A NOTE ABOUT THE SURVEYING POSITION OF OUTLET FLOW IN CASCADE EXPERIMENTS

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Summary

Experiments were done to find the closest traverse position where the cascade performance calculated from the simple mean of measured values obtained from the wake traverse of cascade (we term it "conventional value") are close to the one which is obtained at the position fully distant from cascade where the wake becomes uniform by mixing (we term it "exact value"). As a result of these experiments it was found that the closest traverse position above mentioned is such one as being distant from blade trailing edge at least about one-third chord length downstream along the streamline. It seems that this result serves as a good reference to other cascade conditions too. Experiments were done on two cases, that is, with exit side walls and without them. From this experiment it was also found that cascade exit flow is influenced by the mixing with the open air when side walls are not used.

1. Introduction

In cascade experiment the wake traverse position should be determined carefully to obtain reliable measured values. It has been ascertained experimentally untill now that cascade performance obtained from the simple mean of measured value in the direction of cascade (we term it "conventional value") becomes nearly constant if the traverse position is more distant than about one chord length from the trailing edge.

Before the consideration of the traverse position we must decide what is the true cascade performance. Otsuka[1] and Kawasaki[2)3)] suggested that we should use the value obtained at such a position as being fully distant from cascade where the wake becomes uniform by mixing (we term it "exact value"). Although the exact value calculated from measurements close to the blade trailing edge gives nearly constant value, it requires considerable labour to calculate them. It may be recommended, therefore, to do traverse at the position where the "conventional value" approaches fully to the "exact" one and use the former in place of the latter.

In this research work we did cascade experiment in which the distance from blade trailing edge to the traverse position was changed to find a traverse position being as close to the blade as possible (considering a narrow space between blade rows in the axial-flow turbo-machine). Cascades of compressor type were employed in the experiment. Measurements were repeated with and without exit side walls to examine the difference of cascade performance between these two exit flow types, where the cascade exit flow is separated from the open air by a

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exit side walls in one type, and released directly into the open air in another.

2. Symbols

AR: aspect ratio, $AR=2B/C$
a : blade pitch
$2B$: blade span
C : chord length
D : displacement thickness, $D=\int_0^a\left(1-\frac{V_2}{V_2^*}\right)dt$
h_1, h_2, h_3: see Fig. 4
Δh : total pressure loss, $\Delta h_{\text{mean}}=\frac{1}{a}\int_0^a(p_{t\,1\text{CL}}-p_{t\,2})dt$
l : distance from blade trailing edge to the downstream traverse position along blade lower surface
M : momentum thickness, $M=\int_0^a\frac{V_2}{V_2^*}\left(1-\frac{V_2}{V_2^*}\right)dt$
p : static pressure
p_d : dynamic pressure, $p_{d\,2\,\text{mean}}=p_{t\,2\,\text{mean}}-p_2$
p_t : total pressure, $p_{t\,2\,\text{mean}}=\frac{1}{a}\int_0^a p_{t\,2}dt$
Re : Reynolds number, $Re=V_1C/\nu$
t : distance along traverse line in cascade direction
V : air speed
α : stagger angle
α_i : attack angle
γ : flow angle measured from axial direction, $\gamma_{2\,\text{mean}}=\frac{1}{a}\int_0^a\gamma_2 dt$
ε : turning angle, $\varepsilon=\gamma_1-\gamma_{2\,\text{mean}}$
ζ : total pressure loss coefficient, $\zeta_{2\,\text{mean}}=\Delta h_{\text{mean}}/p_{d\,2\,\text{mean}}$, $\zeta_{2\,\infty}=\Delta h_{2\,\infty}/p_{d\,2\,\infty}$
ν : kinematic viscosity

Subscripts
1 : before cascade
2 : behind cascade (at traverse position)
2∞: uniform flow at infinite downstream of cascade
$*$: main flow outside of wake
CL : center of span

3. Theory

We suppose that wakes of blades at cascade exit become uniform at infinite downstream as a result of mixing. Assuming the blade span being long enough, and this mixing process being considered to be two dimensional, we apply momentum theory to this flow. Because the solution was already obtained by Kawasaki[2)3)], only the result is shown as follows. (For the convenience of readers we write the procedure to obtain these results in Appendix A.)

$$\varepsilon_{2\infty}=\varepsilon+\left(\frac{g^2}{f}-1\right)\sin\gamma_2\cos\gamma_2 \tag{1}$$

$$\frac{\Delta h_{2\infty}}{\frac{1}{2}\rho V_{2\infty}^2} = \frac{g^2-f}{(fg)^2}[-2(g^2-f)^2\sin^6\gamma_2+(g^2-f)(2g^2-6f+1)\sin^4\gamma_2$$

$$+2f(2g^2-3f+1)\sin^2\gamma_2]+\frac{g^2-2f+1}{g^2} \qquad (2)$$

where $f=1-\frac{D}{a}-\frac{M}{a}$, $g=1-\frac{D}{a}$,

$$D=\int_0^a\left(1-\frac{V_2}{V_2^*}\right)dt \qquad \text{(displacement thickness of wake)},$$

$$M=\int_0^a\left(\frac{V_2}{V_2^*}\right)\left(1-\frac{V_2}{V_2^*}\right)dt \qquad \text{(momentum thickness of wake)}.$$

Assuming particularly that $D/a \ll 1$, $M/a \ll 1$, we obtain

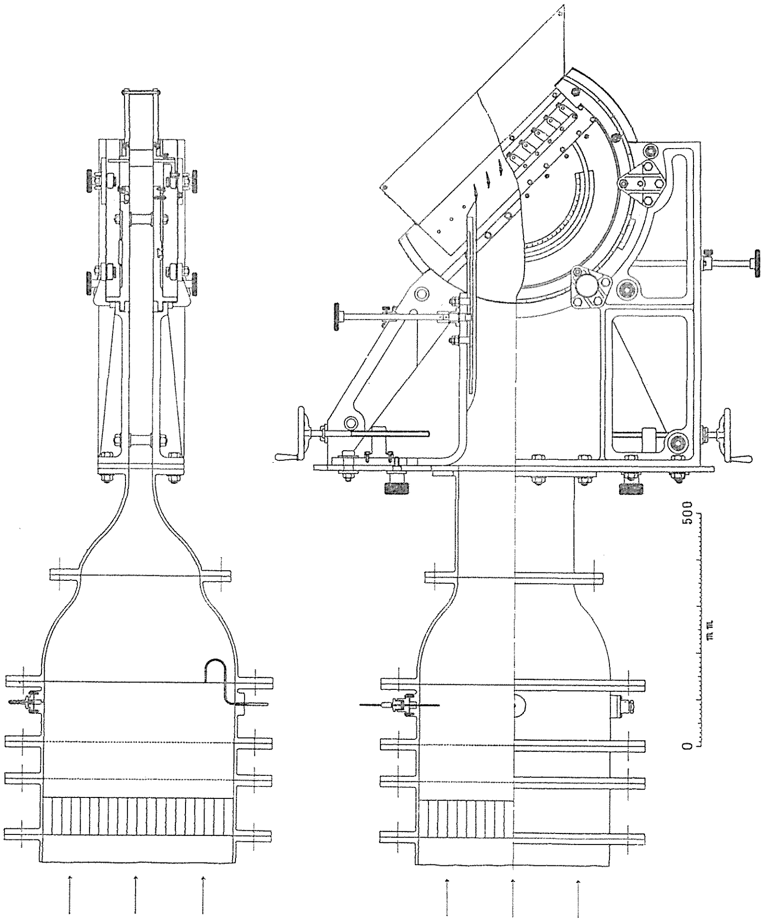


FIG. 1

$$\varepsilon_{2\infty}=\varepsilon-\left(\frac{D}{a}-\frac{M}{a}\right)\sin\gamma_2\cos\gamma_2 \tag{1'}$$

$$\frac{\Delta h_{2\infty}}{\frac{1}{2}\rho V_{2\infty}^2}=2\frac{M}{a} \tag{2'}$$

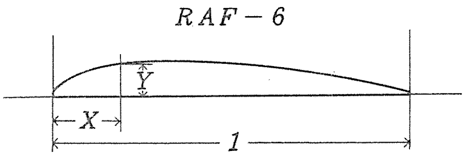
As aforesaid we term these values as "exact values" of turning angle and total pressure loss coefficient respectively.

4. Apparatus and Procedure

The cascade wind tunnel used was a high-speed one which belongs to the laboratory of one of the authors. The schematic drawing is illustrated in Fig. 1. The blade profile is RAF-6 and its dimensions are 50 mm in span, and 48 mm in chord length, and its material is stainless steel. Profile data are illustrated in Fig. 2. Exit side walls are 115 or 70 mm wide in axial direction and made of plastic. Total pressure probe and yaw probe are of 0.7 mm in diameter, and the latter is of arrow head type with the total pressure tube at the center of it. Static pressure probe is of 1 mm in diameter. Symbols concerning cascade arrangement are illustrated in Fig. 3.

Scopes of experimental conditions are as follows:

$AR=1.042$
$a/C=1.042$
$\alpha=30°,\ 50°.$
$\alpha_i=5°.$
With and without exit side walls.



X	Y
0.025	0.041
0.050	0.059
0.100	0.079
0.200	0.095
0.300	0.100
0.400	0.099
0.500	0.095
0.600	0.087
0.700	0.074
0.800	0.056
0.900	0.035
leading edge radius	0.010
trailing edge radius	0.0077

FIG. 2

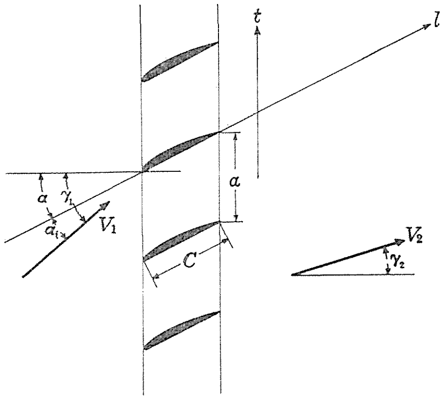
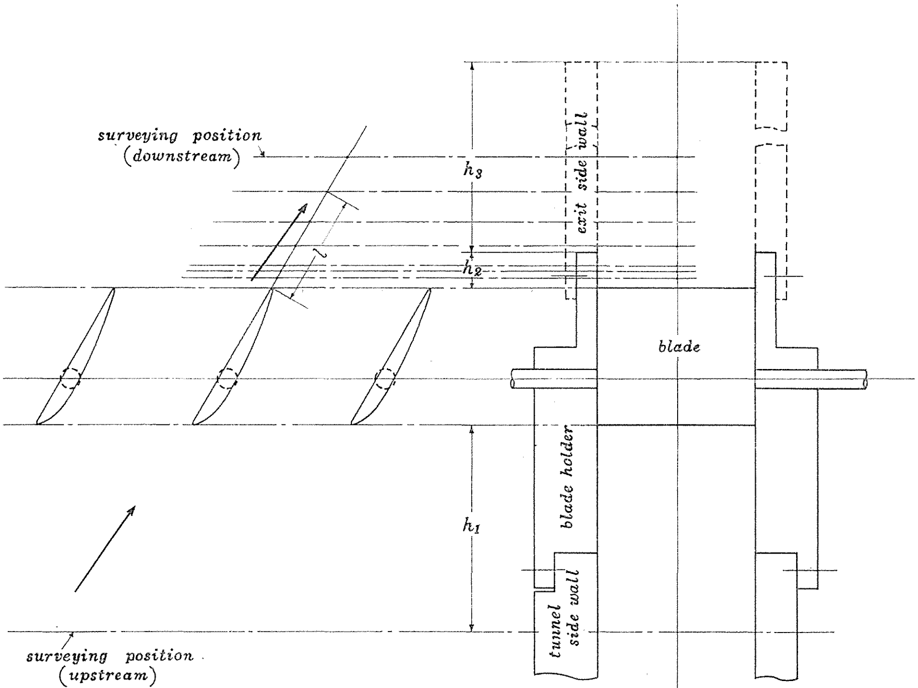


FIG. 3



α	h_1	h_2	h_3	l
30°	65	11	115	3.5, 6, 8, 15
50	68	18	70	24, 35, 48

FIG. 4

On these conditions, the traverse position was changed as follows:

$$l = 3.5,\ 6,\ 8,\ 15,\ 24,\ 35,\ 48\ \text{mm}.$$

5 ~ 7 blades were used depending upon conditions.

Spanwise velocity distribution were measured at the position of h_1 upstream of the cascade as illustrated in Fig. 4. One typical example of this is illustrated in Fig. 5. No qualitative difference is recognized in other cases. Dynamic pressure at the center section of cascade inlet was kept at 300 mmAq, and inlet velocity was therefore about 72 m/sec and Reynolds number was about 2.3×10^5 throughout the experiment.

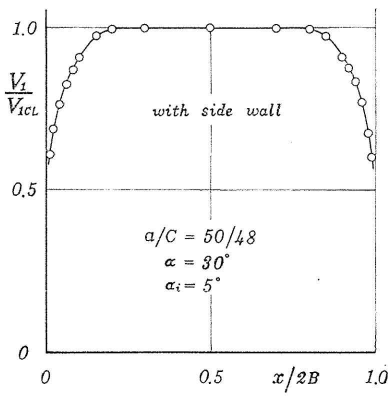


FIG. 5

Exit flow traverses, ranging over some 1.2 pitches, were done about the exit flow angle and the total pressure at midspan position.

Traversing positions in upstream and downstream of cascade, and dimensions of exit side walls are illustrated in Fig. 4.

5. Results and Considerations

Turning angle and loss coefficient calculated from measured values at each traverse position are illustrated in Fig. 6 and 7.

1) Turning angle:

Provided the traverse position is distant more than 15 mm from the blade

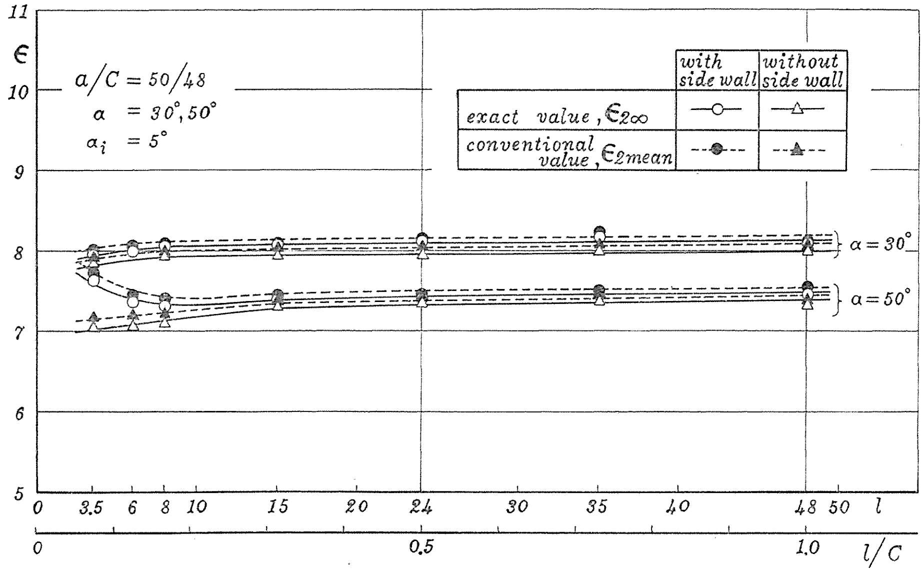


FIG. 6

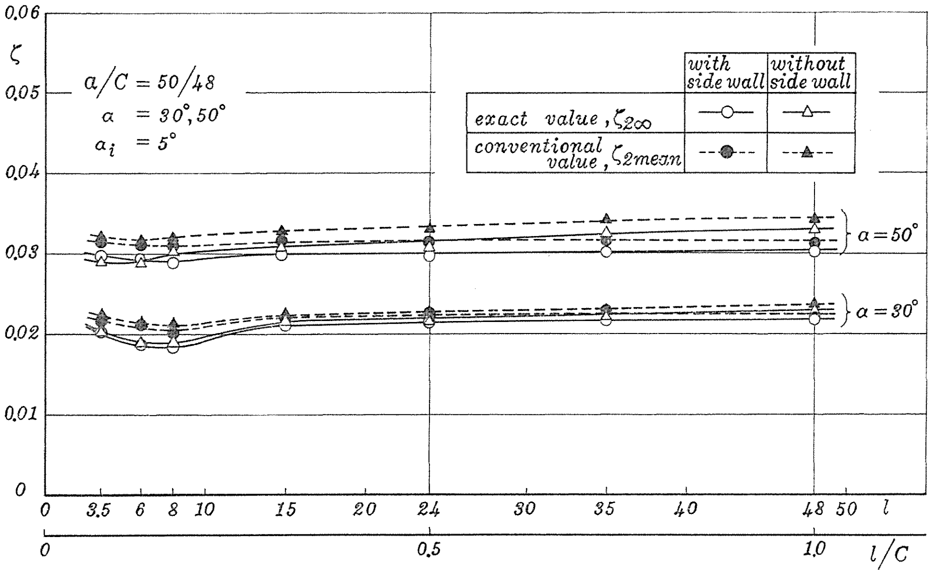


FIG. 7

trailing edge, "exact" and "conventional" values obtained approach fixed ones respectively, and the difference between these two is within 0.1 degree (within an experimental error) whether exit side walls are used or not. The case in which side walls are not used gives the turning angle smaller over all traverse positions than that those are used. This is probably due to the spanwise contraction of the exit flow in the case without side walls.

2) *Loss coefficient:*

Provided the traverse position is more than 15 mm distant from the blade trailing edge, "exact" and "conventional" values approach fixed ones respectively, and the difference between these two is 0.001–0.0015 (within experimental errors) in the case in which side walls are used. But when side walls are not used the loss coefficient increases gradually with the distance from blade trailing edge. This is probably due to the mixing of the open air with the flow released immediately into the former.

Although all "exact values" at each traverse position are expected from the theory to have the equal value, it indicates some divergence near the blade trailing edge. Perhaps this is due to the fact that the dimension of the yaw-probe is not sufficiently small to show correctly the change of flow condition in a narrow wake near the trailing edge.

6. Conclusions

1) When exit side walls are used, "conventional values" obtained at the mid-span traverse position distant more than 15 mm ($l/C=0.3$) from the blade trailing edge are close to the ones which are to be obtained at the position fully distant from cascade where the flow becomes uniform by mixing.

2) It seems that measured values obtained without exit side walls are influenced by the mixing of exit flow with the open air, but coincide with the ones obtained with walls within experimental errors over the range of $l/C=0.3\sim0.5$. Accordingly it is recommended to do the traverse in the range above mentioned to remove an effect of the open air when walls are not used. But it is not clear wheather the range of l/C above mentioned is suitable for the cascade conditions different from those in this report.

3) In this experiment no attempt was made to examine the spanwise distribution of various data, the effect of blade profile, and that of aspect ratio, etc. Further experiments are needed on these cases, and these are future subjects.

References

1) S. Otsuka: Axial Flow Compressors and Turbines, Text of the 54th Educational Meeting of the Japan Society of Mechanical Engineers, 1954 (in Japanese).

2) T. Kawasaki and T. Sato: Tests on Compressor Cascades of the NACA 6409 Section, Transactions of the Japan Society of Mechanical Engineers, Vol. 21, No. 108, 1955 (in Japanese with English abstract).

3) T. Kawasaki: On the Wake Traverse of a Blade in Cascade in Compressible Flow, Transactions of the Japan Society of Mechanical Engineers, Vol. 23, No. 125, 1957 (in Japanese with English abstract).

Appendix (A)

Derivation of Equations (1) and (2) in Chapter 3

Let us consider the streamlines m-m' and n-n' passing through a cascade center section as illustrated in Fig. 8. Let us make use of the following assumptions.

(1) The flow is two dimensional and steady, and the fluid is incompressible.

(2) At position 1 (before cascade) the flow is uniform.

At position 2 (behind cascade) the flow direction and the static pressure are uniform but the air velocity changes as illustrated in Fig. 8.

At position 2 ∞ the wake becomes uniform again.

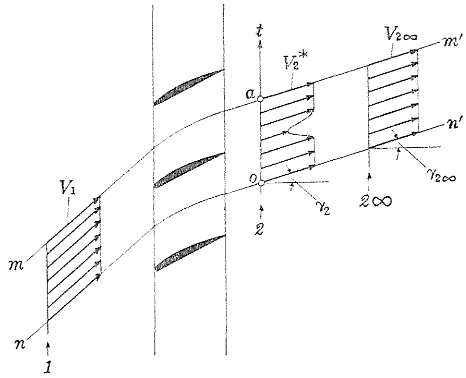


FIG. 8

Bernoulli's equation is

$$\left.\begin{aligned} p_{t1} &= p_1 + \frac{1}{2}\rho V_1^2 \\ &= p_2 + \frac{1}{2}\rho V_2^{*2} = p_2 + \frac{1}{2}\rho V_2^2 + \Delta h_2 \\ &= p_{2\infty} + \frac{1}{2}\rho V_{2\infty}^2 + \Delta h_{2\infty} \end{aligned}\right\} \tag{A-1}$$

Let us apply the continuity and momentum equations to the stream tube between position 2 and 2 ∞. The continuity equation is

$$\rho\int_0^a V_2 \cos\gamma_2 dt = \rho a V_{2\infty} \cos\gamma_{2\infty} \tag{A-2}$$

The momentum equation in the cascade direction is

$$\rho\int_0^a V_2 \cos\gamma_2 \cdot V_2 \sin\gamma_2 dt = \rho a V_{2\infty} \cos\gamma_{2\infty} \cdot V_{2\infty} \sin\gamma_{2\infty} \tag{A-3}$$

and in the axial direction is

$$(p_2 - p_{2\infty})a = -\rho\int_0^a (V_2 \cos\gamma_2)^2 dt + \rho a (V_{2\infty} \cos\gamma_{2\infty})^2 \tag{A-4}$$

We put γ_2=const. from assumptions, and introduce the displacemt thickness,

$$D = \int_0^a \left(1 - \frac{V_2}{V_2^*}\right) dt,$$

and the momentum thickness,

$$M=\int_0^a \frac{V_2}{V_2^*}\left(1-\frac{V_2}{V_2^*}\right)dt,$$

of the wake. Then from equation (A-2), we have

$$\frac{V_{2\infty}}{V_2^*}=\frac{\cos\gamma_2}{\cos\gamma_{2\infty}}\left(1-\frac{D}{a}\right) \tag{A-5}$$

From (A-3),

$$\left(\frac{V_{2\infty}}{V_2^*}\right)^2=\left(1-\frac{M}{a}-\frac{D}{a}\right)\frac{\cos\gamma_2\sin\gamma_2}{\cos\gamma_{2\infty}\sin\gamma_{2\infty}} \tag{A-6}$$

From (A-4),

$$\frac{p_{2\infty}-p_2}{\frac{1}{2}\rho V_2^{*2}}=2\cos^2\gamma_2\left[\left(1-\frac{D}{a}-\frac{M}{a}\right)-\left(1-\frac{D}{a}\right)^2\right] \tag{A-7}$$

Substituting (A-7) into (A-1), we get

$$\frac{\varDelta h_{2\infty}}{\frac{1}{2}\rho V_2^{*2}}=-2\cos^2\gamma_2\left[\left(1-\frac{D}{a}-\frac{M}{a}\right)-\left(1-\frac{D}{a}\right)^2\right]+1-\left(\frac{V_{2\infty}}{V_2^*}\right)^2 \tag{A-8}$$

Writing $\varDelta\gamma_2=\gamma_2-\gamma_{2\infty}$, we can consider $\varDelta\gamma_2$ to be small. Then we get

$$\begin{aligned}\cot\gamma_{2\infty}&=\cot(\gamma_2-\varDelta\gamma_2)\\&=\varDelta\gamma_2+\varDelta\gamma_2\cdot\cot^2\gamma_2+\cot\gamma_2\end{aligned}$$

and

$$\frac{\cot\gamma_{2\infty}}{\cot\gamma_2}=1+\left(\cot\gamma_2+\frac{1}{\cot\gamma_2}\right)\varDelta\gamma_2 \tag{A-9}$$

From equations (A-5) and (A-6)

$$\frac{\tan\gamma_2}{\tan\gamma_{2\infty}}=\frac{\left(1-\frac{D}{a}\right)^2}{\left(1-\frac{D}{a}-\frac{M}{a}\right)} \tag{A-10}$$

From equations (A-9) and (A-10)

$$\varDelta\gamma_2=\left(\frac{g^2}{f}-1\right)\sin\gamma_2\cos\gamma_2 \tag{A-11}$$

where $f=1-\frac{D}{a}-\frac{M}{a}$, $g=1-\frac{D}{a}$

Therefore

$$\varepsilon_{2\infty} = \varepsilon + \Delta\gamma_2 = \varepsilon + \left(\frac{g^2}{f} - 1\right)\sin\gamma_2\cos\gamma_2 \tag{A-12}$$

When D/a and M/a are small, we have

$$\Delta\gamma_2 = -\left(\frac{D}{a} - \frac{M}{a}\right)\sin\gamma_2 \operatorname{con}\gamma_2 \tag{A-11'}$$

and

$$\varepsilon_{2\infty} = \varepsilon - \left(\frac{D}{a} - \frac{M}{a}\right)\sin\gamma_2\cos\gamma_2 \tag{A-12'}$$

From equation (A-5) we have

$$\frac{V_{2\infty}}{V_2^*} = \frac{\cos\gamma_2}{\cos\gamma_2 + \sin\gamma_2 \cdot \Delta\gamma_2}\left(1 - \frac{D}{a}\right) \tag{A-13}$$

Writing as

$$\frac{\Delta h_{2\infty}}{\frac{1}{2}\rho V_{2\infty}^2} = \frac{\Delta h_{2\infty}}{\frac{1}{2}\rho V_2^{*2}}\left(\frac{V_2^*}{V_{2\infty}}\right)^2$$

and substituting equations (A-8), (A-11), and (A-13) into this equation, and after readjustment we have

$$\frac{\Delta h_{2\infty}}{\frac{1}{2}\rho V_{2\infty}^2} = \frac{g^2 - f}{(fg)^2}[-2(g^2 - f)^2\sin^6\gamma_2 + (g^2 - f)(2g^2 - 6f + 1)\sin^4\gamma_2$$
$$+ 2f(2g^2 - 3f + 1)\sin^2\gamma_2] + \frac{g^2 - 2f + 1}{g^2} \tag{A-14}$$

When D/a and M/a are small we have

$$\frac{\Delta h_{2\infty}}{\frac{1}{2}\rho V_{2\infty}^2} = 2\frac{M}{a} \tag{A-14'}$$