A STUDY ON THE PROGRESSIVE SIGNAL SYSTEM IN THE URBAN AREA

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(Received May 30, 1968)

Introduction

In the urban area, traffic conditions of the street are exceedingly congested and traffic delays chronic. To save these conditions in a large city the grade separation system which is used mainly for the urban expressway has been recently adopted, but this system is not enough to cover all the urban area. Therefore, to keep the smooth traffic flow, the signal control by the at-grade intersection is still helpful.

Up to now, *fixed-time signal system* and *vehicle actuated signal system* have been employed for the signal control system of the at-grade intersection. As modern urban traffic congestion seems to widen from point to line, or from line to area, it is impossible to control sufficiently these congestion with these two systems. To response this demand, traffic engineers have found the coordinated control system.

It is important to see the traffic conditions at system and network for all the moment and control the signal suited for its conditions. For this purpose in Japan the coordinated control system has been recently taken notice. The aim of the coordinated system decreases the interference between signals by setting the appropriate offset and recovers the each intersection's ability to these maximum capacity. Then the method of the coordinated system is to survey the traffic volume at the point which is seemed to be representing the system area and decide the optimum signal setting.

There are some mesures of optimulity, but in this paper the through band width is taken for the mesure of the optimulity. Therefore, to seek the maximal band width is our purpose. In Chap. (1), the equal band width whose assumption is that the traffic volumes between both directions are always same is described. In Chap. (2), the unequal band width is described, where the traffic volume between each direction is supposed to be different.

As for the application of this theory to the actual problem, the optimum offset is computed with respect to "SEIBU-KANJOSEN" or "Western ring road" in Nagoya City. For this computation the digital computer (HITAC 5020) is used.

Chapter 1. The Equal Band Width System for the Progressiveness

1–1. Relation between progressive system and the through band width

The traffic flows through progressive traffic signals is generally showed by a time-distance diagram. It can been seen that for a one-way street it is relatively easy to calculate the optimum offsets with the assumption that the cycle time is the same for all the intersections and the progressive speed is constant through the system.

But for a two-way traffic flows the decision of the optimum offsets is no longer easy and in order to make a decision it is neccesary to optimise some values. Therefore, the problem is what is taken for the measure of the most effective system. Average time for the traffic to pass through the system or average delay for all intersections or the numbers of stops made by through traffic or the through band are taken as the measure. Up to now, the usual practice using a time-distance diagram has been to maximise the band width. But this is not satisfactory because the traffic flows through width of the band is not of uniform intensity.

So there is no allowance that the delay allways becomes minimum when the through band width is maximized. The decision of the effective through band width is important problem after this. Leaving the relation between the fluctuation of band width and minimum delay as unknown, the through band width is used for the measure of the optimulity.

In this paper the author decided the offsets that maximized the through band width, but it is a matter of cource to study the new measure in stead of the through band width.

The more important notation used in Fig. 1 are as follows:

S_i: signal with the subscript increasing in the outbound direction;
C: cycle length of the signals (seconds);
r_i: red time of S_i on the street under study (cycles);
$b(\bar{b})$: outbound (inbound) band width (cycles);
$t_{ij}(\bar{t}_{ij})$: travel time from S_i to S_j in the outbound (inbound) direction (cycles);
θ_{ij}: offset of S_i and S_j measured as the time from the center of a red of S_i to the next center of red of S_j (cycles);
f: the trajectory forming the front edge of the band;
r: the trajectory forming the rear edge of the band;

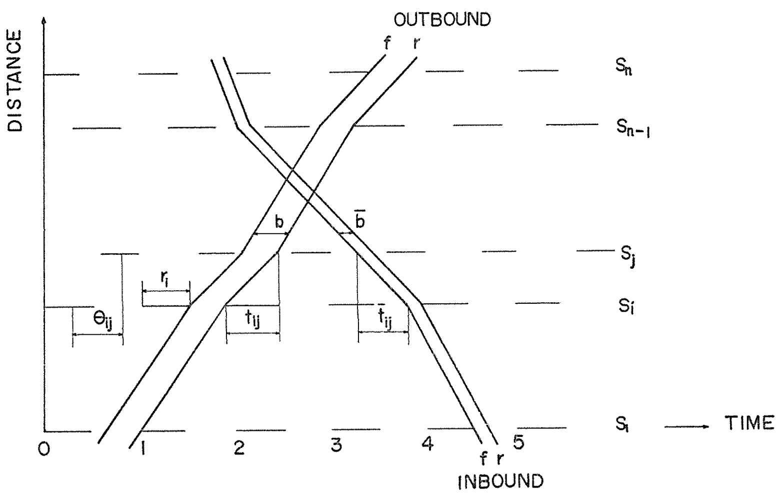


FIG. 1. Time-distance diagram showing outbound and inbound through bands.

Note that any time quantity can be expressed in cycles by dividing by C.

1–2. Half-integer synchronization

Certain signal forms the ultimate limitation of band width and will be called critical signals. A sinals S_i is said to be critical signal if one side of S_i's red touches the green band in the other direction. All critical signals can be divided into two groups; Group 1 consists of signals whose reds touch the front of the outbound and the rear of the inbound. Group 2 consists of signals whose reds touch the front of the inbound and the rear of the outbound.

In Fig. 2, suppose two signals S_i and S_j are in the same group, say group 1. From Fig. 2 a,

$$1/2\ r_i + t_{ij} = 1/2\ r_j + \theta_i + \text{INTEGER}$$

From Fig. 2 b,

$$1/2\ r_i - \bar{t}_{ij} = 1/2\ r_j - \theta_{ij} + \text{INTEGER}$$

Consequently,

$$\theta_{ij} = 1/2(t_{ij} + \bar{t}_{ij}) + 1/2\ \text{INTEGER} \tag{1}$$

By convention,

$$0 \leqq \theta_{ij} \leqq 1.$$

Therefore it may be seen that Eq. (1) has two solutions.

A more explicit expression for the two possible values of θ_{ij} can be developed. Let

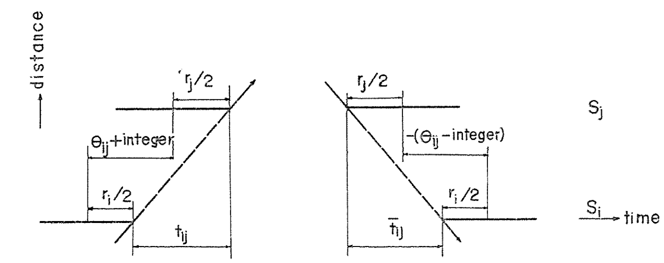


FIG. 2. Time-distance diagram for group 1.

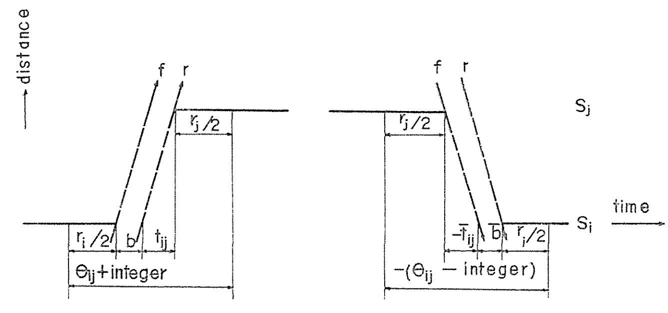


FIG. 3. Time-distance diagram for group 2.

$$\delta_{ij} = 0 \text{ or } 1/2.$$

Now Eq. (1) becomes

$$\theta_{ij} = \text{man}\,[1/2(t_{ij} + \bar{t}_{ij}) + \delta_{ij}] \tag{2}$$

The phasing represented by Eq. (2) will be called *half-integer synchronization.*

Next, in Fig. 3 S_i and S_j are from different group. Say group 2. From Fig. 3 a,

$$1/2\, r_i + b + t_{ij} + 1/2\, r_j = \theta_{ij} + \text{INTEGER} \tag{3}$$

From Fig. 3 b,

$$1/2\, r_i + \bar{b} + \bar{t}_{ij} + 1/2\, r_j = -\theta_{ij} + \text{INTEGER}$$

But $b = \bar{b}$ for the equal band width, therefore

$$\theta_{ij} = 1/2(t_{ij} + \bar{t}_{ij}) + 1/2 \text{ INTEGER} \tag{4}$$

This is the same as Eq. (1) and implies that S_i and S_j have the *half-integer synchronization.*

From above consideration, about all critical signals if $b = \bar{b}$ is supposed, the offset may be decided by the *half-integer synchronization.*

1–3. Synchronization for maximal equal band width

The result of the previous section shows that a synchronization for maximal equal band width can be found by examing the *half-integer synchronization.* Moreover it suffices to examine only the outbound direction.

Let

- b greatest outbound band width under *half-integer synchronization* if S_i's red touches the front of the outbound band.
- B the value of one of the maximal equal band width.

If S_i's red touches the front of the outbound band, the situation is in Fig. 4. Take as an origin for measurements the right side of S_i's red. The trajectory that touches the right side of S_i's red passes S_i at a time that will be denoted u_{ij}.

From Fig. 4 this is seen to be

$$\text{man}\,[\theta_{ij} + r_j/2 - r_i/2 - t_{ij}]$$

When this expression is zero, $u_{ij} = 1$ is accomplished.

$$u_{ij} = 1 - [\text{man} - \theta_{ij} - r_j/2 + r_i/2 + t_{ij}]$$

From Eq. (2) this is denoted

$$u_{ij}(\delta_{ij}) = 1 - \text{man}\,[1/2(r_i - r_j) + 1/2(t_{ij} - \bar{t}_{ij}) - \delta_{ij}]$$

The trajectory that touches the left side of S_j's red passes S_i at $u_{ij} - r_j$. So the best δ_{ij} is identified by

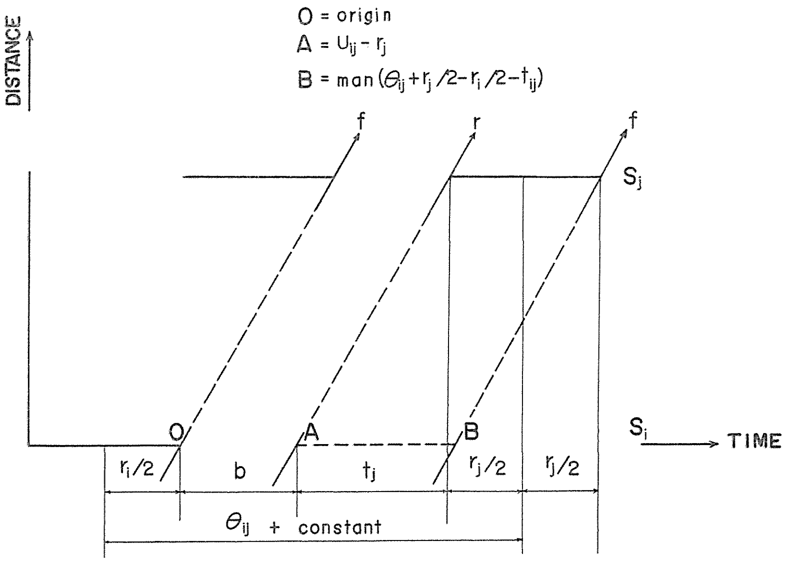


FIG. 4. Time-distance diagram.

$$\max[u_{ij}(0) - r_j,\ u_{ij}(1/2) - r_i]$$

Therefore

$$b_i = \min_j \max_{\delta=0,1/2} [u_{ij}(\delta) - r_j]$$

And

$$B = \max_i b_i$$

So the maximal equal band width is

$$B = \max_i \min_j \max_{\delta=0,1/2} [u_{ij}(\delta) - r_j] \qquad (5)$$

1-4. Advance survey for the progression of the SEIBU-KANJOSEN

The SEIBU-KANJOSEN is thought not only the main route in Nagoya City, but also the important route which connects between the National Highway No. 1 and the National Highway No. 22 and leads to the MEISHIN expressway and TOMEI expressway. The road network around Nagoya City is shown in Fig. 5.

The section of the progression is 7.2 km long as shown in Fig. 6. This is the interval between TAIHEI-DORI and NISHISHOBOSHOMAE and there are the eighteen signal intersections. In those, NISHISHOBOSHOMAE intersection and KAMISARA intersection and TAIHEIDORI intersection and SHINYOSHI intersection are important.

The element which are necessary for the analysis of the progression are 1) cycle length 2) red-green split 3) pregressive speed. To obtain these elements the traffic advance survey was carried out at the four intersection above.

1) cycle length

The result of the traffic survey is as shown in Table 1. Traffic counter and

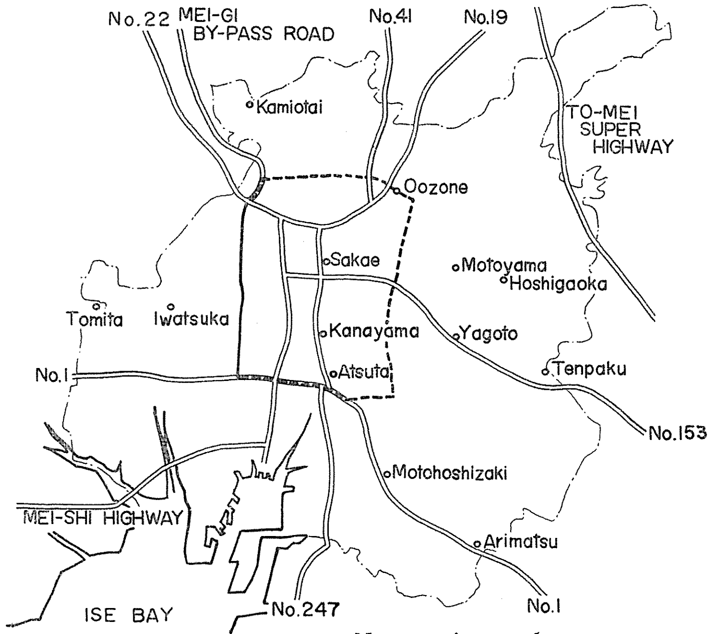
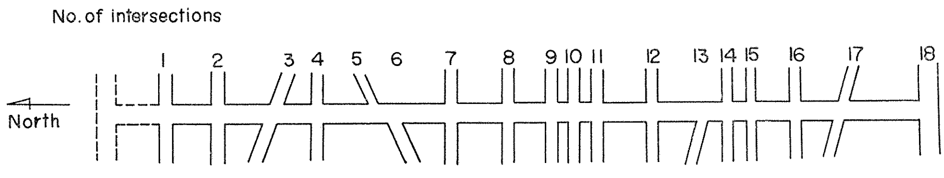


FIG. 5. The road network around Nagoya City.



1. Taiheidōri-4
2. Matsubakoen
3. Shinyoshi
4. Satsukidori
5. Kogane-minami
6. Kogane-kita
7. Koganedori
8. Taikodori
9. Noritake-3
10. Noritake-2
11. Noritake
12. Noritakehondori
13. Sakosansaro
14. Sako-kita
15. Sako-minami
16. Kamisara
17. Koseidori-4
18. Nishishoboshomae

FIG. 6. Schematic configuration of "Seibu kanjo sen" or "Westerm ring road".

Miki Behavior Recorder are used for this survey. The theoretical cycle length is calculated by the help of the next equation.

$$C=\frac{34200}{3600-2.1\,(q_1+q_2)} \tag{6}$$

Let

TABLE 1. Traffic Volumes on the Main four Intersections

Intersection	Approach	Total vehicles (V.P.H)	Left turns (%)	Right turns (%)	Heavy vehicles (%)	Adjusted vehicles (V.P.H)
NISHISHOBOSHOMAE	south	1604	0.64	0.36	0.11	1724
	west	1141	0.57	0.43	0.11	1273
	east	1202	0.68	0.32	0.24	1309
KAMISARA	south	1628	0.23	0.14	0.14	1700
	north	1654	0.31	0.08	0.10	1700
	west	1226	0.10	0.25	0.17	1328
	east	1650	0.14	0.35	0.10	1769
TAIKODORI	south	1271	0.08	0.14	0.15	1273
	north	1224	0.13	0.09	0.10	1281
	west	1084	0.15	0.12	0.05	1117
	east	1052	0.15	0.16	0.06	1088
SHINYOSHI	south	1160	—	0.39	0.15	1260
	north	960	—	0.01	0.17	987
	east	651	0.94	—	0.15	666

survey; 8, Dec. 1965

TABLE 2. Comparisom of Cycle Cength (sec.)

	TAIKODORI	KAMISARA
Theoretical cycle length	35	67
Actual cycle length	60	100

TABLE 3. Green-Red Split on the Study Route

No. of intersections	1	2	3	4	5	6	7	8	9
green-split (%)	70	60	68	60	70	70	60	44	70
red-split (%)	30	40	32	40	30	30	40	56	30
No. of intersections	10	11	12	13	14	15	16	17	18
green-split (%)	70	70	60	70	70	70	40	70	50
red-split (%)	30	30	40	30	30	30	60	30	50

C=cycle length (second)
q_1q_2=hourly traffic volume per a lane (The suffix show the number of the approach).

The result of the computation is as shown in Table 2 and the value listing is far short against the values used now. Considering both the theoretical values and the values used now, the cycle length which is used for the analysis of the progressive problem is decided between 60 seconds and 120 seconds.

2) green-red split

Only one split is used about each intersection regardless of the cycle length. It is not necessary right to adopt the values used now, but in this study the ratio of the traffic volume per lane is used for the main four intersections and the intersections which are not surveyed is calculated by the split used now. The result of the computation is shown in Table 3.

3) progressive speed

The progressive speed was obtained by the floating method. The result is as shown in Table 4. From this result it is clear that the velocity is slow at the section where the distance between adjacent intersection is short. Therefore, the average speed obtained is a little smaller than the actual value. The progressive speed which is used for the analysis of this problem is dicided between 30 km/hr and 60 km/hr.

TABLE 4. Result of Floating Method

No. of intersections (Direction ↑)	Distance (m)	t_w (sec)	t_w+t_a (sec)	(vehicles) X	Y	q (v.p.s)	$\bar{t}$ (sec)	V (m/sec)	v (km/sec)
1–2	705	61.8	118.0	60.7	−1.0	0.504	63.8	11.1	40.0
2–3	510	51.0	101.0	74.3	−1.7	0.719	53.4	9.6	35.0
3–4	375	39.2	81.0	45.3	0	0.559	39.2	9.6	35.0
4–5	150	15.5	33.7	27.3	−1.7	0.760	17.7	8.5	31.0
5–6	170	33.2	58.0	36.7	−2.0	0.594	36.6	4.6	17.0
6–7	490	51.7	97.7	48.5	−0.8	0.488	53.3	9.2	33.0
7–8	445	42.0	84.6	39.7	−0.3	0.466	42.6	10.4	37.0
8–9	155	19.3	40.3	26.8	−0.3	0.658	19.8	7.8	28.0
9–10	210	27.8	52.0	24.7	−0.7	0.462	29.3	7.2	26.0
10–11	285	34.0	68.6	74.3	−2.0	1.054	35.9	7.9	28.0
11–12	445	48.3	103.1	54.3	1.0	0.536	46.4	9.6	35.0
12–13	710	72.3	123.9	51.5	0.5	0.420	71.1	10.0	36.0
13–14	305	36.2	64.2	36.7	2.5	0.611	32.1	9.5	34.0
14–15	695	63.2	124.2	43.0	1.5	0.358	59.0	11.8	43.0
15–16	400	43.3	83.1	38.0	1.3	0.473	40.6	9.9	36.0
16–17	585	56.0	118.4	52.5	1.7	0.458	51.8	11.3	41.0
17–18	590	49.7	113.7	60.3	−1.3	0.518	52.2	11.3	41.0
Total	7225						744.8		

Average over-all speed = 34.9 km/sec.

TABLE 4. (*Continued*)

No. of intersections (Direction ↓)	Distance (m)	t_w (sec)	t_w+t_a (sec)	(vehicles) X	Y	q (v.p.s)	$\bar{t}$ (sec)	V (m/sec)	v (km/sec)
1–2	705	56.6	118.4	55.4	−0.8	0.465	58.3	12.1	44.0
2–3	510	50.0	101.0	57.4	−1.0	0.558	51.8	9.8	35.0
3–4	375	41.8	81.0	52.6	0	0.649	41.8	9.0	32.0
4–5	150	18.2	33.7	18.8	0.2	0.563	17.8	8.4	30.0
5–6	170	25.2	58.4	33.6	−1.6	0.548	28.1	6.0	21.0
6–7	490	46.0	97.7	38.0	−2.6	0.362	53.2	9.2	33.0
7–8	445	42.6	84.6	35.8	−1.4	0.407	46.0	9.7	35.0
8–9	155	21.0	40.3	21.2	1.2	0.556	18.9	8.2	30.0
9–10	210	24.2	52.0	18.2	−1.6	0.319	29.2	7.2	26.0
10–11	285	34.6	68.6	31.4	−2.0	0.429	39.3	7.3	26.0
11–12	445	54.8	103.1	83.2	−2.0	0.788	57.3	7.8	28.0
12–13	710	51.6	123.9	43.8	−2.6	0.333	59.4	11.9	43.0
13–14	305	28.0	64.2	35.0	−1.8	0.517	31.5	9.7	35.0
14–15	695	61.0	124.2	37.0	−2.4	0.279	69.6	10.0	36.0
15–16	400	39.8	83.1	26.2	−1.4	0.298	44.5	9.0	32.0
16–17	585	62.4	118.4	75.2	−2.4	0.615	66.3	8.8	32.0
17–18	590	64.0	113.7	65.3	−1.3	0.562	66.3	8.8	32.0
Total	7225						779.3		

Average over-all speed = 33.4 km/sec.

1-5. The use of the digital computer

The vehicle-actuated progressive signal appoints several cycle length and offset according to the fluctuation of the traffic volume and velocity. For this purpose it is decided that the cycle length is between 60 seconds and 120 seconds and the progressive speed is between 30 km/hr and 60 km/hr. Moreover the split and the distance between adjacent intersection is fixed.

In the computation of the optimum offset and through band width, the digital computer (HITAC 5020) was used. The result of the computation is in Fig. 7, which shows the relation between the through band and VT.

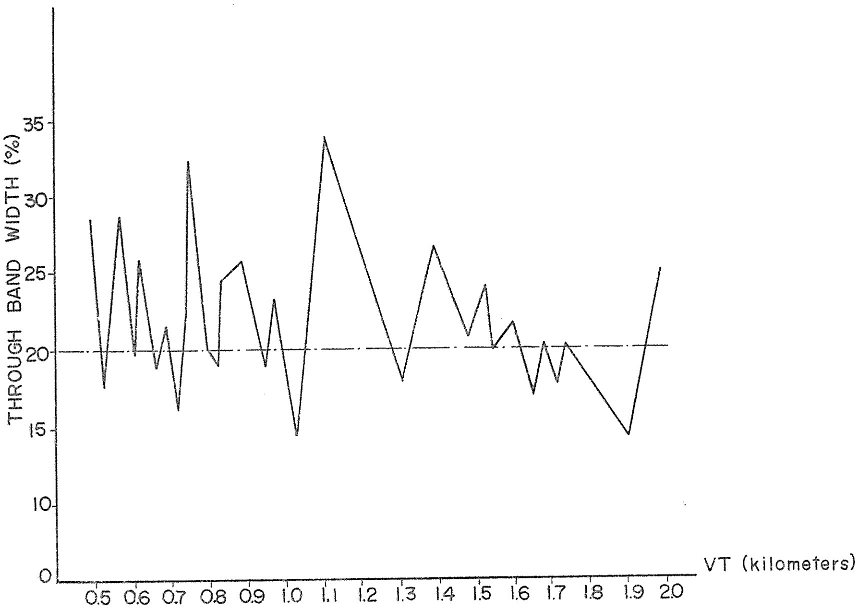


FIG. 7. Relationship between through band width and VT.

1-6. The choice of the optimum offset

From Fig. 7 it is clear that the through band width fluctuates with the fluctuation of VT. So the optimum offset must be chosen with caution. This is, if large through band width which is calculated decreases rapidly by a silght change of VT or a slight fluctuation of the progressive speed, it is not good idea to adopt this value as the optimum offset.

These argument becomes clear to explain with two examples; VT=1111 m and VT=758 m. In example, the effective range of the through band is taken more than 20%.

example 1. VT=1111 m (4000 km·sec/hr).

From Fig. 7 the effective range of VT is 1070 m<VT<1290 m or 3825 (km·sec/hr)<VT<4644 (km·sec/hr). If cycle length is 100 seconds, the effective range of the progressive speed is as follows,

$$38.3 \text{ (km/hr)} < VT < 46.4 \text{ (km/hr)},$$

Therefore the effective range of the progressive speed is between (40−2) km/hr and (40+3.5) km/hr.

example 2. VT=758 m (2728 km·sec/hr)

From Fig. 7 the effective range of VT is 740 m<VT<800 m or 2664 (km·sec·/hr)<VT<2880 (km·sec/hr). If cycle length is 100 seconds, the effective range of the progressive speed is as follows,

$$26.6\ (\text{km/hr}) < V < 28.8\ (\text{km/hr})$$

Therefore effective range of the progressive speed is between (27.3−0.7) km/hr and (27.3+1.5) km/hr.

Comparison example 1 and 2 shows that in example 2 the offset cannot correspond with the fluctuation of the velocity at all, but in example 1 the offset can correspond to a certain extent.

Six kind of optimum offsets can be prepared for the vehicle actuated progressive signal in SEIBU-KANJOSEN. As for the condition of the optimum offset, the progressive speed is taken between 40 km/hr and 50 km/hr and the cycle length is taken between 90 seconds and 100 seconds. So the optimum offsets are chosen as Table 5.

TABLE 5. Optimum Offset

VT (m)	VT (km·sec/hr)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
889	3200	0.0	0.5	0.0	0.5	0.5	0.5	0.5	0.0	0.5	0.5	0.5	0.0	0.5	0.0	0.0	0.5	0.0	0.0
1111	4000	0.5	0.0	0.5	0.0	0.5	0.0	0.5	0.0	0.0	0.5	0.5	0.0	0.5	0.5	0.5	0.0	0.5	0.0
1400	5040	0.5	0.0	0.5	0.0	0.5	0.5	0.0	0.5	0.5	0.0	0.0	0.0	0.5	0.5	0.0	0.0	0.5	0.0

Therefore the offset time for the vehicle actuated progressive signal on SEIBUKANJOSEN is decided as follows.

equal offset	three kinds (Table 5)
preferred offset	two kinds
simultaneous offset	one kind

Chapter 2. The Unequal Band Width System for the Progressiveness

2-1. Introduction

The fundamental assumption used in Chap. 1, *EQUAL BAND WIDTH*, is that the traffic volumes between both direction are always same or can be considered as the same. Therefore the maximal through band width obtained can be adopted on the case of having same traffic volumes at the both directions. But in the actual traffic condition this assumption is not always applicable. For traffic volumes are sometimes different between the inbound and outbound.

PREFERED OFFSET SYSTEM has been one of the analytical method for these problem. But this method ignors the direction which has small traffic volumes, so the total time through the progressive section does not always become minimum. Therefore except the traffic volumes between both direction differ extremely, this method dose not rational or optimal. To solve these question, in this chapter the unequal offset system is described, whose procedure is to shift band width calculated in Chap. 1 from one direction to the other.

2-2. Platoon length

Increase of traffic volumes tend to increase traffic congestion. After taking the progressive system, individual vehicle does not run at random but forms the regular platoon. In the case of same traffic volume between both direction, the traffic congestion may be small in the condition of forming platoon than in the random condition. As progressiveness aims at relieving the traffic condition, for the analysis of the unequal offset it may be better to use platoon length of both direction instead of traffic volume.

The through band width determines the maximal size of platoon for which stops can be avoided and increase in band width usually tend to decrease both stops and delay. But a platoon may arise in the case which turning traffic is entering the main street at the intersection. So cars in this arriving platoon must stop or slow down to wait being releasing. And these differnt platoon speeds decrease the efficiency of the progressiveness.

2-3. Shifting procedures

If the platoon length in outbound direction is larger than the platoon length in inbound direction, it may be possible to shift band width from outbound direction to inbound direction. So it is natural to increase outbound band width to $b \geqq B$, or, if B (maximal equal band width) is negative, then to $b \geqq 0$. But it is possible to increase b to the smallest value of green time; then $b=g$. From this argument it is concluded that b can be increased from max $(0, B)$ to any value less than or equal to g.

Therefore outbound band width b is max $(0, B) \leqq b \leqq g$. As b is increased, $\bar{b}$ is decreased. So,

$$\bar{b} = \max(2B - b,\ 0)$$

Next consider first the shift to obtain b. From Fig. 8 the front of the outbound band is pushed left to the position $u=B-b$ (See the dashed line in the outbound portionof Fig. 8). Therefore the appropriate phase shift for S_j is to the left by an amount:

$$\alpha_j = \max[(u_{cj} - 1) - (B - b),\ 0]$$

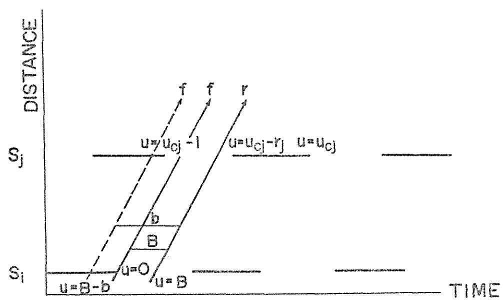


FIG. 8. Widening the outbound band from B to b.

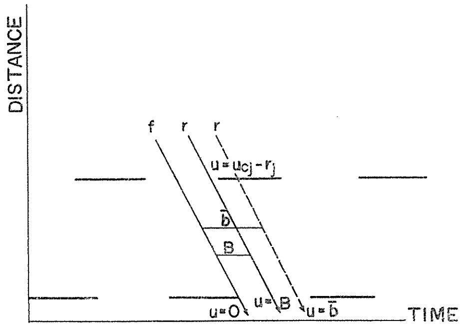


FIG. 9. Widening the inbound band from B to $\bar{b}$.

Then $b<\bar{b}$ is accomplished from Fig. 9, the appropriate phase for S_j is to the right by an amount:

$$\alpha_j = \max[\bar{b} - (u_{cj} - r_j),\ 0]$$

2-4. Partitioning of the through band width

The notation used here as follwws;

$L(\bar{L})$ the platoon length in the outbound (inbound) direction (seconds).

$P(\bar{P}) = L/T(\bar{L}/T)$ the platoon rate (cycle).

Whenver $P=\bar{P}$, maximal equal band width are proposed. Otherwise we proceed as follows: If $P+\bar{P} \leqq 2B$, as the sum of the band width of both sirection, there may be enough band width to accomodate both platoon. Computation may be carried out to make the width propotional to platoon rate:

a) if $P>\bar{P}$,

$$b = \min\ (g,\ 2BP/(P+\bar{P}))$$
$$\bar{b} = \max\ (2B - b,\ 0)$$

b) if $P<\bar{P}$,

$$b = \max\ (2B - \bar{b},\ 0)$$
$$\bar{b} = \min\ (g,\ 2B\bar{P}/(P+\bar{P}))$$

If $P+\bar{P}>2B$, the sum of the platoon rate of both drection, there may not be enough band width to accomodate both platoon.

a) if $P>\bar{P}$,

$$b = \min\ (g,\ P)$$
$$\bar{b} = \max\ (2B - b,\ 0)$$

However if $P>2B$, then $\bar{b} = g$.

b) if $P<\bar{P}$,

$$b = \max\ (2B - \bar{b},\ 0)$$
$$\bar{b} = \min\ (g,\ \bar{P})$$

TABLE 6. Equal Offset and Unequal Offset

(Equal offset)

TA	VA	VT	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	BB
70	40	777	0	0.50	0.50	0	0.50	0	0	0.50	0	0	0.50	0	0.50	0	0	0.50	0	0	0.24

(Unequal offset)

LO	LI	PO	PI	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	BO	BI
60	50	0.86	0.71	0.88	0.50	0.37	0.84	0.50	0	0	0.50	0	0	0.50	0	0.50	0.92	0	0.45	0	0	0.40	0.08
50	40	0.71	0.57	0.88	0.50	0.37	0.84	0.50	0	0	0.50	0	0	0.50	0	0.50	0.92	0	0.45	0	0	0.40	0.08
40	30	0.57	0.43	0.88	0.50	0.37	0.84	0.50	0	0	0.50	0	0	0.50	0	0.50	0.92	0	0.45	0	0	0.40	0.08
30	20	0.43	0.29	0.88	0.50	0 37	0.84	0.50	0	0	0.50	0	0	0.50	0	0.50	0.92	0	0.45	0	0	0.40	0.08
20	10	0.29	0.14	0.94	0.50	0.44	0.92	0.50	0	0	0.50	0	0	0.50	0	0.50	0	0	0.50	0	0	0.32	0.16
10	20	0.14	0.29	0	0.50	0.50	0	0.43	0	0	0.43	0	0.92	0.50	0	0.50	0	0	0.47	0.97	0.96	0.16	0.32
20	30	0.29	0.43	0	0.50	0.50	0	0.36	0.96	0	0.35	0	0.84	0.50	0	0.44	0	0	0.40	0.89	0.89	0.08	0.32
30	40	0.43	0.57	0	0.50	0.50	0	0.36	0.96	0	0.35	0	0.84	0.50	0	0.44	0	0	0.40	0.89	0.89	0.08	0.32
40	50	0.57	0.71	0	0.50	0.50	0	0.36	0.96	0	0.35	0	0.84	0.50	0	0.44	0	0	0.40	0.89	0.89	0.08	0.32
50	60	0.71	0.86	0	0.50	0.50	0	0.36	0.96	0	0.35	0	0.84	0.50	0	0.44	0	0	0.40	0.89	0.89	0.08	0.32

TA: cycle length (second), VA: velocity (km/sec), LO(LI): outbound (inbound) platoon length (second), PO(PI): outbound (inbound) platoon rate (cycle), BB: maximum equal band width (%), BO(BI): outbound (inbound) band width (%).

TABLE 7. Unequal Offset Respect to Three Optimum Offsets

VT (%)	p (%)	$\bar{p}$ (%)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
889	0.75	0.63	0.82	0.43	0	0.50	0.37	0.50	0.47	0.89	0.34	0.50	0.50	0	0.50	0.99	0	0.43	0	0.85
	0.38	0.25	0.85	0.50	0	0.50	0.40	0.50	0.49	0.91	0.36	0.50	0.50	0	0.50	0	0	0.46	0	0.87
	0.25	0.13	0.93	0.50	0	0.50	0.48	0.50	0.50	0.99	0.44	0.50	0.50	0	0.50	0	0	0.50	0	0.95
	0.13	0.25	0	0.50	0	0.50	0.50	0.50	0.50	0	0.50	0.50	0.48	0.93	0.43	0	0	0.49	0	0
	0.25	0.38	0	0.50	0.99	0.50	0.50	0.43	0.50	0	0.50	0.50	0.40	0.85	0.35	0	0.99	0.41	0.99	0
	0.63	0.75	0	0.50	0.97	0.48	0.50	0.40	0.50	0.97	0.50	0.50	0.37	0.82	0.32	0	0.96	0.39	0.97	0
1111	0.75	0.63	0.50	0	0.50	0	0.50	0	0.50	0.91	0	0.50	0.50	0.98	0.47	0.50	0.50	0.95	0.50	0
	0.38	0.25	0.50	0	0.50	0	0.50	0	0.50	0.94	0	0.50	0.50	0	0.49	0.50	0.50	0.97	0.50	0
	0.25	0.13	0.50	0	0.50	0	0.50	0	0.50	0.91	0	0.50	0.50	0.98	0.47	0.50	0.50	0.95	0.50	0
	0.13	0.25	0.50	0.95	0.46	0	0.48	0	0.50	0	0.91	0.50	0.50	0	0.50	0.50	0.46	0.97	0.50	0.92
	0.25	0.38	0.50	0.97	0.48	0	0.50	0	0.50	0	0.94	0.50	0.50	0	0.50	0.50	0.48	0.99	0.50	0.95
	0.63	0.75	0.50	0.95	0.46	0	0.48	0	0.50	0	0.91	0.50	0.50	0	0.50	0.50	0.46	0.97	0.50	0.92
1400	0.67	0.56	0.50	0	0.50	0.88	0.42	0.50	0	0.33	0.50	0.82	0.93	0	0.50	0.50	0.83	0.94	0.46	0.86
	0.33	0.22	0.50	0	0.50	0.94	0.49	0.50	0	0.40	0.50	0.88	0	0	0.50	0.50	0.89	0	0.50	0.93
	0.22	0.11	0.50	0	0.50	0.98	0.50	0.50	0	0.44	0.50	0.93	0	0	0.50	0.50	0.93	0	0.50	0.97
	0.11	0.22	0.43	0.96	0.50	0	0.50	0.50	0	0.50	0.50	0	0	0.93	0.50	0.50	0	0.98	0.50	0
	0.22	0.33	0.38	0.91	0.50	0	0.50	0.50	0.99	0.50	0.50	0	0	0.89	0.50	0.47	0	0.94	0.50	0
	0.56	0.67	0.32	0.85	0.47	0	0.50	0.48	0.92	0.50	0.45	0	0	0.82	0.50	0.40	0	0.88	0.50	0

However if $P>2B$, then $\bar{b}=g$.

Summarizing the above argument,

If $P>\bar{P}$, outbound band width (b) is ranged by $\max(0, B) \leqq b \leqq g$.

The phase shift for S_j is; $\alpha_j = \max(u_{ij} - 1 + b - B, 0)$

Then inbound band width ($\bar{b}$) is; $\bar{b} = \max(2B - b, 0)$

If $P<\bar{P}$, inbound band width ($\bar{b}$) is ranged by $\max(0, B) \leqq \bar{b} \leqq g$.

The phase shift for S_j is; $\alpha_j = \max(\bar{b} + r_j - u_{cj}, 0)$

Then outbound band width (b) is $b = \max(2B - \bar{b}, 0)$

At last, adjusted phase for S_j is

$$\theta'_{cj} = \text{man}(\theta_{cj} - \alpha_j)$$

2-5. The use of the digital computer

By using the method described, it is possible to calculate the shifting amount and obtain the offset time. But complication happens when calculation must be carried out for all the groups of cycle time, split time, and progressive speed. Therefore the digital computer is quite useful.

If input data include following variables; cycle length, the position of each signal, split times, velocity and platoon rate, it is possible to calculate the optimum offset.

Next consideration are deduced from the result of the computation. Table 6 shows the unequal offset for the case which is calculated in Chap. 1. When the sum of the platoon rate of both directions is more than the maximal through band width (24%), partioned through band width is in proportion to the platoon rate. But otherwise, partitioned through band width whose platoon rate is greater has an advantage over the other.

Table 7 shows the unequal offset with respect to three optimum offsets which are calculated in Chap. 1.

Conclusion

The following conclusions can be drawn from the result of this work.

(1) The maximal equal through band width is computed, then this band width is partitioned off two parts in proportion to the platoon rate of each direction.

(2) In searching for the optimum offset, it is convenient to use the *VT-B* diagram.

(3) In the case of the unequal offset, when the sum of the platoon rate for both direction is greater than two times of the maximal through band width; $P+\bar{P} \geqq 2B$, the direction whose traffic volume is greater has an advantage over the other direction.

In this study the progression for one route is described, but hereafter the progression for the network problem must be studied. The authors will make a study of this problem after this.

Acknowledgment

The authors are deelpy grateful to Dr. Hiroshi Takata for his advice and helpful criticism during the preparation of this report. The authors would also like to thank Mr. Kenzo Endo for his good assistance, and also wishes to acknowledge all the members in the Mōri's Laboratory for their useful discussions and suggestions throughout this study.

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