RESEARCH REPORTS

A STUDY ON THE HYDRAULIC LOSS OF SPIRALLY COILED TUBES

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1. Introduction

Tubular coils are widely used for heat exchangers and the other engineering apparatus, but little information is available on the hydraulic resistance in such coils. It is plausible that secondary flow caused by the centrifugal force acting on the fluid in coils produces an increase in friction loss over that obtained in an equivalent straight pipe.

A number of treaties on simply curved tubes (tubes curved in one plane) have been published in the past[1)-7)]. And friction coefficients of these tubes were found by Dean[3)4)] and also by White[5)] to be a function of $Re(d/D)^{1/2}$ for laminar flow, and by Ito[6)] to be a function of $Re(d/D)^2$ for turbulent flow. It is found by one of the present authors[8)] that when $Re(d/D)^{1/2}$ and $Re(d/D)^2$ are replaced by $Re(d/D_1)^{1/2}$ and $Re(d/D_1)^{1/2}$, respectively, similar relations can be applied to the friction coefficients of tubular coils winded three-dimensionally when $d/D_1 \leqq 1/6$, $\theta < 70°$, which cover almost every field of engineering practice.

This paper describes the law of hydraulic resistance of spirally coiled tubes and provides an experimental formula for the hydraulic resistance.

Nomenclature

d = inside diameter of tube under test conditions
D = diameter of a coil
D_1 = twice the radius of curvature of tube coiled helically or spirally (Eqs. (1), (2))
g = acceleration of gravity

h_0=loss of head over the test sector, ①-②, when tube is held straight
h_n=loss of head over the test sector, ①-②, when tube is curved in coils
l_n=length of tube in coiled parts (Fig. 2)
L=length of tube between the test sector (Fig. 2)
n=number of turns of coils
Re=Reynolds number, Vd/ν
Rec=critical Reynolds number, $(Vd/\nu)_c$
t=spiral pitch of coils (Fig. 1)
V=mean flow velocity
α=angle of cone vertex (Fig. 1)
θ=pitch angle (Fig. 1)
λ_0=friction coefficient of straight tube
λ_n=mean value of friction coefficient of coiled part of tube
Λ_n=mean value of friction coefficient of coiled tube including the straight parts before and after coils
ν=kinematic viscosity

2. Dimension and Form of Tubes, and Equations to Predict Experimental Results

2.1. Form and Dimension of Test Tubes

To facilitate the investigations, flexible vinyl tubes reinforced by steel wires were employed. Change of the inside diameter and the sectional form of the tubes, together with the hydraulic roughness of the inside surface, were carefully checked by the method as used in the foregoing research[8]. The investigation was performed with hydraulically smooth tubes. Dimension and form of the coiled tubes are given in Fig. 1 and also Tables 1 and 2.

Fig. 1-A shows an Archimedean spiral, of which radius of curvature differs for each turn of coils. Let the length of the coiled tubes be l_n, then the mean value of the radius of curvature for the tube is

$$\rho_1 = l_n/2\pi n$$

where l_n includes the length of a cylindrically coiled part before the Archimedean

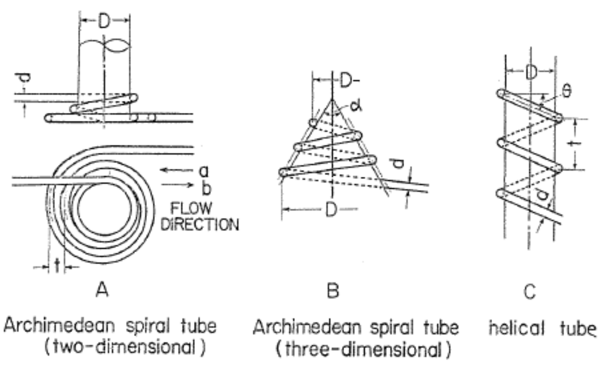

FIG. 1. Configurations of coils

TABLE 1. Dimension of Spirally Coiled Tubes

No.	d (mm)	t (mm)	Di (mm)	Do (mm)	l_n (mm)	D_1 (mm)	D_1/d	n
1 a	10.20	26	326.2	66.2	3293.4	174.7	17.1	6
1 b	10.20	26	66.2	326.2	3293.4	174.7	17.1	6
2 a	10.20	26	351.2	91.2	3763.9	199.7	19.6	6
2 b	10.20	26	91.2	351.2	3763.9	199.7	19.6	6
3 a	10.20	26	376.2	116.2	4234.6	224.7	22.0	6
3 b	10.20	26	116.2	376.2	4234.6	224.7	22.0	6
4 a	10.20	51	576.2	66.2	5262.3	279.2	27.3	6
4 b	10.20	51	66.2	576.2	5262.3	279.2	27.3	6
5 a	10.20	76	674.2	66.2	4874.2	310.3	30.4	5
5 b	10.20	76	66.2	674.2	4874.2	310.3	30.4	5
6 a	10.20	76	699.2	91.2	5265.0	335.2	32.9	5
6 b	10.20	76	91.2	699.2	5265.0	335.2	32.9	5
7 a	10.20	76	724.2	116.2	5656.3	360.1	35.3	5
7 b	10.20	76	116.2	724.2	5656.3	360.1	35.3	5
8	10.20	64	172.1	492.1	5227.7	332.8	32.6	5
9	10.20	48	135.1	423.1	5254.9	278.8	27.3	6

No. 1 ~ No. 7: two-dimensionally coiled tubes (Fig. 1-A)
No. 8, No. 9: three-dimensionally coiled tubes (Fig. 1-B)

TABLE 2. Dimension of Helically Coiled Tubes (Fig. 1-C)

No.	d (mm)	t (mm)	D (mm)	D_1 (mm)	$\theta°$	l_1 (mm)	l_1/d	D/d	D_1/d
I	9.99	50	32.1	40.0	26.5	112.6	11.2	3.20	4.00
II	10.06	75	32.0	49.8	36.8	125.4	12.5	3.18	4.95
III	10.06	100	32.0	63.7	44.8	141.8	14.1	3.18	6.33
IV	9.95	200	105.4	143.9	31.2	386.8	38.4	10.6	14.5
V	10.20	436	130.2	278.1	46.8	597.8	58.6	12.8	27.3
VI	10.20	508	130.2	331.0	51.2	652.2	64.0	12.8	32.5
VII	10.20	540	130.2	357.1	52.9	677.4	66.4	12.8	35.0

spiral.

Putting $2\rho_1 = D_1$, this yields to

$$D_1 = l_n/\pi n \tag{1}$$

A three-dimensionally coiled Archimedean spiral is shown in Fig. 1-B. For this spiral tube, Eq. (1) is also available. Change of the vertex angle gives a different type of coils, namely, when $\alpha=180°$, this spiral yields to a two-dimensionally coiled Archimedean spiral (Fig. 1-A), and when $\alpha=0°$, a helically coiled spiral is obtained (Fig. 1-C). To obtain an intermediate form, α is taken to be 60° and the tests were carried out only with this spiral tube. Fig. 1-C shows a helically coiled tube. Hydraulic loss in this tube was already investigated[5]. For reference, however, some new experiments were carried out and the results were compared with those of the spirally coiled tubes. A mean value of the radius of curvature for a helically coiled tube is given by

$$D_1 = D + (t^2/\pi D) \tag{2}$$

2. 2. Friction Coefficients

Hydraulic resistance of tubular coils can be found by measuring the head loss h_n between sections ① and ② in Fig. 2. The measuring section ① is located 50 diameters (L_1) upstream of the coils, and the section ② is provided 160 diameters or more (L_2) downstream of the coils. Thus, at these sections the effects of coils on flow are perfectly eliminated.

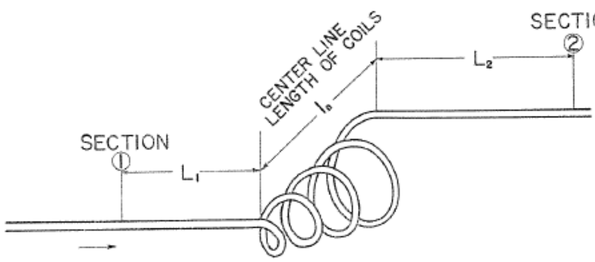

FIG. 2. Measuring sections of tubular coils.

With measured value of h_n, the gross coefficient of flow resistance over the test sector is derived by

$$\Lambda_n = h_n/(L/d)(V^2/2g) \tag{3}$$

When the tube over the test sector is held in a straight line and the loss of head between sections ① and ② is measured to be h_0, the friction coefficient of the straight tube is given by

$$\lambda_0 = h_0/(L/d)(V^2/2g) \tag{4}$$

Assuming that the straight parts before and after the coils, $L_1 + L_2$, have the same friction factor as that obtained in a wholly straight tube, the mean friction coefficient of coiled part is derived by[8]

$$\lambda_n = (\Lambda_n - \lambda_0)(L/l_n) + \lambda_0 \tag{5}$$

When the water in a straight channel enters a curved tube, a secondary flow will set in and it grows up to a saturated condition. The unsaturated length in the tubular coils is found by the other experiment[8] to be rather short. If the number of turns of coils n is greater than 4 or 5, the effects of the unsaturated length on the friction coefficient of coils can be negligible, and the mean value of the friction coefficient for the coils including the unsaturated length is nearly equal to the friction coefficient for the saturated part of coils.

3. Experimental Results and Discussion

3.1. Friction Coefficients and Form of Tubular Coils

The previous investigation[8] on helically coiled tubes shows that when the diametral ratio D_1/d is held constant, the friction coefficient λ_n is independent of the pitch t and pitch angle θ of the coils. The same situation is anticipated for a spirally coiled tube. The friction coefficients of tubes with various forms but with the same diametral ratio D_1/d are compared in Fig. 3. The plots of λ_n fall almost into a single curve.

By reversing the flow direction, flow conditions, especially, those in the inlet length will be altered. To check this effect the results for reversed flow directions are also plotted in Fig. 3, where any appreciable difference can not be seen. The letter a or b in the indication of experimental numbers, for example, in No. 6 a or No. 6 b, shows the flow direction as indicated in Fig. 1.

In Fig. 3, the line a–a′ shows the friction factor of a straight tube calculated by the Hagen-Poiseuille formula (for laminar flow) and the line b–b′ is one calculated by the Blasius formula (for turbulent smooth surface). The discontinuity of $\lambda_n - Re$ curve observed at about the Reynolds number 4×10^3 will probably be attributable to the assumption on Eq. (5). The flow downstream of the coils is affected considerably by the secondary flow developed in coils, and the flow resistance will also be altered. A depression of λ_n observed at about $Re=8\times10^3$ in Fig. 3 (point A) exhibits an onset of turbulence in the coils. This depression is accentuated by the increase of D_1/d.

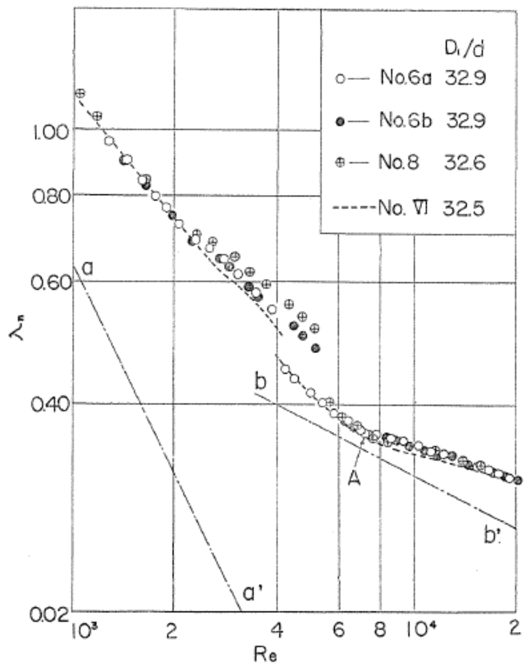

FIG. 3. Comparison of λ_n for different coils with constant value of D_1/d.

3.2. Friction Coefficients for Laminar Flow

Similary with a simply curved tube, the diametral ratio D_1/d for spirally coiled tubes is a governing factor for the flow resistance as seen in the preceeding section. The resistance coefficient λ_n multiplied by $(D_1/d)^{1/2}$ is plotted against $Re(d/D_1)^{1/2}$ in Fig. 4. The plots fall fairly well into a single curve $t-t'$ within laminar flow conditions. The points where the plots diverge from this curve show an onset of turbulence in spiral coils. The centrifugal force acting on the fluids in coils is increased by reducing the diametral ratio D_1/d and delays the transition from laminar to turbulent flow.

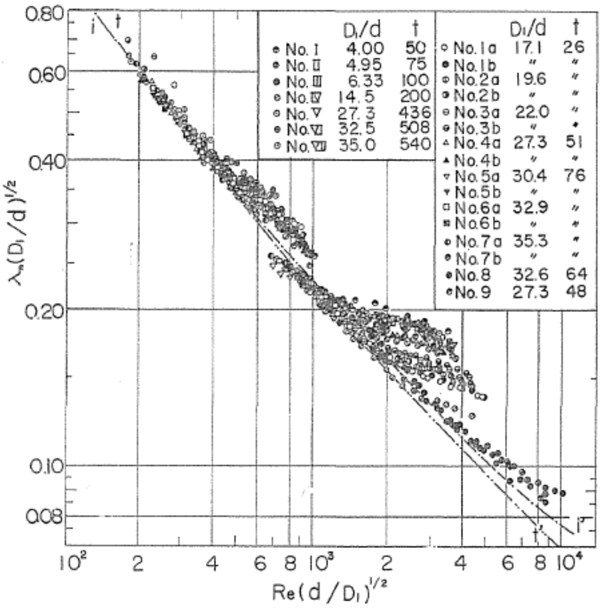

FIG. 4. Relations between $\lambda_n(D_1/d)^{1/2}$ and $Re(d/D_1)^{1/2}$ for laminar flow.

The curve $i-i'$ in the figure shows the relationship given by

$$\lambda_n(D_1/d)^{1/2} = \frac{1376}{\{1.56+\log Re(d/D_1)^{1/2}\}^{5.73}} \tag{7}$$

where

$$13.5 < Re(d/D_1)^{1/2} < 2\times 10^3$$

This is a modified expression of Ito's equation for a simply curved tube under laminar flow conditions. The curve $t-t'$ is an empirical formula given by

$$\lambda_n(D_1/d)^{1/2} = \frac{1376}{\{1.56+\log Re(d/D_1)^{1/2}\}^{5.76}} \tag{8}$$

3.3. Friction Coefficients for Turbulent Flow

The values $\lambda_n(D_1/d)^{1/2}$ for turbulent flow are plotted against $Re(d/D_1)^2$ in Fig.

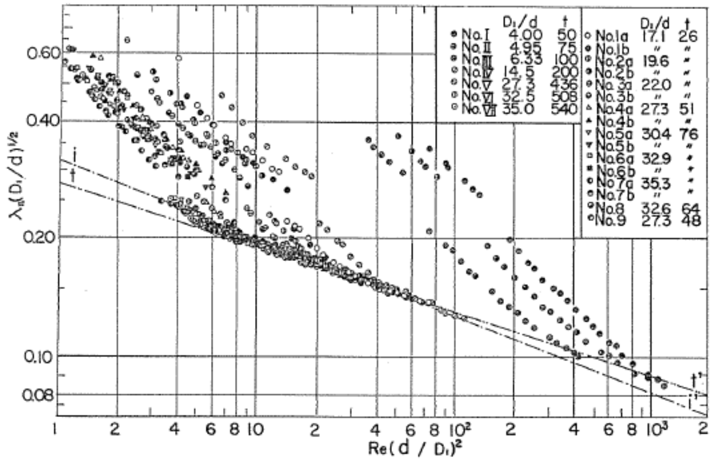

FIG. 5. Relation between $\lambda_n(D_1/d)^{1/2}$ and $Re(d/D_1)^2$ for turbulent flow.

5. The plots lie fairly well on a curve $t-t'$ within turbulent flow. At the points diverging from this curve, the transition from laminar to turbulent occurs. The curve $i-i'$ in Fig. 5 represents the following relationship

$$\lambda_n(D_1/d)^{1/2} = \frac{0.316}{\{Re(d/D_1)^2\}^{1/5}} \tag{9}$$

where

$$Re(d/D_1)^2 > 6$$

Eq. (9) is a modified expression of Ito's, and the curve $t-t'$ represents the following empirical formula

$$\lambda_n(D_1/d)^{1/2} = \frac{0.273}{\{Re(d/D_1)^2\}^{1/5}} \tag{10}$$

where

$$Re(d/D_1)^2 > 15$$

3.4. *Critical Reynolds Number*

The Reynolds number at which the flow changes from laminar to turbulent

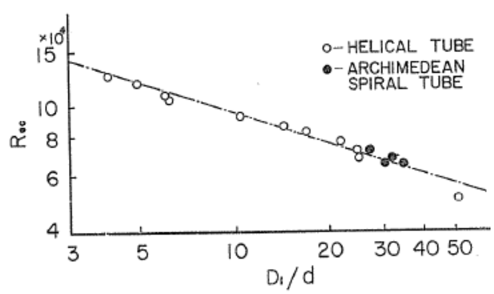

FIG. 6. Critical Reynolds number and diametral ratio D_1/d.

are shown in relation to the ratio (D_1/d) in Fig. 6. The solid line $a-a'$ denotes the following relationship

$$Rec = 2(D_1/d)^{0.32} \times 10^4 \tag{11}$$

which is also a modified expression of Ito's equation. The test results verify approximately this relationship.

4. Conclusion

The tests on the hydraulic resistance of spirally coiled tubes for $D_1/d > 17$, and of helically coiled tubes for $D_1/d \geqq 4$ lead to the following conclusions.

(1) The factors governing the hydraulic resistance of these tubes are $Re(d/D_1)^{1/2}$ for laminar flow and $Re(d/D_1)^2$ for turbulent flow.

(2) The critical Reynolds number characterising the flow conditions in coils is increased by decreasing the ratio (D_1/d).

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