HEAT AND MASS TRANSFER FROM A ROTATING DISK IN AN OPEN ENVIRONMENT

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Introduction

Many reports of theoretical and experimental researches on heat and mass transfer from a rotating disk have been presented for the past 20 years.

C. Wagner[1] gave the famous results of heat transfer from the rotating disk of infinite radius theoretically using von Kármán's equation[2]. K. Millsaps[3], N. Riley[4], D. R. Olander[5], E. M. Sparrow[6], J. P. Hartnett[7] and C. L. Tien[8] reported results of their researches on heat transfer from a rotating disk of infinite radius.

L. A. Maroti *et al.*[9], S. S. Koong *et al.*[10], J. W. Mitchell[11], R. L. Young[12], E. C. Cobb *et al.*[13] and R. Izumi[14] reported their experimental results. In the case of a rotating disk in still air, heat transfer rate increases with increase of rotating speed of disk: n rpm.

The flow of air on the disk transfers from laminar state to turbulent, and its transition point was reported as follows

$$R_{er} = \frac{v_e \cdot r_0}{\nu} = 2 \times 10^5$$

where r_0 m; radius of rotating disk, ν m²/h; kinematic viscosity of air.

Average Nusselt number Num for the rotating disk in laminar range was given theoretically as

$$N_{um} = \frac{\alpha_m \cdot r_0}{\lambda} = a \cdot R_{er}^{0.5} \tag{1}$$

where α_m kcal/m²h°C; average heat transfer coefficient, λ kcal/mh°C; thermal conductivity of air,

a;	0.34	by Wagner
	0.329	by Ostlach
	0.35	by Millsaps

Cobb *et al.* gave their experimental results[13] in the form of

$$N_{um} \propto R_{er}^{0.5}$$

Young[12] gave

$$N_{um} = 1.36 \ R_{er}^{0.4} \tag{2}$$

and Izumi[14] presented

$$N_{um} = 1.67 \ R_{er}^{0.335} \tag{3}$$

for the range of $R_{er} = 1.5 \times 10^2 \sim 1.5 \times 10^4$.

The difference between (2) and (3) is about 2 per cent at $R_{er} = 3.8 \times 10^4$. In turbulent flow range

$$N_{um} = b \cdot R_{er}^{0.8} \tag{4}$$

was given theoretically[15]

where b; 0.012 by Davies
0.018 by Hartnett *et al.*

Prof. Kreith and others[16] reported the mass transfer rate from a disk in open air, mainly in turbulent region and in other report[17] they discussed the effects of gap between the confronted stationary disk and the rotating disk upon the mass transfer of their experiment with a rotating naphthalene disk.

The heat transfer accompanying with mass transfer is an important problem for the study in the chemical engineering. E. M. Sparrow[6] reported about a rotating disk with condensation. The authors present the experimental data on heat and mass transfer from a horizontally rotating disk in open air for laminar flow range.

Experimental apparatus and procedure

Figs. 1 and 2 show the schematic views of apparatus. Naphthalene ① is cast on a brass saucer ② and a disk of naphthalene was heated by electric heater of Ni-Cr wire ③.

A plate of bakelite of 10 mm thick ④ was placed between two saucers ② and ⑤, and in the lower saucer the dummy heater ⑥ was inserted and adjusted to keep the temperature of the lower saucer as same as the upper saucer.

4 thermocouples in the disk and 2 thermocouples on the saucer were led to the electronic recorder through slip rings which were set in the lower part of the shaft.

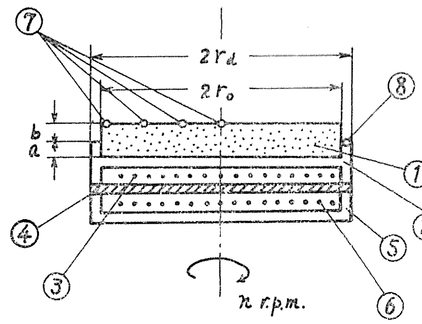

FIG. 1

Automatic temperature recorder was 6 points recorder with electronic tubes and had the accuracy of 0.3°C for 0∼5 mV range.

Mean surface temperature of the disk ϑ_{sm} is calculated as

$$\vartheta_{sm} = \frac{\int_A \vartheta dA}{\int_A dA}$$

where ϑ°C is the measured temperature on the surface of the disk, dA m^2 is the differential area of the surface.

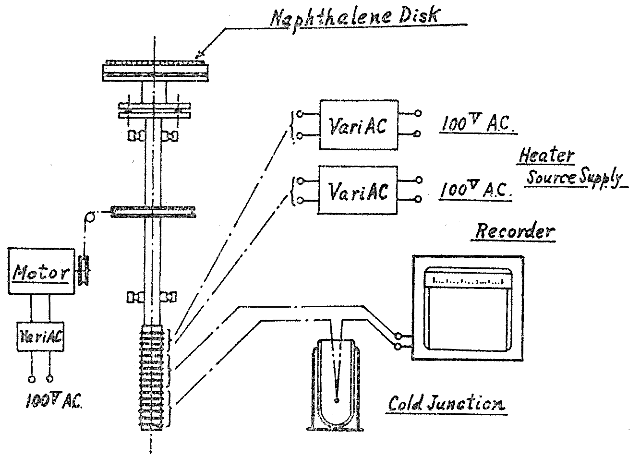

FIG. 2

Getting ϑ_{sm}, average heat transfer coefficient α_m kcal/m²h°C is given as follows

$$\alpha_m = \frac{0.86\ VI - q_s - q_r}{A(\vartheta_{sm} - \vartheta_m)} \tag{5}$$

where q_s kcal/h is the heat loss by convection which transfers from the surface of cylinder ⑧, q_r kcal/h is the heat loss by radiation from the whole surfaces and V volt and I amp. are the input voltage and current in the electric heater coil respectively.

ϑ_∞°C is the room temperature and A m² is the area of the surface of the disk. During test periods the temperature difference of two saucers was adjusted to keep in 1~2°C. The weight of sublimated naphthalene is calculated by measuring the decrement of thickness of naphthalene with dial gauge in accuracy of 1/250 mm and multiplying with specific weight of naphthalene.

The average mass transfer coefficient k_{cm} and the average Sherwood number Sh_m are expressed as followings

$$\left.\begin{aligned} k_{cm} &= \overline{m} \cdot R_v \cdot T_s / p_v \cdot A \qquad \text{m/h} \\ Sh_m &= \frac{k_{cm} \cdot r_0}{D_v} \end{aligned}\right\} \tag{6}$$

where $\overline{m}$ kg/h; the mass transfer rate of naphthalene,
R_v ; ideal gas constant for naphthalene vapor,
T°K ; the mean abusolute temperature of the disk surface in the testing period,
D_v m²/h; diffusivity of naphthalene vapor into air for mass transfer
P_v kg/m²; partial vapor pressure of naphthalene at the surface temperature ϑ_{sm}.

Every measurement was taken under the steady conditions of the temperature of disk in order to keep constant rate of mass transfer $\overline{m}$. The disk was driven by pulley. The rotating speed of the disk was in the range of 100 to 600 rpm.

The rotational Reynolds number was $Rer = v_\theta \cdot r_0 / \nu = 1.92 \times 10^2 \sim 4.2 \times 10^4$, thus flow around the disk might be considered to be in laminar range.

The properties of naphthalene are as follows

melting point	80.7°C
ideal gas constant	$R_v = 6.615$ kgm/kg°K
molecular weight	128.16 kg/kmol
latent heat for sublimation	$r = 133$ kcal/kg
specific weight of cast naphthalene	$\gamma = 1.12 \pm 0.001$ gr/cm³
	1.1008 (measured by anthors)

diffusivity of naphthalene vapor into air[18]

$$D_v = 0.0513\left(\frac{T_\infty}{273}\right)^2 \cdot \frac{760}{p_0} \quad \mathrm{cm^2/s} \tag{7}$$

partial pressure of vapor of naphthalene P_v in mmHg

$$\log p_v = 11.55 - \frac{3765}{T_v} \qquad (8)^{19)},$$

$$\log p_v = 11.7797 - \frac{3812.34}{T_v} - 0.02593 \log_{10} T_v \qquad (9)^{20)}$$

where T_∞ and T_v°K are the absolute temperature of air and vapor of naphthalene. The three disk $2r_0 = 35.4$, 90 and 180 mm were tested. The height "a" of the side wall of the upper saucer was 10 mm and b/a were 0, 0.3, 0.6, 1.0, 1.3 and 1.6. Authors studied about effects of b/a upon the mass transfer rate.

Experimental results and its consideration

Fig. 3 indicates an example of the relation of mass transfer rate $\bar{m}$ of naphthalene to time of testing period. The mass transfer rate was observed to be nearly constant after 20 min. since the test began. $\bar{m}$ depends upon the rotating speed of the disk n, and the temperature of the surface of the disk ϑ_s. In every experiment the deviation of experimental results were within 5.2% from the mean

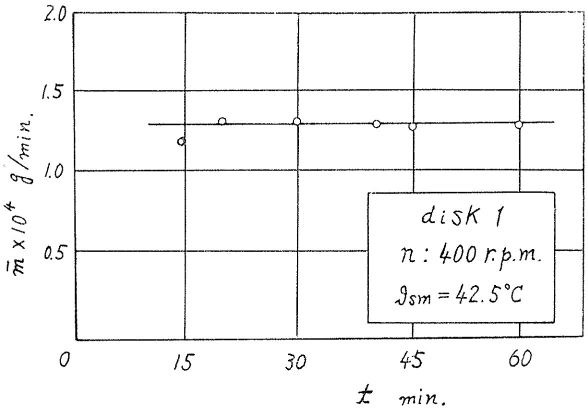

FIG. 3

value. The time of test period was taken as 1 hour, so that the experimental error should be kept minimum.

Fig. 4 illustrates the profile of the surface of naphthalene disk at the end of test. In case of $c=3$ mm, fairly larger amount of mass transfer rate was observed near the center and the outskirts of the disk, comparing with intermediate radii.

The reason of this phenomenon is considered as follows; near the outskirts of the disk, at $r=r_0$, the high peripheral speed and thermal influences of side wall of the saucer make the mass transfer rate increase. Near the center of disk, $r=0$, swirling flow of fresh air pours down to the center of disk surface from ambient air and makes the vapor pressure gradient so high that the mass transfer increases.

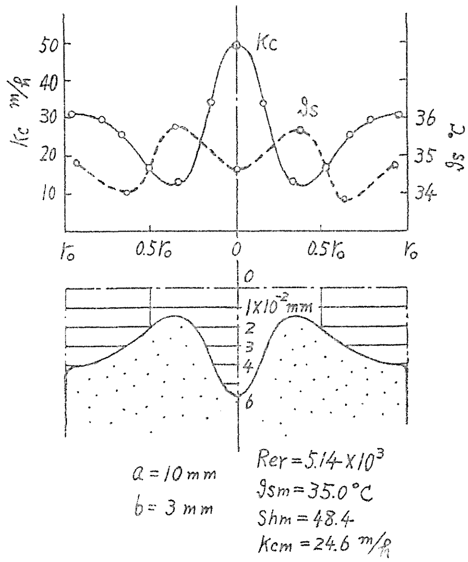

FIG. 4

A convex shape of the disk surface near $r=0.5\ r_0$ was caused by a stagnation of naphthalene vapor which hindered the diffusion of sublimated naphthalene.

The local mass transfer coefficient k_c at various radii of the disk which are shown in Fig. 4 was obtained as follows

$$\left.\begin{aligned} \overline{m} &= \gamma \cdot A \cdot d \qquad \text{kg/h} \\ k_c &= \frac{R_v \cdot T_s}{p_v \cdot A} \cdot \gamma \cdot A \cdot d = \frac{R_v \cdot T_s}{p_v} \gamma \cdot d \qquad \text{m/h} \\ k_{cm} &= \int_A k_c \cdot dA \Big/ \int_A dA \qquad \text{m/h} \end{aligned}\right\} \tag{10}$$

where γ kg/m³; specific weight of cast naphthalene
A m² ; area of the surface of a disk
d m/h ; the lost depth of sublimated naphthalene.

As shown in this figure, at $r=0$, maximum loss of naphthalene. $k_c=50$ m/h appeared and at $r=0.3\,r_0$, minimum coefficient of mass transfer $k_c=10$ m/h was measured. k_c increases with increase of r, and at $r=r_0$, $k_c=30$ m/h was observed.

Fig. 5 shows the result in the case of $b=0$, large amount of sublimation appeared near the center of disk and at the outskirts of disk. Along the direction of radius, uniform value of k_c was observed. Experiments were executed by varying b/a.

In Fig. 6, average Sherwood numbers were plotted for various b/a, somewhat large value of Sh_m was observed at small ratio of b/a, but its deviation was only 7~8%. It can be considered that b/a does not effect so much.

In Fig. 7, average mass transfer codfficients k_{cm} against temperature difference between temperature of surface of the disk and ambient air $\Delta\vartheta=\vartheta_{sm}-\vartheta_\infty$ were

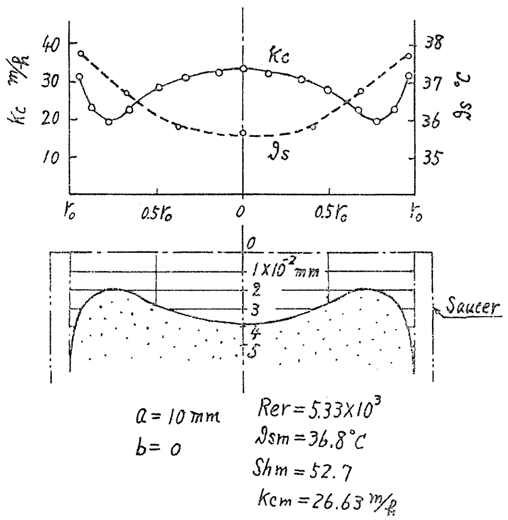

FIG. 5

indicated.

For the disk 1 of small radius rotating with speeds less than $n=200$ rpm, an axial flow of air is not induced, and the natural convection gives much effect. Sh_m gradually increases with increase of $\Delta\vartheta$, and beyond $\Delta\vartheta=20$°C, this tendency is weakened and at 200 rpm, $\Delta\vartheta=20$°C, saturated value of k_{cm} is 27 m/h. For the revolutions $n=400$ and 600 rpm the trends of saturation were obtained at lower temperature difference and $\Delta\vartheta=15$°C, k_{cm} is 35 and 40.5 m/h respectively.

For the disk 2 of large radius, as shown in Fig. 7 (b), the tendency of saturation occured at much lower temperature difference. For $\Delta\vartheta=$ 20°C and $n=200$, 400 and 600 rpm, k_{cm} saturates at 20.5, 25.5 and 32 m/h respectively. The relation of heat transfer coefficient against temperature difference has the same tendency.

All experiments were carried out keeping the temperature difference over 20°C. The relation of average Sherwood number Kreith *et al.* gave the following equation putting $P_r=S_c$ in 2.4, and they proclaimed that the calculated values by this equation coincided well with the experimental values in the range of $R_{er}>$ 3.6×10^4

$$N_{um}=Sh_m=0.67\,R_{er}^{0.5} \tag{11}$$

Their experimental data are held between the former and the following equation which Pohlhausen *et al.* have gotten for a rotating plate in laminar flow range

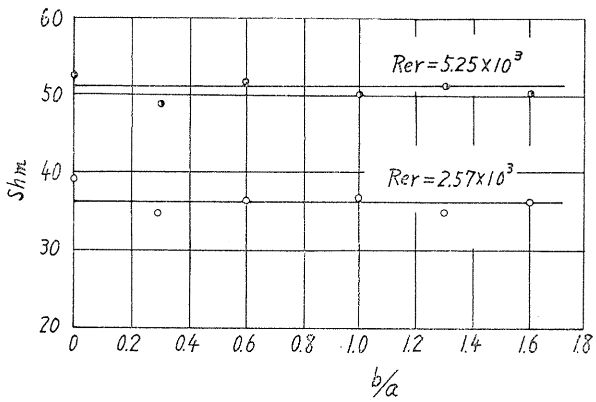

FIG. 6

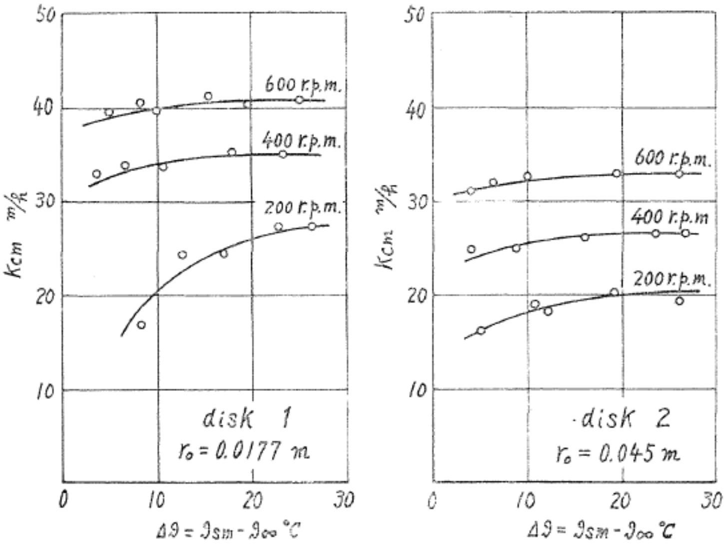

FIG. 7

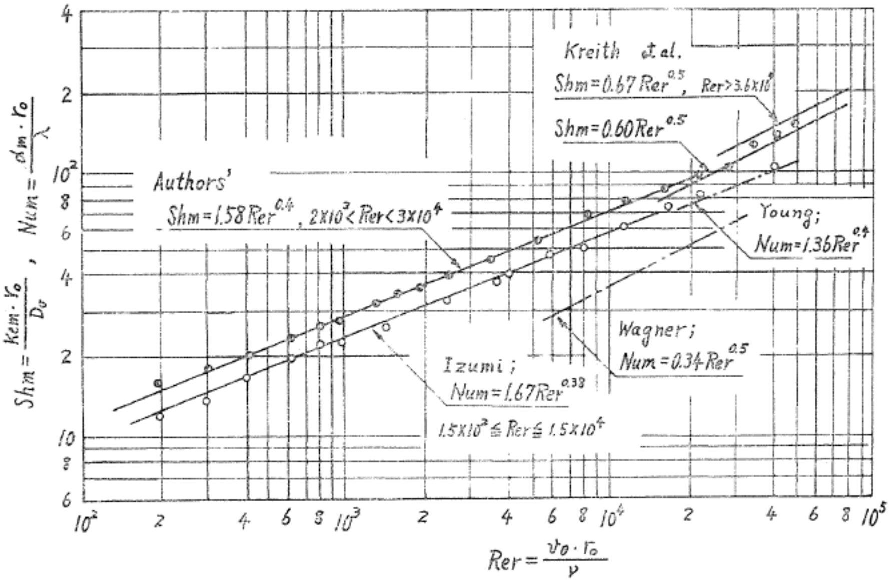

FIG. 8

$$Sh_m = 0.60\, R_{er}^{0.5} \tag{12}$$

As above stated in the theoretical analysis about a infinite diameter disk by Kreith and Cobb *et al.*, N_{um} and Sh_m were given in the form of proportion to $R_{er}^{0.5}$. Young and Izumi reported $N_{um} \propto R_{er}^{0.4}$ on the heat transfer of rotating disk of finite radius in still air.

Authors' experimental values of mass transfer were shown by half black dots in Fig. 8. In the range of $R_{er}>3.5\times10^4$, these points coincide with the Kreith's data. But in the range of $R_{er}<3.0\times10^4$ it seems to be difficult to express the experimental data in proportion to $R_{er}^{0.5}$. For the disk of finite diameter, surface flow is in laminar range, to which weak axial flow is added, so the mass transfer k_{cm}, as in Fig. 4 indicated, becomes large and Sh_m is larger than the value expressed by $Sh_m \propto R_{er}^{0.5}$.

From these experimental values, authors present following expression for $1.95\times10^2<R_{er}<3\times10^4$ with the maximum deviation of $\pm5\%$

$$Sh_m = 1.58\ R_{er}^{0.4} \tag{13}$$

The authors' experimental data of heat transfer were plotted with white dots and these points are included between two equations of Young and Izumi and N_{um} is confirmed to be proportional to $R_{er}^{0.4}$.

Heat transfered from the surface of the disk includes heat required for sublimation of naphthalene. The rate of mass transfer $\overline{m}$ is estimated as $4.8\times10^{-5}\sim4\times10^{-4}$ kg/h and the latent heat of sublimation is 133 kcal/kg and heat transfered by sublimation is less than 1% of the whole heat transfered. It may be almost negligible.

In this authors' experiment, the similarity between Sherwood number and Nusselt number is observed in the range of $R_{er}=2\times10^2\sim3\times10^4$ and the ratio of k_{cm}/α_m is about 1.35.

Conclusion

Here authors summarize the results of experiment over the heat and mass transfer for a naphthalene disk of finite radius rotating in an open environment of air.

(1) Heat transfer accompanying with mass transfer from a naphthalene disk, N_{um} can be expressed to be proportional to $R_{er}^{0.4}$ as reported formerly by Young and Izumi.

(2) The equation of mass transfer, $Sh_m \propto R_{er}^{0.5}$ by Kreith *et al.* can be usefull only for the range of $R_{er}>3.6\times10^4$. Authors present the equation $Sh_m=1.58\ R_{er}^{0.4}$ in the range of $2\times10^2<R_{er}<3\times10^4$.

(3) As in Figs. 4 and 5, the surface of naphthalene disk is not uniform in the direction of radius by influence of air flow on the surface of the disk. The local mass transfer coefficient near the center of a disk is different from that at outskirts of the disk.

(4) The similarity between heat and mass transfer was observed in this case, and k_{cm}/α_m was about 1.35.

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