ON THE CHARACTERISTICS OF TWO-DIMENSIONAL SLIDE PLATE OF GROUND EFFECT MACHINES WITH SMALL ATTACK ANGLE

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Summary

Forces and moment produced by the motion of an inclined flat plate, which is supported by the ejection of compressed gas from the centre, are investigated. The clearance between the plate and the ground is assumed to be very small compared with the length of the plate moving with a consant velocity along the ground, and, therefore, the theory on flow with low Reynolds number can be applied. The coefficients of lift, drag and pithing moment about the centre of plate are expressed by functions with two sets of parameters, one of which represents a relative angle of inclination and another parameter which consists of pressure coefficient of jet and reduced Reynolds number of moving flat plate.

1. Introduction

G.E.M. (Ground Effect Machines) can be divided into two categories[1], *i.e.* air cushions vehicles of pressure type and air lubricated vehicles of bearing type. The latter can support the weight of vehicles with comparatively small area of air bearings. An example of practical application to the high-speed rail-way is shown in Fig. 1. Lubricating flow through bearing plate has been investigated by many authors, for example, by Sommerfeld[2], Rayleigh[3] and Michell[4]. The characteristic of the circular plate nozzle and two-dimensional plate nozzle is shown in Fig. 2, and we observe a difference of the pressure distribution between them. In the present paper, the bearing force and moment by a set of two-dimensional slide plates with air lubrication is investigated. The essential features of this type of motion is shown by the model of a slide plate moving on the ground surface in Fig. 3.

It is found that the small inclination of sliding plate gives an important effect on the characteristics of the whole system.

Considering two-dimensional flows we fixed the axes on the sliding plate and the ground is assumed to move with a constant velocity in the opposite direction. Along the ground the x-axis is taken in the direction of motion, and the y-axis is taken in the normal to it as shown in Fig. 3.

The height of the clearance between the sliding plate and the ground is assumed to be very small as compared with the length of plate[5].

G. I. Taylor and P. G. Saffman[6] have been demonstrated that the effect of comressibility may become very important even at very low Reynolds number, in spite of the fact that the Mach number may be very low. For the simplicity, however, in this paper the case of two-dimensional incompressible steady flow[7] is investigated.

2. Notations

a : half length of plate
b : half width of slit
d : plate width
C_D^* : reduced drag coefficient
C_M : pitching moment coefficient
D : drag
G : parallel force on plate
h_m : mean height
L : lift
p : local pressure
p_b : jet pressure
r : radial distance
R_r : reduced Reynolds number
U_m: mean velocity of clearance
x, y: coordinates
A, B: integral coefficients
C : integral constant
C_D : drag coefficient
C_L : lift coefficient
C_P : pressure coefficient
F : normal force on plate
h : local height
l : plate length
M : pitching moment
p_a : atmospheric pressure
Q : volume of flow
R_e : Reynolds number
U : velocity of ground
u, v : velocity components
x_i : distance from origin $i=1, 2, 3, 4, c$
σ : attack angle of plate
ρ : density
η : parameter $a\sigma/h_m$
P^*: parameter $(p-p_a)/(p_b-p_a)$
μ : viscosity
ν : kinematic viscosity
κ : parameter $C_p R_r$
S : parameter $(x-x_c)/a$

The positive senses of forces, moment and angle are indicated in Fig. 4.

3. Basic Equation

For the case of incompressible two-dimensional steady flow in the x, y plane, the Navier Stokes' equations are expressed by

$$\left.\begin{aligned} u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y} &= -\frac{1}{\rho}\frac{dp}{dx}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \\ u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y} &= -\frac{1}{\rho}\frac{dp}{dy}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \end{aligned}\right\} \tag{1}$$

and the equation of continuity is

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2}$$

The typical viscous term in the equation of motion for the x-direction is $u\partial^2 u/\partial y^2$. The ratio of inertia and viscous forces is estimated into the following form:

$$\frac{\text{inertia force}}{\text{viscous force}}=\frac{O(\rho u\partial u/\partial x)}{O(\mu\partial^2 u/\partial y^2)}=\frac{\rho U^2/l}{\mu U/h^2}\fallingdotseq\frac{\rho U_m l}{\mu}\left(\frac{h_m}{l}\right)^2=R_r$$

The inertia force can be neglected comparing to the viscous forces if the reduced Reynolds number are sufficiently small.

In the equation in x-direction, $\partial^2 u/\partial x^2$ can be neglected comparing to $\partial^2 u/\partial y^2$, because the former is smaller than the latter by a factor of the order $(h/l)^2$.

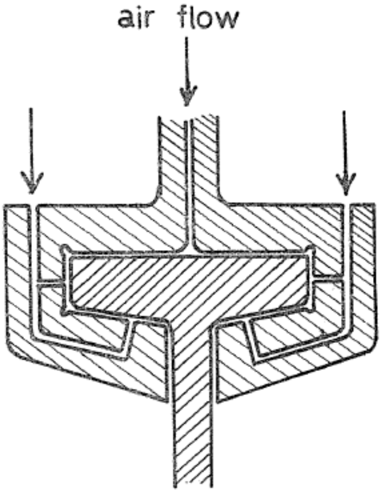

FIG. 1. Standard type.

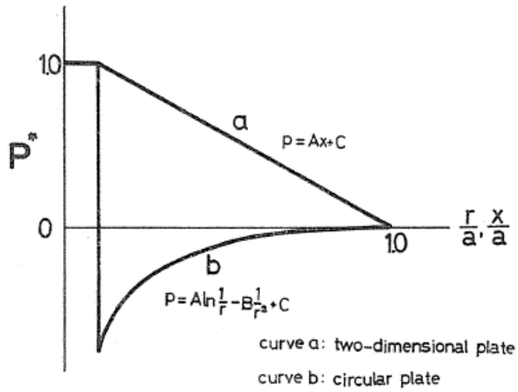

FIG. 2. Pressure distribution.

This equation can be simplified as $\mu\partial^2u/\partial y^2=\partial p/\partial x$. Considering the very small value of v, y component of motion can reduce to $\partial p/\partial y=0$ as given in boundary layer flows. With these simplifications the differential equation (1) can be reduced to

$$\mu\partial^2 u/\partial y^2 = dp/dx \tag{3}$$

and the equation of continuity can be replaced by the condition that the volume flow in any section should be constant.

$$Q = \int_0^h u dy = \text{const.} \tag{4}$$

4. Velocity and Pressure Distributions

The boundary condition are

$$\left.\begin{array}{l} u = U \text{ at } y=0, \quad u=0 \text{ at } y=h \\ p = p_a \text{ at } x=x_1, \quad p=p_b \text{ at } x_2 \leq x \leq x_3 \\ p = p_a \text{ at } x=x_4. \end{array}\right\} \tag{5}$$

The solution of equation (3) which satisfies the boundary conditions (5) is given by

$$u = U\left(1-\frac{y}{h}\right) - \frac{h^2}{2\mu}\frac{dp}{dx}\frac{y}{h}\left(1-\frac{y}{h}\right) \tag{6}$$

where dp/dx denotes the pressure gradient, which must be determined in such a way as to satisfy the continuity equation (4), and the boundary conditions for pressure.

Inserting (6) into (4) we obtain

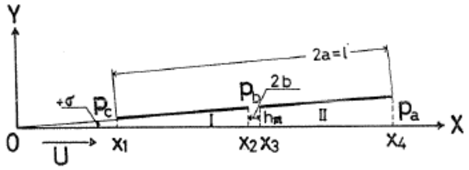

FIG. 3. Two-dimensional sliding plate.

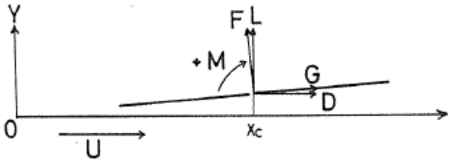

FIG. 4. Three components of the forces.

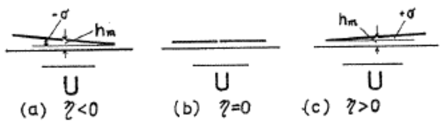

FIG. 5. Plate attitudes.

$$Q = \frac{Uh}{2} - \frac{h^3}{12\,\mu}\frac{dp}{dx} \tag{7}$$

or

$$\frac{dp}{dx} = 12\,\mu\left(\frac{U}{2\,h^2} - \frac{Q}{h^3}\right) \tag{8}$$

Inserting the relation $h = \sigma x$, equation (8) can be integrated into the following form:

$$p = \frac{6\,\mu Q}{\sigma^2 x^2} - \frac{6\,\mu U}{\sigma^2 x} - p_0 \tag{9}$$

where p_0 denotes the integral constant.

i) REGION I: $x_1 \leq x \leq x_2$

Boundary conditions for pressure are

$$p = p_a \ \text{at} \ x = x_1$$
$$p = p_b \ \text{at} \ x = x_2$$

The solutions of equations (9), (7) and (6) are expressed by

$$p - p_a = \frac{(x^2 - x_1^2)x_2^2}{(x_2^2 - x_1^2)x^2}(p_b - p_a) - \frac{6\,\mu U(x_2 - x)(x - x_1)}{\sigma^2(x_2 + x_1)x^2} \tag{10}$$

$$Q = \sigma U \frac{x_1 x_2}{x_1 + x_2} + \frac{\sigma^3 x_2^1 x_2^2}{6\,\mu(x_1^2 - x_2^2)}(p_b - p_a) \tag{11}$$

$$u = \left\{3\,U - \frac{6\,\mu x_1 x_2}{(x_1 + x_2)x} + \frac{\sigma^2 x_2^1 x_2^2}{\mu(x_2^2 - x_1^2)x}(p_b - p_a)\right\}\frac{y}{\sigma x}\left(1 - \frac{y}{\sigma x}\right) + U\left(1 - \frac{y}{\sigma x}\right) \tag{12}$$

When the plate is at rest, equations (10) to (12) are simplified by putting $U=0$.

$$p - p_a = \frac{(x^2 - x_1^2)x_2^2}{(x_2^2 - x_1^2)x^2}(p_b - p_a) \tag{13}$$

$$Q=\frac{\sigma^3 x_1^2 x_2^2}{6\,\mu(x_1^2-x_2^2)}\,(p_b-p_a) \tag{14}$$

$$u=-\frac{\sigma^2 x_1^2 x_2^2}{\mu(x_2^2-x_1^2)x}\,(p_b-p_a)\,\frac{y}{\sigma x}\left(1-\frac{y}{\sigma x}\right) \tag{15}$$

ii) REGION II: $x_3 \leq x \leq x_4$

Boundary conditions for pressure are

$$p=p_b \text{ at } x=x_3$$
$$p=p_a \text{ at } x=x_4$$

The solutions of equations (9), (7) and (6) are

$$p-p_a=\frac{(x_4^2-x^2)x_3^2}{(x_4^2-x_3^2)x^2}\,(p_b-p_a)-\frac{6\,\mu U(x_4-x)(x-x_3)}{\sigma^2(x_4+x_3)x^2} \tag{16}$$

$$Q=\sigma U\frac{x_3+x_4}{x_3 x_4}+\frac{\sigma^3 x_3^2 x_4^2}{6\,\mu(x_4^2-x_3^2)}\,(p_b-p_a) \tag{17}$$

$$u=-\left\{3\,U-\frac{6\,U x_3 x_4}{(x_3+x_4)x}-\frac{\sigma^2 x_3^2 x_4^2}{\mu(x_4^2-x_3^2)}\,(p_b-p_a)\right\}\frac{y}{\sigma x}\left(1-\frac{y}{\sigma x}\right)+U\left(1-\frac{y}{\sigma x}\right) \tag{18}$$

The equations (10) and (16) are plotted in Fig. (6) to (9). When the plate is at rest the equations (16) to (18) are simplified by putting $U=0$ as follow:

$$p-p_a=\frac{(x_4^2-x^2)x_3^2}{(x_4^2-x_3^2)x^2}\,(p_b-p_a) \tag{19}$$

$$Q=\frac{\sigma^3 x_3^2 x_4^2}{6\,\mu(x_4^2-x_3^2)}\,(p_b-p_a) \tag{20}$$

$$u=\frac{\sigma^2 x_3^2 x_4^2}{\mu(x_4^2-x_3^2)x}\,(p_b-p_a)\,\frac{y}{\sigma x}\left(1-\frac{y}{\sigma x}\right) \tag{21}$$

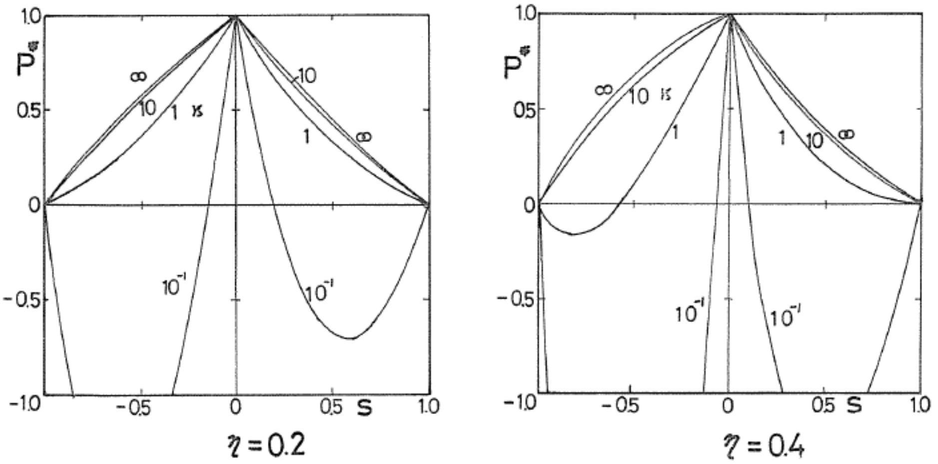

FIG. 6, Pressure distribution.

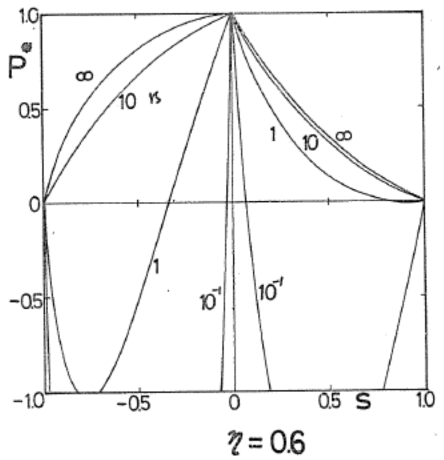

FIG. 7. Pressure distribution.

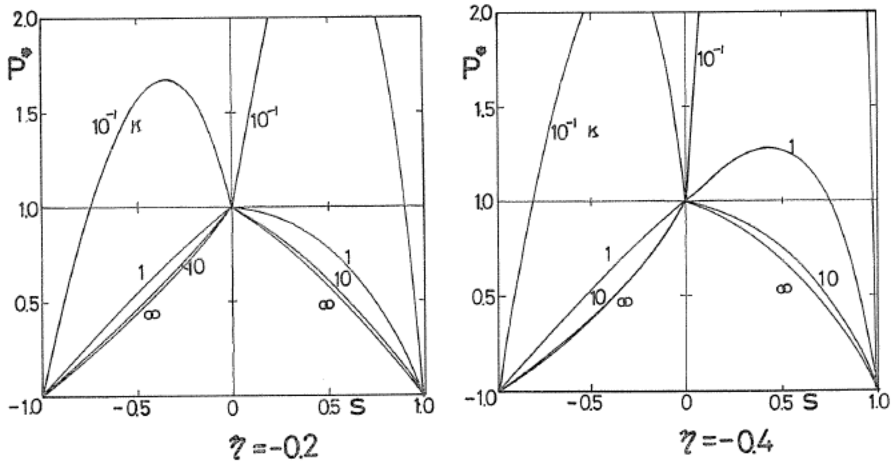

FIG. 8. Pressure distribution.

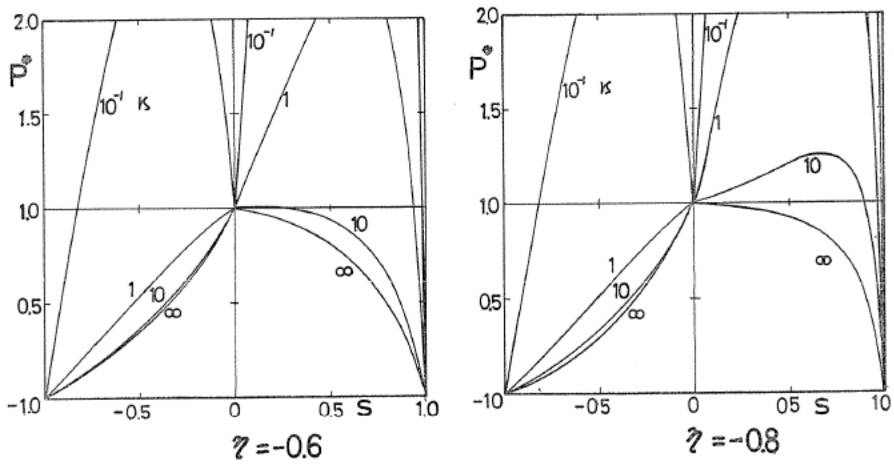

FIG. 9. Pressure distribution.

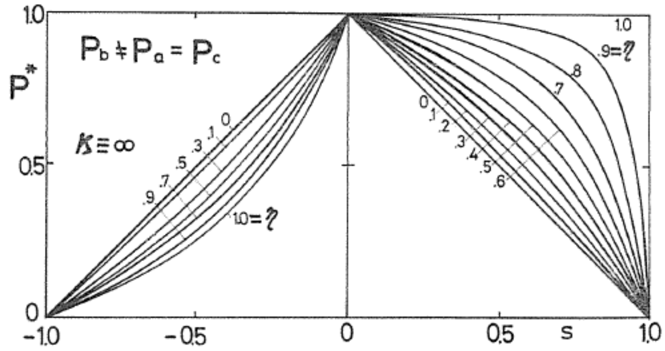

FIG. 10. Pressure distribution: $U=0$.

The equations (13), (19) are plotted in Fig. (10). When we have no excess pressure in the plenum chamber the equations (10) to (12) are simplified by putting $p_b=p_a$.

$$p-p_a=-\frac{6\,\mu U(x_2-x)(x-x_1)}{\sigma^2(x_2+x_1)x} \tag{22}$$

$$Q=\sigma U\frac{x_1x_2}{x_1+x_2} \tag{23}$$

$$u=\left\{-3\,U+\frac{6\,Ux_1x_2}{(x_1+x_2)x}\right\}\frac{y}{\sigma x}\left(1-\frac{y}{\sigma x}\right)+U\left(1-\frac{y}{\sigma x}\right) \tag{24}$$

and the equations (16) to (18) are simplified

$$p-p_a=-\frac{6\,\mu U(x_4-x)(x-x_3)}{\sigma^2(x_4+x_3)x^2} \tag{25}$$

$$Q=\sigma U\frac{x_3x_4}{x_3+x_4} \tag{26}$$

$$u=-\left\{3\frac{(x_3+x_4)x-2\,x_3x_4}{(x_3+x_4)x}\frac{y}{\sigma x}-1\right\}U\left(1-\frac{y}{\sigma x}\right) \tag{27}$$

5. Lift, Drag and Pitching Moment of the Plate

The resultant force from pressure can be obtained by integration of equations (10) and (16).

$$F=\int_{x_1}^{x_2}(p-p_a)dx+\int_{x_3}^{x_4}(p-p_a)dx$$
$$=\left\{\frac{x_2-x_1}{x_2+x_1}x_2-\frac{x_4-x_3}{x_4+x_3}x_3\right\}(p_a-p_a)-\frac{6\,\mu U}{\sigma^2}\left\{\ln\frac{x_2x_4}{x_1x_3}-2\left(\frac{x_2-x_1}{x_2+x_1}+\frac{x_4-x_3}{x_4+x_3}\right)\right\} \tag{28}$$

The resultant force from shearing stress can be calculated in a similar manner:

$$G=-\int_{x_1}^{x_2}\mu(\partial u/\partial y)_{y=h}dx-\int_{x_3}^{x_4}\mu(\partial u/\partial y)_{y=h}dx$$
$$=-\sigma\left\{\frac{x_1x_2}{x_1+x_2}-\frac{x_3x_4}{x_3+x_4}\right\}(p_b-p_a)-\frac{2\,\mu U}{\sigma}\left\{\ln\frac{x_2x_4}{x_1x_3}-3\left(\frac{x_2-x_1}{x_2+x_1}-\frac{x_4-x_3}{x_4+x_3}\right)\right\} \tag{29}$$

Situations of four edges, x_1 to x_4 are given by the use of plate mean height h_m and angle of inclination σ.

$$\left.\begin{array}{ll} x_1 = h_m/\sigma - a - b, & x_3 = h_m/\sigma - b \\ x_2 = h_m/\sigma + b, & x_4 = h_m/\sigma + a + b \end{array}\right\} \quad (30)$$

Substituting (30) into (28) and (29), we obtain

$$F = \frac{2(a-b)\{2h_m^2 - b\sigma^2(a+b)\}}{4h_m^2 - \sigma^2(a+b)^2}(p_a - p_a) + \frac{6\mu U}{\sigma^2}\left\{\frac{8\sigma h_m(a-b)}{4h_m^2 - \sigma^2(a+b)^2} - \ln\frac{(h_m + a\sigma)(h_m - b\sigma)}{(h_m - a\sigma)(h_m + b\sigma)}\right\} \quad (31)$$

$$G = -\frac{2\sigma(a+b)(ab\sigma^2 - h_m^2)}{4h_m^2 - \sigma^2(a+b)^2}(p_b - p_a) - \frac{2\mu U}{\sigma^2}\left\{\ln\frac{(h_m + a\sigma)(h_m - b\sigma)}{(h_m - a\sigma)(h_m + b\sigma)} - \frac{12\sigma h_m(a-b)}{4h_m^2 - \sigma^2(a+b)^2}\right\} \quad (32)$$

Considering the small magnitude of the angle of inclination and $b \ll a$, lift and drag are calculated from the resultant forces of pressure and of shearing force as follows:

$$L = F - \sigma G = \frac{4ah_m^2}{4h_m^2 - a^2\sigma^2}(p_b - p_a) - \frac{6\mu U}{\sigma^2}\left\{\ln\frac{h_m + a\sigma}{h_m - a\sigma} - \frac{8a\sigma h_m}{4h_m^2 - a^2\sigma^2}\right\} \quad (33)$$

$$D = -\sigma F - G = \frac{2a\sigma h_m^2}{4h_m^2 - a^2\sigma^2}(p_b - p_a) + \frac{4\mu U}{\sigma}\left\{\ln\frac{h_m + a\sigma}{h_m - a\sigma} - \frac{6a\sigma h_m}{4h_m^2 - a^2\sigma^2}\right\} \quad (34)$$

Pitching moment is calculated by the following integrations,

$$\begin{aligned} M &= \int_{x_1}^{x_2}(x_c - x)(p - p_a)dx - \int_{x_3}^{x_4}(x_c - x)(p - p_a)dx \\ &= \frac{h_m^2}{\sigma^2(2h_m - a\sigma)}\left\{\frac{(h_m - a\sigma)^2}{a\sigma}\ln\frac{h_m}{h_m - a\sigma} + \frac{3}{2}a\sigma - h_m\right\}(p_b - p_a) \\ &- \frac{h_m^2}{\sigma^2(2h_m + a\sigma)}\left\{\frac{(h_m + a\sigma)^2}{a\sigma}\ln\frac{h_m + a\sigma}{h_m} - \frac{3}{2}a\sigma - h_m\right\}(p_b - p_a) \\ &- \frac{6\mu U}{\sigma^2(2h_m - a\sigma)}\left\{\frac{h_m}{\sigma}(3h_m - 2a\sigma)\ln\frac{h_m}{h_m - a\sigma} - a\left(3h_m - \frac{a\sigma}{2}\right)\right\} \\ &- \frac{6\mu U}{\sigma^2(2h_m + a\sigma)}\left\{\frac{h_m}{\sigma}(3h_m + 2a\sigma)\ln\frac{h_m + a\sigma}{h_m} - a\left(3h_m + \frac{a\sigma}{2}\right)\right\} \end{aligned} \quad (35)$$

where positive pitching moment is taken in clockwise direction as shown in Fig. 4.

6. Lift, Drag and Pitching Moment Coefficients

The coefficients of lift, drag and pitching moment are defined, respectively

$$C_L = \frac{L}{(p_b - p_a)\cdot 2a} \quad C_D = \frac{D}{(p_b - p_a)\cdot 2a} \quad C_M = \frac{M}{(p_b - p_a)\cdot(2a)^2} \quad (36)$$

Substituting equations (33) to (35) into equations (36), we obtain

$$C_L = \frac{2}{4-\eta} - \frac{1}{\kappa}\frac{12}{\eta^2}\left\{\ln\frac{1+\eta}{1-\eta} - \frac{8\eta}{4-\eta^2}\right\} \tag{37}$$

$$C_D^* = -\frac{\eta}{4-\eta^2} + \frac{8}{\kappa\cdot\eta}\left\{\ln\frac{1+\eta}{1-\eta} - \frac{6\eta}{4-\eta^2}\right\} \tag{38}$$

$$\begin{aligned} C_M =& \frac{1}{4\eta^2(2-\eta)}\left\{\frac{(1-\eta)^2}{\eta}\ln\frac{1}{1-\eta} - 1 + \frac{3}{2}\eta\right\} \\ & - \frac{1}{4\eta^2(2+\eta)}\left\{\frac{(1+\eta)^2}{\eta}\ln(1+\eta) - 1 - \frac{3}{2}\eta\right\} \\ & - \frac{1}{\kappa}\left[\frac{6}{\eta^2(2-\eta)}\left\{\frac{3-2\eta}{\eta}\ln\frac{1}{1-\eta} - 3 + \frac{\eta}{2}\right\}\right. \\ & \left. + \frac{6}{\eta^2(2+\eta)}\left\{\frac{3+2\eta}{\eta}\ln(1+\eta) - 3 - \frac{\eta}{2}\right\}\right] \end{aligned} \tag{39}$$

where the pressure coefficient $C_P = (p_b - p_a)/\frac{1}{2}\rho U^2$, Reynolds number $Re = Ul/\nu$ and the parameters $\eta = a\sigma/h_m$, and $\kappa = C_P Rr$. The equations (37) to (39) are plotted in Fig. (11) to (13).

When the plate is at rest where $U=0$, these coefficients become, respectively

$$C_L = 2/(4-\eta^2) \tag{40}$$

$$C_D^* = -\eta/(4-\eta^2) \tag{41}$$

$$\begin{aligned} C_M =& \frac{1}{4\eta^2(2-\eta)}\left\{\frac{(1-\eta)^2}{\eta}\ln\frac{1}{1-\eta} - 1 + \frac{3\eta}{2}\right\} \\ & - \frac{1}{4\eta^2(2+\eta)}\left\{\frac{(1+\eta)^2}{\eta}\ln(1+\eta) - 1 - \frac{3\eta}{2}\right\} \end{aligned} \tag{42}$$

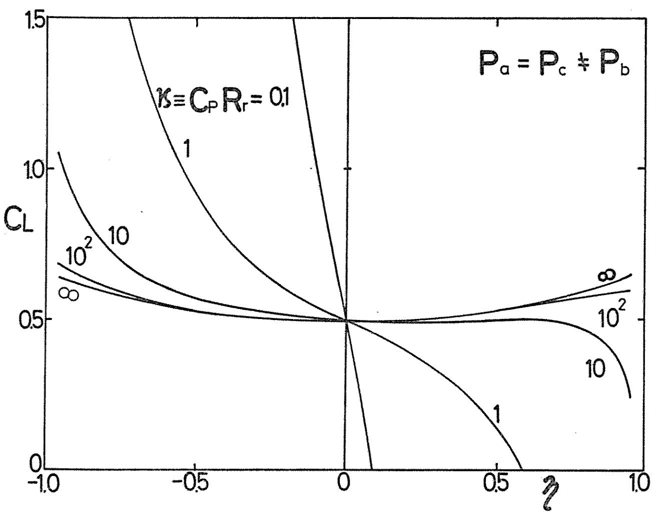

FIG. 11. Lift coefficient of a plate.

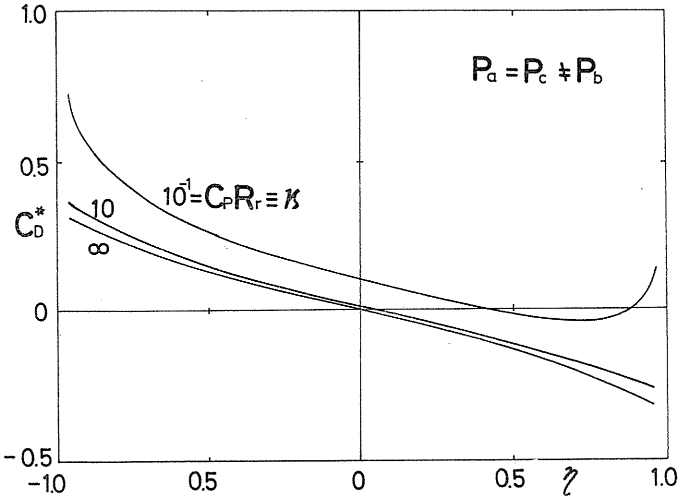

FIG. 12. Drag coefficient of a plate.

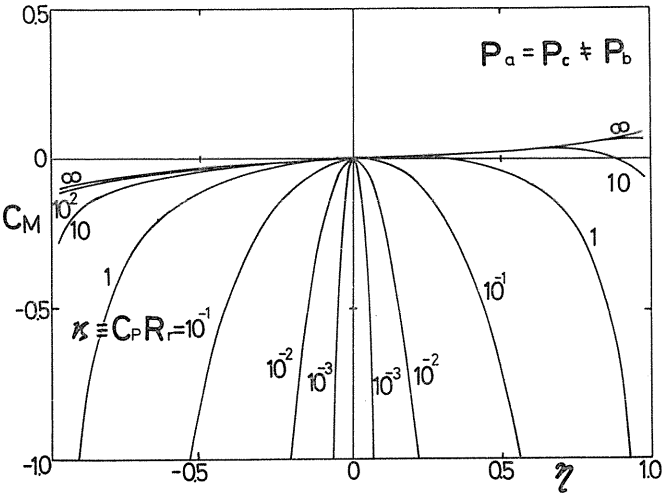

FIG. 13. Moment coefficient of a plate.

In the case of zero angle of inclination, $\sigma = 0$, these coefficients become

$$C_L = 1/2, \qquad C_D^* = 4/\kappa, \qquad C_M = 0 \tag{43}$$

When the pressure of plenum chamber $p_b = p_a$, the present definition of coefficients can not be used. Alternative coefficients are defined, respectively

$$C_L = L/\tfrac{1}{2}\rho U^2(2\,a), \qquad C_D = D/\tfrac{1}{2}\rho U^2(2\,a), \qquad C_M = M/\tfrac{1}{2}\rho U^2(2\,a)^2 \tag{44}$$

Substituting equations (33) to (35) into equations (44), we obtain

$$C_L = \frac{12}{Rr\eta^2}\left(\ln\frac{1+\eta}{1-\eta} - \frac{8\eta}{4-\eta^2}\right) \tag{45}$$

$$C_D^* = \frac{8}{Rr\eta}\left(\ln\frac{1+\eta}{1-\eta} - \frac{6\eta}{4-\eta^2}\right) \tag{46}$$

$$C_M = \frac{1}{Rr}\Big[\frac{6}{\eta^2(2-\eta)}\left\{\frac{3+2\eta}{\eta}\ln\frac{1}{1-\eta} - 3 - \frac{\eta}{2}\right\}$$

$$+ \frac{6}{\eta^2(2+\eta)}\left\{\frac{3+2\eta}{\eta}\ln(1+\eta) - 3 - \frac{\eta}{2}\right\}\Big] \tag{47}$$

When $\eta=1$ the leading edge is attached to the ground, and the trailing edge is attached in $\eta=-1$. The attitudes of the sliding plate are shown in Fig. 5 whih respect to the sign of parameter η. The coefficients of forces and moment are calculated as a unique function of two parameters, η and κ, as shown in Figs. 6, 7 and 8. In Fig. 7 the reduced drag coefficient $C_D^* = C_D/(h_m/a)$ is used in place of the usual drag coefficient. It is found that coefficients for $\kappa=10^4$ coincide with the values at $\kappa=\infty$.

For the sake of practical use two examples are picked up from these results. Choosing $U=5$ m/s, $a=2.5$ m, $d=1$ m, $h_m=2$ mm, $p_b-p_a=1000$ kg/m^2, $\nu=1.5\times10^{-5}$ m^2/s, wehave the Reynolds number referred to the plate length $Re=1.67\times10^6$, the reduced Reynolds number $Rr=0.267$, and the parameter $\kappa=1.7\times10^2$.

i) *Example 1* (a case of head up)

$\sigma=-0.0005$ ($\eta=-0.625$), $C_L=0.54$, $L=2{,}700$ kg
$C_D^*=0.18$, $D=0.27$ kg
$C_M=-0.05$, $M=-1{,}250$ kg-m

ii) *Example 2* (a case of head down)

$\sigma=0.0005$ ($\eta=0.625$), $C_L=0.54$, $L=2{,}700$ kg
$C_D^*=-0.18$, $D=-0.27$ kg
$C_M=0.05$, $M=1{,}250$ kg-m

7. Conclusions

1) *The case of negative attack angle* ($\eta>0$, and plate nose down)

a) *In low speed motion*

As the magnitude of attack angle of the plate increases, or as the mean height of the plate decreases, the lift coefficient increases gradually, the drag coefficient decreases down to the negative value in high η, and the pitching moment coefficient decreases gradually.

b) *In high speed motion*

As the attack angle of the plate increases, or as the mean height of the plate decreases, the lift coefficient decreases and the drag coefficient decreases initially and then increases, and the pitching moment coefficient increases rapidly in the direction of nose down.

2) The case of positive attack angle ($\eta<0$, and plate nose up)

As the attack angle of the plate increases, or as the mean height of the plate decreases, the coefficients of lift, drag and pitching moment are all increases. When the parameter κ is small compared with 1, these coefficients show a great change.

3) The range of application of the present calculation

The present theory can be applied to the case of extremely small value of reduced Reynolds number, $Rr \ll 1$. When the mean height of plate is taken in a small value, it means that the plate runs with comparatively high speed. Since the leading edge and the trailing edge attach to the ground at $\eta=\pm 1$ respectively, the parameter should be limited in the range of $-1 \leq \eta \leq 1$.

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