A CONTROL SYSTEM IMPROVING ITS CONTROL DYNAMICS BY LEARNING

KAHEI NAKAMURA

Automatic Control Laboratory Nagoya University, Japan

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1. Introduction

In this paper, is proposed a learning control system or a real time dynamic optimization system which improves its dynamic character by itself through seeking the optimal or sub-optimal control by an iterative approach. This research aimes to develop a learning controller which is applicable to control the plant operating repeatedly in the same environments, such as the start-up control of plant or the batch process control. That is, the objective of this learning controller is to search the best dynamic beharvior when it is required to shift from a steady state (arbitrary fixed starting state) to another steady state (given final state) in the repetitive control of the plant, which is to be considered to have an a priori non-linear structure with unknown parameters.

All the signals such as control input, plant output, and desired output are expressed by sequential vectors (composed of time samples of signals), and the learning control process is described as an iterative one in the sequential vector space[1]. Then, the criterion function is the quadratic distance between the current point and the goal in the vector space. The unknown controlled plant which is essentially non-linear can be considered to be linear approximately in each small scale step of the learning process. Repeating the determination of subgoal, the estimation of unknown parameters and the evaluation of the increment of control in each stage of the iterative procedure, the system approaches gradually to the aimed goal and finally reaches to the sub-optimal control which exists in the range with specified small distance from the desired goal. Since the control experience in each stage is utilized in the linealized operation in the succeeding stages, this procedure is worthy to call learning process. This means that the learning controller have learned the way to improve the dynamic (transient) response of the original control system progressively.

2. Problem Statement

Assume that the unknown plant can be simulated by a non-linear system with properly preselected structure such as n-th order differential equation with unknown parameter and operates in the noiseless environment.

Let the desired performance of the control system or the goal point in $\boldsymbol{x}$-space (sequential vector space of plant output) be $\boldsymbol{x}_d$, and the pre-fixed staring point be $\boldsymbol{x}^\circ$. The criterion function in quadratic form is expressed as $J=\|\boldsymbol{x}_d-\boldsymbol{x}\|$. Then the present problem is to search the best control or the admissible control in u-space (sequential vector space of control), causing the plant output colsest to the goal x_d, in other words, is to get the control which minimizes J under specified constraint on control if it is, through the linearized successive operation in each small-scale stage of the iterative produre.

3. Description of Search Principle (cf. Fig. 1)

In this section, the proposed iterative learning approach is explained refering to Fig. 1. Initially assume that the arbitrary selected control u^0 (for instance step function control with the height to cause the desired steady state of plant output) causes the plant output of x^0. Call the association of (u^0, x^0) as the initial situation of the iterative improving approach.

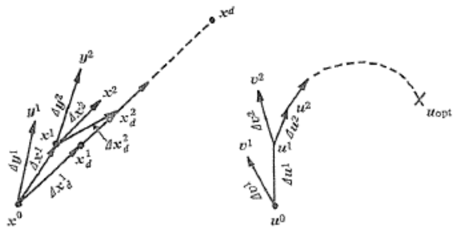

FIG. 1. Vector space deseription of learning process.

First Stage: The operations in this stage are to set a sub-goal x_d^1 near x^0 and to get the control u^1 which causes x_d^1 or its approximation. Since it is reasonable to take the sub-goal on the limited line $\overline{x_0 x_d}$, the following relation is provided:

$$x_d^1 = x^0 + \alpha_1(x_d - x^0) \qquad \alpha < 1 \tag{1}$$

First, select an appropriate small variation of control Δv^1 and observe the plant output y^1 caused by the control $v^1 = u^0 + \Delta v^1$. The resultant variation of plant output is $\Delta y^1 = y^1 - x^0$. If the variation of control situation $(\Delta v^1, \Delta y^1)$ is existed in the neighborhood of (u^0, x^0) with relevant small distance in the product space $U \times X$, then the minor variation of plant behavior can be regarded as it has occurred under the approximately linearized condition, which is expressed by the relation (2).

$$\Delta y^1 = H^1(x^0)\Delta v^1 \tag{2}$$

where $H^1(x^0)$ is designated Sequential Transfer Matrix or S.T.M.[1] in the first stage, and is evaluated by the procedure described Section 4. Since the evaluated H^1 is available in the neighborhood of x^0, the control variation required to attain the subgoal x_d^1 is derived by the relation (3).

$$\Delta u^1 = [H^1(x^0)]^{-1}\Delta x_d^1 \tag{3}$$

where $\qquad \Delta x_d^1 = x_d^1 - x^0$

By applying the new control $u_1 = u^0 + u\Delta^1$, the system reaches at x^1 which is located near x_d^1.

Second Stage: The sub-goal in the second stage x_d^2 is to be selected on the line $\overline{x_d^1 x_d}$ as to satisfy the relation (4).

$$x_d^2 = x_d^1 + \alpha_2(x_d - x_d^1) \quad \alpha_2 < 1 \tag{4}$$

Since it is reasonable that the $H^1(x^0)$ evaluated in the first stage is still valid approximately in the vicinity of x_d^1, the approximate increment of control required to reach at x_d^2 is calculated by Eq. (5).

$$\Delta v^2 = [H^1(x^0)]^{-1} \Delta x_d^2 \tag{5}$$

where $$\Delta x_d^2 = x_d^2 - x^1$$

The usage of $H^1(x^0)$ in Eq. (5) may, of course, cause some considerable error. If the plant output caused by $v^2 = u^1 + \Delta v^2$ is denoted by y^2, the caused output varation is $\Delta y^2 = y^2 - x^1$. The S.T.M. of the second stage $H^2(x^1)$ is calculated by the same way as in the first stage.

That is $$\Delta y^2 = [H^2(x^1)]\Delta v^2 \tag{6}$$

By applying this evaluated $H^2(x^1)$ to

$$\Delta u^2 = [H^2(x^1)]^{-1} \Delta x_d^2, \tag{7}$$

the control variation Δu^2 to cause the desired variation of output Δx_d^2 can be calculated. By applying to the plant the control $u^2 = u^1 + u\Delta^2$, the system reaches at x^2 located near x_d^2.

Succeeding Stages: The same procedures as the second stage is repeated in the succeeding stages. Passing through $x^3, x^4, \ldots$ which are located near x_d^3, $x_d^4, \ldots$ respectively, the system approaches the final goal x_d. This iterative process finishes when it entered in the domain with small distance from x_d. Fig. 2 is the block digagram for the above iterative procedure.

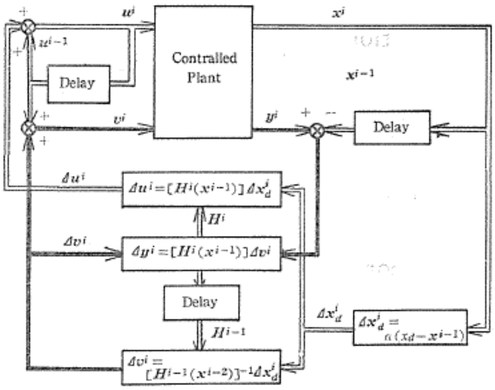

FIG. 2. Flow diagram for learning process ($i \geq 2$).

4. Determination of Linearized S. T. M.

A system of n-th degree nonlinear differential equation subjected by a single input is generally expressed by Eq. (8).

$$f(x, \dot{x}, \ldots, \overset{(n)}{x}) = g(u) \tag{8}$$

The variational equation of (8) comes to Eq. (9).

$$\Delta\overset{(n)}{x}\frac{\partial}{\partial \overset{(n)}{f}}f(x, \dot{x}, \ldots, \overset{(n)}{x}) + \Delta\overset{(n-1)}{x}\frac{\partial}{\partial \overset{(n-1)}{f}}f(x, \dot{x}, \ldots, \overset{(n)}{x}) + \cdots$$
$$+ \Delta\dot{x}\frac{\partial}{\partial \dot{x}}f(x, \dot{x}, \ldots, \overset{(n)}{x}) + \Delta x\frac{\partial}{\partial x}f(x, \dot{x}, \ldots, \overset{(n)}{x}) = \frac{\partial}{\partial u}g(u)\Delta u$$

$$\Delta\overset{(n)}{x} + \alpha_1(x, \dot{x}, \ldots, \overset{(n)}{x})\Delta\overset{(n-1)}{x} + \cdots$$
$$+ \alpha_{n-1}(x, \dot{x}, \ldots, \overset{(n)}{x})\Delta\dot{x} + \alpha_n(x, \dot{x}, \ldots, \overset{(n)}{x})\Delta x = k(u)\Delta u \tag{9}$$

Now, assume that coefficients α_i's be linear functions of $x, \dot{x}, \ldots, \overset{(n)}{x}$,

$$\alpha_j(x, \dot{x}, \ldots, \overset{(n)}{x}) = c_j(t) + \sum_{l=0}^{n}\gamma_{jl}\overset{(l)}{x}(t), \tag{10}$$

and $k(u)$ be a power-expanded polinominal of u,

$$k(u) = \sum_{l=0}^{L}k_l u^l \tag{11}$$

respectively. Then, Eq. (9) is rewritten as

$$\Delta\overset{(n)}{x}(t) + \sum_{j=1}^{n}\left\{c_j(t) + \sum_{l=0}^{n}\gamma_{jl}\overset{(l)}{x}(t)\right\}\Delta\overset{(n-j)}{x}(t) = \left(\sum_{l=0}^{L}k_l u^l\right)\Delta u(t). \tag{12}$$

If we select the dimension of sequence vector as a finite number N, and introduce a shift matrix S

$$S = \begin{pmatrix} 0 & 1 & 0 & & 0 \\ 0 & 0 & 1 & & \cdot \\ 0 & 0 & 0 & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & 1 \\ 0 & 0 & \cdots & \cdot & 0 \end{pmatrix} \quad \text{NXN matrix,} \tag{13}$$

the following relations can be easily derived.

$$\dot{x} = [1 - S]x, \tag{14}$$

$$\Delta\dot{x} = [1 - S]\Delta x \tag{15}$$

$$\text{sequential vector of } c_j(t)\Delta\overset{(n-j)}{x}(t) = [c_j' R]\Delta\overset{(n-j)}{x} \tag{16}$$

$$\text{sequential vector of } \gamma_{jl}\overset{(l)}{x}(t)\Delta\overset{(n-j)}{x}(t) = [\gamma_{jl}[1 - S]^l x]' R\Delta\overset{(n-j)}{x} \tag{17}$$

$$\text{sequential vector of } u^l(t)\Delta u(t) = u_l' R\Delta u \tag{18}$$

where $\boldsymbol{c}$ and u_l are sequential vectors of $c(t)$ and $\{u(t)\}^l$ respectively and

$$R = [e_1 e_1' + e_2 e_2' + \cdots + e_N e_N']. \quad e: \text{unit vector} \tag{19}$$

By applying Eq. (14) ~ (18) to Eq. (12), the following vector equation is obtained under the assumtion that $c_j(t) = c_j$ (constant).

$$\overset{(n)}{\Delta} x + \sum_{j=1}^{n}\Big[c_j\,\mathbf{1} + \sum_{l=1}^{n}\gamma_{jl}[\mathbf{1}-S]^l x\Big]' R \overset{(n-j)}{\Delta} x = \Big(\sum_{l=0}^{L} k_l u_l'\Big) R\Delta u$$

$$\Big[[\mathbf{1}-S]^n + \sum_{j=1}^{n} M_j(x,\, c_j,\, \gamma_{jl})[\mathbf{1}-S]^{(n-j)}\Big]\Delta x = K(u,\, k_l)\Delta u \tag{20}$$

Where
$$M_j = \Big[c_j\,\mathbf{1} + \sum_{l=1}^{n}\gamma_{jl}[\mathbf{1}-S]^l x\Big]' R. \tag{21}$$

$\boldsymbol{M}_i$, $i=1, 2, \ldots, n$ are, in all, $N\times N$ digonal matrices, and their elements are dependent on $\boldsymbol{x}$ (variational base point of output) and $j\times(n+1)$ parameters c_j, γ_{jl}, $l=1, 2, \ldots, n$. K is also a $N\times N$ diagonal matrix and related to u (variational base point of control) and $(L+1)$ parameters k_l, $l=0, 1, 2, \ldots, L$.

Put as

$$H = \Big[[\mathbf{1}-S]^n + \sum_{j=1}^{n} M_j(x,\, c_j,\, \gamma_{jl})[\mathbf{1}-S]^{(n-j)}\Big]^{-1} \tag{22}$$

A linear equation

$$\Delta x = H\Delta u \tag{23}$$

can be derived.

Eq. (23) is a linearized relation of small-scale stage and the matrix of Eq. (22) is the linearized S.T.M. being valid around the control situation $(\boldsymbol{u},\, \boldsymbol{x})$. The expression of Eq. (23) is not valid when the inverse matrix Eq. (22) does not exist*. However, since the present problem is to calculate of elements of $\boldsymbol{H}$ or c_j's, γ_{jl}'s and k_l's for given $(\boldsymbol{u},\, \boldsymbol{x})$, it is enough to use Eq. (20) containing no inverse matrix, instead of Eq. (23). From Eq. (23), N scalar equations are derived, each of which contains $\{j(n+1)+L+1\}$ parameters. If N is selected equal to or larger than number of total parameters, all parameters, can be calculated by solving these simultaneous equations.

5. Convergence of Learning Process

Convergency of the series $\boldsymbol{x}^0, \boldsymbol{x}^1, \boldsymbol{x}^2, \ldots$ is dependent upon the approximation degree of the evaluated $\boldsymbol{H}^i$, $i=1, 2, \ldots$. If $\boldsymbol{H}^i$ is related to the real S.T.M. in i-th stage $\boldsymbol{G}^i$ by

$$G^i = [\mathbf{1} - \Delta^i]H^i,$$

then the following expression is true.

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* The discussion developed here is also correspondent to answer for the question by Prof. K. S. Narendra at the Seminar, on the singularity of $\boldsymbol{H}$.

$$x^i = x^{i-1} + G^i \Delta u^i = x^{i-1} + G^i [H^i]^{-1} \Delta x_d^i \tag{24}$$

$$= x^{i-1} + [1 - \Delta^i](x_d^i - x^{i-1}) = \Delta^i (x^{i-1} - x_d^i) + x_d^i \tag{25}$$

While, as for the series x^0, x_d^1, x_d^2, . . . , the following relation is valid.

$$x_d^i = x_d^{i-1} + \alpha_i (x_d - x_d^{i-1}) = x_d + (1 - \alpha_i)(x_d^{i-1} - x_d)$$

Repeating to use this relation, Eq. (26) can be derived.

$$x_d^i = x_d + \prod_{j=1}^{i} (1 - \alpha_j)(x^0 - x_d) \tag{26}$$

Applying Eq. (26) to Eq. (25), the next relation is derived

$$x^i - x_d = \Delta(x^{i-1} - x_d) + \prod_{i=1}^{i} (1 - \alpha_j)(x^0 - x_d)$$

From this recurrence relation, Eq. (27) follows.

$$x^i - x_d = \Big[\prod_{m=1}^{i} \Delta^m + \sum_{l=0}^{i-1} \prod_{k=0}^{l-1} \Delta^{i-k} [1 - \Delta^{i-l}] \prod_{j=1}^{i-l} (1 - \alpha_j) \Big] (x^0 - x_d) \tag{27}$$

When there is a Δ which satisfy the relation $\|\Delta x\| > \|\Delta x^i\|$, $i = 1, 2, \ldots$ and it is possible to put $\alpha = \alpha_i$, $i = 1, 2, \ldots$, next relation is correct.

$$\|x_i - x_d\| \leq \Big\| \Big[[\Delta]^i + [1 - \Delta] \sum_{l=0}^{i-1} [\Delta]^l (1 - \alpha)^{i-l} \Big] (x^0 - x_d) \Big\|$$

$$= \Big\| [\Delta]^i \Big[\Delta + [1 - \Delta][1 - (1-\alpha)\Delta^{-1}]^{-1} [1 - \{(1-\alpha)\Delta^{-1}\}^i] \Big] (x^0 - x_d) \Big\| \tag{28}$$

Since all components of Δ are usually smaller than 1, the next relation is derived for large number of i.

$$x_i - x_d = \Delta^i [(1 - \alpha)\Delta^{-1} + \alpha](x^0 - x_d) \tag{29}$$

Then,

$$\lim_{i \to \infty} x^i = x_d \tag{30}$$

For general $\alpha_j < 1$, the convergence of x^i is gurranteed for noise free case.

6. Discussions

There are several problems to be discussed.

(A) Constraints on Control

The constraint on control is usually norm constraint or saturation constraint. For these cases, the following strategy is hopeful. Set a hypersphere with specified radius or a hypercube with specified size whose origin is x_d in x-space. When the control trajectory of learning procedure once crossed the hypersphere or hypercube, the best control whose corresponding point in x-space is colsest to x_d should be searched on the surface of control constraint sphere or cube, by the relevant searching procedure such as, for example, the optimum gradient method.

(B) Effect of Noise*

In this paper, only a noise free case is treated. For a noisy case, the effect of noise contained in plant output may appear on the following two points: One is the effect on evaluation of H, and the other is that on the appreciation of approaching trajectory. The former problem in which measured values of Δx and Δu are contaminated by noise, is easily solved by taking N as enough larger than for noise free case and by solving the simultaneous equations through the least mean square method. The latter problem is the effect of the fluctuation of x and u by noisy signal, then is involved in the case of (C).

(C) Modification for Stochastic Envionment**

Under stochastic (randomly varying) enviroment, the initial stationary state, starting point (u_0^0, x^0), passing point (u^i, x^i) and final point (u^N, x^N) on the trjectory may be fluctuated randomly. Then, $J_i=\|x_d-x^i\|$ must be appreciated in stochastic average. So the approaching process turns reasonably to an algorithm by stochastic approximation method, research on which is remained for future.

(D) Prolem on Controllability

Is it always possible to make a system starting at arbitrary $\boldsymbol{x}^0$ pass through an arbitrary x_d?*** In order to realize a $\boldsymbol{x}_d$, it must be guaranteed that any $\boldsymbol{x}^0$ must be transformed to a given $\boldsymbol{x}_d$ through the system dynamics S (Transforming character of the system), in other words, Transformation T: $\boldsymbol{x}_0 \rightarrow \boldsymbol{x}_d$ must involves S. This concerns with controllability problem. For the control of unknown plant, it is of course impossible to predict the controllability of this sense. Therefore, when the desired point is considered to be out of the set which is a transform of $\boldsymbol{x}^0$ by system dynamics, author's procedure can only lead the system to an admissible point closest to the desired point.

7. Conclusions

The learning control scheme offered here aimed to apply to the process control of batch type or the repeating start-stop process. Basic structure of the scheme has described and main problem accompanied has outlined to be discussed. However, there are several problems to be studied in future such as, modifications for stochastic enviroment, extention to multi-demensional system, investigation from the view point of Dynamic Programing and so on.

Finally the author express his great gratitudes to the researchers in Automatic Control Laboratory, Nagoya University, who have discussed about this work.

Referenece

1) K. Nakamura: Sequential Vector Space Description of Descrete Control System, Memoirs of Faculty of Engng. Nagoya University, vol. 20, No. 1, May 1968.
2) K. Nakamura, Y. Shibata and M. Oda: A learning Control System Improving its Dynamic Behavior, Res. Rept. of Aut. Cont. Lab., Nagoya University, vol. 15, April 1968 (in Japanese).

* This discussion is developed as author's answer to Prof. K. S. Narendra's question at the Seminar on a noisy case.

** This is the discussion induced by Prof. K. S. Narendra's question at the Seminar.

*** This is the problem proposed by Dr. J. M. Mendel at the Seminar.