ : unit weight of backfill,

α_1: the angle of the backface of wall from vertical plane

δ : the angle of wall friction and

K_{PE}: the coefficient of the passive earth pressure during earthquake.

The value of α_1 is considered positive if the backface of wall is inclined to the right as shown in Fig. 3 (a). If γ equals 1.0, equation 4 becomes Eq. 5;

$$K_{PE} = \frac{2 P_{PE} \cos \delta}{H_1^2 \cos \alpha_1}. \tag{5}$$

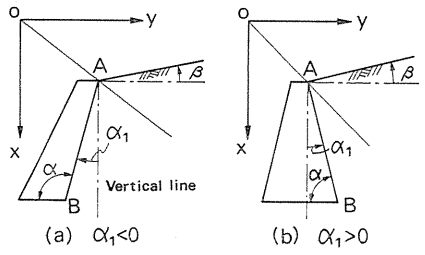

FIG. 3. Inclined angle of the backface of wall.

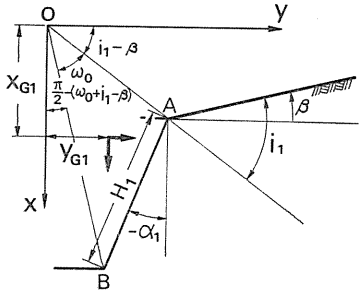

FIG. 4. Determination of side lengths of △OBA.

2.2. Calculation in the Case Where Sliding Surface is Convex

Substituting the following equation into Eq. 1, the resultant force of passive earth pressure can be given by the minimum value of P_{PE} for various values of ω_0 which is shown in Fig. 1.

(1) Calculation of moment M_1

The side lengths of △OBA are given by the following equations;

$$\left.\begin{aligned} \overline{AB} &= H_1, \\ \overline{OA} &= \frac{\cos(\omega_0 + i_1 - \beta + \alpha_1)}{\sin\omega_0} \cdot H_1, \\ \overline{OB} &= \frac{\cos(\beta - i_1 - \alpha_1)}{\sin\omega_0} \cdot H_1, \end{aligned}\right\} \tag{6}$$

where H_1: the length of backface of wall,

β : the inclined angle of backfill surface from the horizontal, and

i_1, ω_0: the angles shown in Fig. 4.

Then, the coordinates (x, y) of edge points of △OBA are as follows;

O: $(0, 0)$,

A: $(\overline{OA}\sin(i_1-\beta),\ \overline{OA}\cos(i_1-\beta))$,

B: $(\overline{OB}\sin(\omega_0-\beta+i_1),\ \overline{OB}\cos(\omega_0-\beta+i_1))$.

The coordinates (x_{G1}, y_{G1}) of the center of gravity in △OBA are as follows;

$$x_{G1} = \frac{1}{3}\left[\overline{OA}\sin(i_1-\beta) + \overline{OB}\sin(\omega_0-\beta+i_1)\right],$$

$$y_{G1} = \frac{1}{3}\left[\overline{OA}\cos(i_1-\beta) + \overline{OB}\cos(\omega_0-\beta+i_1)\right].$$

The area S_{OBA} of △OBA is given by the following equation;

$$S_{OBA} = \frac{1}{2}\overline{OA}\cdot\overline{OB}\sin\omega_0.$$

In conclusion, M_1 is represented by Eq. 7;

$$M_1 = S_{OBA}(y_{G1} - \tan\theta_0 \cdot x_{G1}), \tag{7}$$

where θ_0 is given by Eq. 27.

(2) Calculation of moment M_2

The sliding surface BD in Fig. 1 is given by the following logarithmic spiral equation;

$$r = r_0 e^{\omega \cdot \tan\phi}, \tag{8}$$

where r: the legth of vector shown in Fig. 5,
r_0: the length of vector r for $\omega=0$,
e: base of natural logarithm, and
ω: the angle of vector r from r_0 (expressed in radians), and ω is positive when it is measured counterclockwise.

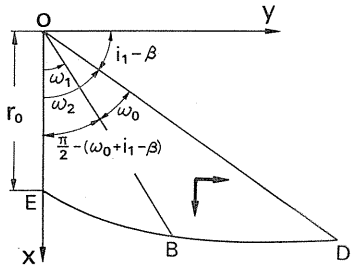

FIG. 5. Determination of moment M_2 of the spiral method (convex downward).

The moment M_2 is given by the next equation;

$$M_2 = M_x - M_y, \tag{9}$$

where $$M_x = \int y dA = \int_{\omega_1}^{\omega_2} \frac{2}{3} r \cdot \sin\omega \cdot \frac{1}{2} r^2 d\omega = \frac{1}{3}\int_{\omega_1}^{\omega_2} r^3 \sin\omega d\omega.$$

Accordingly, the moments M_x and M_y are represented by the following equations;

$$\left.\begin{aligned} M_x &= \frac{1}{3} r_0^3 \frac{1}{1+9\tan^2\phi}\left[e^{3\omega\cdot\tan\phi}(3\tan\phi\sin\omega - \cos\omega)\right]_{\omega_1}^{\omega_2}, \\ M_y &= \frac{1}{3} r_0^3 \frac{1}{1+9\tan^2\phi}\left[e^{3\omega\cdot\tan\phi}(3\tan\phi\cos\omega + \sin\omega)\right]_{\omega_1}^{\omega_2}. \end{aligned}\right\} \tag{10}$$

In the above equations, the values of ω_1, ω_2 and r_0 are as follows;

$$\left.\begin{aligned} \omega_1 &= \angle\text{EOB in Fig. } 5 = \frac{\pi}{2} - (\omega_0 + i_1 - \beta), \\ \omega_2 &= \angle\text{EOD in Fig. } 5 = \frac{\pi}{2} - (i_1 - \beta), \end{aligned}\right\} \tag{11}$$

$$r_0 = \frac{\cos(\beta - i_1 - \alpha_1)}{\sin\omega_0} \cdot H_1 \cdot e^{(\omega_0 + i_1 - \beta - \pi/2)\cdot\tan\phi}. \tag{12}$$

(3) Calculation of moment M_3

In Fig. 1, the lengths $\overline{AD}$ and $\overline{OD}$ are as follows;

$$\left.\begin{aligned} \overline{AD} &= \overline{OD} - \overline{OA}, \\ \overline{OD} &= \overline{OB} \cdot e^{\omega_0\tan\phi} = \frac{\cos(\beta - i_1 - \alpha_1)}{\sin\omega_0} H_1 \cdot e^{\omega_0\tan\phi}. \end{aligned}\right\} \tag{13}$$

where $\overline{OA}$ is given by Eq. 6.

Accordingly, the side lengths of $\triangle$ADF are as follows;

$$\overline{AD} = [\cos(\beta - i_1 - \alpha_1)e^{\omega_0 \cdot \tan\phi} - \cos(\omega_0 + i_1 - \beta + \alpha_1)]\frac{H_1}{\sin\omega_0}$$

$$\overline{AF} = \frac{\cos(i_1 - \beta)}{\cos\beta}\overline{AD}, \text{ and}$$

$$\overline{FD} = \frac{\sin i_1}{\cos\beta}\overline{AD}.$$

The coordinates (x, y) of three vertexes of $\triangle$ADF are as follows;

$$\begin{aligned} &A: [\overline{OA}\sin(i_1 - \beta),\ \overline{OA}\cos(i_1 - \beta)], \\ &D: [\overline{OD}\sin(i_1 - \beta),\ \overline{OD}\cos(i_1 - \beta)], \text{ and} \\ &F: [\overline{OD}\sin(i_1 - \beta) - \overline{FD},\ \overline{OD}\cos(i_1 - \beta)]. \end{aligned}$$

The coordinates (x_{G3}, y_{G3}) of the center of gravity of $\triangle$ADF are as follows;

$$x_{G3} = \frac{1}{3}[(2\overline{OD} + \overline{OA})\sin(i_1 - \beta) - \overline{FD}],$$

$$y_{G3} = \frac{1}{3}[(2\overline{OD} + \overline{OA})\cos(i_1 - \beta)].$$

The area S_{ADF} of $\triangle$ADF is given by the following equation;

$$\begin{aligned} S_{ADF} &= \frac{1}{2}\cdot\overline{AD}\cdot\overline{AF}\sin i_1 \\ &= \frac{1}{2}\overline{AD}^2\frac{\cos(i_1 - \beta)}{\cos\beta}\cdot\sin i_1. \end{aligned}$$

In conclusion, M_3 is obtained by Eq. 14;

$$M_3 = S_{ADF}(y_{G3} - \tan\theta_0 \cdot x_{G3}). \tag{14}$$

(4) Calculation of moment M_4

The moment M_4 is already given by Eq. 3. The arm lengths of l_1 and l_2 in Fig. 1 are as follows;

$$\left.\begin{aligned} l_1 &= \overline{OD}\sin(i_1 - \beta) - \frac{1}{3}\overline{FD}, \text{ and} \\ l_2 &= \overline{OD}\cos(i_1 - \beta). \end{aligned}\right\} \tag{15}$$

The length $\overline{OD}$ is given by Eq. 13. The normal force P_{DN} and the shearing force P_{DS} are given in the next section.

(5) Calculation of arm length l_0 of resultant force P_{PE} from spiral center

The applied point of the resultant force of passive earth pressure on the wall AB is at the height of $H/3$ above B as shown in Fig. 1. Refering Fig. 6, l_0 is obtained by the following equation;

$$l_0 = (\overline{OB} - \overline{BB'})\sin(\omega_0 + i_1 - \beta + \alpha_1 - \delta) \tag{16}$$

where $\overline{BB'} = \frac{1}{3}H_1\frac{\cos\delta}{\sin(\omega_0 + i_1 - \beta + \alpha_1 - \delta)}$,

$\overline{OB}$ is given by Eq. 6.

2.3. Calculation in the Case Where Sliding Surface is Concave

Substituting the following equations into Eq. 2, the resultant force of passive earth pressure can be given by the minimum value of P_{PE} for various values of ω_0 which is shown in Fig. 2.

(1) Calculation of moment M_1

In Fig. 2 where $\beta<0$, the side lengths of $\triangle$ABO are obtained by the following equations;

$$\left.\begin{aligned}\overline{AB} &= H_1\\ \overline{AO} &= \frac{\cos(i_1-\beta+\alpha_1-\omega_0)}{\sin\omega_0}H_1, \text{ and}\\ \overline{BO} &= \frac{\cos(i_1-\beta+\alpha_1)}{\sin\omega_0}H_1.\end{aligned}\right\} \quad (17)$$

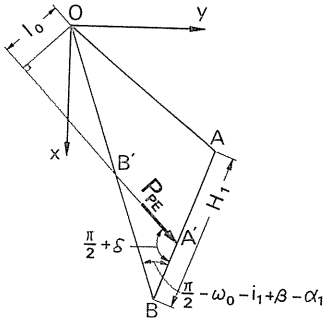

FIG. 6. Determination of the arm length l_0 of P_{PE} from the spiral center O (convex downward).

Then, the coordinates (x_{G1}, y_{G1}) of the center of gravity of $\triangle$ABO are as follows;

$$x_{G1} = -\frac{1}{3}[\overline{AO}\sin(i_1-\beta)+\overline{BO}\sin(i_1-\beta-\omega_0)], \text{ and}$$

$$y_{G1} = -\frac{1}{3}[\overline{AO}\cos(i_1-\beta)+\overline{BO}\cos(i_1-\beta-\omega_0)].$$

The area S_{ABO} of $\triangle$ABO is given by the following equation;

$$S_{ABO} = \frac{1}{2}\cdot\overline{AO}\cdot\overline{BO}\sin\omega_0$$

Therefore, M_1 is obtained by Eq. 18.

$$M_1 = S_{ABO}(|y_{G1}| - \tan\theta_0\cdot|x_{G1}|). \quad (18)$$

(2) Calculation of moment M_2

The sliding surface BD of logarithmic spiral is expressed by Eq. 8 and the moment M_2 is given by Eq. 9 and Eq. 10. In order to calculate M_2, the values of ω_1, ω_2 and r_0 shown in Fig. 7 are obtained by the following equations;

$$\left.\begin{aligned}\omega_1 &= \frac{3\pi}{2}-i_1+\beta,\\ \omega_2 &= \frac{3\pi}{2}-i_1+\beta+\omega_0, \text{ and}\\ r_0 &= \frac{\cos(i_1-\beta+\alpha_1)}{\sin\omega_0}H_1\cdot e^{(i_1-\beta-\omega_0-3\pi/2)\tan\phi}\end{aligned}\right\} \quad (19)$$

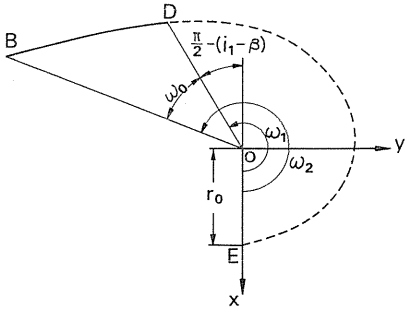

FIG. 7. Determination of moment M_2 of the spiral method (concave downward).

(3) Calculation of moment M_3

In Fig. 2, the side lengths of $\triangle$ADF are given by the following equations;

$$\left.\begin{aligned} \overline{FD} &= \frac{\sin i_1}{\cos\beta}\overline{AD}, \\ \overline{AF} &= \frac{\cos(i_1-\beta)}{\cos\beta}\overline{AD}, \text{ and} \\ \overline{AD} &= \overline{AO}-\overline{DO}. \end{aligned}\right\} \tag{20}$$

where $\overline{AO}$ is given by Eq. 6 and $\overline{DO}$ is given by the following equation;

$$\overline{DO} = \frac{\cos(i_1-\beta+\alpha_1)}{\sin\omega_0} H_1 \cdot e^{-\omega_0 \tan\beta} \tag{21}$$

The center of gravity of $\triangle$ADF is represented by the following coordinates (x_{G3}, y_{G3});

$$x_{G3} = -\frac{1}{3}[(\overline{AO}+2\overline{DO})\sin(i_1-\beta)+\overline{FD}], \text{ and}$$

$$y_{G3} = -\frac{1}{3}(\overline{AO}+2\overline{DO})\cos(i_1-B).$$

The area S_{ADF} of $\triangle$ADF is given as follows;

$$S_{ADF} = \frac{1}{2}\overline{AD}^2\frac{\cos(i_1-\beta)}{\cos\beta}\sin i_1.$$

Therefore, M_3 is given by the following equation;

$$M_3 = S_{ADF}(|y_{G3}| - \tan\theta_0 |x_{G3}|). \tag{22}$$

(4) Calculation of moment M_4

In Fig. 2, the arm lengths of l_1 and l_2 are obtained by the following equations;

$$\left.\begin{aligned} l_1 &= \overline{DO}\sin(i_1-\beta)+\frac{1}{3}\overline{FD}, \\ l_2 &= \overline{DO}\cos(i_1-\beta). \end{aligned}\right\} \tag{23}$$

In the above equations, the length $\overline{DO}$ is given by Eq. 21.

(5) Calculation of arm length l_0 of P_{PE} from spiral center

Refering Fig. 8, l_0 is obtained by the following equation;

$$l_0 = \overline{OA'}\sin(i_1-\beta+\alpha_1-\delta). \tag{24}$$

where $\overline{OA'} = \overline{OA} - \overline{A'A}$,

$$\overline{A'A} = \frac{2}{3}H_1\frac{\cos\delta}{\sin(i_1-\beta+\alpha_1-\delta)}, \text{ and}$$

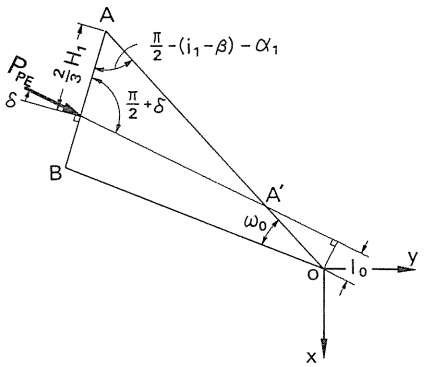

FIG. 8. Determination of the arm length l_0 of P_{PE} (concave downward).

$$\overline{OA} = \frac{\cos(i_1 - \beta + \alpha_1 - \omega_0)}{\sin\omega_0} \cdot H_1.$$

2.4. Passive Earth Pressure on Vertical Section and Angle at Which Sliding Surface Intersects with Backfill Surface

In order to discuss stresses and sliding surface in the Rankine region during earthquake, we use the rectangular coordinates (u, v) and (x, y) as shown in Fig. 9. The stress components σ_u and τ_{uv} of body force including seismic effect of soil mass acting on the u-plane are given by the following equations;

$$\left.\begin{aligned} \sigma_u &= \gamma_0 u \cos\beta_0, \\ \tau_{uv} &= -\gamma_0 u \sin\beta_0. \end{aligned}\right\} \qquad (25)$$

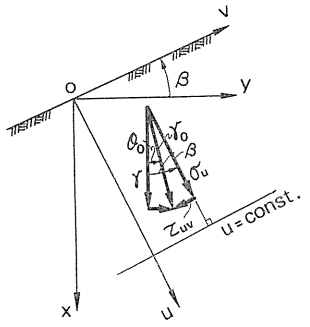

FIG. 9. Stress in the Rankine region during earthquake.

In Eq. 25, γ_0 and β_0 are given by Eq. 26.

$$\left.\begin{aligned} \gamma_0 &= \frac{\gamma}{\cos\theta_0}, \\ \beta_0 &= \beta - \theta_0. \end{aligned}\right\} \qquad (26)$$

where θ_0 is defined with the horizontal acceleration of earthquake, a, and with the acceleration of gravity force, g, as follows;

$$\theta_0 = \tan^{-1} a/g. \qquad (27)$$

In the plastic equilibrium state, the stress components σ_u, σ_v and τ_{uv} can be expressed in terms of two variables $\tilde{\sigma}$ and ϕ_1;

$$\left.\begin{aligned} \sigma_u &= \tilde{\sigma}(1 + \sin\phi\cos 2\phi_1), \\ \sigma_v &= \tilde{\sigma}(1 - \sin\phi\cos 2\phi_1), \text{ and} \\ \tau_{uv} &= -\tilde{\sigma}\sin\phi\sin 2\phi_1. \end{aligned}\right\} \qquad (28)$$

where σ_v is stress component acting on the v-plane,
$\tilde{\sigma}$ is obtained by the Mohr circle corresponding to passive state in earthquake condition as shown in Fig. 10, and

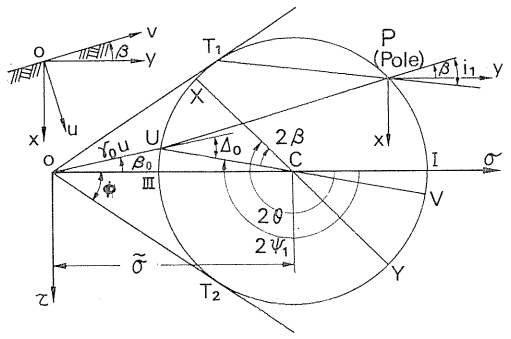

FIG. 10. Mohr's diagram in passive state during earthquake.

ψ_1 is angle between u-plane and major principal plane, and considered positive when it is measured counterclockwise from major principal plane.

We obtain the following equation from Eq. 25 and Eq. 28.

$$\sin(2\psi_1 - \beta_0) - \sin \Delta_0 = 0. \tag{29}$$

where

$$\left.\begin{aligned} &\sin \Delta_0 = \frac{\sin \beta_0}{\sin \phi} \quad \text{and} \\ &|\Delta_0| \leqq \frac{\pi}{2}. \end{aligned}\right\} \tag{30}$$

Accordingly, ψ_1 is represented by the following equation.

$$2\psi_1 = \beta_0 - \Delta_0 - \pi. \tag{31}$$

Further, equating τ_{uv} of Eq. 25 and τ_{uv} of Eq. 28, we obtain the following equation;

$$\tilde{\sigma} = \gamma_0 u \frac{\sin \Delta_0}{\sin 2\psi_1}.$$

The normal stress σ_y and the shearing stress τ_{yx} at $y=0$ are given by the following equations;

$$\left.\begin{aligned} &\sigma_y = \tilde{\sigma}[1 + \sin\phi \cos(\beta + \theta_0 + \Delta_0)] \quad \text{and} \\ &\tau_{yx} = -\tau_{xy} = \tilde{\sigma} \sin\phi \sin(\beta + \theta_0 + \Delta_0). \end{aligned}\right\} \tag{32}$$

where

$$\tilde{\sigma} = \frac{\gamma x \cos\beta}{\cos^2\phi \cos\theta_0}(\cos\beta_0 + \sqrt{\cos^2\beta_0 - \cos^2\phi}).$$

Therefore, P_{DN} and P_{DS} are obtained by the following equations;

$$\left.\begin{aligned} P_{DN} &= \frac{1}{2}\overline{FD}\sigma_y \quad (x = FD) \\ &= \frac{\gamma \overline{FD}^2 \cos\beta}{2\cos^2\phi \cos\theta_0}[1 + \sin\phi \cos(\beta + \theta_0 + \Delta_0)](\cos\beta_0 + \sqrt{\cos^2\beta_0 - \cos^2\phi}), \\ P_{DS} &= \frac{1}{2}\overline{FD}\tau_{yx} \quad (x = FD) \\ &= \frac{\gamma \overline{FD}^2 \cos\beta}{2\cos^2\phi \cos\theta_0} \sin\phi \sin(\beta + \theta_0 + \Delta_0)(\cos\beta_0 + \sqrt{\cos^2\beta_0 - \cos^2\phi}). \end{aligned}\right\} \tag{33}$$

As above mentioned, the sliding surface AD is $+m$ sliding surface in Fig. 1 and Fig. 2. The pole of the Mohr circle in Fig. 10 is the intersection between the Mohr circle and the line which is parallel to the v-axis from the stress point U. The direction of $+m$ sliding surface can be given by the straight line PT_1. The angle i_1 between the backfill surface and the sliding surface AD can be obtained in Fig. 10 and expressed as follows;

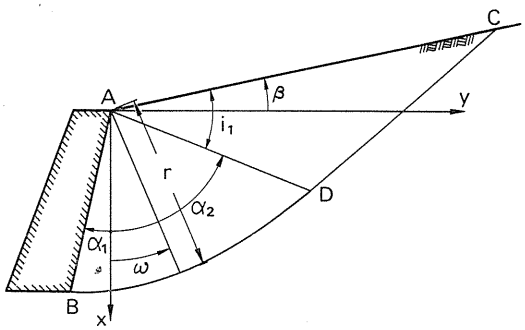

FIG. 11. Explanation of the Sokolovski method.

$$i_1 = \frac{1}{2}(\beta_0 - \Delta_0) + \frac{\pi}{4} - \frac{\phi}{2}. \tag{34}$$

Also refering Fig. 11, the angle α_2 between x-axis and sliding surface AD is given by the following equation;

$$2\alpha_2 = \phi + \frac{\pi}{2} + \beta + \theta_0 + \Delta_0. \tag{35}$$

3. Calculation of Passive Earth Pressure during Earthquake by the Sokolovski Method

Sokolovski[8] has given the active and passive earth pressure of critical equilibrium in the static state assuming that the stress component $\tilde{\sigma}$ was proportional to radius vector r. In order to calculate the passive earth pressure during earthquake with this method, we consider the plane critical equilibrium using the plane-polar coordinates (r, ω) as shown in Fig. 12.

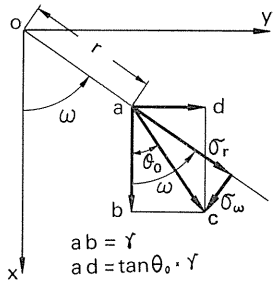

FIG. 12. Stresses in the soil mass during earthquake.

The differential equations of equilibrium are as follows;

$$\left.\begin{aligned} \frac{\partial \sigma_r}{\partial r} + \frac{1}{r}\frac{\partial \tau_{r\omega}}{\partial \omega} + \frac{\sigma_r - \sigma_\omega}{r} &= \gamma_0 \cos(\omega - \theta_0), \\ \frac{\partial \tau_{r\omega}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_\omega}{\partial \omega} + \frac{2\tau_{r\omega}}{r} &= -\gamma_0 \sin(\omega - \theta_0). \end{aligned}\right\} \tag{36}$$

The above equations are in the case when the seismic force acts in the positive direction of y-axis as shown in Fig. 12.

The stress components σ_r, σ_ω and $\tau_{r\omega}$ at critical plastic equilibrium are represented by the following equations;

$$\left.\begin{aligned}\sigma_r &= \tilde{\sigma}(1+\sin\phi\cos 2\psi),\\ \sigma_w &= \tilde{\sigma}(1-\sin\phi\cos 2\psi), \text{ and}\\ \tau_{rw} &= -\tilde{\sigma}\sin\phi\sin 2\psi,\end{aligned}\right\} \tag{37}$$

where ψ is the angle between the major principal plane and r-plane, and it is positive in counterclockwise from major principal plane. The relation between the angles ψ_1, θ and ψ is as follows;

$$\psi = \theta + \omega = \psi_1 - \beta + \omega, \tag{38}$$

where θ is the angle between the major principal plane and x-plane.
Hence, we introduce the following functions according to Sokolovski;

$$\left.\begin{aligned}\psi &= \psi(\omega),\\ \tilde{\sigma} &= \gamma_0 r S(\omega).\end{aligned}\right\} \tag{39}$$

Substituting Eq. 37 into Eq. 36, we obtain the following equations;

$$\left.\begin{aligned}\frac{dS}{d\omega} &= \frac{\sin[2\psi-(\omega-\theta_0)]-S\sin 2\psi}{\cos 2\psi-\sin\phi},\\ \frac{d\psi}{d\omega}-1 &= \frac{\cos(\omega-\theta_0)-\sin\phi\cos[2\psi-(\omega-\theta_0)]-S\cos^2\phi}{-2S\sin\phi\,(\cos 2\psi-\sin\phi)}.\end{aligned}\right\} \tag{40}$$

The above equations are given S and ψ which vary along the curved sliding surface BD when the soil body is failed along the sliding surface BDC as shown in Fig. 11. The region $\triangle$ADC is the passive Rankine zone and is given already in section 2.4. Accordingly, the values of θ, ψ, ω and S along the sliding surface AD are defined by the following equations;

$$\left.\begin{aligned}\theta &= \frac{1}{2}(\beta_0-\Delta_0)-\beta-\frac{\pi}{2},\\ \psi &= -\left(\frac{\pi}{4}-\frac{\phi}{2}\right),\\ \omega &= \alpha_2 = \frac{1}{2}\left(\phi+\frac{\pi}{2}+\beta+\theta_0+\Delta_0\right), \text{ and}\\ S &= \frac{\cos(\alpha_2-\beta)}{\cos^2\phi}(\cos\beta_0+\sqrt{\sin^2\phi-\sin^2\beta_0}).\end{aligned}\right\} \tag{41}$$

Now, the angle δ of wall friction with which the passive earth pressure acts on the back surface AB of the wall, is considered positive when it acts as shown in Fig. 1 and Fig. 2. Parameters θ, ψ, S and ω along the back surface AB of the wall are given as follows;

$$\left.\begin{aligned}\theta &= \frac{1}{2}(\delta+\Delta'-\pi)-\alpha_1,\\ \psi &= \frac{1}{2}(\delta+\Delta'-\pi),\\ S &= S_0, \text{ and}\\ \omega &= \alpha_1.\end{aligned}\right\} \tag{42}$$

where

$$\sin \Delta' = \frac{\sin \delta}{\sin \phi}.$$

The variables ω, ψ and S along the surface AD are known. Along the surface AB, two variables ω and ψ are known, but S_0 is unknown. Therefore, we assume the value of S_0, and compute the variables S and ψ along the surface AD with the recurrence formula of Eq. 43. In order to get S_0, the calculation must be repeated until the computed values become equal to those of Eq. 41 along the surface AD with various assumption of S_0.

$$S_{i+1} = S_i + \frac{\sin(2\psi_i - \omega_i + \theta_0) - S_i \sin 2\psi_i}{\cos 2\psi_i - \sin \phi} \cdot \Delta\omega,$$

$$\psi_{i+1} = \psi_i + \left[\frac{\cos(\omega_i - \theta_0) - \sin\phi \cos(2\psi_i - \omega_i + \theta_0) - S_i \cos^2 \phi}{-2 S_i \sin\phi (\cos 2\psi_i - \sin\phi)} + 1 \right] \cdot \Delta\omega. \tag{43}$$

Thus, the coefficient K_{PE} of the passive earth pressure during earthquake is obtained by the following equation;

$$K_{PE} = \frac{S_0 \cos\delta (\cos\delta + \sqrt{\sin^2\phi - \sin^2\delta})}{\cos\theta_0 \cos\alpha_1}. \tag{44}$$

Also, the equation of sliding surface is given as follows;

$$r = r_1 \exp\left[- \int_{\alpha_1}^{\omega} \cot(\psi - \mu) d\omega \right]. \tag{45}$$

The term μ is given by $\left(\frac{\pi}{4} - \frac{\phi}{2}\right)$. The solution of Eq. 45 is not a logarithmic spiral because ψ varies with ω.

We know the next three conditions by comparing θ of Eq. 41 with that of Eq. 42.

The slding surface BD is convex, if

$$2\alpha_1 < \delta + \Delta' + \beta + \theta_0 + \Delta_0. \tag{46.a}$$

The sliding surface BD is straight, if

$$2\alpha_1 = \delta + \Delta' + \beta + \theta_0 + \Delta_0. \tag{46.b}$$

The sliding surface BD is concave, if

$$2\alpha_2 > 2\alpha_1 > \delta + \Delta' + \beta + \theta_0 + \Delta_0. \tag{46.c}$$

If $\alpha_1 \geqq \alpha_2$, it is easy to obtain the earth pressure as the wall exists in the passive Rankine zone.

The Sokolovski method above mentioned can not be easily used for the case of Eq. 46. c, because the discontinuous line appears in the backfill. Hence, the authors show the method of calculation of the passive earth pressure during earthquake for this case with the Sokolovski method. The values of ψ and S on the discontinuous line AR in Fig. 13 are given by the following equations[8];

$$\tan\psi = \frac{1-\sin\phi}{1+\sin\phi}\cot\psi_{1-1}, \qquad (47)$$

$$S = S_1\frac{\sin 2\psi_{1-1}}{\sin 2\psi}. \qquad (48)$$

The value of ψ_{1-1} is given by the next equation for the region I in Fig. 13;

$$\psi_{1-1} = \theta_1 + \alpha_3, \qquad (49)$$

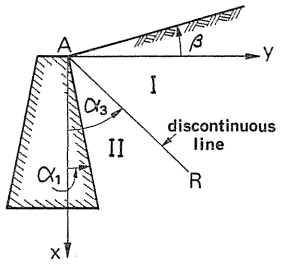

FIG. 13. Discontinuous line and angle of α_3.

where α_3 is angle between x-axis and discontinuous line AR as shown in Fig. 13. The value of θ_1 is the angle between the x-axis and the major principal axis in the region I, and is shown in Eq. 41 as θ. And also S_1 is given by the next equation;

$$S_1 = \frac{\cos(\alpha_3-\beta)}{\cos^2\phi}(\cos\beta_0 + \sqrt{\sin^2\phi-\sin^2\beta_0}). \qquad (50)$$

ψ and S are obtained by Eq. 47 and Eq. 48, assuming the value of α_3.

Using the values of ψ and S on the discontinuous line AR, we can calculate the values of ψ and S on the wall which is inclined α_1 to the vertical. In order to determine the value of S_n on the wall, the next recurrence formula are used until the calculated value of ψ_n on the wall becomes equal to the given ψ.

$$S_{i+1} = S_i - \frac{\sin(2\psi_i-\omega_i+\theta_0) - S_i\sin 2\psi_i}{\cos 2\psi_i - \sin\phi}\Delta\omega,$$

$$\psi_{i+1} = \psi_i - \left[\frac{\cos(\omega_i-\theta_0) - \sin\phi\cos(2\psi_i-\omega_i+\theta_0) - S_i\cos^2\phi}{-2S_i\sin\phi(\cos 2\psi_i-\sin\phi)} + 1\right]\Delta\omega. \qquad (51)$$

The value of α_1' corresponding to $\psi_n=\psi$ is obtained by the next equations;

$$\alpha_1' = \alpha_3 - n\Delta\omega, \qquad (52.a)$$

where

$$\Delta\omega = \frac{1}{3000}(\alpha_3-\alpha_1). \qquad (52.b)$$

The relationship between α_3 and α_1' is shown in Fig. 14 as an example. Since the value of α_1' must satisfy the next equation, the value of α_3 is determined as shown in Fig. 14.

$$\alpha_1' = \alpha_1. \qquad (53)$$

The values of K_{PE} obtained with this method are shown in Tables A-3, A-4 and A-5. However, the value of $\Delta\omega$ in Eq. 52.b gives the very small error to the calculation of S on the wall. The sliding surface obtained by the solution of the discontinuous line corresponds to the concave curve obtained by the logarithmic spiral method as shown in these Tables. The value of K_{PE} obtained by the discontinuous line is smaller 3.0% at the maximum than that of the spiral method.

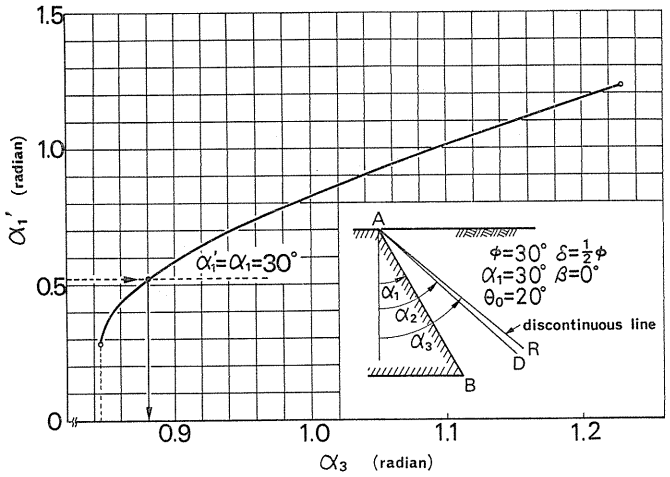

FIG. 14. Relationship between α_1' and α_3.

4. Comparison of Results by the Logarithmic Spiral Method and by Other Methods

4.1. Coefficient K_{PE} of Passive Earth Pressure

The authors calculated the values of K_{PE} in the spiral method with a pitch of $\Delta\omega=0.2°$. In the Sokolovski method, the variables of S and ϕ along the surface AD which were obtained from the surface AB were calculated down to four places of decimals so as to agree with the values of S and ϕ along the surface AD in Eq. 41. The value of $\Delta\omega$ in the recurrence formula Eq. 43 is taken as $\Delta\omega=(\alpha_2-\alpha_1)/1000$ in the case of $\delta=1/2\cdot\phi$ and as $\Delta\omega=(\alpha_2-\alpha_1)/3000$ in the case of $\delta=2/3\cdot\varphi$ and $\delta=\phi$. In this article, the authors give the coefficients of passive earth pressure during earthquake corresponding to the magnitude of the angle of the wall friction δ.

(1) The case of $\delta=0°$

The comparison of the values of K_{PE} which are obtained by the spiral method and the Mononobe method in the case of $\delta=0°$ and $\alpha_1=\beta=0°$ is shown in Fig. 15. The values of K_{PE} of the Mononobe method are nearly equal to those of the spiral method. In Fig. 16, the sliding surfaces which are obtained by the Sokolovski method and the spiral method are shown in this case. In Fig. 17, and Fig. 18, the values of K_{PE} which are obtained by the Mononobe method and the spiral method are shown for various value of α_1 and β in the case of $\delta=0°$. The value of K_{PE} of the Mononobe method is greater than that of the spiral method as decreasing in the value of α_1, or as

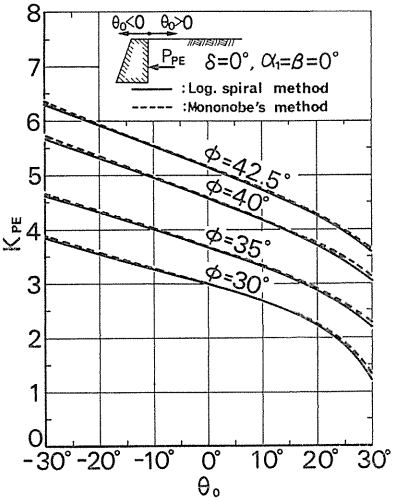

FIG. 15. Comparison between K_{PE} of the spiral method and that of the Mononobe method in the case of $\alpha_1=\beta=0°$ and $\delta=0°$.

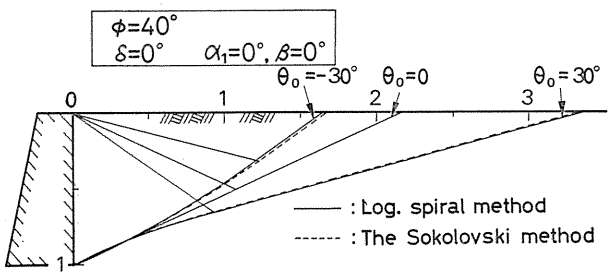

FIG. 16. Sliding surfaces obtained by the logarithmic spiral method and the Sokolovski method in the case of $\alpha_1=\beta=0°$ and $\delta=0°$.

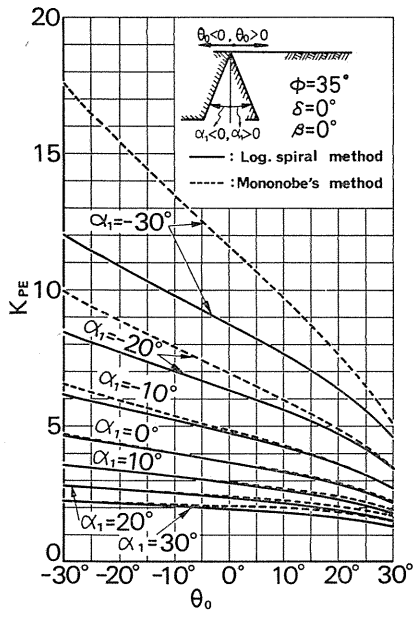

FIG. 17. Comparison between K_{PE} of the spiral method and that of the Mononobe method for various α_1 and θ_0 in the case of $\delta=0°$ and $\beta=0°$.

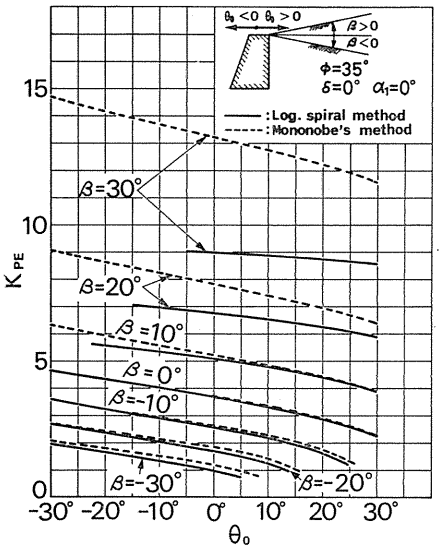

FIG. 18. Comparison between K_{PE} of the spiral method and that of the Mononobe method for various β and θ_0 in the case of $\delta=0°$ and $\alpha_1=0°$.

increasing in the value of β. For example, in the case of $\phi=35°$, $\alpha_1=-30°$ and $\theta_0=20°$, the value of K_{PE} of the Mononobe method is greater than that of the spiral method by 27%, and in the case of $\phi=35°$, $\beta=30°$ and $\theta_0=20°$, the former is greater than the latter by 39%.

(2) The case of $\delta=\pm 1/2\cdot\phi$ and $\delta=2/3\cdot\phi$

In the case of $\delta=1/2\cdot\phi$ and $\alpha_1=\beta=0°$, the values of K_{PE} and ω_0 are shown in Table A-1 for many values of θ_0 including $\theta_0<0$. The value of ω_0 is necessary to draw the sliding surface. The value of K_{PE} are graphically shown in Fig. 19. The values of K_{PE} of the spiral method coincide well with those of the Sokolovski method. In the case of $\delta=2/3\cdot\phi$ and $\alpha_1=\beta=0°$, the values of K_{PE} are shown

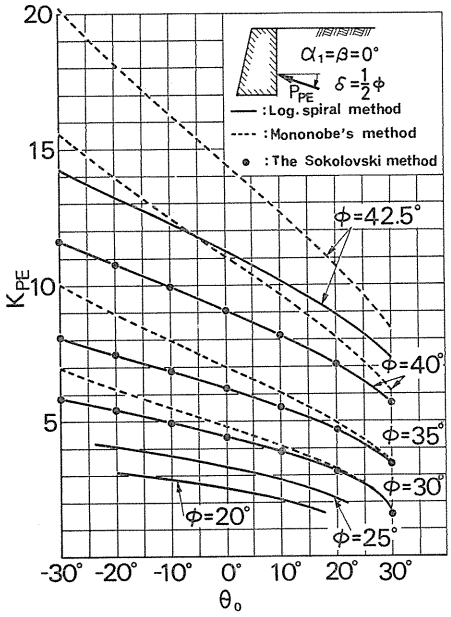

FIG. 19. Valeue of K_{PE} obtained by the spiral method, the Mononobe method and the Sokolovski method in the case of $\delta=1/2\cdot\phi$ and $\alpha_1=\beta=0°$.

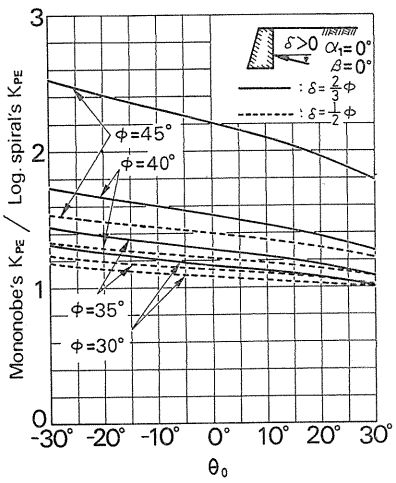

FIG. 20. Ratios of K_{PE} of the Mononobe method versus that of the spiral method in the case of $\alpha_1=0°$ and $\beta=0°$ under the conditions of $\delta=1/2\cdot\phi$ and $\delta=2/3\cdot\phi$.

in Table A-2, for many values of θ_0 including $\theta_0<0$.

In Fig. 20, the ratio of K_{PE} of the Mononobe method to that of the spiral method are shown in the case of $\delta=1/2\cdot\phi$ and $\delta=2/3\cdot\phi$. This figure represents that this ratio increases as the value of δ increases. For an example, in the case of $\delta=1/2\cdot\phi=20°$ and $\theta_0\geqq 20°$, the value of K_{PE} of the Mononobe method is greater 10% to 20% than that of the spiral method, and in the case of $\delta=2/3\cdot\phi=26.7°$ and $\theta_0\geqq 0°$, the former is greater 30% to 50% than the latter. It is remarkable that the coefficients K_{PE} of the Mononobe method approach to those of the spiral method as the value of θ_0 increases. In Fig. 21, the ratios of K_{PE} of the spiral method in the case of $\delta=1/2\cdot\phi$ and $\delta=2/3\cdot\phi$ to that in the case of $\delta=0°$ are shown under the condition of $\alpha_1=\beta=0°$. In the case of $\delta=1/2\cdot\phi=20°$ and $\theta_0\geqq 0°$, the values of K_{PE} are 1.8 to 2.2 times of the values for $\delta=0°$, and in the case of $\delta=2/3\cdot\phi=26.7°$, the values of K_{PE} are 2.2 to 2.4 times of the values for $\delta=0°$.

The calculation of the passive earth pressure in the case of $\delta=1/2\cdot\phi$ and $\delta=2/3\cdot\phi$ is very important for the design of retaining walls, because the practical angle of wall friction is almost considered as $\delta\leqq 2/3\cdot\phi$. Hence, in Table A-3 and Table A-4, the values of K_{PE} of the spiral method and the Sokolovski method for α_1 or β is not zero and $\theta_0\geqq 0°$ are shown in the case of $\delta=0°$, $\delta=1/2\cdot\phi$, $\delta=2/3\cdot\phi$ and $\phi=20°\sim 45°$. In these Tables, the values of ω_0 in parentheses indicate the cases of the concave curves. Further, the values of K_{PE} and ω_0 in the case of $\delta=1/2$ $\phi=20°$ and $\beta=20°$ are shown in Fig. 22 and Fig. 23 for various values of α_1 and θ_0. In Fig. 22, the comparison of K_{PE} of the Sokolovski method to that

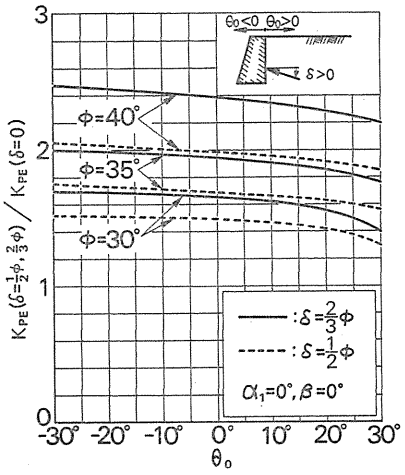

FIG. 21. Ratios of K_{PE} for $\delta=1/2\cdot\phi$ or $\delta=2/3\cdot\phi$ versus K_{PE} for $\delta=0°$ by the spiral method.

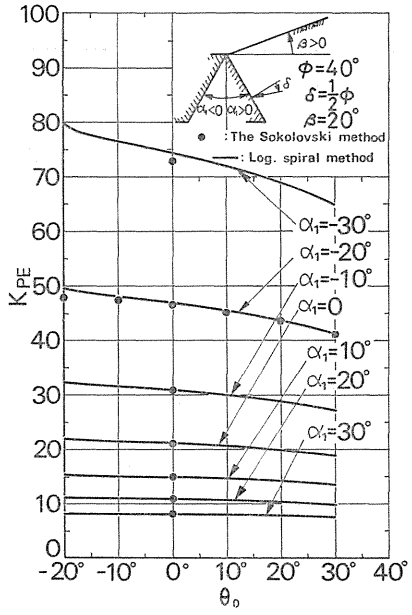

FIG. 22. Values of K_{PE} for various values of α_1 and θ_0 in the case of $\beta=20°$ and $\delta=1/2\cdot\phi=20°$.

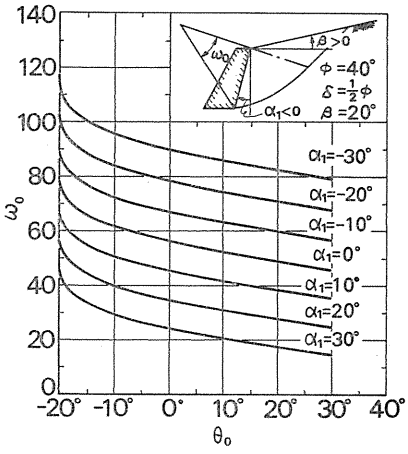

FIG. 23. Values of ω_0 for various values of α_1 and θ_0 in the case of $\beta=20°$ and $\delta=1/2\cdot\phi=20°$.

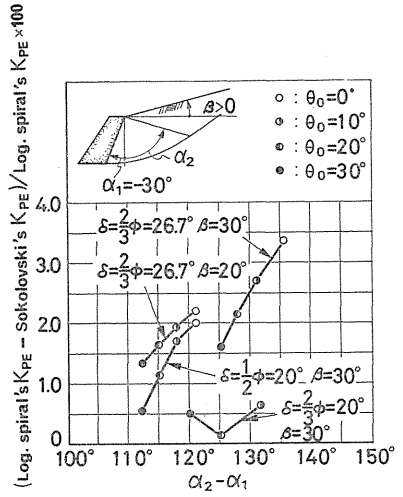

FIG. 24. Percent difference between K_{PE} of the spiral method and that of the Sokolovski method for various values of $(\alpha_2-\alpha_1)$ in the cases of $\delta=2/3\cdot\phi$ and $\delta=1/2\cdot\phi$.

of the spiral method is shown.

The differences of K_{PE} of the spiral method and the Sokolovski method vary with $(\alpha_2-\alpha_1)$ and δ. In Fig. 24, the percent difference of K_{PE} of the methods are shown for the cases of $\delta=2/3\cdot\phi=26.7°$, $\delta=2/3\cdot\phi=20°$ and $\delta=1/2\cdot\phi=20°$ under the condition of $\alpha_1=-30°$, $\beta=20°$ and $\theta_0=0°\sim30°$. In this figure, it is obtained

that the maximum percent difference of K_{PE} of the methods is 3.4% for the case of $\alpha_1 \geqq -30°$, $\beta \leqq 30°$, $\delta \leqq 2/3 \cdot \phi$ and $(\alpha_2 - \alpha_1) \leqq 136°$.

(3) The case of $\delta = \phi$

In general, the passive earth pressure along the plane sliding surface of soil mass is calculated with the assumption of $\delta = \phi$. The values of K_{PE} and ω_0 of the spiral method are shown in Fig. 25 and Fig. 26 in the case of $\delta = \phi = 30°$ and $\beta = 0°$ for various values of α_1. In Fig. 25, the values of K_{PE} by the Sokolovski method are also shown, comparing with K_{PE} of the spiral method. The ratios of K_{PE} of the spiral method to that of the Sokolovski method are shown in Fig. 27 in the case of $\delta = \phi = 30°$ and $\beta = 0°$. From this figure, it can be understood that these ratios increase with decreasing in the value of α_1. The value of K_{PE} of the spiral method is greater 21% than that of the Sokolovski method in the case of $\theta_0 = 20°$ and $\alpha_1 = -70°$. And in the case of $\theta_0 = 20°$ and $\alpha_1 = -50°$, the former is greater 6% than the latter. However, in the case of $\alpha_1 \geqq 0°$, the difference of the values of K_{PE} by the methods does not exceed 2.0%.

In Table A-6, the values of K_{PE} in the case of $\delta = \phi$ are shown. The differ-

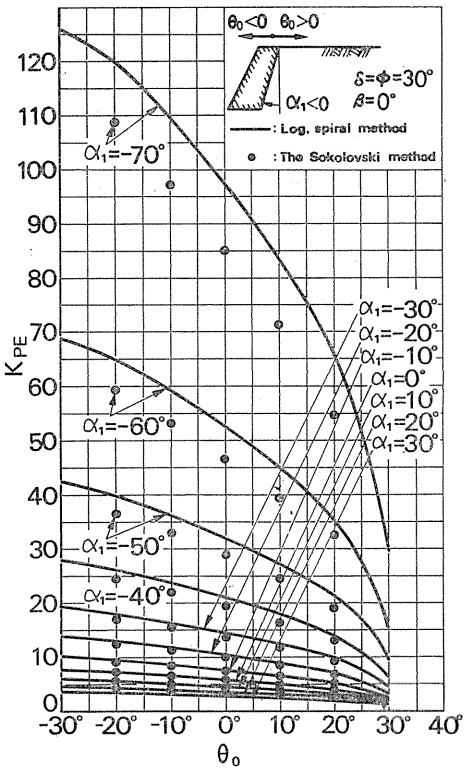

FIG. 25. Values of K_{PE} for various values of α_1 and θ_0 in the case of $\delta = \phi = 30°$ and $\beta = 0°$.

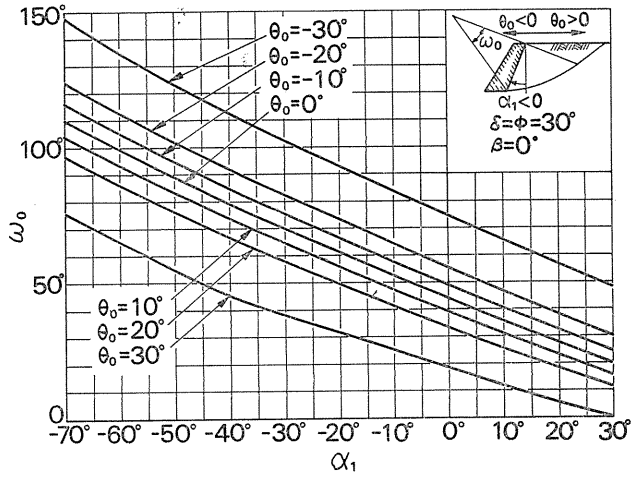

FIG. 26. Values of ω_0 for various values of α_1 and θ_0 in the case of $\delta = \phi = 30°$ and $\beta = 0°$.

ence of these K_{PE} by the methods for the case of $\delta=\phi$ is larger than that for the case of $\delta=1/2\cdot\phi$ and $\delta=2/3\cdot\phi$. The spiral method does not satisfy the boundary conditions at the backface of wall. But, the difference of K_{PE} between the spiral method and the Sokolovski method for the case of $\delta\leqq 2/3\cdot\phi$ is so small in practice that the passive earth pressure against retaining walls can be obtained by the spiral method with enough accuracy.

4.2. Sliding Surface

(1) Relationship between sliding surface, angle of wall friction and angle of internal friction

In Fig. 28, the sliding surfaces obtained by the spiral method are shown for various angles of wall friction δ in the case of $\alpha_1=\beta=0°$, $\phi=40°$ and $\theta_0=0$. In this figure, we can know that the sliding surface extends far away from the wall as increasing in the angle of wall friction. In Fig. 29, the sliding surfaces obtained by the spiral method are shown for various angles of internal friction ϕ in the case of $\alpha_1=\beta=0°$ and $\theta_0=0°$. The sliding surface extends far away from the wall and becomes longer as increasing in the angle of internal friction.

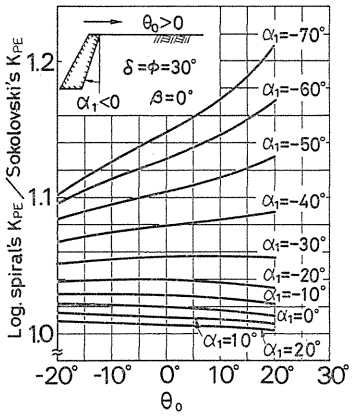

FIG. 27. Ratios of K_{PE} of the spiral method versus that of the Sokolovski method for various values of α_1 and θ_0 in the case of $\delta=\phi=30°$ and $\beta=0°$.

(2) Relationship between sliding surface and seismic force

In Fig. 30, the sliding surfaces of the spiral method, the Sokolovski method and the Mononobe method are shown in the case of $\alpha_1=\beta=0°$ and $\delta=1/2\cdot\phi=20°$. As increasing in the value of θ_0 which shows the intensity of seismic force, the ratio of the straight part to the curved part of the sliding surface becomes larger and the sliding surface extends far away from the wall. The sliding surface of the Mononobe method approaches to that of the spiral method as the value of θ_0 increases. The sliding surfaces of the spiral method are almost the same with those of the Sokolovski method for $\alpha_1=0°$ and $\beta=0°$.

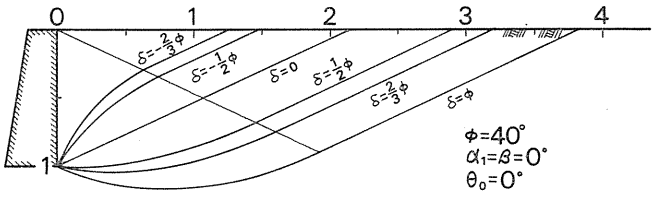

FIG. 28. Comparison of the sliding surfaces for various values of δ in the case of $\phi=40°$, $\alpha_1=\beta=0°$ and $\theta_0=0°$.

(3) Relationship between sliding surface and inclination of ground surface

In Fig. 31, the sliding surface obtained by the spiral method are shown for

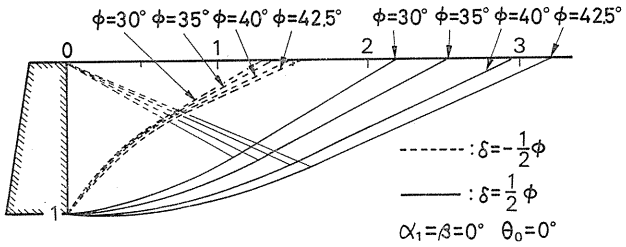

FIG. 29. Comparison of the sliding surfaces for various values of ϕ in the case of $\alpha_1=\beta=0°$ and $\theta_0=0°$.

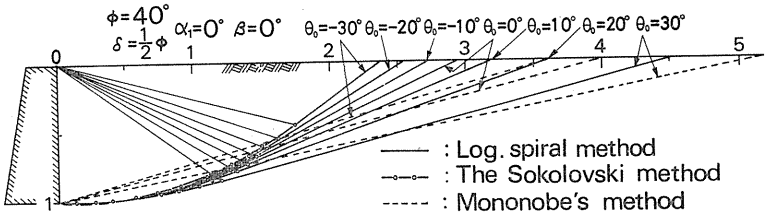

FIG. 30. Comparison of the sliding surfaces for various values of θ_0 in the case of $\delta=1/2\cdot\phi=20°$ and $\alpha_1=\beta=0°$.

various inclination angle β of ground surface in the case of $\phi=40°$, $\alpha_1=0°$ and $\theta_0=0°$. The ratio of the curved part to the straight part of sliding surface becomes larger as increasing in the value of β.

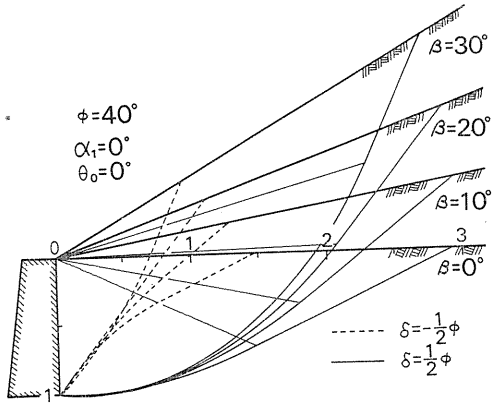

FIG. 31. Comparison of the sliding surfaces for various values of β in the case of $\phi=40°$, $\alpha_1=0°$ and $\theta_0=0°$.

(4) Relationship between sliding surface and inclined angle of wall

In Fig. 32, the sliding surfaces obtained by the spiral method and the Sokolovski method are shown for $\alpha_1=-30°$, $\alpha_1=0°$ and $\alpha_1=30°$ in the case of $\beta=20°$, $\delta=1/2\cdot\phi=20°$ and $\theta_0=0°$. The sliding surface becomes longer and the ratio of the curved part to the straight part of the sliding surface becomes larger as decreasing in the value of α_1. Even in the case of $\alpha_1=-30°$, the sliding surfaces of the spiral method do not practically differ from those of the Sokolovski method, and as shown before, the percent difference of K_{PE} of the

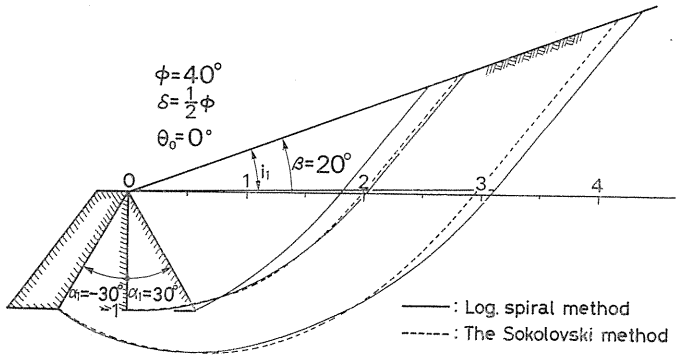

FIG. 32. Sliding surfaces for various values of α_1 in the case of $\delta=1/2\cdot\phi=20°$, $\beta=20°$ and $\theta_0=0°$.

methods is 2.0% in the case of $\alpha_1=-30°$.

(5) Comparison between sliding surface of the spiral method and that of Sokolovski method

In Fig. 33 the sliding surfaces obtained by the spiral method and the Sokolovski method in the case of $\beta=0°$, $\alpha_1=-70°$ and $\delta=\phi=40°$ are graphically shown. In this case, one sliding surface is the backface of the wall and the other sliding surface must intersect with backface of the wall at the angle of $2\mu=\pi/2-\phi$. The Sokolovski method satisfy this condition, but the spiral method does not satisfy this condition in Fig. 33.

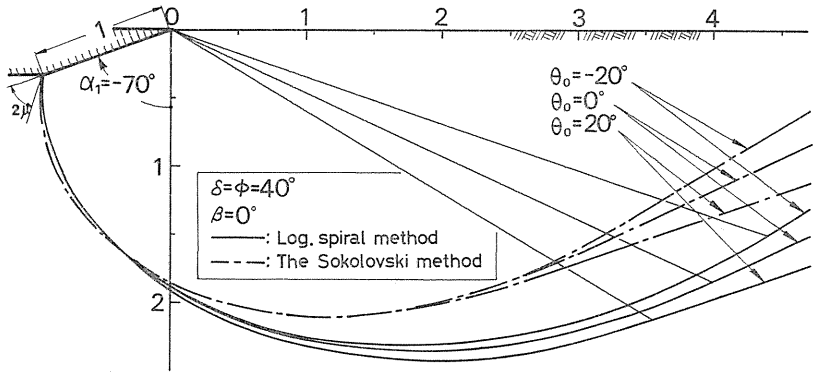

FIG. 33. Comparison between the sliding surface of the spiral method and that of the Sokolovski method in the case of $\delta=\phi=40°$.

4.3. Comparison between the Author's Results, Mayer-Vorfelder's Ones and Terzaghi's Ones

Mayer-Vorfelder (1970)[2] obtained the horizontal coefficients of the passive earth pressure by means of the spiral method. The horizontal coefficient λ_{ph} by Mayer-Vorfelder can be converted to the coefficient K_P used in this paper with the following equation;

$$K_P = \frac{\cos\delta \cdot \cos\alpha_1}{\cos(\delta-\alpha_1)}\lambda_{ph}. \tag{54}$$

The value of K_P for $\delta=1/2\cdot\phi=20°$ by Mayer-Vorfelder are shown with the author's values in Fig. 34. In this case, Mayer-Vorfelder's results are coincide with the author's ones. But, in the case of $\delta=\phi=40°$ and $\alpha_1=-30°$, the value of Mayer-Vorfelder is larger about 40% in comparison with the author's value as shown in Fig. 35.

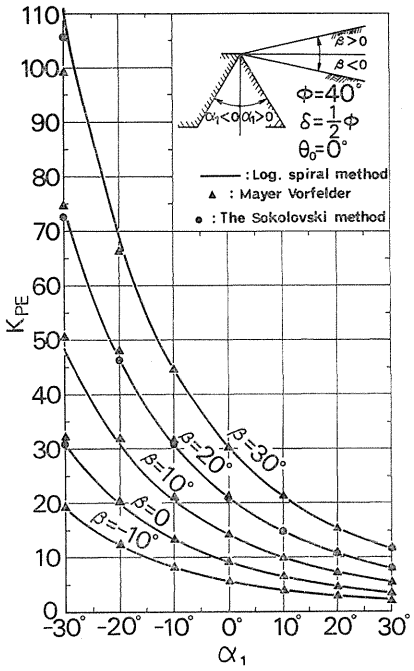

FIG. 34. Comparison between values of K_{PE} of the spiral method, those of Mayer-Vorfelder and those of the Sokolovski method in the case of $\delta=1/2\cdot\phi=20°$.

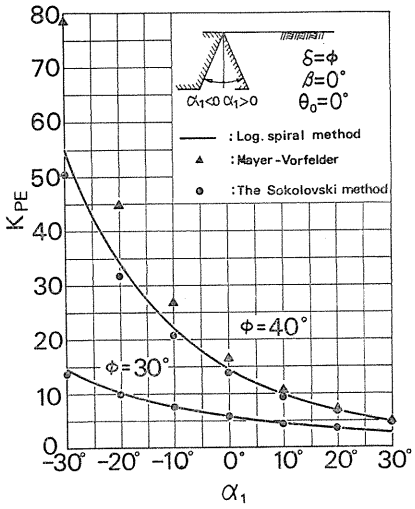

FIG. 35. Comparison between the values of K_{PE} of the spiral method, those of Mayer-Vorfelder and those of the Sokolovski method in the case of $\delta=\phi$.

Then, the authors compared their results by the spiral method with the values given by Terzaghi (1943)[9] as shown in Fig. 36. The values of K_P according to Terzaghi in Fig. 36 were calculated with his values of N_γ by the following equation;

$$K_P = \cos^2\phi\left(\frac{2N_\gamma}{\tan\phi}+1\right). \tag{55}$$

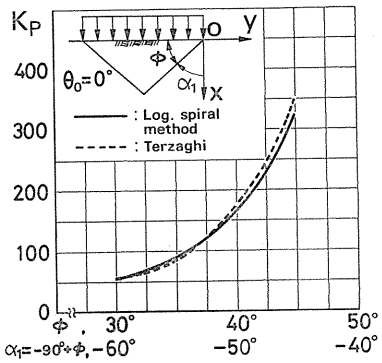

FIG. 36. Comparison between value of K_{PE} of the spiral method and that of N_γ according to Terzaghi.

In the calculation of K_P with Eq. 55, the inclined angle of wall is given as $\alpha_1=$

$\phi-90°$. Fig. 36 shows a good agreement of Terzaghi's values with the author's ones. The difference of both results is 7% in the case of $\delta=\phi=40°$ and $\alpha_1=-50°$. Mayer-Vorfelder's results are greater than those of the Sokolovski method and our logarithmic spiral method in the case of $\delta=\phi=40°$, because his calculation is based on the "Gleichgewichts-Verfahren"[2] suggested by him.

5. Conclusions

We compared the coefficients of the passive earth pressure obtained by the Sokolovski method, the logarithmic spiral method and the Mononobe method. Considering the rationality of the sliding surface, the Sokolovski method is best of all. Because, the Sokolovski method satisfies three conditions of equilibrium, *i.e.* $\Sigma H=0$, $\Sigma V=0$ and $\Sigma M=0$, in the sliding soil mass and boundary conditions of the Rankine region and of the wall. The spiral method satisfies only the condition of $\Sigma M=0$ of the sliding soil mass and the boundary condition of the Rankine region. The Mononobe method satisfies conditions of $\Sigma H=0$ and $\Sigma V=0$ of the sliding soil mass, but does not satisfy boundary conditions of the Rankine region and of the wall.

The logarithmic spiral method is calculated more easily than the Sokolovski method, and the values of K_{PE} of the spiral method are almost same with those of the Sokolovski method in the case of $\delta\leqq 2/3\cdot\phi$ and $\alpha_1\geqq-30°$. Therefore, the values of K_{PE} for design of retaining wall can be calculated practically by the spiral method.

The results obtained by the authors are summarized as follows.

(1) Equation 46 gives the shape of sliding surface of the Sokolovski method. This equation is useful to distinguish between the convex sliding surface and concave sliding surface of the spiral method.

(2) The value of K_{PE} of the Mononobe method is reasonable only for the condition of $\delta=0°$ and $\alpha_1=\beta=0°$. In the case of $\delta=0°$ and α_1 or β is not zero, the value of K_{PE} of the Mononobe method differs considerably from that of the spiral method. For an example, in the case of $\phi=35°$, $\theta_0=20°$, $\alpha_1=-30°$ and $\beta=0°$, the former is greater 27% than the latter.

(3) As increasing in the value of δ under the same conditions of α_1, β, ϕ and θ_0, the value of K_{PE} of the Mononobe method becomes greater than that of the spiral method. For an instance, the former is greater 30% to 50% than the latter in the case of $\delta=2/3\cdot\phi=26.7°$, $\alpha_1=\beta=0°$ and $\theta_0=0°$.

(4) As increasing in the value of θ_0 under the condition of $\alpha_1=\beta=0°$ and $\delta\fallingdotseq 0°$, the values of K_{PE} of the Mononobe method and the spiral method approach gradually to that of the Sokolovski method.

(5) As increasing in the value of δ and $(\alpha_2-\alpha_1)$, the value of K_{PE} of the spiral method becomes greater than that of the Sokolovski method. However, even in the case of $\delta=2/3\cdot\phi=26.7°$, $\alpha_1=-30°$, $\beta=30°$ and $(\alpha_2-\alpha_1)=135.5°$, the percent difference of K_{PE} of the methods is only 3.4%.

(6) In the case of $\delta=\phi=30°$, the value of K_{PE} of the spiral method is greater about 6% than that of the Sokolovski method for the conditon of $\alpha_1=-30°$, $\beta=0°$, $\theta_0=20°$ and $(\alpha_2-\alpha_1)=78.4°$. Further, for the condition of $\alpha_1=-70°$, $\beta=0°$, $\theta_0=20°$ and $(\alpha_2-\alpha_1)=118.4°$, the former is greater about 21% than the latter.

(7) In the case of $\delta=\phi=40°$, the value of K_P of Mayer-Vorfelder is greater

40% than that of the spiral method under the condition of $\alpha_1=-30°$, $\beta=0°$ and $\theta_0=0°$, and Terzaghi's value of K_P which is calculated from his value of N_T is greater 7% than that of the spiral method under the condition of $\alpha_1=-50°$, $\beta=0°$ and $\theta_0=0°$.

(8) As increasing in the value of θ_0, the sliding surface extends far away from the wall and becomes longer.

The authors wish to thank Professor K. Ueshita for his encouragement and co-operation. These calculations were made with FACOM 230-60 Computer at Nagoya University Computing Center.

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Appendix

Appendix 1. Mononobe Equation

The Mononobe equation assums that the sliding surface is consisted of a straight line and is calculated by the following equations;

$$P_{PE}=\frac{1}{2}(1-k_1)\gamma H^2\frac{K_{PE}}{\sin\alpha\cos\delta},$$

$$K_{PE}=\frac{\sin^2(\alpha+\theta_0-\phi)\cdot\cos\delta}{\cos\theta_0\sin\alpha\cdot\sin(\alpha+\theta_0+\delta)\left[1-\sqrt{\frac{\sin(\phi+\delta)\cdot\sin(\phi+\beta-\theta_0)}{\sin(\alpha+\theta_0+\delta)\cdot\sin(\alpha+\beta)}}\right]^2},$$

$$\theta_0=\frac{k}{1-k_1}, \tag{A-1}$$

where P_{PE} is the resultant force due to the passive earth pressure during earthquake, K_{PE} is the passive earth pressure coefficient during earthquake, k is the coefficient of horizontal earthquake intensity and k_1 is the coefficient of vertical earthquake intensity, respectively. In this study, θ_0 is given by Eq. 27.

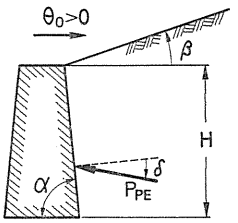

FIG. A-1. The Mononobe equation.

Appendix 2. Accuracy of Calculation

In the calculation of the Sokolovski method, there may be two kinds of errors. One is relating to the number of division of $(\alpha_2-\alpha_1)$. The other is caused by the approximate solution in the case of $\delta=\phi$. The authors divided the value of $(\alpha_2-\alpha_1)$ into 1000 segments for the finite differential method in the case of $\delta=1/2\cdot\phi$ and compared the results of the 1000-segment computation with those of the 4000-segment computation or the 6000-segment one. The values of K_{PE} computed for above 4000 or 6000 segment are almost the same regardless of the number of division. In Table A-7, the errors of the 1000-segment computations are shown. The values of K_{PE} become smaller as increasing in the number of division and decreasing in the value of $(\alpha_2-\alpha_1)$. The maximum error of the 1000-segment computaion is 0.6% in the case of $\delta=1/2\cdot\phi$. However, in the case of $\delta=2/3\cdot\phi$ and $\delta=\phi$, we computed the value of K_{PE} not for 1000-segment but for 3000-segment. In Fig. A-2, the errors for various divisions are shown in the

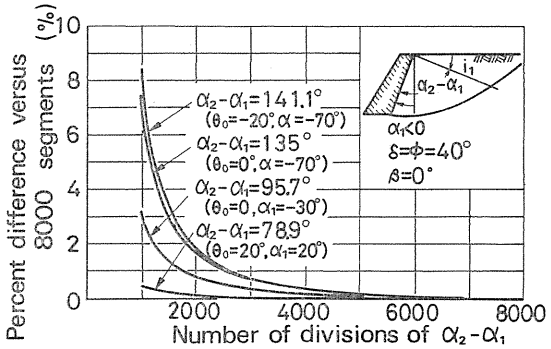

FIG. A-2. Percent difference of K_{PE} of the Sokolovski method computed with less division of $(\alpha_2-\alpha_1)$ versus that computed with 8000 division in the case of $\delta=\phi=40°$.

case of $\delta=\phi=40°$. As shown in this figure, the result of the 3000-segment computation is larger 0.78% than that of the 8000-segment one in the case of $(\alpha_2-\alpha_1)=141.1°$.

In the case of $\delta=\phi$, the values of S and ψ at the backface of wall are discontinuous. Hence, in place of real backface of wall, the authors used the assumed backface which was inclined $\Delta\omega'$ to the real backface according to Sokolovski. S', ψ' and ω on the assumed backface of wall are given by the extended Sokolovski method using the following equations;

$$\left.\begin{aligned}&(\psi'+\mu)^2=\frac{1}{2}\cot\phi\left[1-\frac{\cos(\phi-\alpha_1+\theta_0)}{S_0\cos\phi}\right](\omega-\alpha_1),\\&S'=S_0[1+2(\psi'+\mu)\tan\phi],\end{aligned}\right\}\qquad\text{(A-2)}$$

where $\omega=\alpha_1+\Delta\omega'$.

The value of S_0 on the real backface of wall is obtained by the method described relating to Eq. 43. In the case of $\delta=\phi$, the values of K_{PE} are calculated under the condition that $\Delta\omega'$ equals to 0.001° and $\alpha_2-(\alpha_1+\Delta\omega')$ is divided into 3000 segments. In Table A-8, the percent differences of K_{PE} of $\Delta\omega'=1°$ and 0.1° versus that of $\Delta\omega'=0.001°$ are presented. The error by a large value of $\Delta\omega'$ seems to be compensated with the error of finite differential method. However, if we consider the shape of sliding surface near the wall, we must use the very small value of $\Delta\omega'$.

Next, we consider the errors of K_{PE} by the spiral method. The authors obtained the values of K_{PE} for various values of $\Delta\omega$ and the percent differences of these values of K_{PE} from those of $\Delta\omega=0.01°$ in Table A-9. From this Table, we can understand that the values of K_{PE} in the case of $\Delta\omega=0.2°$ are satisfactory.

Appendix 3. Passive Earth Pressure of Cohesive Soils and Passive Earth Pressure of Soils under Uniform Surcharge

If equation 46.a or 46.b is satisfied, we can use the foregoing results foi medium possesing both internal friction and cohesion according to Sokolovski. In Fig. 1, assuming that the backfill posseses the uniform surcharge, q on the line AC and both internal friction and cohesion, the normal and tangetial stresses on the wall are obtained by the next equations.

$$\left.\begin{aligned}&\sigma_\omega=\gamma xK_{PE}+p_{qE}\qquad\text{and}\\&\tau_{\omega r}=-(\gamma xK_{PE}+p_{qE}+c\cot\phi)\tan\delta,\end{aligned}\right\}\qquad\text{(A-3)}$$

where c is cohesion, ϕ is the angle of internal friction of the backfill and δ is the angle of wall friction.

γxK_{PE} is the normal pressure on the wall for an ideal granular medium ($c=0$) possesing weight but without the surcharge on the boundary of the fill ($q=0$). The values of K_{PE} are shown in Tables 1~6. p_{qE} is the normal pressure on the wall for a soil medium lacking weight ($\gamma=0$).

In Fig. 1, the point of A is a singular point, and is replaced by a characteristic, $\xi=\xi_0$. Along this sliding surface, $\tilde{\sigma}$ is given by the next formula.

$$\tilde{\sigma}=Ce^{2\psi_1\tan\phi},\qquad\text{(A-4)}$$

where C is constant, ψ_1 is the angle between u-plane and major principal plane. The normal and tangential stresses on $\overline{AC}$ are given by the following equations.

$$\left.\begin{aligned}&\sigma_u = q_E \cos\beta_0 \text{ and}\\&\tau_{uv} = -q_E \sin\beta_0,\end{aligned}\right\} \tag{A-5}$$

where q_E equals $q/\cos\theta_0$, β_0 and θ_0 are shown in Eqs. 26 and 27 respectively. Denoting the reduced load as q_1 on $\overline{AC}$ (v-axis) and the inclined angle to the normal line of $\overline{AC}$ as δ_1, the following equations can be obtained.

$$\left.\begin{aligned}&\sigma_u + c\cot\phi = q_E\cos\beta_0 + c\cot\phi = q_1\cos\delta_1 \quad \text{and}\\&\tau_{uv} = -q_E\sin\beta_0 = q_1\sin\delta_1,\end{aligned}\right\} \tag{A-6}$$

where

$$\tan\delta_1 = -\frac{q_E\sin\beta_0}{q_E\cos\beta_0 + c\cot\phi}. \tag{A-7}$$

Also, we can obtained the next equations considering Mohr's circle.

$$\left.\begin{aligned}&q_1\cos\delta_1 = \tilde{\sigma}(1+\sin\phi\cos 2\psi_1) \text{ and}\\&q_1\sin\delta_1 = -\tilde{\sigma}\sin\phi\sin 2\psi_1.\end{aligned}\right\} \tag{A-8}$$

Therefore, ψ_1 and $\tilde{\sigma}$ are given by the following equations.

$$\left.\begin{aligned}&2\psi_1 = -\delta_1 + \Delta_1 - \pi,\\&\tilde{\sigma} = q_1\frac{\sin\Delta_1}{\sin(\Delta_1-\delta_1)},\\&\sin\Delta_1 = \frac{\sin\delta_1}{\sin\phi} \text{ and}\\&\frac{\pi}{2} \geqq \Delta_1 \geqq -\frac{\pi}{2}.\end{aligned}\right\} \tag{A-9}$$

The normal pressure p_{qE} on the backface of wall is given as follows, using the reduced stress p_1.

$$\left.\begin{aligned}&\sigma_\omega + c\cot\phi = p_{qE} + c\cot\phi = p_1\cos\delta \quad \text{and}\\&\tau_{\omega r} = -\tau_{r\omega} = -p_1\sin\delta,\end{aligned}\right\} \tag{A-10}$$

where

$$\left.\begin{aligned}&p_1\cos\delta = \tilde{\sigma}(1-\sin\phi\cos 2\phi) \quad \text{and}\\&p_1\sin\delta = -\tilde{\sigma}\sin\phi\sin 2\phi.\end{aligned}\right\} \tag{A-11}$$

and ϕ is the angle between the major principal plane and the backface of wall. In order to calculate the passive earth pressure against the wall the next relations can be derived.

$$\left.\begin{aligned} &2\psi = \delta + \Delta' - \pi, \\ &\bar{\sigma} = \frac{p_1 \sin \Delta'}{\sin(\delta + \Delta')} \quad \text{and} \\ &\sin \Delta' = \frac{\sin \delta}{\sin \phi}, \end{aligned}\right\} \tag{A-12}$$

where $\psi = \psi_1 + \alpha_1 - \beta$.
Therefore, using the equations A-12, the value of ψ_1 on the wall can be derived as follows.

$$2\psi_1 = \delta + \Delta' - \pi - 2\alpha_1 + 2\beta. \tag{A-13}$$

Using the equations A-4, A-9, A-12, A-13, the next equation can be obtained.

$$p_1 = q_1 \frac{(\cos\delta_1 + \sqrt{\cos^2\delta_1 - \cos^2\phi})(\cos\delta + \sqrt{\cos^2\delta - \cos^2\phi})}{1 - \sin^2\phi} \cdot \exp[(\delta + \Delta' + \delta_1 - \Delta_1 - 2\alpha_1 + 2\beta)\tan\phi]. \tag{A-14}$$

If $\beta = 0°$, $\theta = 0°$ and $\delta_1 = 0°$, the values of Δ_1 equals zero and the following equation can be derived by equation A-14.

$$p_q + c\cot\phi = (q + c\cot\phi)\frac{\cos\delta(\cos\delta + \sqrt{\cos^2\delta - \cos^2\phi})}{1 - \sin\phi} \cdot \exp[(\delta + \Delta' - 2\alpha_1)\tan\phi]. \tag{A-15}$$

This equation is the same with that derived by Sokolovski. The value of p_1/q_1 is obtained by equation A-14 and the value of p_{qE} can be determined by the following equation.

$$p_{qE} = \frac{p_1}{q_1}\left(\frac{\cos\delta}{\cos\delta_1}\right)(q_E\cos\beta_0 + c\cot\phi) - c\cot\phi. \tag{A-16}$$

Thus, σ_ω and $\tau_{\omega r}$ against the wall can be obtained by equation A-3. If $\tau_{\omega r}$ is negative, the tangential stress on the backface of wall acts upwards.

In Fig. A-3, the values of p_1/q_1 are shown for vrious values of ϕ in the case of $\alpha_1 = \beta = 0°$ and $\delta = 1/2 \cdot \phi$. In Fig. A-4, the values of p_{qE} are shown for specified cohesion in the case of $\alpha_1 = \beta = 0°$, $\delta = 1/2 \cdot \phi = 15°$ and $q = 2t/m^2$. The comparison between the results of equation A-3 and the more exact solution are shown in Fig. A-5 and Fig. A-6. This exact solution can be obtained by use of the finite difference method for (c, ϕ) materials. In Fig. A-5, the values of σ_ω are shown for various values of α_1 in the case of $\beta = 0°$, $\theta_0 = 10°$, $\delta = 1/2 \cdot \phi = 10°$, $c = 1t/m^2$ and $q = 1t/m^2$. In Fig. A-6, the values of σ_ω are shown for various values of β in the case of $\alpha_1 = 0°$, $\theta_0 = 10°$, $\delta = 1/2 \cdot \phi = 10°$, $c = 1t/m^2$ and $q = 0$. The results of equation A-3 yield the exact values or values rather less than the exact values. However, in the case of $|\beta_0| > \phi$, σ_ω and $\tau_{\omega r}$ must be calculated by not Eq. A-3 but the exact method.

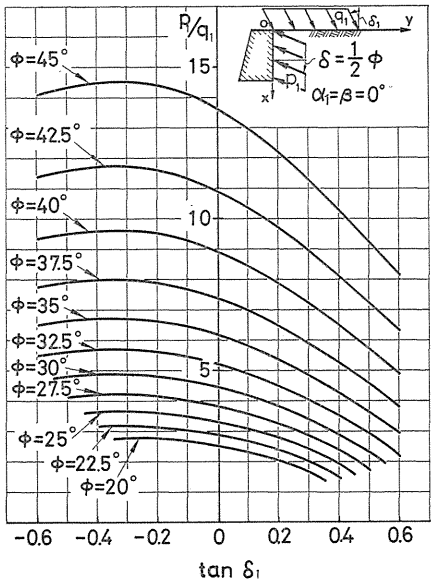

FIG. A-3. Values of p_1/q_1 for various values of ϕ in the case of $=1/2\cdot\phi$ and $\alpha_1=\beta=0°$.

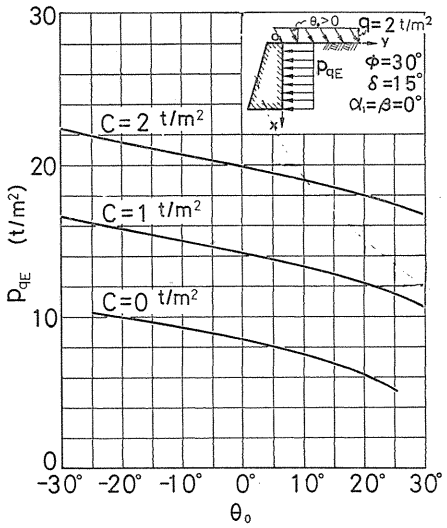

FIG. A-4. Values of p_{qE} for specified cohesion in the case of $\alpha_1=\beta=0°$, $\delta=1/2\cdot\phi=15°$ and $q=2t/m^2$.

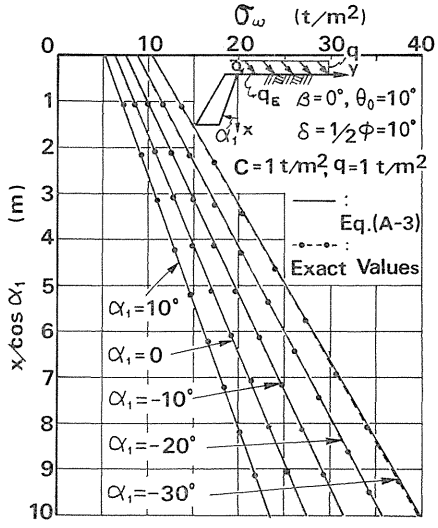

FIG. A-5. Comparison between values of σ_ω of Eq. A-3 and those of the exact method in the case of $\beta=0°$, $\theta_0=10°$, $\delta=1/2\cdot\phi=10°$, $c=1t/m^2$ and $q=1t/m^2$.

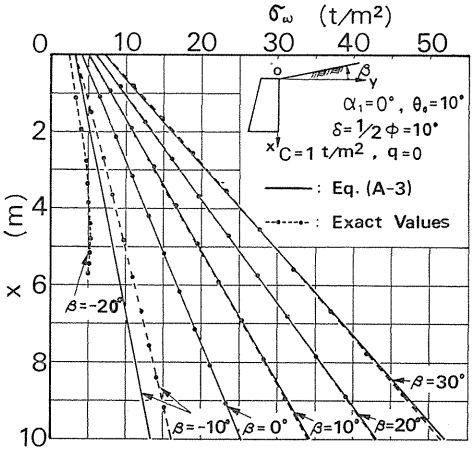

FIG. A-6. Comparison between values of σ_ω of Eq. A-3 and those of the exact method in the case of $\alpha_1=0°$, $\theta_0=10°$, $\delta=1/2\cdot\phi=10°$, $c=1t/m^2$ and $q=0$.

TABLE A-1. Values of K_{PE} and ω_0 of the spiral method and the Sokolovski method for various values of θ_0 in the case of $\alpha_1=\beta=0°$ and $\delta=1/2\cdot\phi$

θ_0	K_{PE} / ω_0 (degrees) / ϕ		20.0°	22.5°	25.0°	27.5°	30.0°	32.5°	35.0°	37.5°	40.0°	42.5°	45.0°
-30°	Log. spiral method	K_{PE}	-----	-----	-----	-----	5.865	6.845	8.078	9.633	11.632	14.257	17.785
		ω_0	-----	-----	-----	-----	58.8	46.9	43.4	41.4	39.9	39.1	38.6
	Sokolovski	K_{PE}					5.833		8.064		11.611		
-20°	Log. spiral method	K_{PE}	3.098	3.529	4.044	4.659	5.404	6.319	7.459	8.903	10.764	13.215	16.518
		ω_0	60.2	45.5	41.1	38.6	37.0	35.8	35.2	34.8	34.7	34.7	35.0
	Sokolovski	K_{PE}					5.401		7.455		10.754		
-10°	Log. spiral method	K_{PE}	2.833	3.227	3.696	4.260	4.947	5.793	6.851	8.195	9.933	12.227	15.328
		ω_0	31.7	30.5	30.0	29.8	29.6	29.9	30.1	30.4	30.8	31.5	32.2
	Sokolovski	K_{PE}					4.945		6.848		9.929		
0°	Log. spiral method	K_{PE}	2.520	2.879	3.308	3.825	4.458	5.239	6.220	7.469	9.089	11.234	14.139
		ω_0	20.6	21.5	22.3	23.2	24.0	24.9	25.7	26.8	27.6	28.7	29.7
	Sokolovski	K_{PE}					4.456		6.219		9.086		
10°	Log. spiral method	K_{PE}	2.114	2.446	2.840	3.316	3.896	4.616	5.520	6.674	8.174	10.166	12.873
		ω_0	10.1	12.8	14.8	16.7	18.4	20.0	21.5	23.1	24.4	25.8	27.2
	Sokolovski	K_{PE}					3.895		5.518		8.173		
20°	Log. spiral method	K_{PE}	1.194	1.762	2.173	2.632	3.175	3.838	4.667	5.724	7.099	8.927	11.419
		ω_0	(17.0)	0.4	4.7	8.7	11.8	14.3	16.8	18.9	20.9	22.6	24.6
	Sokolovski	K_{PE}	1.193				3.173		4.666		7.098		
30°	Log. spiral method	K_{PE}	-----	-----	-----	-----	1.567	2.605	3.432	4.420	5.675	7.330	9.584
		ω_0	-----	-----	-----	-----	(7.2)	4.6	9.4	12.9	16.1	18.6	21.2
	Sokolovski	K_{PE}					1.563		3.432		5.674		

TABLE A-2. Values of K_{PE} and ω_0 of the spiral method and the Sokolovski method for various values of θ_0 in the case of $\alpha_1=\beta=0°$ and $\delta=2/3\cdot\phi$

θ_0	K_{PE} / ω_0 (degrees) / ϕ		20.0°	22.5°	25.0°	27.5°	30.0°	32.5°	35.0°	37.5°	40.0°	42.5°	45.0°
-30°	Log. spiral method	K_{PE}	-----	-----	-----	-----	6.510	7.745	9.313	11.343	14.030	17.677	22.766
		ω_0	-----	-----	-----	-----	64.4	53.5	50.4	48.6	47.7	47.1	47.0
	Sokolovski	K_{PE}									14.013		
-20°	Log. spiral method	K_{PE}	3.277	3.779	4.381	5.113	6.017	7.151	8.598	10.480	12.981	16.384	21.147
		ω_0	64.8	50.9	46.9	44.8	43.6	42.8	42.4	42.4	42.7	43.1	43.6
	Sokolovski	K_{PE}									12.952		
-10°	Log. spiral method	K_{PE}	2.999	3.451	3.998	4.668	5.499	6.546	7.887	9.636	11.967	15.149	19.615
		ω_0	37.3	36.5	36.2	36.4	36.6	37.1	37.5	38.2	39.0	39.9	41.0
	Sokolovski	K_{PE}									11.953		
0°	Log. spiral method	K_{PE}	2.660	3.070	3.568	4.180	4.943	5.908	7.147	8.768	10.935	13.903	18.080
		ω_0	26.8	27.9	28.9	30.0	31.0	32.3	33.5	34.8	36.0	37.3	38.5
	Sokolovski	K_{PE}									10.922		
10°	Log. spiral method	K_{PE}	2.221	2.597	3.051	3.610	4.307	5.188	6.324	7.815	9.814	12.561	16.440
		ω_0	16.9	19.6	21.8	23.9	25.8	27.6	29.3	31.1	32.8	34.6	36.2
	Sokolovski	K_{PE}									9.814		
20°	Log. spiral method	K_{PE}	1.236	1.855	2.318	2.848	3.490	4.294	5.325	6.679	8.497	11.003	14.555
		ω_0	(8.0)	7.0	12.3	16.1	19.2	22.1	24.6	27.1	29.3	31.4	33.6
	Sokolovski	K_{PE}	1.236								8.486		
30°	Log. spiral method	K_{PE}	-----	-----	-----	-----	1.688	2.883	3.883	5.122	6.755	8.995	12.174
		ω_0	-----	-----	-----	-----	1.6	12.8	17.6	21.3	24.7	27.6	30.2
	Sokolovski	K_{PE}									6.750		

TABLE A-3 (1). Values of K_{PE} and ω_0 of the spiral method and the Sokolovski method for various values of α_1 in the case of $\beta=0°$, $\delta=0°$ and $\phi=20°\sim32.5°$ (The values of ω_0 in parentheses indicate the case of the concave curve)

ϕ δ β	θ_0	K_{PE} / ω_0 (degrees) / α_1	-30°	-20°	-10°	0°	10°	20°	30°
$\phi=20.0°$ $\delta=0°$ $\beta=0°$	0°	Log. spiral method K_{PE}	3.183	2.699	2.330	2.040	1.806	1.616	1.466
		Log. spiral method ω_0	33.0	22.2	11.2	(0.8)	(11.8)	(24.2)	(38.0)
		Sokolovski K_{PE}					1.805	1.612	1.451
	10°	Log. spiral method K_{PE}	2.613	2.249	1.968	1.744	1.563	1.418	1.315
		Log. spiral method ω_0	20.9	10.5	(0.6)	(11.6)	(24.0)	(37.6)	(52.8)
		Sokolovski K_{PE}				1.744	1.560	1.406	
	20°	Log. spiral method K_{PE}	1.429	1.275	1.150	1.052	0.991	-----	-----
		Log. spiral method ω_0	(5.2)	(16.0)	(28.6)	(43.0)	(59.8)	-----	-----
		Sokolovski K_{PE}		1.275	1.148	1.041			
	30°	Log. spiral method K_{PE}	-----	-----	-----	-----	-----	-----	-----
		Log. spiral method ω_0	-----	-----	-----	-----	-----	-----	-----
		Sokolovski K_{PE}							
$\phi=22.5°$ $\delta=0°$ $\beta=0°$	0°	Log. spiral method K_{PE}	3.720	3.081	2.606	2.240	1.951	1.720	1.538
		Log. spiral method ω_0	33.3	22.3	11.3	(0.9)	(11.7)	(24.1)	(37.7)
		Sokolovski K_{PE}					1.950	1.715	1.522
	10°	Log. spiral method K_{PE}	3.097	2.603	2.229	1.938	1.707	1.524	1.390
		Log. spiral method ω_0	23.2	12.6	1.6	(9.7)	(21.7)	(34.7)	(49.5)
		Sokolovski K_{PE}				1.938	1.705	1.513	
	20°	Log. spiral method K_{PE}	2.174	1.871	1.635	1.447	1.299	1.195	-----
		Log. spiral method ω_0	8.6	(1.9)	(12.7)	(24.7)	(38.3)	(53.7)	-----
		Sokolovski K_{PE}			1.634	1.445	1.289		
	30°	Log. spiral method K_{PE}	-----	-----	-----	-----	-----	-----	-----
		Log. spiral method ω_0	-----	-----	-----	-----	-----	-----	-----
		Sokolovski K_{PE}							
$\phi=25.0°$ $\delta=0°$ $\beta=0°$	0°	Log. spiral method K_{PE}	4.365	3.530	2.923	2.464	2.109	1.831	1.615
		Log. spiral method ω_0	33.3	22.3	11.3	(0.8)	(11.6)	(24.0)	(37.4)
		Sokolovski K_{PE}					2.108	1.825	1.597
	10°	Log. spiral method K_{PE}	3.680	3.015	2.527	2.154	1.864	1.637	1.469
		Log. spiral method ω_0	24.8	14.0	3.2	(8.0)	(19.8)	(32.8)	(47.0)
		Sokolovski K_{PE}				2.154	1.861	1.627	1.437
	20°	Log. spiral method K_{PE}	2.752	2.301	1.962	1.699	1.493	1.339	1.246
		Log. spiral method ω_0	13.7	3.1	(7.6)	(19.2)	(31.8)	(46.0)	(62.0)
		Sokolovski K_{PE}			1.962	1.698	1.487	1.315	
	30°	Log. spiral method K_{PE}	-----	-----	-----	-----	-----	-----	-----
		Log. spiral method ω_0	-----	-----	-----	-----	-----	-----	-----
		Sokolovski K_{PE}							
$\phi=27.5°$ $\delta=0°$ $\beta=0°$	0°	Log. spiral method K_{PE}	5.148	4.060	3.286	2.710	2.283	1.950	1.697
		Log. spiral method ω_0	33.6	22.4	11.4	(0.7)	(11.7)	(23.9)	(37.1)
		Sokolovski K_{PE}					2.282	1.944	1.676
	10°	Log. spiral method K_{PE}	4.389	3.503	2.868	2.396	2.035	1.758	1.552
		Log. spiral method ω_0	26.3	15.3	4.3	(6.9)	(18.5)	(31.1)	(44.9)
		Sokolovski K_{PE}				2.396	2.033	1.748	1.522
	20°	Log. spiral method K_{PE}	3.412	2.772	2.306	1.954	1.684	1.478	1.340
		Log. spiral method ω_0	17.1	6.5	(4.3)	(15.5)	(27.5)	(41.1)	(56.1)
		Sokolovski K_{PE}				1.953	1.678	1.460	
	30°	Log. spiral method K_{PE}	-----	-----	-----	-----	-----	-----	-----
		Log. spiral method ω_0	-----	-----	-----	-----	-----	-----	-----
		Sokolovski K_{PE}							
$\phi=30.0°$ $\delta=0°$ $\beta=0°$	0°	Log. spiral method K_{PE}	6.108	4.689	3.708	3.001	2.475	2.080	1.783
		Log. spiral method ω_0	33.8	22.6	11.4	(0.6)	(11.6)	(23.8)	(36.8)
		Sokolovski K_{PE}					2.474	2.073	1.761
	10°	Log. spiral method K_{PE}	5.258	4.083	3.263	2.668	2.224	1.889	1.641
		Log. spiral method ω_0	27.4	16.4	5.4	(5.8)	(17.4)	(29.8)	(43.2)
		Sokolovski K_{PE}				2.668	2.222	1.879	1.611
	20°	Log. spiral method K_{PE}	4.206	3.318	2.693	2.229	1.883	1.622	1.438
		Log. spiral method ω_0	19.8	9.0	0.2	(12.8)	(24.6)	(37.6)	(52.0)
		Sokolovski K_{PE}				2.229	1.878	1.606	
	30°	Log. spiral method K_{PE}	2.000	1.665	1.398	1.202	1.058	0.977	-----
		Log. spiral method ω_0	0.4	(4.5)	(21.2)	(33.6)	(47.8)	(47.8)	-----
		Sokolovski K_{PE}			1.397	1.197			
$\phi=32.5°$ $\delta=0°$ $\beta=0°$	0°	Log. spiral method K_{PE}	7.296	5.455	4.198	3.323	2.688	2.220	1.874
		Log. spiral method ω_0	34.1	22.7	11.3	(0.7)	(11.5)	(23.7)	(36.5)
		Sokolovski K_{PE}					2.687	2.212	1.850
	10°	Log. spiral method K_{PE}	6.338	4.780	3.723	2.976	2.432	2.030	1.735
		Log. spiral method ω_0	28.4	17.2	6.2	(4.9)	(16.5)	(28.7)	(41.9)
		Sokolovski K_{PE}					2.430	2.019	1.705
	20°	Log. spiral method K_{PE}	5.184	3.965	3.131	2.535	2.097	1.773	1.541
		Log. spiral method ω_0	22.1	11.1	0.7	(10.9)	(22.5)	(34.9)	(48.7)
		Sokolovski K_{PE}				2.534	2.093	1.758	
	30°	Log. spiral method K_{PE}	3.428	2.696	2.182	1.807	1.528	1.326	1.205
		Log. spiral method ω_0	11.2	0.8	(9.9)	(20.9)	(33.1)	(46.9)	(60.5)
		Sokolovski K_{PE}			2.182	1.806	1.520	1.299	

TABLE A-3 (2). Values of K_{PE} and ω_0 of the spiral method and the Sokolovski method for various values of α_1 in the case of $\beta=0°$, $\delta=0°$ and $\phi=35°\sim45°$

ϕ δ β	θ_0	K_{PE} / ω_0 (degrees) / α_1	-30°	-20°	-10°	0°	10°	20°	30°
$\phi=35.0°$ $\delta=0°$ $\beta=0°$	0°	Log. spiral method K_{PE}	8.785	6.360	4.774	3.691	2.925	2.372	1.971
		Log. spiral method ω_0	34.3	22.7	11.3	(1.0)	(11.6)	(23.6)	(36.4)
		Sokolovski K_{PE}					2.923	2.363	1.945
	10°	Log. spiral method K_{PE}	7.696	5.626	4.265	3.329	2.664	2.183	1.835
		Log. spiral method ω_0	29.5	18.1	6.9	(4.0)	(15.8)	(27.8)	(40.8)
		Sokolovski K_{PE}					2.661	2.172	1.805
	20°	Log. spiral method K_{PE}	6.411	4.748	3.644	2.879	2.331	1.933	1.649
		Log. spiral method ω_0	23.8	12.8	1.8	(9.2)	(20.6)	(32.8)	(46.2)
		Sokolovski K_{PE}				2.878	2.328	1.920	1.612
	30°	Log. spiral method K_{PE}	4.613	3.494	2.738	2.205	1.819	1.538	1.349
		Log. spiral method ω_0	15.6	5.0	(5.6)	(16.6)	(28.2)	(41.0)	(53.7)
		Sokolovski K_{PE}				2.204	1.813	1.518	
$\phi=37.5°$ $\delta=0°$ $\beta=0°$	0°	Log. spiral method K_{PE}	10.676	7.479	5.455	4.112	3.189	2.537	2.075
		Log. spiral method ω_0	34.6	22.8	11.4	0.4	(11.5)	(23.5)	(36.1)
		Sokolovski K_{PE}					3.187	2.527	2.047
	10°	Log. spiral method K_{PE}	9.427	6.664	4.906	3.733	2.923	2.349	1.942
		Log. spiral method ω_0	30.3	18.9	7.5	(3.7)	(15.1)	(26.9)	(39.7)
		Sokolovski K_{PE}					2.920	2.338	1.911
	20°	Log. spiral method K_{PE}	7.977	5.706	4.250	3.271	2.590	2.106	1.764
		Log. spiral method ω_0	25.5	14.3	3.3	(7.9)	(19.1)	(31.1)	(44.1)
		Sokolovski K_{PE}				3.271	2.587	2.093	1.728
	30°	Log. spiral method K_{PE}	6.045	4.407	3.343	2.618	2.108	1.742	1.490
		Log. spiral method ω_0	18.7	7.9	(2.9)	(13.7)	(24.9)	(37.1)	(50.7)
		Sokolovski K_{PE}				2.618	2.103	1.726	
$\phi=40.0°$ $\delta=0°$ $\beta=0°$	0°	Log. spiral method K_{PE}	13.115	8.865	6.268	4.600	3.484	2.718	2.185
		Log. spiral method ω_0	34.8	23.0	11.4	(0.4)	(11.6)	(23.4)	(35.8)
		Sokolovski K_{PE}					3.482	2.707	2.155
	10°	Log. spiral method K_{PE}	11.668	7.954	5.673	4.201	3.213	2.531	2.056
		Log. spiral method ω_0	31.0	19.4	8.2	(3.0)	(14.6)	(26.4)	(38.8)
		Sokolovski K_{PE}					3.210	2.518	2.024
	20°	Log. spiral method K_{PE}	10.010	6.897	4.973	3.724	2.879	2.293	1.885
		Log. spiral method ω_0	26.9	15.5	4.3	(6.5)	(18.0)	(29.6)	(42.2)
		Sokolovski K_{PE}				3.724	2.875	2.280	1.851
	30°	Log. spiral method K_{PE}	7.872	5.514	4.042	3.075	2.414	1.952	1.632
		Log. spiral method ω_0	21.5	10.3	(0.8)	(11.4)	(22.6)	(34.4)	(47.4)
		Sokolovski K_{PE}				3.074	2.410	1.937	
$\phi=42.5°$ $\delta=0°$ $\beta=0°$	0°	Log. spiral method K_{PE}	16.316	10.605	7.247	5.260	3.817	2.915	2.302
		Log. spiral method ω_0	35.1	23.1	11.5	(0.5)	(11.5)	(23.1)	(35.5)
		Sokolovski K_{PE}					3.814	2.902	2.269
	10°	Log. spiral method K_{PE}	14.618	9.577	6.600	4.746	3.539	2.729	2.177
		Log. spiral method ω_0	31.8	20.0	8.6	(2.7)	(14.1)	(25.7)	(37.9)
		Sokolovski K_{PE}					3.536	2.716	2.144
	20°	Log. spiral method K_{PE}	12.696	8.399	5.848	4.250	3.204	2.497	2.014
		Log. spiral method ω_0	28.2	16.6	5.2	(5.7)	(16.9)	(28.5)	(40.7)
		Sokolovski K_{PE}					3.200	2.484	1.980
	30°	Log. spiral method K_{PE}	10.276	6.897	4.874	3.594	2.749	2.173	1.779
		Log. spiral method ω_0	23.6	12.2	1.2	(9.7)	(20.7)	(32.1)	(44.5)
		Sokolovski K_{PE}				3.594	2.745	2.159	1.742
$\phi=45.0°$ $\delta=0°$ $\beta=0°$	0°	Log. spiral method K_{PE}	20.600	12.823	8.442	5.829	4.193	3.132	2.427
		Log. spiral method ω_0	35.3	23.1	11.5	0.3	(11.4)	(23.0)	(35.2)
		Sokolovski K_{PE}					4.190	3.117	2.392
	10°	Log. spiral method K_{PE}	18.582	11.652	7.733	5.385	3.909	2.947	2.307
		Log. spiral method ω_0	32.4	20.6	9.0	(2.4)	(13.6)	(25.2)	(37.2)
		Sokolovski K_{PE}					3.906	2.933	2.273
	20°	Log. spiral method K_{PE}	16.318	10.324	6.919	4.867	3.570	2.720	2.152
		Log. spiral method ω_0	29.4	17.6	6.2	(5.0)	(16.0)	(27.4)	(39.4)
		Sokolovski K_{PE}					3.566	2.706	2.118
	30°	Log. spiral method K_{PE}	13.521	8.665	5.885	4.198	3.122	2.410	1.932
		Log. spiral method ω_0	25.4	13.8	2.8	(8.2)	(19.0)	(30.4)	(42.4)
		Sokolovski K_{PE}				4.198	3.118	2.397	1.898

TABLE A-3 (3). Values of K_{PE} and ω_0 of the spiral method and the Sokolovski method for various values of α_1 in the case of $\beta=0°$, $\delta=1/2\cdot\phi$ and $20°\sim32.5°$

ϕ δ β	θ_0	K_{PE} / ω_0 (degrees) / α_1	-30°	-20°	-10°	0°	10°	20°	30°
$\phi=20.0°$ $\delta=1/2\cdot\phi$ $\beta=0°$	0°	Log. spiral method K_{PE}	4.179	3.467	2.934	2.520	2.191	1.921	1.698
		Log. spiral method ω_0	49.4	39.8	30.4	20.6	10.8	0.6	(11.4)
		Sokolovski K_{PE}							1.698
	10°	Log. spiral method K_{PE}	3.388	2.846	2.436	2.114	1.854	1.639	1.462
		Log. spiral method ω_0	38.3	29.1	19.7	10.1	(0.6)	(11.4)	(25.6)
		Sokolovski K_{PE}						1.639	1.457
	20°	Log. spiral method K_{PE}	1.755	1.528	1.345	1.194	1.050	-----	-----
		Log. spiral method ω_0	14.0	5.0	(5.0)	(17.0)	(28.0)	-----	-----
		Sokolovski K_{PE}			1.345	1.193			
	30°	Log. spiral method K_{PE}	-----	-----	-----	-----	-----	-----	-----
		Log. spiral method ω_0	-----	-----	-----	-----	-----	-----	-----
		Sokolovski K_{PE}							
$\phi=22.5°$ $\delta=1/2\cdot\phi$ $\beta=0°$	0°	Log. spiral method K_{PE}	5.132	4.147	3.426	2.879	2.452	2.110	1.832
		Log. spiral method ω_0	50.7	40.9	31.3	21.5	11.5	1.1	(10.5)
		Sokolovski K_{PE}							1.832
	10°	Log. spiral method K_{PE}	4.229	3.455	2.883	2.446	2.100	1.822	1.595
		Log. spiral method ω_0	41.2	31.8	22.4	12.8	2.6	(8.5)	(21.7)
		Sokolovski K_{PE}						1.822	1.592
	20°	Log. spiral method K_{PE}	2.886	2.407	2.046	1.762	1.533	1.347	1.214
		Log. spiral method ω_0	27.4	18.4	9.2	0.4	(12.1)	(26.1)	(44.7)
		Sokolovski K_{PE}					1.533	1.343	
	30°	Log. spiral method K_{PE}	-----	-----	-----	-----	-----	-----	-----
		Log. spiral method ω_0	-----	-----	-----	-----	-----	-----	-----
		Sokolovski K_{PE}							
$\phi=25.0°$ $\delta=1/2\cdot\phi$ $\beta=0°$	0°	Log. spiral method K_{PE}	6.364	5.001	4.029	3.308	2.757	2.327	1.984
		Log. spiral method ω_0	51.9	41.9	32.1	22.3	12.3	1.7	(9.6)
		Sokolovski K_{PE}							1.983
	10°	Log. spiral method K_{PE}	5.318	4.219	3.429	2.840	2.387	2.029	1.743
		Log. spiral method ω_0	43.8	34.2	24.6	14.8	4.8	(6.0)	(18.6)
		Sokolovski K_{PE}						2.029	1.741
	20°	Log. spiral method K_{PE}	3.892	3.139	2.590	2.173	1.847	1.587	1.385
		Log. spiral method ω_0	33.1	23.9	14.5	4.7	(5.8)	(18.0)	(33.8)
		Sokolovski K_{PE}					1.847	1.586	
	30°	Log. spiral method K_{PE}	-----	-----	-----	-----	-----	-----	-----
		Log. spiral method ω_0	-----	-----	-----	-----	-----	-----	-----
		Sokolovski K_{PE}							
$\phi=27.5°$ $\delta=1/2\cdot\phi$ $\beta=0°$	0°	Log. spiral method K_{PE}	7.976	6.089	4.775	3.825	3.118	2.577	2.155
		Log. spiral method ω_0	53.2	43.0	33.0	23.2	13.2	2.6	(8.7)
		Sokolovski K_{PE}							2.154
	10°	Log. spiral method K_{PE}	6.749	5.194	4.106	3.316	2.723	2.267	1.909
		Log. spiral method ω_0	46.1	36.3	26.5	16.7	6.7	(4.1)	(16.1)
		Sokolovski K_{PE}						2.267	1.908
	20°	Log. spiral method K_{PE}	5.160	4.024	3.221	2.632	2.184	1.836	1.564
		Log. spiral method ω_0	37.5	27.9	18.3	8.7	(1.7)	(13.1)	(27.1)
		Sokolovski K_{PE}						1.835	1.557
	30°	Log. spiral method K_{PE}	-----	-----	-----	-----	-----	-----	-----
		Log. spiral method ω_0	-----	-----	-----	-----	-----	-----	-----
		Sokolovski K_{PE}							
$\phi=30.0°$ $\delta=1/2\cdot\phi$ $\beta=0°$	0°	Log. spiral method K_{PE}	10.122	7.493	5.712	4.458	3.547	2.867	2.350
		Log. spiral method ω_0	54.4	44.0	34.0	24.0	13.8	3.4	(8.0)
		Sokolovski K_{PE}				4.456			2.349
	10°	Log. spiral method K_{PE}	8.662	6.456	4.957	3.896	3.122	2.542	2.098
		Log. spiral method ω_0	48.4	38.2	28.4	18.4	8.4	(2.2)	(14.0)
		Sokolovski K_{PE}				3.895			2.097
	20°	Log. spiral method K_{PE}	6.837	5.151	3.997	3.175	2.569	2.110	1.758
		Log. spiral method ω_0	41.0	31.2	21.4	11.8	1.6	(9.4)	(22.4)
		Sokolovski K_{PE}				3.173		2.110	1.754
	30°	Log. spiral method K_{PE}	3.025	2.375	1.912	1.567	1.304	1.107	1.000
		Log. spiral method ω_0	21.6	12.4	3.0	(7.2)	(19.4)	(35.2)	(59.0)
		Sokolovski K_{PE}				1.563	1.302		
$\phi=32.5°$ $\delta=1/2\cdot\phi$ $\beta=0°$	0°	Log. spiral method K_{PE}	13.031	9.336	6.904	5.239	4.063	3.208	2.573
		Log. spiral method ω_0	55.7	45.3	35.1	24.9	14.7	4.1	(7.1)
		Sokolovski K_{PE}							
	10°	Log. spiral method K_{PE}	11.269	8.120	6.042	4.616	3.604	2.864	2.313
		Log. spiral method ω_0	50.4	40.2	30.0	20.0	9.8	(0.3)	(12.1)
		Sokolovski K_{PE}							2.312
	20°	Log. spiral method K_{PE}	9.121	6.629	4.978	3.838	3.024	2.424	1.975
		Log. spiral method ω_0	44.3	34.3	24.3	14.3	4.3	(6.5)	(18.7)
		Sokolovski K_{PE}							1.972
	30°	Log. spiral method K_{PE}	5.839	4.330	3.317	2.605	2.087	1.700	1.413
		Log. spiral method ω_0	33.6	24.0	14.4	4.6	(5.7)	(17.7)	(32.5)
		Sokolovski K_{PE}					2.087	1.698	

TABLE A-3 (4). Values of K_{PE} and ω_0 of the spiral method and the Sokolovski method for various values of α_1 in the case of $\beta=0°$, $\delta=1/2\cdot\phi$ and $\phi=35°\sim45°$

ϕ δ β	θ_0	K_{PE} / ω_0 (degrees) / α_1	-30°	-20°	-10°	0°	10°	20°	30°
$\phi=35.0°$ $\delta=1/2\cdot\phi$ $\beta=0°$	0°	Log. spiral method K_{PE}	17.056	11.800	8.446	6.220	4.692	3.612	2.831
		Log. spiral method ω_0	56.9	46.3	36.1	25.7	15.5	4.9	(6.2)
		Sokolovski K_{PE}				6.219			2.831
	10°	Log. spiral method K_{PE}	14.895	10.353	7.451	5.520	4.190	3.247	2.561
		Log. spiral method ω_0	52.5	42.1	31.7	21.5	11.3	0.9	(10.6)
		Sokolovski K_{PE}				5.518			2.561
	20°	Log. spiral method K_{PE}	12.311	8.616	6.249	4.667	3.572	2.791	2.220
		Log. spiral method ω_0	47.2	36.8	26.8	16.8	6.6	(4.0)	(15.8)
		Sokolovski K_{PE}				4.666			2.218
	30°	Log. spiral method K_{PE}	8.667	6.152	4.528	3.432	2.664	2.109	1.702
		Log. spiral method ω_0	39.2	29.2	19.2	9.4	(0.6)	(11.8)	(25.0)
		Sokolovski K_{PE}				3.432		2.109	1.695
$\phi=37.5°$ $\delta=1/2\cdot\phi$ $\beta=0°$	0°	Log. spiral method K_{PE}	22.756	15.162	10.479	7.469	5.468	4.096	3.132
		Log. spiral method ω_0	58.4	47.6	37.2	26.8	16.4	5.8	(5.3)
		Sokolovski K_{PE}							3.132
	10°	Log. spiral method K_{PE}	20.055	13.414	9.313	6.674	4.914	3.704	2.851
		Log. spiral method ω_0	54.5	43.9	33.3	23.1	12.7	2.3	(8.9)
		Sokolovski K_{PE}							2.850
	20°	Log. spiral method K_{PE}	16.876	11.349	7.930	5.724	4.247	3.226	2.502
		Log. spiral method ω_0	49.9	39.5	29.1	18.9	8.7	(1.9)	(13.3)
		Sokolovski K_{PE}							2.501
	30°	Log. spiral method K_{PE}	12.598	8.559	6.049	4.420	3.320	2.552	2.003
		Log. spiral method ω_0	43.5	33.1	22.9	12.9	2.7	(7.9)	(20.1)
		Sokolovski K_{PE}						2.552	1.999
$\phi=40.0°$ $\delta=1/2\cdot\phi$ $\beta=0°$	0°	Log. spiral method K_{PE}	31.044	19.863	13.213	9.089	6.440	4.683	3.485
		Log. spiral method ω_0	59.8	48.8	38.2	27.6	17.2	6.6	(4.4)
		Sokolovski K_{PE}				9.086			3.485
	10°	Log. spiral method K_{PE}	27.598	17.711	11.827	8.174	5.823	4.260	3.191
		Log. spiral method ω_0	56.4	45.6	35.0	24.4	14.0	3.6	(7.6)
		Sokolovski K_{PE}				8.173			3.190
	20°	Log. spiral method K_{PE}	23.592	15.205	10.205	7.099	5.092	3.753	2.832
		Log. spiral method ω_0	52.5	41.7	31.1	20.9	10.5	0.3	(11.2)
		Sokolovski K_{PE}				7.098			2.831
	30°	Log. spiral method K_{PE}	18.367	11.925	8.077	5.675	4.115	3.065	2.339
		Log. spiral method ω_0	47.1	36.5	26.3	16.1	5.9	(4.8)	(16.4)
		Sokolovski K_{PE}				5.674		3.065	2.337
$\phi=42.5°$ $\delta=1/2\cdot\phi$ $\beta=0°$	0°	Log. spiral method K_{PE}	43.466	26.613	16.979	11.234	7.678	5.405	3.906
		Log. spiral method ω_0	61.3	50.1	39.3	28.7	18.1	7.5	(3.5)
		Sokolovski K_{PE}							3.906
	10°	Log. spiral method K_{PE}	38.959	23.909	15.303	10.166	6.983	4.943	3.595
		Log. spiral method ω_0	58.2	47.2	36.4	25.8	15.4	4.8	(6.3)
		Sokolovski K_{PE}							3.595
	20°	Log. spiral method K_{PE}	33.777	20.795	13.366	8.927	6.171	4.399	3.223
		Log. spiral method ω_0	54.8	44.0	33.2	22.6	12.2	1.8	(9.3)
		Sokolovski K_{PE}							3.223
	30°	Log. spiral method K_{PE}	27.169	16.816	10.884	7.330	5.115	3.683	2.726
		Log. spiral method ω_0	50.6	39.6	29.0	18.6	8.4	(2.1)	(13.3)
		Sokolovski K_{PE}							2.725
$\phi=45.0°$ $\delta=1/2\cdot\phi$ $\beta=0°$	0°	Log. spiral method K_{PE}	62.747	36.613	22.312	14.139	9.288	6.306	4.413
		Log. spiral method ω_0	62.7	51.5	40.5	29.7	19.1	8.3	(2.6)
		Sokolovski K_{PE}							
	10°	Log. spiral method K_{PE}	56.683	33.134	20.243	12.873	8.494	5.798	4.083
		Log. spiral method ω_0	60.0	49.0	38.0	27.2	16.6	6.0	(5.0)
		Sokolovski K_{PE}							4.083
	20°	Log. spiral method K_{PE}	49.772	29.162	17.876	11.419	7.577	5.206	3.693
		Log. spiral method ω_0	57.2	46.0	35.2	24.6	14.0	3.4	(7.6)
		Sokolovski K_{PE}							3.692
	30°	Log. spiral method K_{PE}	41.113	24.178	14.899	9.584	6.412	4.447	3.184
		Log. spiral method ω_0	53.6	42.4	31.6	21.2	10.6	(0.4)	(10.8)
		Sokolovski K_{PE}							3.183

TABLE A-3 (5). Values of K_{PE} and ω_0 of the spiral method and the Sokolovski method for various values of α_1 in the case of $\beta=0°$, $\delta=2/3\cdot\phi$ and $\phi=20°\sim32.5°$

ϕ δ β	θ_0	K_{PE} / ω_0 (degrees) / α_1	-30°	-20°	-10°	0°	10°	20°	30°
$\phi=20.0°$ $\delta=2/3\cdot\phi$ $\beta=0°$	0°	Log. spiral method K_{PE}	4.491	3.701	3.113	2.660	2.300	2.007	1.764
		Log. spiral method ω_0	54.2	45.0	35.8	26.8	17.6	8.2	(2.5)
		Sokolovski K_{PE}							
	10°	Log. spiral method K_{PE}	3.631	3.028	2.575	2.221	1.937	1.703	1.506
		Log. spiral method ω_0	43.3	34.5	25.7	17.0	7.5	0.3	(15.1)
		Sokolovski K_{PE}							1.506
	20°	Log. spiral method K_{PE}	1.854	1.602	1.401	1.236	1.098	-----	-----
		Log. spiral method ω_0	19.8	11.6	2.8	(8.0)	(22.2)	-----	-----
		Sokolovski K_{PE}				1.236	1.096		
	30°	Log. spiral method K_{PE}	-----	-----	-----	-----	-----	-----	-----
		Log. spiral method ω_0	-----	-----	-----	-----	-----	-----	-----
		Sokolovski K_{PE}							
$\phi=22.5°$ $\delta=2/3\cdot\phi$ $\beta=0°$	0°	Log. spiral method K_{PE}	5.588	4.482	3.677	3.070	2.598	2.223	1.919
		Log. spiral method ω_0	55.7	46.3	37.1	27.9	18.7	9.1	0.5
		Sokolovski K_{PE}							
	10°	Log. spiral method K_{PE}	4.595	3.723	3.083	2.597	2.215	1.909	1.658
		Log. spiral method ω_0	46.4	37.4	28.4	19.6	10.2	(0.4)	(11.4)
		Sokolovski K_{PE}							1.657
	20°	Log. spiral method K_{PE}	3.115	2.574	2.169	1.855	1.602	1.395	1.232
		Log. spiral method ω_0	33.0	24.6	16.0	7.0	(3.1)	(15.5)	(33.3)
		Sokolovski K_{PE}						1.394	
	30°	Log. spiral method K_{PE}	-----	-----	-----	-----	-----	-----	-----
		Log. spiral method ω_0	-----	-----	-----	-----	-----	-----	-----
		Sokolovski K_{PE}							
$\phi=25.0°$ $\delta=2/3\cdot\phi$ $\beta=0°$	0°	Log. spiral method K_{PE}	7.029	5.479	4.379	3.568	2.953	2.475	2.094
		Log. spiral method ω_0	57.1	47.5	38.1	28.9	19.5	9.9	(0.8)
		SOkolovski K_{PE}							
	10°	Log. spiral method K_{PE}	5.865	4.610	3.715	3.051	2.544	2.147	1.828
		Log. spiral method ω_0	49.4	40.0	30.8	21.8	12.6	2.8	(8.4)
		Sokolovski K_{PE}							1.828
	20°	Log. spiral method K_{PE}	4.274	3.411	2.787	2.318	1.954	1.664	1.431
		Log. spiral method ω_0	38.9	30.1	21.3	12.3	2.7	(8.2)	(22.4)
		Sokolovski K_{PE}						1.664	1.427
	30°	Log. spiral method K_{PE}	-----	-----	-----	-----	-----	-----	-----
		Log. spiral method ω_0	-----	-----	-----	-----	-----	-----	-----
		Sokolovski K_{PE}							
$\phi=27.5°$ $\delta=2/3\cdot\phi$ $\beta=0°$	0°	Log. spiral method K_{PE}	8.950	6.771	5.263	4.180	3.379	2.770	2.297
		Log. spiral method ω_0	58.6	49.0	39.4	30.0	20.6	11.0	1.0
		Sokolovski K_{PE}							
	10°	Log. spiral method K_{PE}	7.566	5.764	4.513	3.610	2.938	2.424	2.022
		Log. spiral method ω_0	51.9	42.3	33.1	23.9	14.5	4.7	(6.1)
		Sokolovski K_{PE}							2.022
	20°	Log. spiral method K_{PE}	5.768	4.446	3.521	2.848	2.341	1.948	1.638
		Log. spiral method ω_0	43.5	34.3	25.3	16.1	6.7	(3.5)	(15.9)
		Sokolovski K_{PE}						1.948	1.636
	30°	Log. spiral method K_{PE}	-----	-----	-----	-----	-----	-----	-----
		Log. spiral method ω_0	-----	-----	-----	-----	-----	-----	-----
		Sokolovski K_{PE}							
$\phi=30.0°$ $\delta=2/3\cdot\phi$ $\beta=0°$	0°	Log. spiral method K_{PE}	11.558	8.474	6.396	4.943	3.895	3.120	2.531
		Log. spiral method ω_0	60.2	50.2	40.6	31.0	21.6	11.8	1.8
		Sokolovski K_{PE}							
	10°	Log. spiral method K_{PE}	9.888	7.290	5.537	4.307	3.416	2.752	2.246
		Log. spiral method ω_0	54.4	44.6	35.2	25.8	16.2	6.6	(3.8)
		Sokolovski K_{PE}							2.246
	20°	Log. spiral method K_{PE}	7.790	5.797	4.445	3.490	2.793	2.269	1.865
		Log. spiral method ω_0	47.4	37.8	28.6	19.2	10.0	0.4	(11.4)
		Sokolovski K_{PE}							1.865
	30°	Log. spiral method K_{PE}	3.396	2.625	2.085	1.688	1.385	1.152	1.000
		Log. spiral method ω_0	28.4	19.6	10.8	1.6	(9.2)	(23.6)	(59.0)
		Sokolovski K_{PE}					1.385	1.148	
$\phi=32.5°$ $\delta=2/3\cdot\phi$ $\beta=0°$	0°	Log. spiral method K_{PE}	15.168	10.757	7.870	5.908	4.531	3.539	2.806
		Log. spiral method ω_0	61.7	51.7	41.9	32.3	22.7	12.9	2.7
		Sokolovski K_{PE}							
	10°	Log. spiral method K_{PE}	13.120	9.346	6.873	5.188	4.003	3.145	2.507
		Log. spiral method ω_0	56.8	46.8	37.2	27.6	18.0	8.4	(2.1)
		Sokolovski K_{PE}							
	20°	Log. spiral method K_{PE}	10.610	7.612	5.642	4.294	3.340	2.644	2.123
		Log. spiral method ω_0	50.9	41.1	31.5	22.1	12.7	2.9	(8.1)
		Sokolovski K_{PE}							2.122
	30°	Log. spiral method K_{PE}	6.760	4.935	3.723	2.883	2.277	1.827	1.487
		Log. spiral method ω_0	40.4	31.2	22.0	12.8	3.2	(7.3)	(20.7)
		Sokolovski K_{PE}						1.827	1.483

TABLE A-3 (6). Values of K_{PE} and ω_0 of the spiral method and the Sokolovski method for various values of α_1 in the case of $\beta=0°$, $\delta=2/3\cdot\phi$ and $\phi=35°\sim45°$

ϕ δ β	θ_0	K_{PE} / ω_0 (degrees)	α_1 -30°	-20°	-10°	0°	10°	20°	30°
$\phi=35.0°$ $\delta=2/3\cdot\phi$ $\beta=0°$	0°	Log. spiral method K_{PE}	20.281	13.883	9.823	7.147	5.324	4.047	3.130
		Log. spiral method ω_0	63.3	53.1	43.3	33.5	23.7	13.9	3.7
		Sokolovski K_{PE}							
	10°	Log. spiral method K_{PE}	17.722	12.174	8.651	6.324	4.737	3.621	2.815
		Log. spiral method ω_0	59.1	48.9	39.1	29.3	19.7	9.9	(0.6)
		Sokolovski K_{PE}							
	20°	Log. spiral method K_{PE}	14.647	10.116	7.234	5.325	4.017	3.093	2.421
		Log. spiral method ω_0	54.0	44.0	34.2	24.6	15.2	5.4	(5.2)
		Sokolovski K_{PE}							2.421
	30°	Log. spiral method K_{PE}	10.290	7.189	5.206	3.883	2.967	2.311	1.829
		Log. spiral method ω_0	46.2	36.4	27.0	17.6	8.2	(1.8)	(13.6)
		Sokolovski K_{PE}							1.828
$\phi=37.5°$ $\delta=2/3\cdot\phi$ $\beta=0°$	0°	Log. spiral method K_{PE}	27.705	18.263	12.467	8.768	6.329	4.672	3.518
		Log. spiral method ω_0	65.0	54.8	44.6	34.8	24.8	15.0	4.8
		Sokolovski K_{PE}							
	10°	Log. spiral method K_{PE}	24.442	16.155	11.066	7.815	5.668	4.207	3.184
		Log. spiral method ω_0	61.3	51.1	41.1	31.1	21.3	11.5	1.3
		Sokolovski K_{PE}							
	20°	Log. spiral method K_{PE}	20.581	13.657	9.401	6.679	4.875	3.642	2.774
		Log. spiral method ω_0	56.9	46.9	36.9	27.1	17.5	7.7	(2.9)
		Sokolovski K_{PE}							
	30°	Log. spiral method K_{PE}	15.357	10.270	7.136	5.122	3.779	2.853	2.194
		Log. spiral method ω_0	50.7	40.7	30.9	21.3	11.7	1.9	(8.9)
		Sokolovski K_{PE}							2.193
$\phi=40.0°$ $\delta=2/3\cdot\phi$ $\beta=0°$	0°	Log. spiral method K_{PE}	38.803	24.564	16.130	10.935	7.626	5.453	3.987
		Log. spiral method ω_0	66.6	56.2	46.0	36.0	26.0	16.0	5.8
		Sokolovski K_{PE}				10.922			
	10°	Log. spiral method K_{PE}	34.543	21.910	14.427	9.814	6.873	4.939	3.629
		Log. spiral method ω_0	63.4	53.0	42.8	32.8	23.0	13.0	2.8
		Sokolovski K_{PE}				9.814			
	20°	Log. spiral method K_{PE}	29.564	18.806	12.430	8.497	5.984	4.327	3.199
		Log. spiral method ω_0	59.7	49.3	39.3	29.3	19.5	9.7	(1.0)
		Sokolovski K_{PE}				8.486			
	30°	Log. spiral method K_{PE}	23.034	14.729	9.802	6.755	4.801	3.503	2.614
		Log. spiral method ω_0	54.7	44.5	34.5	24.7	14.9	5.1	(5.4)
		Sokolovski K_{PE}				6.750			2.614
$\phi=42.5°$ $\delta=2/3\cdot\phi$ $\beta=0°$	0°	Log. spiral method K_{PE}	55.950	33.908	21.348	13.903	9.336	6.446	4.564
		Log. spiral method ω_0	68.5	23.8	47.5	37.3	27.1	17.1	6.9
		Sokolovski K_{PE}							
	10°	Log. spiral method K_{PE}	50.234	30.485	19.232	12.561	8.466	5.871	4.177
		Log. spiral method ω_0	65.6	55.0	44.6	34.6	24.4	14.4	4.2
		Sokolovski K_{PE}							
	20°	Log. spiral method K_{PE}	43.622	26.524	16.781	11.003	7.452	5.197	3.719
		Log. spiral method ω_0	62.4	51.8	41.6	31.4	21.4	11.6	1.4
		Sokolovski K_{PE}							
	30°	Log. spiral method K_{PE}	35.144	21.442	13.632	8.995	6.138	4.316	3.115
		Log. spiral method ω_0	58.2	47.8	37.6	27.6	17.6	7.8	(2.5)
		Sokolovski K_{PE}							
$\phi=45.0°$ $\delta=2/3\cdot\phi$ $\beta=0°$	0°	Log. spiral method K_{PE}	83.474	48.253	29.014	18.080	11.645	7.734	5.284
		Log. spiral method ω_0	70.1	59.5	48.9	38.5	28.3	18.3	7.9
		Sokolovski K_{PE}							
	10°	Log. spiral method K_{PE}	75.553	43.711	26.323	16.440	10.622	7.083	4.862
		Log. spiral method ω_0	67.6	57.0	46.6	36.2	26.0	16.0	5.8
		Sokolovski K_{PE}							
	20°	Log. spiral method K_{PE}	66.471	38.505	23.235	14.555	9.443	6.328	4.368
		Log. spiral method ω_0	65.0	54.2	43.8	33.6	23.4	13.4	3.2
		Sokolovski K_{PE}							
	30°	Log. spiral method K_{PE}	55.026	31.942	19.340	12.174	7.947	5.365	3.733
		Log. spiral method ω_0	61.4	50.8	40.4	30.2	20.2	10.2	(0.4)
		Sokolovski K_{PE}							

TABLE A-4 (1). Values of K_{PE} and ω_0 of the spiral method and the Sokolovski method for various values of β in the case of $\alpha_1=0°$, $\delta=0°$ and $\phi=20°\sim32.5°$ (The values of ω_0 in parentheses indicate the case of the concave curve)

ϕ δ α_1	θ_0	K_{PE} / ω_0 (degrees) / β	-30°	-20°	-10°	0°	10°	20°	30°
$\phi=20.0°$ $\delta=0°$ $\alpha_1=0°$	0°	Log. spiral method K_{PE}	-----	-----	1.516	2.040	2.530	-----	-----
		Log. spiral method ω_0	-----	-----	(24.0)	(0.8)	23.3	-----	-----
		Sokolovski K_{PE}			1.513				
	10°	Log. spiral method K_{PE}	-----	-----	0.931	1.744	2.322	2.867	3.482
		Log. spiral method ω_0	-----	-----	(59.6)	(11.6)	11.2	34.5	79.2
		Sokolovski K_{PE}				1.744			
	20°	Log. spiral method K_{PE}	-----	-----	-----	1.052	2.019	2.672	3.307
		Log. spiral method ω_0	-----	-----	-----	(43.0)	0.5	22.2	45.7
		Sokolovski K_{PE}				1.041			
	30°	Log. spiral method K_{PE}	-----	-----	-----	-----	-----	2.354	3.133
		Log. spiral method ω_0	-----	-----	-----	-----	-----	10.7	33.0
		Sokolovski K_{PE}							
$\phi=22.5°$ $\delta=0°$ $\alpha_1=0°$	0°	Log. spiral method K_{PE}	-----	1.056	1.656	2.240	2.820	3.414	-----
		Log. spiral method ω_0	-----	(53.7)	(21.7)	(0.9)	21.3	49.3	-----
		Sokolovski K_{PE}			1.653				
	10°	Log. spiral method K_{PE}	-----	-----	1.221	1.938	2.598	3.257	3.961
		Log. spiral method ω_0	-----	-----	(38.3)	(9.7)	11.3	32.5	61.1
		Sokolovski K_{PE}			1.211	1.938			
	20°	Log. spiral method K_{PE}	-----	-----	-----	1.447	2.288	3.050	3.838
		Log. spiral method ω_0	-----	-----	-----	(24.7)	1.6	22.5	43.7
		Sokolovski K_{PE}				1.445			
	30°	Log. spiral method K_{PE}	-----	-----	-----	-----	1.747	2.728	3.657
		Log. spiral method ω_0	-----	-----	-----	-----	(12.7)	12.8	33.3
		Sokolovski K_{PE}					1.747		
$\phi=25.0°$ $\delta=0°$ $\alpha_1=0°$	0°	Log. spiral method K_{PE}	-----	1.182	1.808	2.464	3.151	3.879	-----
		Log. spiral method ω_0	-----	(46.0)	(19.8)	(0.8)	19.6	43.5	-----
		Sokolovski K_{PE}			1.805				
	10°	Log. spiral method K_{PE}	-----	-----	1.403	2.154	2.913	3.713	4.597
		Log. spiral method ω_0	-----	-----	(31.8)	(8.0)	11.3	31.0	55.3
		Sokolovski K_{PE}			1.397	2.154			
	20°	Log. spiral method K_{PE}	-----	-----	-----	1.699	2.594	3.493	4.473
		Log. spiral method ω_0	-----	-----	-----	(19.2)	3.2	22.5	42.4
		Sokolovski K_{PE}				1.698			
	30°	Log. spiral method K_{PE}	-----	-----	-----	-----	2.096	3.164	4.287
		Log. spiral method ω_0	-----	-----	-----	-----	(7.6)	14.2	33.5
		Sokolovski K_{PE}					2.096		
$\phi=27.5°$ $\delta=0°$ $\alpha_1=0°$	0°	Log. spiral method K_{PE}	-----	1.306	1.974	2.710	3.529	4.427	-----
		Log. spiral method ω_0	-----	(41.1)	(18.5)	(0.7)	18.4	39.8	-----
		Sokolovski K_{PE}		1.289	1.972				
	10°	Log. spiral method K_{PE}	-----	-----	1.582	2.396	3.276	4.248	5.366
		Log. spiral method ω_0	-----	-----	(27.5)	(6.9)	11.4	29.8	51.6
		Sokolovski K_{PE}			1.577	2.396			
	20°	Log. spiral method K_{PE}	-----	-----	-----	1.954	2.945	4.015	5.240
		Log. spiral method ω_0	-----	-----	-----	(15.5)	4.5	22.6	41.4
		Sokolovski K_{PE}				1.953			
	30°	Log. spiral method K_{PE}	-----	-----	-----	-----	2.464	3.678	5.051
		Log. spiral method ω_0	-----	-----	-----	-----	(4.3)	15.5	33.8
		Sokolovski K_{PE}							
$\phi=30.0°$ $\delta=0°$ $\alpha_1=0°$	0°	Log. spiral method K_{PE}	-----	1.432	2.157	3.001	3.965	5.073	-----
		Log. spiral method ω_0	-----	(37.6)	(17.4)	(0.6)	17.6	37.2	-----
		Sokolovski K_{PE}		1.418	2.155				
	10°	Log. spiral method K_{PE}	-----	-----	1.769	2.668	3.695	4.882	6.301
		Log. spiral method ω_0	-----	-----	(24.6)	(5.8)	11.4	29.0	49.0
		Sokolovski K_{PE}			1.765	2.668			
	20°	Log. spiral method K_{PE}	-----	-----	0.960	2.229	3.351	4.637	6.175
		Log. spiral method ω_0	-----	-----	(47.8)	(12.8)	5.4	22.6	40.6
		Sokolovski K_{PE}			0.941	2.229			
	30°	Log. spiral method K_{PE}	-----	-----	-----	1.202	2.875	4.289	5.987
		Log. spiral method ω_0	-----	-----	-----	(33.6)	(1.8)	16.4	34.0
		Sokolovski K_{PE}				1.197			
$\phi=32.5°$ $\delta=0°$ $\alpha_1=0°$	0°	Log. spiral method K_{PE}	-----	1.565	2.359	3.323	4.471	5.844	7.597
		Log. spiral method ω_0	-----	(34.9)	(16.5)	(0.7)	16.7	35.0	59.9
		Sokolovski K_{PE}		1.553	2.357				
	10°	Log. spiral method K_{PE}	-----	1.095	1.971	2.976	4.184	5.640	7.449
		Log. spiral method ω_0	-----	(46.9)	(22.5)	(4.9)	11.5	28.3	46.8
		Sokolovski K_{PE}			1.967				
	20°	Log. spiral method K_{PE}	-----	-----	1.387	2.535	3.825	5.382	7.327
		Log. spiral method ω_0	-----	-----	(33.1)	(10.9)	6.2	22.7	39.9
		Sokolovski K_{PE}			1.380	2.534			
	30°	Log. spiral method K_{PE}	-----	-----	-----	1.807	3.323	5.024	7.144
		Log. spiral method ω_0	-----	-----	-----	(20.9)	0.7	17.4	34.3
		Sokolovski K_{PE}				1.806			

TABLE A-4 (2). Values of K_{PE} and ω_0 of the spiral method and the Sokolovski method for various values of β in the case of $\alpha_1=0°$, $\delta=0°$, and $\phi=35°\sim45°$

ϕ δ β	θ_0	K_{PE} / ω_0 (degrees) / β	-30°	-20°	-10°	0°	10°	20°	30°
ϕ=35.0° δ=0° α_1=0°	0°	Log. spiral K_{PE}	-----	1.707	2.584	3.691	5.064	6.770	8.995
		method ω_0	-----	(32.8)	(15.8)	(1.0)	15.9	33.2	54.8
		Sokolovski K_{PE}		1.695	2.581				
	10°	Log. spiral K_{PE}	-----	1.271	2.191	3.329	4.757	6.554	8.877
		method ω_0	-----	(41.0)	(20.6)	(4.0)	11.3	27.5	45.2
		Sokolovski K_{PE}		1.255	2.187				
	20°	Log. spiral K_{PE}	-----	-----	1.651	2.879	4.381	6.284	8.764
		method ω_0	-----	-----	(28.2)	(9.2)	6.9	22.9	39.3
		Sokolovski K_{PE}			1.646	2.878			
	30°	Log. spiral K_{PE}	-----	-----	-----	2.205	3.870	5.914	8.593
		method ω_0	-----	-----	-----	(16.6)	2.0	18.3	34.3
		Sokolovski K_{PE}				2.204			
ϕ=37.5° δ=0° α_1=0°	0°	Log. spiral K_{PE}	1.117	1.860	2.835	4.112	5.762	7.896	10.769
		method ω_0	(50.9)	(31.1)	(15.1)	0.4	15.4	31.8	51.6
		Sokolovski K_{PE}		1.848	2.832				
	10°	Log. spiral K_{PE}	-----	1.440	2.434	3.733	5.436	7.669	10.677
		method ω_0	-----	(37.1)	(19.1)	(3.7)	11.4	27.0	44.0
		Sokolovski K_{PE}		1.427	2.431				
	20°	Log. spiral K_{PE}	-----	-----	1.913	3.271	5.041	7.387	10.582
		method ω_0	-----	-----	(24.9)	(7.9)	7.7	23.0	38.8
		Sokolovski K_{PE}			1.908	3.271			
	30°	Log. spiral K_{PE}	-----	-----	-----	2.618	4.515	7.007	10.432
		method ω_0	-----	-----	-----	(13.7)	3.3	18.9	34.6
		Sokolovski K_{PE}				2.618			
ϕ=40.0° δ=0° α_1=0°	0°	Log. spiral K_{PE}	1.224	2.205	3.116	4.600	6.593	9.282	13.039
		method ω_0	(47.4)	(29.6)	(14.6)	(0.6)	14.8	30.9	49.1
		Sokolovski K_{PE}		2.013	3.113				
	10°	Log. spiral K_{PE}	-----	1.623	2.706	4.201	6.246	9.046	12.980
		method ω_0	-----	(34.4)	(18.0)	(3.0)	11.4	26.6	42.9
		Sokolovski K_{PE}		1.601	2.702				
	20°	Log. spiral K_{PE}	-----	-----	2.191	3.724	5.829	8.754	12.916
		method ω_0	-----	-----	(22.6)	(6.5)	8.2	23.0	38.6
		Sokolovski K_{PE}			2.187	3.724			
	30°	Log. spiral K_{PE}	-----	-----	1.079	3.075	5.285	8.363	12.801
		method ω_0	-----	-----	(37.6)	(11.4)	4.3	19.6	34.8
		Sokolovski K_{PE}			1.071	3.074			
ϕ=42.5° δ=0° α_1=0°	0°	Log. spiral K_{PE}	1.334	2.205	3.433	5.260	7.593	11.011	15.991
		method ω_0	(44.5)	(28.5)	(14.1)	(0.5)	14.5	29.9	47.1
		Sokolovski K_{PE}	1.306	2.193	3.429				
	10°	Log. spiral K_{PE}	-----	1.795	3.010	4.746	7.222	10.767	15.980
		method ω_0	-----	(32.1)	(16.9)	(2.7)	11.5	26.3	42.1
		Sokolovski K_{PE}		1.784	3.007				
	20°	Log. spiral K_{PE}	-----	1.208	2.495	4.250	6.782	10.468	15.965
		method ω_0	-----	(39.5)	(20.7)	(5.7)	8.6	23.1	38.3
		Sokolovski K_{PE}		1.195	2.491				
	30°	Log. spiral K_{PE}	-----	-----	1.693	3.594	6.216	10.070	15.907
		method ω_0	-----	-----	(27.7)	(9.7)	5.4	20.0	35.1
		Sokolovski K_{PE}			1.689	3.594			
ϕ=45.0° δ=0° α_1=0°	0°	Log. spiral K_{PE}	1.449	2.402	3.792	5.829	8.808	13.202	19.904
		method ω_0	(42.4)	(27.4)	(13.6)	0.3	14.0	29.0	45.6
		Sokolovski K_{PE}		2.390	3.788				
	10°	Log. spiral K_{PE}	-----	1.992	3.355	5.385	8.412	12.955	19.966
		method ω_0	-----	(30.4)	(16.0)	(2.4)	11.5	26.0	41.4
		Sokolovski K_{PE}		1.981	3.351				
	20°	Log. spiral K_{PE}	-----	1.449	2.833	4.867	7.947	12.652	20.029
		method ω_0	-----	(35.2)	(19.0)	(5.0)	9.0	23.3	38.2
		Sokolovski K_{PE}		1.439	2.830				
	30°	Log. spiral K_{PE}	-----	-----	2.090	4.198	7.354	12.251	20.061
		method ω_0	-----	-----	(24.0)	(8.2)	6.2	20.6	35.3
		Sokolovski K_{PE}			2.086	4.198			

TABLE A-4 (3). Values of K_{PE} and ω_0 of the spiral method and the Sokolovski method for various values of β in the case of $\alpha_1=0°$, $\delta=1/2\cdot\phi$ and $\phi=20°\sim32.5°$

ϕ δ α_1	θ_0	K_{PE} / ω_0 (degrees) / β	-30°	-20°	-10°	0°	10°	20°	30°
$\phi=20.0°$ $\delta=1/2\cdot\phi$ $\alpha_1=0°$	0°	Log. spiral K_{PE}	-----	-----	1.850	2.520	3.220	-----	-----
		method ω_0	-----	-----	0.2	20.6	41.5	-----	-----
		Sokolovski K_{PE}							
	10°	Log. spiral K_{PE}	-----	-----	1.007	2.114	2.923	3.709	4.479
		method ω_0	-----	-----	(33.0)	10.1	30.4	51.3	91.6
		Sokolovski K_{PE}							
	20°	Log. spiral K_{PE}	-----	-----	-----	1.194	2.498	3.428	4.356
		method ω_0	-----	-----	-----	(17.0)	19.7	40.0	61.3
		Sokolovski K_{PE}				1.193			
	30°	Log. spiral K_{PE}	-----	-----	-----	-----	1.437	2.974	4.108
		method ω_0	-----	-----	-----	-----	(4.8)	29.1	49.4
		Sokolovski K_{PE}							
$\phi=22.5°$ $\delta=1/2\cdot\phi$ $\alpha_1=0°$	0°	Log. spiral K_{PE}	-----	-----	2.021	2.879	3.747	4.625	-----
		method ω_0	-----	-----	2.8	21.5	40.5	65.7	-----
		Sokolovski K_{PE}							
	10°	Log. spiral K_{PE}	-----	-----	1.440	2.446	3.413	4.414	5.444
		method ω_0	-----	-----	(12.1)	12.8	31.3	50.3	76.3
		Sokolovski K_{PE}			1.440				
	20°	Log. spiral K_{PE}	-----	-----	-----	1.762	2.957	4.099	5.311
		method ω_0	-----	-----	-----	0.4	22.4	41.1	60.3
		Sokolovski K_{PE}							
	30°	Log. spiral K_{PE}	-----	-----	-----	-----	2.162	3.615	5.040
		method ω_0	-----	-----	-----	-----	9.2	32.0	50.7
		Sokolovski K_{PE}							
$\phi=25.0°$ $\delta=1/2\cdot\phi$ $\alpha_1=0°$	0°	Log. spiral K_{PE}	-----	1.402	2.298	3.308	4.388	5.540	-----
		method ω_0	-----	(18.0)	5.0	22.3	40.0	61.5	-----
		Sokolovski K_{PE}		1.400					
	10°	Log. spiral K_{PE}	-----	-----	1.736	2.840	4.014	5.293	6.688
		method ω_0	-----	-----	(5.8)	14.8	32.1	50.0	71.9
		Sokolovski K_{PE}			1.736				
	20°	Log. spiral K_{PE}	-----	-----	-----	2.173	3.519	4.941	6.535
		method ω_0	-----	-----	-----	4.7	24.6	41.9	60.2
		Sokolovski K_{PE}							
	30°	Log. spiral K_{PE}	-----	-----	-----	-----	2.741	4.419	6.241
		method ω_0	-----	-----	-----	-----	14.5	34.2	51.9
		Sokolovski K_{PE}							
$\phi=27.5°$ $\delta=1/2\cdot\phi$ $\alpha_1=0°$	0°	Log. spiral K_{PE}	-----	1.621	2.622	3.825	5.179	6.694	-----
		method ω_0	-----	(13.1)	6.7	23.2	39.8	59.0	-----
		Sokolovski K_{PE}		1.620					
	10°	Log. spiral K_{PE}	-----	-----	2.052	3.316	4.757	6.408	8.298
		method ω_0	-----	-----	(1.5)	16.7	33.2	50.0	69.6
		Sokolovski K_{PE}							
	20°	Log. spiral K_{PE}	-----	-----	-----	2.632	4.214	6.013	8.128
		method ω_0	-----	-----	-----	8.7	26.5	43.2	60.2
		Sokolovski K_{PE}							
	30°	Log. spiral K_{PE}	-----	-----	-----	-----	3.413	5.444	7.814
		method ω_0	-----	-----	-----	-----	18.5	36.3	53.2
		Sokolovski K_{PE}							
$\phi=30.0°$ $\delta=1/2\cdot\phi$ $\alpha_1=0°$	0°	Log. spiral K_{PE}	-----	1.863	3.008	4.458	6.168	8.170	-----
		method ω_0	-----	(9.4)	8.4	24.0	39.8	57.6	-----
		Sokolovski K_{PE}		1.863		4.456			
	10°	Log. spiral K_{PE}	-----	0.915	2.392	3.896	5.689	7.840	10.416
		method ω_0	-----	(35.2)	1.8	18.4	34.0	50.2	68.2
		Sokolovski K_{PE}		0.903		3.895			
	20°	Log. spiral K_{PE}	-----	-----	1.183	3.175	5.088	7.397	10.236
		method ω_0	-----	-----	(19.4)	11.8	28.4	44.2	60.6
		Sokolovski K_{PE}			1.182	3.173			
	30°	Log. spiral K_{PE}	-----	-----	-----	1.567	4.238	6.771	9.904
		method ω_0	-----	-----	-----	(7.2)	21.6	38.4	54.4
		Sokolovski K_{PE}				1.563			
$\phi=32.5°$ $\delta=1/2\cdot\phi$ $\alpha_1=0°$	0°	Log. spiral K_{PE}	-----	2.141	3.472	5.239	7.423	10.092	13.315
		method ω_0	-----	(6.5)	10.0	24.9	40.1	56.6	78.9
		Sokolovski K_{PE}		2.141					
	10°	Log. spiral K_{PE}	-----	1.404	2.817	4.616	6.877	9.712	13.259
		method ω_0	-----	(17.7)	4.3	20.0	35.1	50.5	67.4
		Sokolovski K_{PE}		1.403					
	20°	Log. spiral K_{PE}	-----	-----	1.894	3.838	6.203	9.212	13.076
		method ω_0	-----	-----	(5.7)	14.3	30.0	45.3	60.9
		Sokolovski K_{PE}			1.894				
	30°	Log. spiral K_{PE}	-----	-----	-----	2.605	5.281	8.518	12.735
		method ω_0	-----	-----	-----	4.6	24.3	40.2	55.7
		Sokolovski K_{PE}							

TABLE A-4 (4). Values of K_{PE} and ω_0 of the spiral method and the Sokolovski method for various values of β in the case of $\alpha_1=0°$, $\delta=1/2\cdot\phi$ and $\phi=35°\sim45°$

ϕ δ α_1	θ_0	K_{PE} / ω_0 (degrees) / β	-30°	-20°	-10°	0°	10°	20°	30°
ϕ=35.0° δ=1/2·ϕ α_1=0°	0°	Log. spiral K_{PE}	-----	2.464	4.038	6.220	9.040	12.640	17.188
		method ω_0	-----	(4.0)	11.5	25.7	40.5	56.2	75.6
		Sokolovski K_{PE}		2.464		6.219			
	10°	Log. spiral K_{PE}	-----	1.743	3.330	5.520	8.412	12.205	17.155
		method ω_0	-----	(11.8)	6.6	21.5	36.1	50.9	67.0
		Sokolovski K_{PE}		1.743		5.518			
	20°	Log. spiral K_{PE}	-----	-----	2.418	4.667	7.650	11.638	16.985
		method ω_0	-----	-----	(0.8)	16.8	31.7	46.5	61.5
		Sokolovski K_{PE}				4.666			
	30°	Log. spiral K_{PE}	-----	-----	-----	3.432	6.632	10.863	16.648
		method ω_0	-----	-----	-----	9.4	26.8	42.1	57.1
		Sokolovski K_{PE}				3.432			
ϕ=37.5° δ=1/2·ϕ α_1=0°	0°	Log. spiral K_{PE}	1.503	2.849	4.736	7.469	11.164	16.092	22.603
		method ω_0	(20.1)	(1.9)	12.9	26.8	40.8	56.0	73.6
		Sokolovski K_{PE}	1.500						
	10°	Log. spiral K_{PE}	-----	2.109	3.960	6.674	10.435	15.592	22.620
		method ω_0	-----	(7.9)	8.7	23.1	37.2	51.4	66.8
		Sokolovski K_{PE}		2.109					
	20°	Log. spiral K_{PE}	-----	-----	2.983	5.724	9.562	14.948	22.490
		method ω_0	-----	-----	2.9	18.9	33.3	47.8	62.2
		Sokolovski K_{PE}							
	30°	Log. spiral K_{PE}	-----	-----	-----	4.420	8.419	14.075	22.183
		method ω_0	-----	-----	-----	12.9	29.1	43.9	58.4
		Sokolovski K_{PE}							
ϕ=40.0° δ=1/2·ϕ α_1=0°	0°	Log. spiral K_{PE}	1.754	3.211	5.613	9.089	14.012	20.882	30.377
		method ω_0	(16.4)	0.3	14.2	27.6	41.4	56.1	72.5
		Sokolovski K_{PE}	1.752			9.086			
	10°	Log. spiral K_{PE}	-----	2.533	4.750	8.174	13.157	20.308	30.493
		method ω_0	-----	(4.8)	10.5	24.4	38.2	52.2	67.1
		Sokolovski K_{PE}		2.533		8.173			
	20°	Log. spiral K_{PE}	-----	1.094	3.700	7.099	12.144	19.573	30.451
		method ω_0	-----	(21.2)	5.9	20.9	35.0	48.8	63.2
		Sokolovski K_{PE}		1.092		7.098			
	30°	Log. spiral K_{PE}	-----	-----	1.611	5.675	10.838	18.584	30.223
		method ω_0	-----	----	(9.2)	16.1	31.1	45.6	59.8
		Sokolovski K_{PE}			1.610	5.674			
ϕ=42.5° δ=1/2·ϕ α_1=0°	0°	Log. spiral K_{PE}	2.045	3.769	6.732	11.234	17.923	27.709	41.887
		method ω_0	(13.3)	2.0	15.4	28.7	42.3	56.5	71.9
		Sokolovski K_{PE}	2.043						
	10°	Log. spiral K_{PE}	1.258	3.043	5.758	10.166	16.906	27.052	42.190
		method ω_0	(21.9)	(2.1)	12.4	25.8	39.3	53.1	67.5
		Sokolovski K_{PE}	1.254						
	20°	Log. spiral K_{PE}	-----	1.893	4.603	8.927	15.713	26.212	42.321
		method ω_0	-----	(9.7)	8.4	22.6	36.4	50.3	64.1
		Sokolovski K_{PE}		1.892					
	30°	Log. spiral K_{PE}	-----	-----	2.893	7.330	14.195	25.086	42.261
		method ω_0	-----	-----	0.8	18.6	33.2	47.2	61.3
		Sokolovski K_{PE}							
ϕ=45.0° δ=1/2·ϕ α_1=0°	0°	Log. spiral K_{PE}	2.388	4.466	8.189	14.139	23.442	37.750	59.537
		method ω_0	(10.8)	3.6	16.6	29.7	43.0	56.8	71.8
		Sokolovski K_{PE}	2.387						
	10°	Log. spiral K_{PE}	1.605	3.550	7.072	12.873	22.215	37.003	60.187
		method ω_0	(16.6)	0.4	14.0	27.2	40.5	54.0	68.0
		Sokolovski K_{PE}	1.603						
	20°	Log. spiral K_{PE}	-----	2.505	5.772	11.419	20.785	36.046	60.650
		method ω_0	-----	(5.4)	10.6	24.6	38.0	51.5	65.2
		Sokolovski K_{PE}		2.505					
	30°	Log. spiral K_{PE}	-----	-----	3.997	9.584	18.987	34.760	60.927
		method ω_0	-----	-----	5.2	21.2	35.2	49.0	62.7
		Sokolovski K_{PE}							

TABLE A-4 (5). Values of K_{PE} and ω_0 of the spiral method and the Sokolovski method for various values of β in the case of $\alpha_1=0°$, $\delta=2/3\cdot\phi$ and $\phi=20°\sim32.5°$

ϕ δ α_1	θ_0	K_{PE} / ω_0 (degrees) / β	-30°	-20°	-10°	0°	10°	20°	30°
$\phi=20.0°$ $\delta=2/3\cdot\phi$ $\alpha_1=0°$	0°	Log. spiral K_{PE}	-----	-----	1.862	2.660	3.425	-----	-----
		method ω_0	-----	-----	7.7	26.8	46.7	-----	-----
		Sokolovski K_{PE}							
	10°	Log. spiral K_{PE}	-----	-----	1.032	2.221	3.102	3.966	4.783
		method ω_0	-----	-----	(22.0)	16.9	36.0	56.1	94.8
		Sokolovski K_{PE}			1.038				
	20°	Log. spiral K_{PE}	-----	-----	-----	1.236	2.639	3.659	4.684
		method ω_0	-----	-----	-----	(8.0)	25.7	45.0	65.5
		Sokolovski K_{PE}				1.236			
	30°	Log. spiral K_{PE}	-----	-----	-----	-----	1.474	3.162	4.414
		method ω_0	-----	-----	-----	-----	2.8	34.5	54.2
		Sokolovski K_{PE}							
$\phi=22.5°$ $\delta=2/3\cdot\phi$ $\alpha_1=0°$	0°	Log. spiral K_{PE}	-----	1.232	2.131	3.070	4.030	5.000	-----
		method ω_0	-----	(15.5)	10.4	27.9	45.9	70.3	-----
		Sokolovski K_{PE}		1.231					
	10°	Log. spiral K_{PE}	-----	-----	1.506	2.597	3.663	4.776	5.914
		method ω_0	-----	-----	(3.0)	19.6	37.1	55.5	80.3
		Sokolovski K_{PE}							
	20°	Log. spiral K_{PE}	-----	-----	-----	1.855	3.162	4.429	5.784
		method ω_0	-----	-----	-----	7.0	28.6	46.3	65.1
		Sokolovski K_{PE}							
	30°	Log. spiral K_{PE}	-----	-----	-----	-----	2.293	3.894	5.487
		method ω_0	-----	-----	-----	-----	16.0	37.6	55.7
		Sokolovski K_{PE}							
$\phi=25.0°$ $\delta=2/3\cdot\phi$ $\alpha_1=0°$	0°	Log. spiral K_{PE}	-----	1.470	2.449	3.568	4.778	6.072	-----
		method ω_0	-----	(8.2)	12.6	28.9	45.8	66.5	-----
		Sokolovski K_{PE}		1.469					
	10°	Log. spiral K_{PE}	-----	-----	1.816	3.051	4.362	5.804	7.372
		method ω_0	-----	-----	2.9	21.8	38.3	55.4	76.5
		Sokolovski K_{PE}							
	20°	Log. spiral K_{PE}	-----	-----	-----	2.318	3.811	5.412	7.217
		method ω_0	-----	-----	-----	12.3	31.0	47.7	65.0
		Sokolovski K_{PE}							
	30°	Log. spiral K_{PE}	-----	-----	-----	-----	2.950	4.828	6.893
		method ω_0	-----	-----	-----	-----	21.3	40.2	57.1
		Sokolovski K_{PE}							
$\phi=27.5°$ $\delta=2/3\cdot\phi$ $\alpha_1=0°$	0°	Log. spiral K_{PE}	-----	1.720	2.829	4.180	5.719	7.446	-----
		method ω_0	-----	(3.5)	14.5	30.0	46.0	64.4	-----
		Sokolovski K_{PE}		1.720					
	10°	Log. spiral K_{PE}	-----	-----	2.178	3.610	5.243	7.130	9.290
		method ω_0	-----	-----	6.9	23.9	39.4	55.6	74.4
		Sokolovski K_{PE}							
	20°	Log. spiral K_{PE}	-----	-----	-----	2.848	4.631	6.685	9.116
		method ω_0	-----	-----	-----	16.1	33.1	49.0	65.4
		Sokolovski K_{PE}							
	30°	Log. spiral K_{PE}	-----	-----	-----	-----	3.731	6.040	8.766
		method ω_0	-----	-----	-----	-----	25.3	42.5	58.6
		Sokolovski K_{PE}							
$\phi=30.0°$ $\delta=2/3\cdot\phi$ $\alpha_1=0°$	0°	Log. spiral K_{PE}	-----	1.923	3.289	4.943	6.917	9.237	-----
		method ω_0	-----	0.6	16.4	31.0	46.2	63.4	-----
		Sokolovski K_{PE}							
	10°	Log. spiral K_{PE}	-----	-----	2.600	4.307	6.370	8.868	11.862
		method ω_0	-----	-----	10.0	25.8	40.6	56.2	73.4
		Sokolovski K_{PE}							
	20°	Log. spiral K_{PE}	-----	-----	1.257	3.490	5.682	8.362	11.676
		method ω_0	-----	-----	(9.2)	19.2	35.2	50.4	66.2
		Sokolovski K_{PE}			1.257				
	30°	Log. spiral K_{PE}	-----	-----	-----	1.688	4.712	7.643	11.306
		method ω_0	-----	-----	-----	1.6	28.6	44.6	60.2
		Sokolovski K_{PE}							
$\phi=32.5°$ $\delta=2/3\cdot\phi$ $\alpha_1=0°$	0°	Log. spiral K_{PE}	-----	2.247	3.856	5.908	8.471	11.619	15.368
		method ω_0	-----	3.3	18.2	32.3	46.7	62.8	84.1
		Sokolovski K_{PE}							
	10°	Log. spiral K_{PE}	-----	1.510	3.111	5.188	7.838	11.189	15.385
		method ω_0	-----	(7.3)	12.7	27.6	41.9	56.7	73.0
		Sokolovski K_{PE}		1.510					
	20°	Log. spiral K_{PE}	-----	-----	2.042	4.294	7.055	10.610	15.201
		method ω_0	-----	-----	3.4	22.1	37.2	51.9	66.9
		Sokolovski K_{PE}							
	30°	Log. spiral K_{PE}	-----	-----	-----	2.883	5.985	9.801	14.819
		method ω_0	-----	-----	-----	12.8	31.5	46.8	61.9
		Sokolovski K_{PE}							

TABLE A-4 (6). Values of K_{PE} and ω_0 of the spiral method and the Sokolovski method for various values of β in the case of $\alpha_1=0°$, $\delta=2/3\cdot\varphi$ and $\phi=35°\sim45°$

ϕ δ β	θ_0	K_{PE} / ω_0 (degrees) / β		-30°	-20°	-10°	0°	10°	20°	30°
ϕ=35.0° δ=2/3·ϕ α_1=0°	0°	Log. spiral method	K_{PE}	-----	2.634	4.563	7.147	10.523	14.853	20.276
			ω_0	-----	5.8	19.7	33.5	47.5	62.6	81.0
		Sokolovski	K_{PE}							
	10°	Log. spiral method	K_{PE}	-----	1.910	3.744	6.324	9.782	14.352	20.323
			ω_0	-----	(2.0)	15.2	29.3	43.3	57.5	73.0
		Sokolovski	K_{PE}							
	20°	Log. spiral method	K_{PE}	-----	-----	2.663	5.325	8.880	13.687	20.162
			ω_0	-----	-----	8.2	24.6	39.1	53.3	67.7
		Sokolovski	K_{PE}							
	30°	Log. spiral method	K_{PE}	-----	-----	-----	3.883	7.676	12.769	19.787
			ω_0	-----	-----	-----	17.6	34.2	48.9	63.3
		Sokolovski	K_{PE}							
ϕ=37.5° δ=2/3·ϕ α_1=0°	0°	Log. spiral method	K_{PE}	1.646	3.107	5.462	8.768	13.289	19.348	27.301
			ω_0	(8.9)	7.9	21.5	34.8	48.2	62.8	79.6
		Sokolovski	K_{PE}	1.645						
	10°	Log. spiral method	K_{PE}	-----	2.257	4.545	7.815	12.413	18.765	27.425
			ω_0	-----	2.3	17.5	31.1	44.6	58.4	73.2
		Sokolovski	K_{PE}							
	20°	Log. spiral method	K_{PE}	-----	-----	3.395	6.679	11.360	17.997	27.326
			ω_0	-----	-----	11.7	27.1	41.1	54.8	68.8
		Sokolovski	K_{PE}							
	30°	Log. spiral method	K_{PE}	-----	-----	-----	5.122	9.979	16.945	26.996
			ω_0	-----	-----	-----	21.3	36.9	51.1	65.0
		Sokolovski	K_{PE}							
ϕ=40.0° δ=2/3·ϕ α_1=0°	0°	Log. spiral method	K_{PE}	1.960	3.696	6.624	10.935	17.110	25.764	37.662
			ω_0	(5.4)	9.9	23.0	36.0	49.2	63.1	78.9
		Sokolovski	K_{PE}	1.960			10.922			
	10°	Log. spiral method	K_{PE}	-----	2.780	5.581	9.814	16.060	25.086	37.942
			ω_0	-----	5.3	19.5	32.8	46.0	59.4	73.7
		Sokolovski	K_{PE}				9.814			
	20°	Log. spiral method	K_{PE}	-----	1.206	4.316	8.497	14.811	24.196	37.976
			ω_0	-----	(10.2)	14.9	29.3	42.8	56.2	70.0
		Sokolovski	K_{PE}		1.206		8.486			
	30°	Log. spiral method	K_{PE}	-----	-----	1.794	6.755	13.197	22.982	37.760
			ω_0	-----	-----	0.4	24.7	39.3	53.0	66.8
		Sokolovski	K_{PE}				6.750			
ϕ=42.5° δ=2/3·ϕ α_1=0°	0°	Log. spiral method	K_{PE}	2.336	4.446	8.159	13.903	22.531	35.204	53.468
			ω_0	(2.5)	11.8	24.6	37.3	50.1	63.9	78.5
		Sokolovski	K_{PE}							
	10°	Log. spiral method	K_{PE}	1.405	3.433	6.951	12.561	21.253	34.418	54.042
			ω_0	(10.5)	8.0	21.6	34.6	47.5	60.7	74.5
		Sokolovski	K_{PE}	1.405						
	20°	Log. spiral method	K_{PE}	-----	2.070	5.521	11.003	19.744	33.383	54.341
			ω_0	-----	0.6	17.8	31.4	44.6	57.9	71.3
		Sokolovski	K_{PE}							
	30°	Log. spiral method	K_{PE}	-----	-----	3.417	8.995	17.819	31.973	54.373
			ω_0	-----	-----	10.4	27.6	41.6	55.0	68.5
		Sokolovski	K_{PE}							
ϕ=45.0° δ=2/3·ϕ α_1=0°	0°	Log. spiral method	K_{PE}	2.565	5.419	10.238	18.080	30.470	49.587	78.527
			ω_0	(0.6)	13.6	26.0	38.5	51.2	64.6	78.8
		Sokolovski	K_{PE}							
	10°	Log. spiral method	K_{PE}	1.848	4.276	8.810	16.440	28.883	48.684	79.654
			ω_0	(5.6)	10.4	23.4	36.2	48.9	61.8	75.2
		Sokolovski	K_{PE}	1.848						
	20°	Log. spiral method	K_{PE}	-----	2.846	7.151	14.555	27.024	47.484	80.470
			ω_0	-----	5.0	20.2	33.6	46.6	59.5	72.6
		Sokolovski	K_{PE}							
	30°	Log. spiral method	K_{PE}	-----	-----	4.894	12.174	24.675	45.840	81.014
			ω_0	-----	-----	14.8	30.2	43.8	57.0	70.1
		Sokolovski	K_{PE}							

TABLE A-5. Values of K_{PE} and ω_0 of the spiral method and the Sokolovski method in the case of $\alpha_1=\beta=0°$ and $\delta=-1/2\cdot\phi$

θ_0	K_{PE} / ω_0 (degrees)		ϕ 20.0°	22.5°	25.0°	27.5°	30.0°	32.5°	35.0°	37.5°	40.0°	42.5°	45.0°
-30°	Log. spiral method	K_{PE}	------	------	------	------	2.071	2.128	2.190	2.247	2.298	2.344	2.383
		ω_0	------	------	------	------	9.2	(5.9)	(11.6)	(15.9)	(19.2)	(22.1)	(24.4)
	Sokolovski	K_{PE}						2.126	2.182	2.231	2.275	2.312	2.343
-20°	Log. spiral method	K_{PE}	------	1.800	1.869	1.938	2.005	2.068	2.129	2.186	2.239	2.287	2.330
		ω_0	------	(1.0)	(5.8)	(10.5)	(14.2)	(17.8)	(19.8)	(22.1)	(24.0)	(25.9)	(27.4)
	Sokolovski	K_{PE}			1.869	1.935	1.998	2.058	2.113	2.166	2.213	2.256	2.294
-10°	Log. spiral method	K_{PE}	1.644	1.714	1.782	1.850	1.916	1.981	2.044	2.104	2.160	2.213	2.262
		ω_0	(12.3)	(15.3)	(17.8)	(19.7)	(21.6)	(23.3)	(24.8)	(26.1)	(27.4)	(28.7)	(29.8)
	Sokolovski	K_{PE}	1.642	1.710	1.776	1.842	1.905	1.967	2.026	2.083	2.136	2.185	2.230
0°	Log. spiral method	K_{PE}	1.526	1.595	1.665	1.734	1.803	1.871	1.938	2.003	2.065	2.124	2.180
		ω_0	(24.3)	(25.1)	(25.8)	(26.5)	(27.4)	(28.1)	(28.8)	(29.5)	(30.2)	(30.9)	(31.6)
	Sokolovski	K_{PE}	1.520	1.588	1.656	1.723	1.790	1.856	1.921	1.983	2.043	2.100	2.153
10°	Log. spiral method	K_{PE}	1.351	1.428	1.504	1.580	1.656	1.731	1.805	1.877	1.947	2.015	2.080
		ω_0	(36.1)	(34.7)	(33.8)	(33.1)	(32.8)	(32.7)	(32.8)	(32.9)	(33.0)	(33.3)	(33.6)
	Sokolovski	K_{PE}	1.341	1.417	1.493	1.568	1.642	1.716	1.788	1.859	1.928	1.995	2.058
20°	Log. spiral method	K_{PE}	0.906	1.130	1.247	1.348	1.443	1.534	1.623	1.709	1.792	1.873	1.950
		ω_0	(66.9)	(49.9)	(44.8)	(41.7)	(39.6)	(38.1)	(37.2)	(36.5)	(36.0)	(35.7)	(35.6)
	Sokolovski	K_{PE}		1.112	1.232	1.335	1.430	1.521	1.608	1.694	1.776	1.856	1.933
30°	Log. spiral method	K_{PE}	------	------	------	------	0.883	1.177	1.323	1.447	1.559	1.665	1.765
		ω_0	------	------	------	------	(47.0)	(47.9)	(44.0)	(41.7)	(40.0)	(38.9)	(38.0)
	Sokolovski	K_{PE}						1.164	1.311	1.435	1.548	1.653	1.753

TABLE A-6 (1). Values of K_{PE} and ω_0 of the spiral method and the Sokolovski method for various values of α_1 in the case of $\delta=\phi$, $\beta=0°$ and $\phi=25°$, 27.5°

ϕ δ β	θ_0	K_{PE} / ω_0 (degrees) / α_1	-70°	-60°	-50°	-40°	-30°	-20°	-10°	0°	10°	20°	30°
ϕ=25.0° δ=25.0° β=0°	-30°	Log. spiral K_{PE}	------	------	------	------	------	------	------	------	------	------	------
		method ω_0	------	------	------	------	------	------	------	------	------	------	------
		Sokolovski K_{PE}											
	-20°	Log. spiral K_{PE}	55.215	31.875	20.733	14.436	10.538	7.988	6.232	4.956	4.002	3.274	2.707
		method ω_0	125.7	114.3	103.5	93.5	83.9	75.1	66.1	57.3	48.7	40.3	32.3
		Sokolovski K_{PE}	50.021	29.235	19.331	13.701	10.161	7.782	6.105	4.879	3.959	3.251	
	-10°	Log. spiral K_{PE}	50.126	28.824	18.690	12.994	9.489	7.211	5.646	4.508	3.657	3.007	2.499
		method ω_0	113.4	102.6	92.2	82.4	73.2	64.8	56.0	47.4	39.2	31.2	23.4
		Sokolovski K_{PE}	43.988	25.825	17.156	12.219	9.107	7.010	5.528	4.441	3.622	2.989	
	0°	Log. spiral K_{PE}	44.100	25.275	16.355	11.366	8.313	6.341	4.988	4.003	3.265	2.698	2.254
		method ω_0	104.9	94.1	84.1	74.3	65.5	57.5	49.1	40.7	32.7	25.1	17.7
		Sokolovski K_{PE}	37.602	22.201	14.833	10.627	7.967	6.168	4.891	3.951	3.239	2.687	
	10°	Log. spiral K_{PE}	36.870	21.053	13.594	9.446	6.927	5.312	4.207	3.400	2.791	2.321	1.949
		method ω_0	96.4	85.8	75.8	66.4	57.8	50.6	42.4	34.4	26.8	19.4	12.6
		Sokolovski K_{PE}	30.315	17.777	12.164	8.652	6.639	5.179	4.136	3.364	2.775	2.315	
	20°	Log. spiral K_{PE}	26.933	15.273	9.819	6.819	5.025	3.891	3.120	2.550	2.116	1.775	1.501
		method ω_0	84.7	74.3	64.3	55.1	47.1	41.1	33.5	26.1	19.1	12.3	6.1
		Sokolovski K_{PE}	20.630	12.510	8.573	6.291	4.823	3.813	3.082	2.534	2.110	1.774	
	30°	Log. spiral K_{PE}	------	------	------	------	------	------	------	------	------	------	------
		method ω_0	------	------	------	------	------	------	------	------	------	------	------
		Sokolovski K_{PE}											
ϕ=27.5° δ=27.5° β=0°	-30°	Log. spiral K_{PE}	------	------	------	------	------	------	------	------	------	------	------
		method ω_0	------	------	------	------	------	------	------	------	------	------	------
		Sokolovski K_{PE}											
	-20°	Log. spiral K_{PE}	80.437	45.040	28.428	19.218	13.625	10.032	7.611	5.895	4.644	3.712	3.003
		method ω_0	124.2	113.0	102.4	92.2	82.6	73.6	64.6	55.8	47.2	38.8	30.6
		Sokolovski K_{PE}	73.065	41.256	26.394	18.133	13.055	9.721	7.426	5.788	4.586	3.682	
	-10°	Log. spiral K_{PE}	73.190	40.828	25.697	17.345	12.297	9.071	6.902	5.365	4.244	3.408	2.770
		method ω_0	114.6	103.8	93.6	83.6	74.2	65.6	56.8	48.0	39.6	31.6	23.8
		Sokolovski K_{PE}	64.761	36.679	23.548	16.237	11.736	8.776	6.733	5.272	4.196	3.385	
	0°	Log. spiral K_{PE}	64.842	36.062	22.652	15.279	10.842	8.020	6.127	4.783	3.802	3.067	2.505
		method ω_0	107.6	97.0	86.8	77.0	67.8	59.4	50.8	42.4	34.2	26.4	18.8
		Sokolovski K_{PE}	56.047	31.868	20.545	14.229	10.332	7.763	5.985	4.709	3.766	3.052	
	10°	Log. spiral K_{PE}	54.991	30.486	19.112	12.888	9.161	6.804	5.227	4.105	3.282	2.663	2.186
		method ω_0	100.3	89.9	79.9	70.3	61.3	53.3	44.9	36.7	28.9	21.3	14.3
		Sokolovski K_{PE}	46.230	26.435	17.142	11.763	8.728	6.598	5.118	4.051	3.258	2.654	
	20°	Log. spiral K_{PE}	42.001	23.175	14.488	9.767	6.965	5.210	4.041	3.204	2.585	2.114	1.747
		method ω_0	91.3	80.9	70.9	61.5	52.9	45.7	37.7	29.9	22.5	15.5	8.9
		Sokolovski K_{PE}	33.705	19.480	12.770	8.995	6.642	5.073	3.973	3.173	2.573	2.111	
	30°	Log. spiral K_{PE}	------	------	------	------	------	------	------	------	------	------	------
		method ω_0	------	------	------	------	------	------	------	------	------	------	------
		Sokolovski K_{PE}											

TABLE A-6 (2). Values of K_{PE} and ω_0 of the spiral method and the Sokolovski method for various values of α_1 in the case of $\delta=\phi$, $\beta=0°$ and $\phi=30°$, 32.5°

ϕ δ β	θ_0	K_{PE} / ω_0 (degrees) / α_1	-70°	-60°	-50°	-40°	-30°	-20°	-10°	0°	10°	20°	30°
ϕ=30.0° δ=30.0° β=0°	-30°	Log. spiral K_{PE}	125.806	68.642	42.301	27.900	19.264	13.783	10.154	7.640	5.850	4.547	3.578
		method ω_0	147.4	134.8	123.4	112.6	102.4	92.8	83.6	74.4	65.4	56.6	48.0
		Sokolovski K_{PE}											
	-20°	Log. spiral K_{PE}	119.769	64.936	39.707	26.021	17.889	12.775	9.409	7.087	5.438	4.240	3.350
		method ω_0	123.8	112.8	102.2	92.0	82.4	73.2	64.2	55.2	46.6	38.0	29.8
		Sokolovski K_{PE}	108.745	59.295	36.637	24.393	17.026	12.039	9.142	6.937	5.359	4.200	
	-10°	Log. spiral K_{PE}	109.282	59.042	36.010	23.562	16.194	11.579	8.549	6.458	4.974	3.894	3.090
		method ω_0	116.2	105.4	95.0	85.0	75.6	66.6	57.6	48.8	40.4	32.2	24.2
		Sokolovski K_{PE}	97.083	53.050	32.896	21.939	15.360	11.142	8.305	6.328	4.907	3.863	
	0°	Log. spiral K_{PE}	97.480	52.518	31.969	20.900	14.371	10.296	7.626	5.783	4.472	3.517	2.803
		method ω_0	110.0	99.4	89.2	79.4	70.0	61.2	52.4	43.8	35.6	27.6	19.8
		Sokolovski K_{PE}	84.962	46.547	28.947	19.370	13.610	9.911	7.418	5.675	4.420	3.494	
	10°	Log. spiral K_{PE}	83.756	44.998	27.342	17.866	12.297	8.836	6.574	5.010	3.895	3.079	2.467
		method ω_0	104.0	93.6	83.4	73.6	64.4	56.0	47.4	39.0	30.8	23.2	15.8
		Sokolovski K_{PE}	71.450	39.282	24.512	16.486	11.640	8.518	6.408	4.928	3.858	3.064	
	20°	Log. spiral K_{PE}	66.193	35.432	21.482	14.030	9.677	6.990	5.239	4.024	3.153	2.510	2.024
		method ω_0	96.6	86.4	76.2	66.6	57.6	49.6	41.2	33.2	25.4	18.2	11.2
		Sokolovski K_{PE}	54.678	30.256	19.013	12.881	9.164	6.759	5.128	3.970	3.131	2.503	
	30°	Log. spiral K_{PE}	29.292	15.441	9.253	6.012	4.173	3.077	2.376	1.878	1.508	1.225	1.000
		method ω_0	75.8	65.4	55.2	45.6	37.6	31.6	25.6	18.6	12.0	5.8	(59.4)
		Sokolovski K_{PE}											
ϕ=32.5° δ=32.5° β=0°	-30°	Log. spiral K_{PE}	195.380	102.957	61.177	38.928	25.960	17.959	12.808	9.346	6.954	5.261	4.037
		method ω_0	135.5	123.9	113.1	102.7	92.7	83.3	73.9	64.7	55.9	47.1	38.5
		Sokolovski K_{PE}											
	-20°	Log. spiral K_{PE}	182.854	95.827	56.671	35.936	23.915	16.536	11.802	8.628	6.436	4.886	3.765
		method ω_0	123.8	113.0	102.4	92.2	82.6	73.2	64.0	55.0	46.2	37.8	29.4
		Sokolovski K_{PE}	165.461	87.062	52.015	33.447	22.597	15.832	11.412	8.417	6.327	4.833	
	-10°	Log. spiral K_{PE}	167.367	87.430	51.581	32.660	21.725	15.033	10.750	7.878	5.896	4.492	3.475
		method ω_0	117.7	106.9	96.5	86.5	76.9	67.7	58.7	49.7	41.1	32.7	24.7
		Sokolovski K_{PE}	148.724	78.365	46.900	30.219	20.465	14.378	10.395	7.692	5.803	4.450	
	0°	Log. spiral K_{PE}	150.278	78.299	46.110	29.167	19.403	13.444	9.638	7.086	5.322	4.071	3.163
		method ω_0	112.5	101.9	91.7	81.7	72.1	63.1	54.3	45.5	36.9	28.9	21.1
		Sokolovski K_{PE}	131.458	69.389	41.608	26.874	18.250	12.861	9.329	7.374	5.247	4.039	
	10°	Log. spiral K_{PE}	130.670	67.910	39.924	25.236	16.796	11.662	8.390	6.194	4.673	3.592	2.803
		method ω_0	107.4	97.0	86.8	76.8	67.4	58.6	49.8	41.2	32.8	25.0	17.4
		Sokolovski K_{PE}	112.386	59.454	35.752	23.164	15.786	11.168	8.136	6.069	4.617	3.569	
	20°	Log. spiral K_{PE}	106.147	54.997	32.268	20.384	13.583	9.465	6.847	5.087	3.864	2.989	2.347
		method ω_0	101.5	91.1	80.9	71.1	61.7	53.1	44.7	36.3	28.3	20.7	13.5
		Sokolovski K_{PE}	89.208	47.374	28.610	18.631	12.767	9.086	6.660	5.000	3.827	2.976	
	30°	Log. spiral K_{PE}	67.933	34.990	20.451	12.906	8.629	6.067	4.450	3.354	2.583	2.024	1.605
		method ω_0	90.8	80.4	70.4	60.6	51.6	43.8	35.8	28.0	20.6	13.6	7.0
		Sokolovski K_{PE}											

TABLE A-6 (3). Values of K_{PE} and ω_0 of the spiral method and the Sokolovski method for various values of α_1 in the case of $\delta=\phi$, $\beta=0°$ and $\phi=35°$, 37.5°

ϕ δ β	θ_0	K_{PE} / ω_0 (degrees) / α_1	-70°	-60°	-50°	-40°	-30°	-20°	-10°	0°	10°	20°	30°
ϕ=35.0° δ=35.0° β=0°	-30°	Log. spiral method K_{PE}	307.917	156.422	89.632	55.041	35.447	23.696	16.344	11.553	8.340	6.132	4.582
		Log. spiral method ω_0	132.8	121.4	110.8	100.6	90.6	81.0	71.6	62.4	53.4	44.6	36.2
		Sokolovski K_{PE}											
	-20°	Log. spiral method K_{PE}	287.298	145.269	82.928	50.785	32.652	21.817	15.059	10.661	7.714	5.690	4.267
		Log. spiral method ω_0	124.4	113.6	103.0	93.0	83.2	73.6	64.4	55.2	46.4	37.8	29.4
		Sokolovski K_{PE}	258.325	131.033	75.514	46.878	30.603	20.742	14.481	10.358	7.563	5.618	
	-10°	Log. spiral method K_{PE}	263.856	133.025	75.772	46.335	29.774	19.902	13.756	9.759	7.081	5.239	3.944
		Log. spiral method ω_0	119.3	108.5	98.1	88.1	78.3	68.9	59.7	50.7	42.1	33.5	25.5
		Sokolovski K_{PE}	233.661	118.625	68.450	42.551	27.797	18.903	13.229	9.490	6.950	5.181	
	0°	Log. spiral method K_{PE}	238.410	119.911	68.185	41.653	26.760	17.902	12.397	8.817	6.418	4.766	3.601
		Log. spiral method ω_0	114.9	104.3	94.1	84.1	74.3	65.1	55.9	47.1	38.5	30.1	22.1
		Sokolovski K_{PE}	208.393	105.912	61.194	38.107	24.973	17.003	11.933	8.585	6.308	4.719	3.584
	10°	Log. spiral method K_{PE}	209.579	105.165	59.703	36.441	23.414	15.685	10.890	7.772	5.679	4.235	3.214
		Log. spiral method ω_0	110.7	100.1	89.9	79.9	70.3	61.1	52.1	43.3	34.9	26.7	19.1
		Sokolovski K_{PE}	180.720	91.984	53.244	33.226	21.832	14.908	10.498	7.582	5.593	4.200	
	20°	Log. spiral method K_{PE}	174.174	87.170	49.397	30.127	19.367	13.004	9.067	6.503	4.778	3.584	2.735
		Log. spiral method ω_0	105.6	95.2	85.0	75.0	65.6	56.6	47.8	39.2	30.8	23.0	15.6
		Sokolovski K_{PE}	147.65	75.315	43.714	27.369	18.054	12.384	8.763	6.361	4.718	3.562	
	30°	Log. spiral method K_{PE}	123.089	61.347	34.668	21.124	13.602	9.182	6.459	4.680	3.475	2.634	2.028
		Log. spiral method ω_0	97.8	87.4	77.4	67.6	58.2	49.4	41.0	32.8	24.8	17.4	10.6
		Sokolovski K_{PE}											
ϕ=37.5° δ=37.5° β=0°	-30°	Log. spiral method K_{PE}	500.414	244.603	134.922	79.801	49.531	31.926	21.249	14.516	10.143	7.231	5.247
		Log. spiral method ω_0	131.6	120.6	110.0	99.8	89.8	80.2	70.8	61.4	52.4	43.6	35.0
		Sokolovski K_{PE}											
	-20°	Log. spiral method K_{PE}	466.596	227.172	124.915	73.712	45.686	29.434	19.599	13.405	9.386	6.709	4.886
		Log. spiral method ω_0	125.2	114.4	104.0	93.8	84.0	74.4	65.0	55.8	46.8	38.0	29.6
		Sokolovski K_{PE}	415.606	202.950	112.650	67.387	42.427	27.758	18.728	12.962	9.170	6.611	
	-10°	Log. spiral method K_{PE}	430.022	208.812	114.589	67.528	41.824	26.948	17.962	12.305	8.635	6.191	4.523
		Log. spiral method ω_0	120.8	110.2	99.8	89.8	79.8	70.4	61.0	52.0	43.0	34.4	26.2
		Sokolovski K_{PE}	378.123	184.757	102.636	61.459	38.745	25.389	17.164	11.907	8.446	6.108	
	0°	Log. spiral method K_{PE}	390.905	189.406	103.772	61.091	37.822	24.380	16.272	11.171	7.860	5.652	4.144
		Log. spiral method ω_0	117.2	106.6	96.4	86.4	76.6	67.0	57.8	48.8	40.0	31.6	23.4
		Sokolovski K_{PE}	340.008	166.240	92.430	55.414	34.986	22.969	15.561	10.082	7.699	5.584	
	10°	Log. spiral method K_{PE}	347.068	167.812	91.803	53.995	33.424	21.562	14.419	9.924	7.006	5.057	3.723
		Log. spiral method ω_0	113.5	103.1	92.9	82.9	73.1	63.7	54.5	45.5	36.9	28.5	20.7
		Sokolovski K_{PE}	298.593	146.118	81.333	48.832	30.889	20.324	13.806	9.632	6.874	5.004	
	20°	Log. spiral method K_{PE}	294.075	141.857	77.475	45.527	28.184	18.208	12.212	8.438	5.984	4.342	3.212
		Log. spiral method ω_0	109.3	98.9	88.7	78.7	69.1	59.7	50.7	41.9	33.3	25.3	17.5
		Sokolovski K_{PE}	249.701	122.353	68.222	41.050	26.035	17.185	11.717	8.209	5.886	4.305	
	30°	Log. spiral method K_{PE}	221.014	106.263	57.902	33.989	21.055	13.645	9.206	6.408	4.581	3.352	2.500
		Log. spiral method ω_0	103.1	92.9	82.7	72.7	63.1	54.1	45.3	36.5	28.3	20.5	13.3
		Sokolovski K_{PE}											

TABLE A-6 (4). Values of K_{PE} and ω_0 of the spiral method and the Sokolovski method for various values of α_1 in the case of $\delta=\phi$, $\beta=0°$ and $\phi=40°$, 42.5°

ϕ δ β	θ_0	K_{PE} / ω_0 (degrees) / α_1	-70°	-60°	-50°	-40°	-30°	-20°	-10°	0°	10°	20°	30°
ϕ=40.0° δ=40.0° β=0°	-30°	Log. spiral method K_{PE}	843.994	396.109	209.845	119.259	71.159	44.114	28.256	18.601	12.545	8.647	6.078
		Log. spiral method ω_0	131.3	120.3	109.9	99.7	89.7	79.9	70.3	61.1	51.9	42.9	34.3
		Sokolovski K_{PE}											
	-20°	Log. spiral method K_{PE}	787.407	368.283	194.585	110.370	65.773	40.752	26.110	17.204	11.622	8.030	5.662
		Log. spiral method ω_0	126.3	115.5	105.5	94.9	85.1	75.3	65.9	56.5	47.5	38.7	30.1
		Sokolovski K_{PE}	692.627	325.045	173.432	99.769	60.442	38.079	24.766	16.541	11.308	7.891	
	-10°	Log. spiral method K_{PE}	728.258	339.822	179.224	101.527	60.458	37.456	24.013	15.842	10.722	7.426	5.252
		Log. spiral method ω_0	122.6	112.0	101.6	91.4	81.6	72.0	62.4	53.2	44.2	35.6	27.2
		Sokolovski K_{PE}	633.692	297.497	158.808	91.422	55.437	34.970	22.776	15.243	10.443	7.306	
	0°	Log. spiral method K_{PE}	665.838	310.077	163.295	92.412	55.003	34.081	21.869	14.450	9.801	6.807	4.829
		Log. spiral method ω_0	119.4	108.8	98.6	88.4	78.6	69.0	59.6	50.4	41.6	32.8	24.6
		Sokolovski K_{PE}	574.098	269.622	144.025	82.970	50.366	31.812	20.754	13.919	9.560	6.706	4.792
	10°	Log. spiral method K_{PE}	596.590	277.300	145.830	82.453	49.058	30.409	19.538	12.936	8.797	6.130	4.365
		Log. spiral method ω_0	116.4	105.8	95.6	85.6	75.6	66.2	56.8	47.6	38.8	30.4	22.2
		Sokolovski K_{PE}	509.776	239.542	128.040	73.842	44.877	28.393	18.562	12.478	8.595	6.048	
	20°	Log. spiral method K_{PE}	513.991	238.414	125.190	70.717	42.066	26.095	16.800	11.155	7.614	5.329	3.812
		Log. spiral method ω_0	112.7	102.3	92.1	82.1	72.3	62.7	53.5	44.5	35.7	27.3	19.5
		Sokolovski K_{PE}	434.686	204.406	109.374	63.160	38.456	24.387	15.987	10.783	7.455	5.268	3.793
	30°	Log. spiral method K_{PE}	403.629	186.713	97.849	55.210	32.843	20.408	13.189	8.803	6.047	4.261	3.070
		Log. spiral method ω_0	107.9	97.5	87.3	77.3	67.7	58.3	49.1	40.1	31.7	23.5	15.9
		Sokolovski K_{PE}											
ϕ=42.5° δ=42.5° β=0°	30°	Log. spiral method K_{PE}	1487.487	668.535	339.230	184.720	105.648	62.804	38.594	24.404	15.832	10.517	7.138
		Log. spiral method ω_0	131.5	120.7	110.1	99.9	89.9	80.1	70.5	61.1	51.9	42.9	34.1
		Sokolovski K_{PE}											
	-20°	Log. spiral method K_{PE}	1389.512	622.652	315.231	171.367	97.901	58.164	35.745	22.617	14.693	9.779	6.656
		Log. spiral method ω_0	127.5	116.7	106.3	96.1	86.3	76.5	66.9	57.5	48.3	39.3	30.7
		Sokolovski K_{PE}	1203.235	541.442	277.043	152.884	88.884	53.777	33.611	21.600	14.226	9.578	
	-10°	Log. spiral method K_{PE}	1289.734	576.726	291.516	158.292	90.364	53.674	32.997	20.897	13.595	9.067	6.188
		Log. spiral method ω_0	124.3	113.7	103.3	93.3	83.3	73.5	64.1	54.7	45.4	36.7	28.1
		Sokolovski K_{PE}	1106.605	498.059	254.949	140.750	81.884	49.584	31.028	19.970	13.177	8.891	
	0°	Log. spiral method K_{PE}	1185.667	529.251	267.163	144.928	82.691	49.112	30.209	19.153	12.482	8.344	5.710
		Log. spiral method ω_0	121.7	111.1	100.9	90.7	80.7	71.1	61.5	52.3	43.1	34.3	25.9
		Sokolovski K_{PE}	1009.454	454.431	232.695	128.535	74.831	45.374	28.417	18.320	12.113	8.192	
	10°	Log. spiral method K_{PE}	1071.323	477.396	240.675	130.447	74.393	44.189	27.202	17.272	11.280	7.561	5.191
		Log. spiral method ω_0	119.0	108.4	98.2	88.0	78.2	68.6	59.0	49.8	40.8	32.0	23.8
		Sokolovski K_{PE}	905.280	407.791	208.836	115.430	67.257	40.813	25.610	16.541	10.963	7.434	
	20°	Log. spiral method K_{PE}	936.530	416.557	209.716	113.562	64.738	38.465	23.710	15.086	9.881	6.648	4.583
		Log. spiral method ω_0	116.0	105.4	95.2	85.2	75.2	65.6	56.2	47.0	38.0	29.4	21.4
		Sokolovski K_{PE}	784.939	353.615	181.250	100.262	58.492	35.550	22.352	14.475	9.622	6.548	
	30°	Log. spiral method K_{PE}	760.567	337.514	169.631	91.754	52.289	31.093	19.211	12.268	8.073	5.462	3.789
		Log. spiral method ω_0	112.0	101.6	91.4	81.4	71.6	62.0	52.6	43.4	34.6	26.2	18.4
		Sokolovski K_{PE}											

TABLE A-6 (5). Values of K_{PE} and ω_0 of the spiral method and the Sokolovski method for various values of α_1 in the case of $\delta=\phi$, $\beta=0°$ and $\phi=45°$

ϕ δ β	θ_0	K_{PE} / ω_0 (degrees) / α_1	-70°	-60°	-50°	-40°	-30°	-20°	-10°	0°	10°	20°	30°
ϕ=45.0°	-30°	Log. spiral method K_{PE}	2761.497	1184.846	573.993	298.480	163.073	92.633	54.418	32.927	20.471	13.053	8.523
		Log. spiral method ω_0	132.0	121.2	110.8	100.6	90.6	80.8	71.2	61.6	52.2	43.2	34.4
		Sokolovski K_{PE}											
	-20°	Log. spiral method K_{PE}	2584.112	1105.882	534.719	277.664	151.548	86.035	50.537	30.592	19.038	12.160	7.958
		Log. spiral method ω_0	128.8	118.0	107.6	97.4	87.6	77.6	68.0	58.6	49.2	40.2	31.4
		Sokolovski K_{PE}											
	-10°	Log. spiral method K_{PE}	2407.157	1028.249	496.477	257.534	140.464	79.717	46.832	28.367	17.673	11.307	7.417
		Log. spiral method ω_0	126.0	115.4	105.2	95.0	85.0	75.2	65.6	56.2	46.8	37.8	29.2
		Sokolovski K_{PE}											
δ=45.0°	0°	Log. spiral method K_{PE}	2224.574	948.737	457.537	237.135	129.263	73.346	43.101	26.126	16.298	10.447	6.870
		Log. spiral method ω_0	123.7	113.3	102.9	92.9	82.9	73.1	63.5	54.1	44.7	35.9	27.3
		Sokolovski K_{PE}											
	10°	Log. spiral method K_{PE}	2025.701	862.586	415.524	215.177	117.238	66.516	39.105	23.728	14.826	9.525	6.281
β=0°		Log. spiral method ω_0	121.4	111.0	100.8	90.6	80.6	70.8	61.4	51.8	42.8	33.8	25.4
		Sokolovski K_{PE}											
	20°	Log. spiral method K_{PE}	1793.635	762.537	366.873	189.824	103.372	58.650	34.506	20.967	13.130	8.460	5.599
		Log. spiral method ω_0	119.0	108.4	98.2	88.2	78.2	68.4	58.8	49.4	40.4	31.6	23.2
		Sokolovski K_{PE}											
	30°	Log. spiral method K_{PE}	1496.336	634.890	305.004	157.645	85.806	48.695	28.687	17.474	10.981	7.107	4.729
		Log. spiral method ω_0	115.6	105.2	95.0	85.0	75.0	65.4	55.8	46.6	37.4	28.8	20.6
		Sokolovski K_{PE}											

TABLE A-7. Percent difference of K_{PE} of the Sokolovski method for various numbers of division of $(\alpha_2-\alpha_1)$ in the case of $\delta=1/2\cdot\phi=20°$

$\alpha_2-\alpha_1$ / S_0, K_{PE} / α_1,β,θ_0 / Numbers of division		54.5°	58.9°	65.0°	75.5°	84.5°	95.0°	105.5°	115.5°	135.5°
		$\alpha_1=0°$ $\beta=0°$ $\theta_0=30°$	$\alpha_1=0°$ $\beta=0°$ $\theta_0=20°$	$\alpha_1=0°$ $\beta=0°$ $\theta_0=0°$	$\alpha_1=0°$ $\beta=0°$ $\theta_0=-30°$	$\alpha_1=-30°$ $\beta=0°$ $\theta_0=30°$	$\alpha_1=-30°$ $\beta=0°$ $\theta_0=0°$	$\alpha_1=-30°$ $\beta=0°$ $\theta_0=-30°$	$\alpha_1=-20°$ $\beta=0°$ $\theta_0=-10°$	$\alpha_1=-30°$ $\beta=0°$ $\theta_0=0°$
1000	S_0	3.524	4.783	6.516	7.211	9.836	19.143	21.705	31.353	65.740
	K_{PE}	5.674	7.098	9.086	11.611	18.288	30.823	40.355	47.243	105.852
4000 ~ 6000	S_0	3.524	4.783	6.514	7.205	9.826	19.110	21.647	*31.243	**65.334
	K_{PE}	5.674	7.098	9.083	11.601	18.269	30.770	40.247	47.077	105.198
Error (%)		0.00	0.00	0.04	0.08	0.10	0.17	0.27	0.35	0.62

* : 5000 segments
** : 6000 segments

TABLE A-8. Percent difference of K_{PE} of the Sokolovski method for $\Delta\omega'=0.001°$ in the case of $\delta=\phi=30°$, $\theta_0=20°$ and $\beta=0°$

$\Delta\omega'$ / K_{PE} Error / α_1		-70°	-60°	-40°	-20°	0°	20°
1°	K_{PE}	53.649	29.699	12.651	6.638	3.895	2.446
	Error (%)	1.88	1.84	1.79	1.79	1.89	2.28
0.1°	K_{PE}	54.513	30.169	12.846	6.743	3.962	2.496
	Error (%)	0.30	0.29	0.27	0.24	0.20	0.28
0.001°	K_{PE}	54.678	30.256	12.881	6.759	3.970	2.503

TABLE A-9. Percent difference of K_{PE} of the spiral method for $\Delta\omega=0.01°$ in the case of $\delta=1/2\cdot\phi=20°$

$\alpha_2-\alpha_1$	$\phi=40°$ $\delta=20°$ / $\Delta\omega$		0.01°	0.02°	0.2°	1.0°
130.0°	$\alpha_1=-20°$ $\beta=20°$ $\theta_0=-20°$	K_{PE}	49.5275	49.5275	49.5281	49.5347
		Error (%)	0	0	0	0.01
102.0°	$\alpha_1=-20°$ $\beta=20°$ $\theta_0=30°$	K_{PE}	41.0258	41.0258	41.0262	41.0297
		Error (%)	0	0	0	0.01
75.5°	$\alpha_1=0°$ $\beta=0°$ $\theta_0=0°$	K_{PE}	11.6322	11.6322	11.6322	11.6334
		Error (%)	0	0	0	0.01
54.5°	$\alpha_1=0°$ $\beta=0°$ $\theta_0=30°$	K_{PE}	5.6747	5.6747	5.6747	5.6753
		Error (%)	0	0	0	0.01