HEURISTICS AND LEARNING CONTROL
(INTRODUCTION TO INTELLIGENT CONTROL)

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(Received June 7, 1971)

1. Introduction

In a phase of engineering development of cybernetics, researches on Artificial Intelligence go on increasing rapidly. Artificial Intelligence includes such pedagogical-psychological functions as learning, education, self-organizing, inductive inference, association, heuristics, concept formation, creation, evolution and so forth, and plays the most serious roles in many problems such as pattern recognition, linguistic analysis, problem-solving, peak-searching, game-playing and so on.

Research on the introduction of learning to control engineering which started about fifteen years ago has grown up favorably and now turned to one of the most remarkable and successful schemes of the trials to introduce the human intelligence to engineering fields. At present, there are found many researches and developments on learning and learning control, including fundamental studies of learning function[31)~34)], construction of learning control system with[35)] or without identification[36)], mathematical analysis of learning processes[37)] and so on. Heuristics is one of the sophisticated artificial intelligences which are very difficult to be analyzed scientifically, but are most fundamental and important from the engineering view point. The authors have done some pioneering works[27)~30)] on heuristics and heuristic control (control with heuristics).

This paper first demonstrates definitions of heuristics, relation among learning, heuristics and intuition, and role of heuristics in control engineering, and next surveys several researches on heuristics and heuristic searching, and last discusses some future problems on heuristics.

2. Definiton and Meaning of Heuristics

2–1. Defintions of Heuristics

The pedagogical-psychological definition of *heuristics or heuristic learning* is expressed in various forms[1)2)]. While the engineering definition of heuristics, although it is not yet so fixed at present, seems to stand on the fairly special concept. By *heuristics* is meant (1) to determine a principle of restricted generalization where by a given machine learns to apply a "generalized" form of a previously successful algorithm to a "similar" problem which is subsequently presented[3)], (2) a principle or device that contributes, on the average, to reduction of search effort in problem solving activity[4)5)], (3) a method which helps in discovering a problem solution by making plausible but failable guesses as to

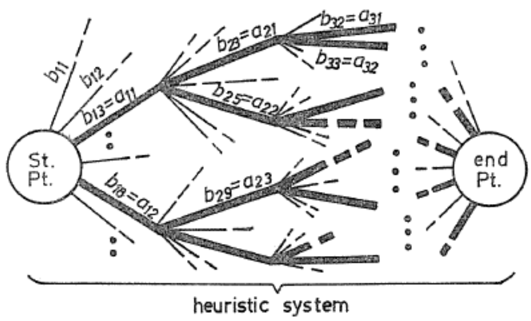

FIG. 1. Heuristic element and heuristic algorithm.

what is the best thing to do next[6].

Now, the authors propose a definition that heuristics is (4) a method which solves a problem using an algorithm (*heuristic algorithm*) which utilizes a series of clues (*heuristic elements*) having a special order of priority[27]. The feature of definition (4) is to be divided into two main concepts of heuristic elements and heuristic algorithm. In Fig. 1, the problem is to search an optimum path to reach at the end point (object, conclusion) from a starting point (initial state, premise). There are so many unknown branches on the way of any path that it is unfeasible to check the whole set of possible paths sequentially. In such a problem, *heuristic elements* mean the branches a_{ij}'s $(i=1, 2, \ldots; j=1, 2, \ldots; J_i)$ which have a higher probability of success of reaching at the end point among the possible branches b_{ik}'s in the i-th interval. On the other hand, a heuristic method means a method of making connection of a series of branches which can pass through from the starting point to the end point under some proper priority of such branches a_{ij}'s with a high probability of success. In practice, the following problems must be considered: What kinds of heuristic elements must be chosen? What kind of heuristic system must be adopted? From which side must the connection be started, from the starting point or the end point? and so on.

2-2. Relation among Learning, Heuristics and Intuition

Learning, Heuristics and Intuition are the three sophisticated native functions of human. The investigation of actual mechanisms of the above human intelligences concerns closely with the problem how human uses the gathered informations and/or the accumulated experiences. The engineering interpretation of the three intelligences may be as follows*: The feature of learning is the active usage of past experiences to improve system behavior. Then, learning is a function relating to time actions like accummulation and utilization of expriences. On the other hand, heuristics is a useful procedure (heuristic algorithm) to extract several relevant factors (heuristic elements) from vast external informations and to make a best path (solution) connecting these heuristic elements in each step of the sequential decision process. In other words, heuristics is a

* This is the opinion of one of the authors induced from questions and discussions made by Professor R. W. McLaren (University of Missouri) and Professor H. Sugiyama (Osaka University) at the seminar.

function concerning spatial factors like the processing of enormous informations and the derivation of spatial solution. Repeating the heuristic processing, man becomes to be able to quicken the heuristic processing and to make a decision quickly, that is, heuristics could be quickened by learning. Moreover, experiences and knowledges obtained by learning will supply more available informations and lead to better probable solutions. Thus, learning can improve the heuristic processing both in speed and accuracy. Heuristics improved by learning may be called *Learned Heuristics.* The extremely quickened heuristics by learning may be able to say *Intuition.* On the other hand, learning is also improved by applying heuristics. Namely, the accumulation and arrangement of informations and experiences can be rationalized by heuristic processing. So that the learning leaded by heuristic decision presents a considerable improvement in the speed of learning and precision of learned results. Learning improved by heuristics may be called *Heuristic Learning.* The few functions mentioned above, learning, heuristics, learned heuristics, heuristic learning, and intuition are, in all, important intelligences of human, and seem to be arranged as above in order of complexity and sophistication. Even if the actual mechanisms of human intelligences, especially of such as intuition, are different from the above description, the above concepts of heuristics and intuition seem to be more convenient from the engineering view point of artificial intelligence. In a decision process, human will probably use these functions in proper way. We can easily guess that human will make the proper use of these functions in a peak-searching process. In game-playing, both heuristics and intuition should be in use to decide a move. A poor chess-player or beginner will be always in use of a slow heuristics to decide a moving policy; on the other hand, a good player will use intuition or learned heuristics more often than an average player. So, a good-player can decide every moves in game-playing very quickly or instaneously.

3. Role of Heuristics in Automatic Control

As been easily guessed from the definitions mentioned in the preceding section, heuristics seems to be a powerful real approach for the search of optimal solution for complex problems which have too many admissible solutions to be ordered one by one and of course to be checked analytically. We can find several applications of heuristic approach in such problems as the solution of mathematical equation[7], the theorem proving problems[8)~13], the search problem of the best strategy in games[14)~18], the solution of pentomino puzzle[19][20], the automatic design of printed network or IC circuit[21)~23] and so on.

In the control problem for complex system with a lot of uncertainty, it is required to learn the association of control choices and a great number of measured samples, and to choose an optimal control from many allowable controls. There seems to be so many kinds of associations in this case that it is not easy to select one optimum control out of the various possible ones. In order to perform this difficult work efficiently, the heuristic approach should provide effective clues on the classification and decision-making which are important to realize recognition or learning. Then the introduction of heuristics or learned heuristics to control ploblem is coming to an important subject. Learning control directed by heuristic learning or learned heuristic may be called *Heuristic Control.*

Sophisticated control schemed by the introduction of artifical intelligence may be named, in general, as *Intelligent Control.*

Major approaches to the design of learning processes include algorithmic approach, heuristic approach, and algorithmic heuristic approach. Most research efforts have been centered, so far, upon the first approach, but the second and third approaches should not be ignored*. Now, the Heuristic Control is going to be highlighted as an interesting and worthy strategy in control.

4. Brief Survey of Sveral Researches on Heuristics and Heuristic Control

4–1. Heuristics Used for Selecting the Important Terms in a Self-Organizing Nonlinear Threshold Network

Mucciardi-Gose[24] proposed "weight size heuristic", which determines the specially important terms (and their weight values) from a lot of terms in a self-organizing network for pattern recognition. The problem is as follows: In the nonlinear threshold network of Fig. 2, assume the input is $X_j=(x_{j1}, x_{j2}, \ldots, x_{jN})$ and choose M linearly-independent terms $f_m(X_j)$, $m=1, 2, \ldots, M$. This is a problem of selecting M proper terms from the 2^N possible linearly-independent terms $(x_{ji}, x_{ji}x_{jk}, x_{ji}x_{jk}x_{jl}, \ldots \ldots, x_{ji}x_{jk}x_{jl}, \ldots, x_{jN})$. The total number of possible combination is $\binom{2^N}{M}$ and it grows very rapidly as the number N increases. Therefore, it is very difficult to select the M proper terms $f_m(X_j)$ among them.

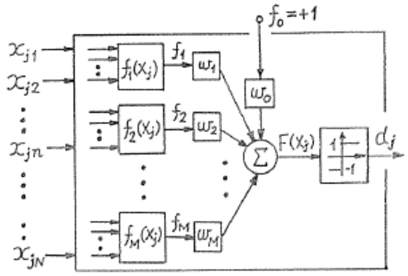

FIG. 2. Nonlinear threshold network.

In the method of weight size heuristic proposed by Mucciardi-Gose, M terms $f_m(X_j)$ are first chosen at random, then the system is trained by a series of J training samples. Comparing the values of w_m's after the training, the M_1 terms with higher values of weight w_m are kept in the network, while the $M_2=(M-M_1)$ terms with lower values of weight are discarded to exchange for the M_2 randomly-chosen new terms. Then, the new network of M terms is trained again. As stated above, if such a process of both training and selecting phases is repeated, then the M desired terms $f_m(X_j)$ can be discovered, and at the same time their weight values can also be decided.

Such a serial process called an *evolutional process* by Mucciardi-Gose includes three important ideas (or subprocesses) of teaching or learning, weight size heuristic, and evolution. In other words, by teaching or learning the network, the weights can be modified; and it leads to the decision of the M terms $f_m(X_j)$ into the M_1 terms to remain and M_2 terms to discard and exchange for M_2 newly chosen terms; after repeating the above procedure, the system attains a gradual evolution.

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* This paragraph is due to the dicsussion after the Professor J. T. Tou's comment in the Seminar.

4-2. Heuristics Used for Approximately Solving an Optimal Control

Thomas-Tou[25)26)] have researched the heuristic method of solving an optimal control. The problem here is "to find an optimum control $\{y_k\}$ in order to shift a system from initial velocity V_0 to a desired final velocity in the specified period N under the condition of minimum fuel consumption". In their experiments, the subject is not informed at all about the plant character. The subject read the current values of six variables (control choice, velocity, difference between the desired and current velocity, fuel comsumption and so on) on meters, and choose heuristically a current control y_k or a series of control $\{y_k\}$ so as to minimize the fuel consumption at the terminal point.

As a result of the experiment, a new mode of heuristic search which a subject evolves was extracted as follows: "A subject pays his attention to the six indications of meters in order to detect if there are any invariants in the nine kinds of factors which are selected beforehand. If there is any one of invariants, a control y_k is chosen according to the invariant. On the other hand, if there are two (or more than two) invariants, y_k is chosen according to the priority designated in advance".

5. Authors' Research on Heuristics Used for Searching an Optimun Point on Multi-dimensional and Multi-modal Hill

5-1. Aim of the Research

The authors have recently tried to clarify the heuristics evolved by a human searcher in a process of searching the highest point (optimum vertex) on two-dimensional and multi-modal hills[27)]. The work aimed to solve the question, "which of the several conventional searching techniques such as (1) random procedure, (2) mixed process of random search and local peak search, and (3) model simulation search is most similar to the human search algorithm?" or "Is the human's procedure quite unique and different from any of them?" Here, the scheme (1) is a method which at first searches n points randmly, then selects the highest trial point among them. Although there are several varieties of the scheme (2), the basic frame of search is as follows. First a point x_i, $i=1, 2, \dots,$ is selected randomly and decided as a base point for a successive local search, which will get an extreme point x_i^m with a value f_i. Next another point x_{i+1} is selected, and the same procedure is repeated till μ times of fail ($f_{i+1} \leqq \max_i \{f_i\}$) are counted. The highest (optimum) point f^0 is got as $\max_i \{f_i\}$. There are also several varieties of (3). An example of them is as follows. First, a global model of a polynomial approximate equation is made for a criterion function from data of local search; then an optimum point of the model is conjectured. After jumping over to a conjectured optimum point, some correcting trials of search are added to stick a true optimum point (Polynomial conjecturing method). In another example, first local search points of four adjacent trials make a local approximate criterion function of a third-order polynomial equation; then a global criterion function is constructed by a piecewise connection of such local functions. A next trial is made at the maximum point of the global function. After the trial, the global function is reconstructed by an addition of the new trial result, and another trial point is selected: in this way, the search process is repeated (method of piecewise cubic approximation). There are also a method of probabilistic model which approximates by probability the intervals between adjacent two points, and soon.

5-2. Method and Conditions of Experiment

In this research, it is planned to extract some heuristics of human though the analysis of experimental search by human subjects performed in the scene shown in Fig. 3. The human subject (searcher) and the experimenter sit at the opposite sides of a masking screen, which prevents the subject from seeing the test hill (criterion function) in front of the experimenter. When the subject selects a trial point $x=(x_1, x_2)$, the experimenter tells him the height of the point on the test hill. The subject writes the height on a recording paper, and selects a next trial point. Repeating the above procedure, the subject gets an optimum point of the test hill which is hidden to the searcher by the screen, but seen by the experimenter, by using tne experience of search trials which are recorded successively on the searching paper. The main goal is to search an optimum point $f(x^m)$. An incidental goal is to keep the value of

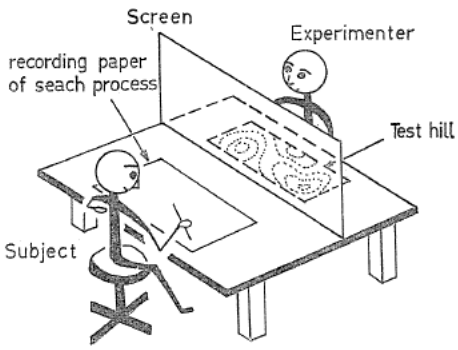


FIG. 3. The scene of experiment.

$$\bar{f}(x_i)=\frac{1}{N}\sum_{i=1}^{N}f(x_i)$$

as big as possible, where $f(\boldsymbol{x}_i)$ is the height of the i-th trial point x_i $(i=1, 2, \ldots, N)$ and $\bar{f}(x_i)$ is the average value of $f(x_i)$. There is no restriction on the selection of trial points $\boldsymbol{x}_i$, trial step width $\Delta\boldsymbol{x}_i$, and trial number N. All a priori informations told to the subject about a test hill are that $f(\boldsymbol{x})$ is a step-wisely continuous and one-valued function on $\boldsymbol{x}$. The number and location of peaks on the test hill are, of course, not informed to the subject.

There are prepared twenty kinds of test hills in which are hidden some regularities (rules) in the number, arrangement, and shape (sharpness) of peaks. Then, the key point for the subject to get the highest peak quickly is to discover the hidden rules as quickly as possible.

5-3. Results of Experiment

One example of experiment is shown in Fig. 4, in which actual equipotential curves (contours) are shown by real line and contours imagined by the human searcher at the end of his search trials are drawn by dotted line. The searcher (YF) tried firstly four global searches (G-mode search), secondly five local searches (L-mode search) around the highest point in the group of first trials, thirdly two convergent searches (C-mode) to comfirm the optimum point with a smaller step width. The search process of this example was finished within eleven trials ($N=11$), while in general cases, N is fairly bigger.

It must be noted that the search technique presented in the examples can be

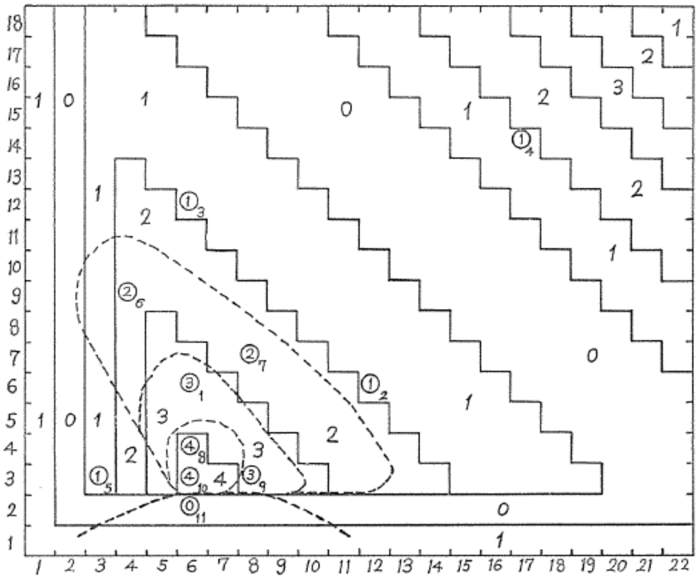


FIG. 4. An example of searching experiment (in case of YF-16).

analyzed in three basic modes (G-mode, L-mode, and C-mode) proposed by authors, and the search process evolved is expressed by the transition among these three modes.

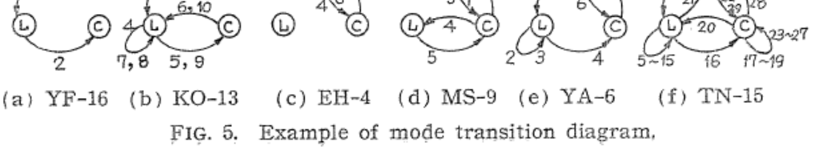
5–4. Discussions

(A) Discussion by Using mode Transition Diagram

Here is proposed a mode transition diagram (abb., *transition diagram*), which is useful to investigate a transition process of a search trial mode. Some examples of the diagram are illustrated in Fig. 5, which indicates any transition between the two search modes by an arrow, on which is written the transition order (1, 2, . . .). For instance, the transition of Fig. 4 is illustrated as Fig. 5 (a) since it is formulated as Eq. (5.1):

$$\text{Transition of the Search Mode of YF-16} = \underbrace{G_1^1 \, L_1^5}_{1} \underbrace{\, C_2^1}_{2} \qquad (5.1)$$

As to the notation of $G_i^{m_i}$, $L_i^{m_i}$, $C_i^{m_i}$, upper suffixes show the number of trials and lower suffixes show the number of group of search trials. The diagrams of Fig. 5 (b) through (f) correspond to another examples. Fig. 5 shows some of



(a) YF-16 (b) KO-13 (c) EH-4 (d) MS-9 (e) YA-6 (f) TN-15

FIG. 5. Example of mode transition diagram.

typical examples of the transition diagrams. In general, however, the mode transition by a human searcher has "a complete flexibility of changing from a specific mode to any of other modes including itself". We call such a type of mode transition "the most flexible type" (cf. Fig. 5 (f)).

(B) Core of Heuristics

As the results of this experimental research, the following heuristics are extracted. They are heuristics in the selection of first trial point (base point), and heuristics in the transition between searching modes. If we think that heuristics is a conscious jump of logical thought, then the tranistion of the search mode in the previous section is just corresponding to the conscious jump. Heuristics appeared in the selection of a base point may be considered as heuristics in a kind of transition from starting condition to base point. The transition to a local search area is especially worthy of being called a *heuristic jump*, two kinds of which extracted as follows: One is a jump (transition) to the vicinity of the base point, which is selected from the G-mode search in the expectation of giving a higher value of the hill. The other is a jump to a point in an unsearched open local area with a comparable extent to that of the most plausible highest peak (a candidate of the optimum point) obtained in the previous process of search.

(C) Heuristics and Learning

In a practical process of search, the appearances of the transition and the ordering of the search mode are deformed variously according to the searching trials (trial experinces), in other words, by learning of a subject. This deformation by learning is just corresponding to the learned heuristics defined in 2-2. The study of such a deforming mechanism from a microscopic point of view is now under investigation at our laboratory.

6. Conclusion

Research of learning control has entered into the second stage where more serious effort should be devoted for the completion of systematic analysis and synthesis of learning process as well as for the high-grade development of the methodological or conceptual research of control.

This paper emphasized that the courageous work for the introduction of more sophisticated intelligence to control engineering should be started just now, and demonstrated authors' opinion on definition and meaning of Heuristics, Heuristic Learning, Learned Heuristic and Intuition, and furthermore introduced authers' research on heuristics evolved by human searcher in the peak-searching process on a two-demensional and multi-modal criterion function. There remains various problems worthy of future works: What kind of difference will appear between heuristics evolved by subjects with and without any sense organ such as visual organ, tactual organ, and so on? What is the adaptability of heuristics for the case of sudden changes of a hill? Although researches on heuristics and heuristic control may be very complex and diffcicult, they will surely bring a epochal aspect on the control of unknown system.

It is sincerely hoped that this brief paper will excite the successive progression of researches in this pioneering field.

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(*: in Japanese)

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