HEAT AND MASS TRANSFER FROM A ROTATING DISK IN A CYLINDER

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Introduction

Recently J. W. Mitchell *et al.*[1] made investigation on heat transfer from a shrouded rotating disk as a model study of turbine wheels. Studies on heat and mass transfer from a rotating disk in a casing are of great importance for practical heat problems of rotating bodies. The design of rotating bodies such as turbine wheels or rotors of electric motor requires knowledge of thermal characteristics for a rotating disk in a closed chamber.

As its basic problem, numerous experimental and theoretical reports on heat transfer from a disk rotating in an open air[2~11] have been published for past 20 years. But the aerodynamic characteristics of flow around the disk rotating in a casing differs from that in open air.

Therefore, the equations based on the characteristics of flow for a free disk cannot predict theoretically the heat and mass transfer for a disk rotating in a casing. The heat transfer of a rotating disk in a closed vessel has not thoroughly been studied.

L. A. Dorfman[12] and J. W. Daily *et al.*[13] reported their experimental results over the flow on a rotating disk in a closed chamber. F. Kreith and others[14] related their experimental results on mass transfer for a rotating disk of naphthalene with a confronted stationary disk and discussed the effects of the spacing between two disks upon mass transfer.

In this paper authors present the experimental data on heat and mass transfer from a rotating disk in a stationary cylinder and compare them with the previous reports by one of this authors[15] in order to make clear the effects of flow on heat and mass transfer.

Experimental apparatus and procedure

Figs. 1, 2 and 3 show the schematic views of apparatus. Naphthalene ① is cast on a brass saucer ② and a disk of naphthalene was heated by electric heater of Ni-Cr wire ③. A plate of bakelite of 10 mm thick ④ was placed between two saucers ③ and ⑤, and in the lower saucer the dummy heater ⑥ was inserted and adjusted to keep the temperature of the lower saucer as same as the upper saucer. 4 thermocouples of Cu-Constantan wire were buried in the naphthalene disk at regular intervals in radial direction to measure the surface temperature. 4 couples in the disk and 2 thermocouples on the saucers were led to the electronic recorder through slip rings which were set in the lower part of the shaft. Automatic temperature recorder was 6 points recorder with electronic tubes and had the accuracy of 0.3°C for 0∼5 mV range.

As shown in Figs. 2 and 3, the disk which was shown in Fig. 1 was placed in the cylinder and at the bottom of the cylinder respectively. The disk rotated horizontally on the axis of cylinder.

In order to analyse the effects of the cylinder upon Sherwood number, the experiments were executed with various diameters of cylinder 2 R of 90, 96, 104, 114, 126, 144, 170 and 190 mm.

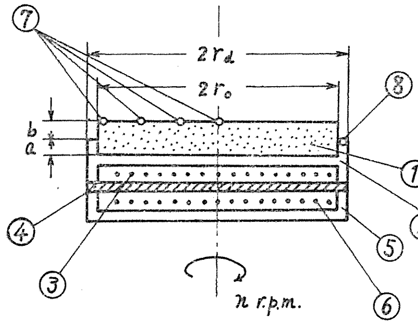

FIG. 1

For Fig. 2 the length of the cylinder of 40 cm were used. In Fig. 3 there is an annulus gap g between the bottom plate of cylinder 10 and the disk. The widths of the annulus gap of 0, 1.5, 3 and 7 mm were tested. For Fig. 3 the length of cylinder of 20, 30, 50 and 100 cm were tested.

Mean surface temperature of the disk ϑ_{sm} is calculated as

$$\vartheta_{sm} = \frac{\int_A \vartheta dA}{\int_A dA} \quad °C$$

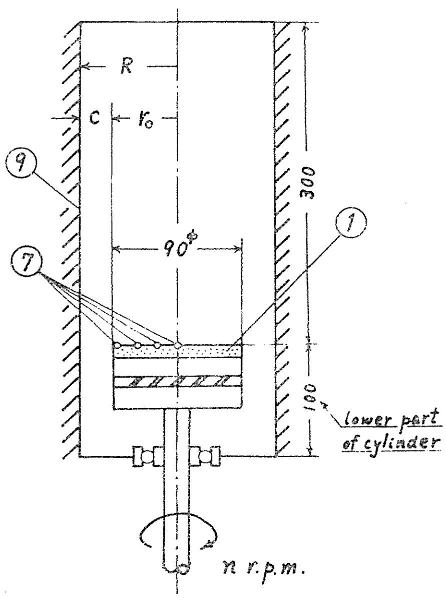

FIG. 2

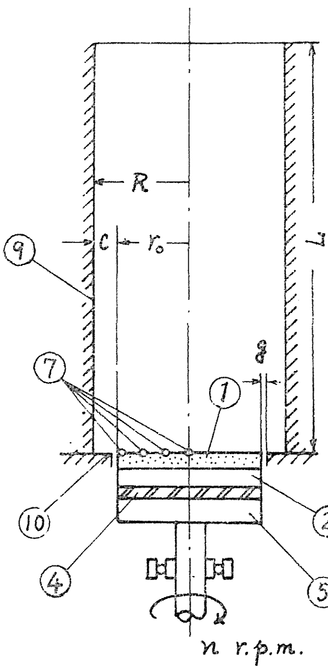

FIG. 3

where ϑ°C is the measured temperature on the surface of the disk, dA m² is the differential area of the surface. Getting ϑ_{sm}, average heat transfer coefficient α_m kcal/m²h°C is given as follows

$$\alpha_m = \frac{0.86\ VI - q_r - q_s}{A(\vartheta_{sm} - \vartheta_\infty)}$$

where q_s kcal/h is the heat loss by convection which transfers from the surface of cylinder ⑧, q_r kcal/h is the heat loss by radiation from the whole surfaces. V volt and I amp. are the input voltage and current in the electric heater coil respectively. ϑ_∞°C is the room temperature and A m² is the area of the surface of the disk.

During the tests the temperature difference of two saucers was adjusted to keep in 1~2°C. The weights of sublimated naphthalene are calculated by multiplying the lost thickness of naphthalene, measured with a dial gauge of accuracy 1/250 mm, by specific weight of naphthalene 1.1008 gr/cm³.

The average mass transfer coefficient k_{cm} and the average Sherwood number Sh_m are expressed as follows

$$k_{cm} = \bar{m} R_v T_s / p_v \cdot A \qquad \text{m/h}$$

$$Sh_m = k_{cm} \cdot r_0 / D_v$$

where $\bar{m}$ kg/h; the mass transfer rate of naphthalene
R_v ; ideal gas constant for naphthalene vapor, 6.615 kgm/kg°K
T_s°K ; the mean abusolute temperature of the disk surface in the testing period.
D_v m²/h; diffusivity of naphthalene vapor into air for mass transfer.
p_v kg/m²; partial vapor pressure of naphthalene at the surface temperature ϑ_{sm}

Every measure was taken under the steady condition in order to keep constant rate of mass transfer $\bar{m}$. The disk was driven by pulley. The revolution of the disk was in the range of 100 to 600 rpm. Flow around the disk might be considered in laminar range.

Experimental results and its consideration

1) A disk rotating in the middle of a cylinder:

The characteristics of flow on the disk rotating in a cylinder differs from that of the rotating disk in an open environment. Radial flow above the surface of disk is suppressed by inner wall of cylinder. Partial pressure of vapor of naphthalene in boundary layer above the surface increases.

In Fig. 4, Sherwood numbers are plotted to the ratios of diameter of disk and cylinder with r.p.m. of the disk are 100~600. Generally, deminutions of the clearance gap between disk and cylinder make Sherwood numbers decrease, but up to the region of $R/r_0 = 1.7$, Sherwood numbers stay constant which correspond to Sherwood number of disk rotating in open air.

Authors observe that Sherwood number of the disk in high r.p.m. is noticeably high at small radius ratio. It might be condidered that the large peripheral speed by rotation of disk makes flow around the outskirt of the disk turbulet and therefore the mass transfer rate increases.

Fig. 5 shows the relations between Sherwood number to the rotational Reynolds number at clearance gap $c=3$, 10, 20, 30 and infinite. For each clearance the mass transfer rate is almost similar in inclination, which increases with r.p.m. of the disk.

For Figs. 4 and 5, decreasing of Sherwood number with the diminutions of clearance gap c or the radius ratio R/r_0 is caused by the conditions that the radial flow is suppressed by the wall of cylinder and the vapor of naphthalene would stagnate. Consequently, in the boundary layer above the surface of the disk the vapor pressure increases and then the pressure gradient $(p_{vs}-p_v)$ between the surface and its environment decreases and therefore the force of diffusion is weakened.

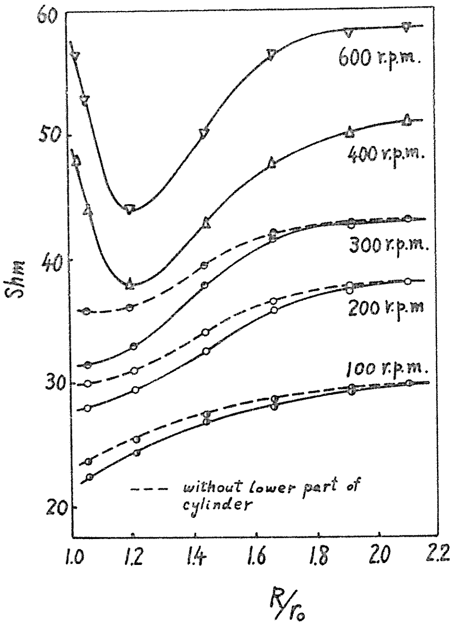

FIG. 4

2) A disk rotating at the bottom of cylinder

Authors observed the pattern of flow in a cylinder by smoke of tetrachrolide titanium induced from upper entrance of the cylinder.

Fig. 6 (a) illustrates the pattern of flow with no clearance between radii of cylinder and disk, namely $R/r_0=1.0$. Radial flow on disk surface turned upwards along the wall of cylinder. Axial flow descended from upper entrance of cylinder towards the surface of the disk in spiral spin state.

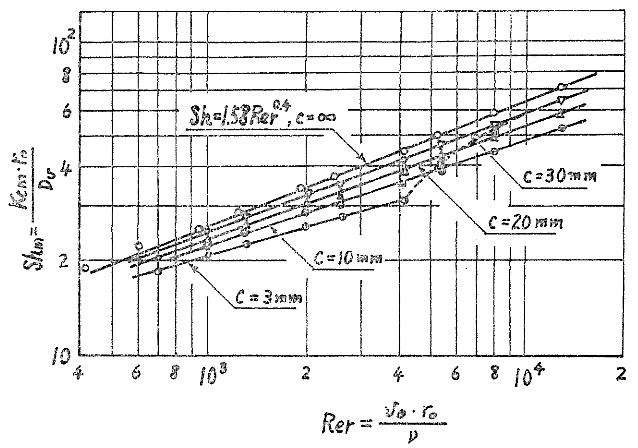

FIG. 5

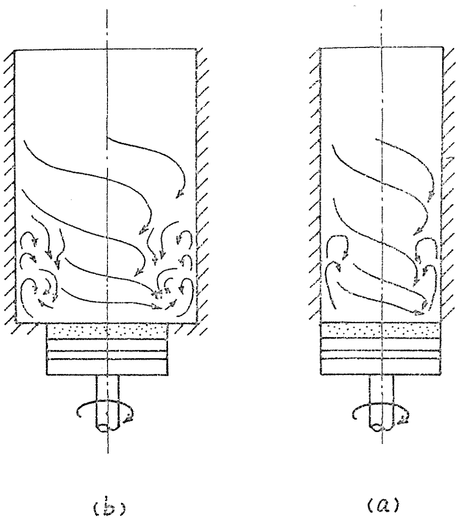

FIG. 6

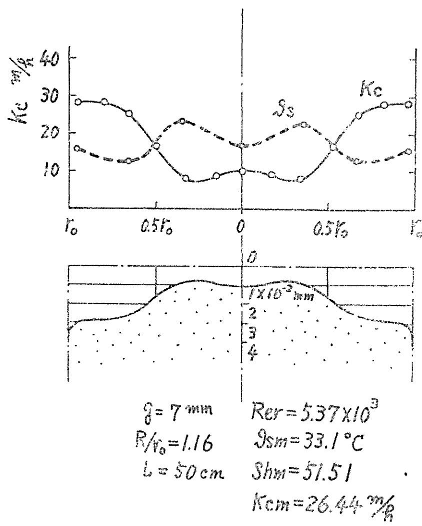

FIG. 7

The disk of naphthalene of 90 mm in diameter was tested. At $n=300$ r.p.m. *viz.* $R_{er}=4\times10^3$ flow ascended up to 7 cm high from the surface of the disk along the inner wall of cylinder. With 20 cm long cylinder, the swirling down flow began at $13\sim15$ cm above the disk at $n=400$ r.p.m. *viz.* $R_{er}=5.3\times10^3$. In the upper part of cylinder flow of air was nearly stagnant and some part of air flowed out of the upper end of the cylinder. The upward flow of air along the wall and the downward flow in the center of cylinder made vortexes in thin boundary layer which extended to the outskirts of disk and produced turbulent state and stimulated the mass transfer. For the case of $R/r_0=1.4$ shown in Fig. 6 (b), in this case upward air flow along the cylinder wall formed thick layer of vortexes and it was observed on the whole bottom area of cylinder. In this case the layer of vortexes did not reach to the surface of the disk and resisted against swirling down flow. These vortexes seem to be so-called Taylor-Goerthor vortexes.

Fig. 7 illustrates the surface of disk of naphthalene after test. It differs from the case of a disk rotating in an open environment, large amount of loss of naphthalene was seen at the outskirts and in the zone $r=0\sim0.35\,r_0$, uniform distribution of k_c was observed. Fig. 8 shows the relations between Sh_m and the radius ratio of R/r_0 in the case of a gap $g=0$.

Decreasing c *viz.* R/r_0 approaching to 1, Sh_m decreases rapidly at $R/r_0<1.7$ to the minimum at $R/r_0=1.3$ and then turns to increase with decreasing of c. This tendency of Sh_m against R/r_0 holds for every r.p.m. of disk and the length of cylinder strengthen this tendency. In this figure $L/2\,r_0=2.22$ ($L=20$ cm) Sh_m does not decrease so much. This phenonenon can be explained by the reason as mentioned in the case (1). The quantity of sublimation is likely to increase at high rotational Reynolds number. Sh_m varies slightly at $n=100$ r.p.m. *i.e.* $R_{er}=1.34\times10^3$ and for $L/2\,r_0=2.22$. While at $n=400$ r.p.m. *i.e.* $R_{er}=5.37\times10^3$ and for $L/2\,r_0=2.22$, Sh_m is affected by influence

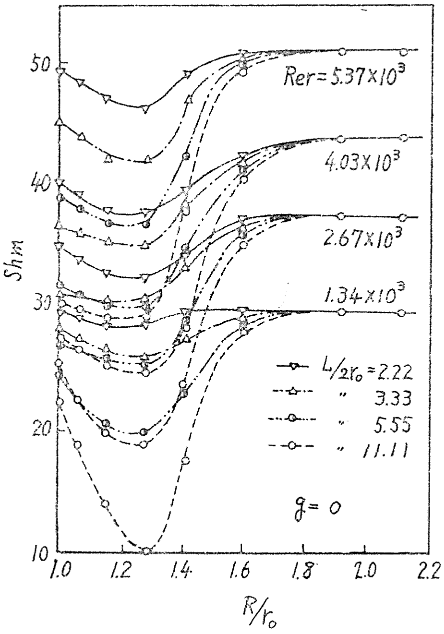

FIG. 8

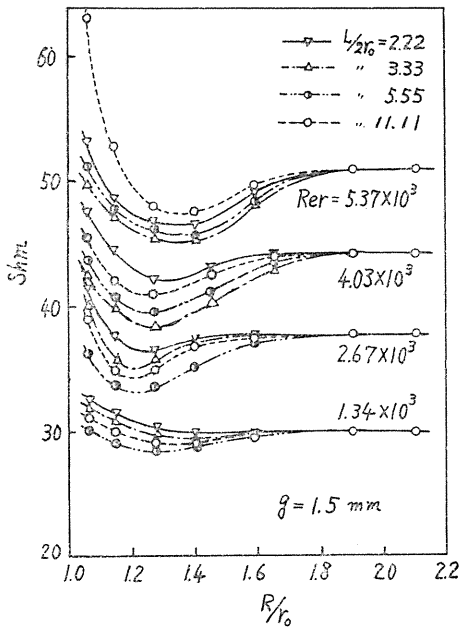

FIG. 9

of wall and deviates 6%. In the domain above $R/r_0 > 1.9$ Sh_m is saturated to the values for the disk rotating in open air and effect of cylinder can be neglected.

Fig. 9 indicates Sh_m against R/r_0 for the disk with annulus gap $g=1.5$ mm between disk and bottom plate of cylinder. Sh_m shows slighter variation than in Fig. 8. The minimum value of Sh_m occures always at $R/r_0=1.3$. The larger the r.p.m. is, the larger variation of Sh_m are seen. In this case, Sh_m is not proportional to the ratio of the length of cylinder and diameter of disk $L/2\,r_0$ especialy at $R_e = 4.03 \times 10^3$ and 5.37×10^3. When there is a gap between cylinder and disk, outward flow through gap occures and in the case of short length of cylinder, downward flow and swirling flow become more vigorous and thus the stagnation of vapor decreases.

Figs. 10 and 11 indicate the results of the mass transfer for the disk with annulus gap $g=3$ and 7 mm in width respectively. For large width of gap, the variations of Sh_m decrease. At $R/r_0=1.3$, Sh_m is minimum as above mentioned. For large annulus gap flow speed on the disk which induced by suction of air into the cylinder turns to decrease and therefore Sh_m decreases. In Fig. 12 Sh_m were plotted again for $R_{er}=2.69 \times 10^3$ and $L/2\,r_0=5.55$ and $g=3$ mm.

On this figure, authors plotted also Nusselt number for the disk rotating in a cylinder, which was calculated under the same condition of experiment.

As this figure shows that the similarity between heat and mass transfer for a disk in a cylinder is established and the ratio of heat and mass transfer coefficients k_{cm}/α_m is about 1.35 which is the same result in authors' previous report.

Authors measured α_m under the condition of supplying constant heat for every experiment. Temperatures on the surface of disk changed susceptively with

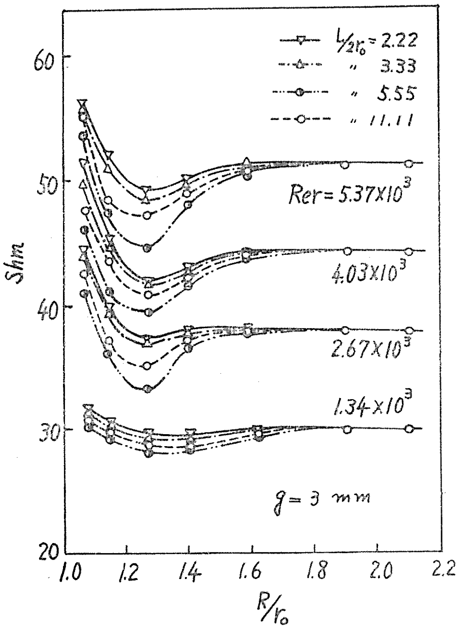

FIG. 10

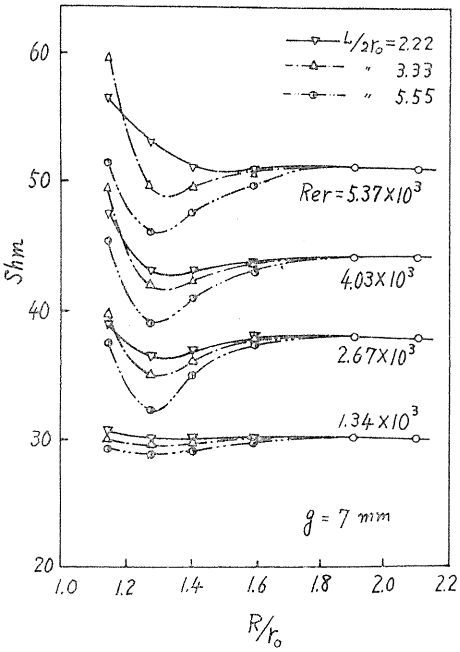

FIG. 11

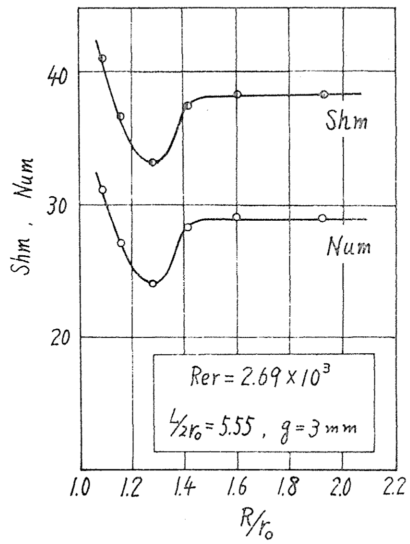

FIG. 12

change of R/r_0. In this measurement α_m or N_{um} had constant value for $R/r_0>1.6$ as described in the case for the disk rotating in an open environment. For $R/r_0<1.6$ the temperature of the surface of the disk becomes high in the influence of wall of cylinder and N_{um} decreases to the minimum value near $R/r_0=1.3$. For an example, the temperature of the surface of the disk at outskirts and air in the boundary layer are 39.2°C and 24°C respectively. At ratio less than $R/r_0=1.3$, the air flow becomes turbulent at outskirts and ϑ_{sm} decreases. The temperatures on the surface of the disk and of air in the boundary layer becomes 38°C and 27°C respectively. Therefore the gradient of temperatures decreases and thus N_{um} increases.

Fig. 13 illustrates the effect of $c=0\sim22$ mm on Sh_m against R_{er} for the disk rotating in a 100 cm long cylinder *i.e.* $L/2r_0=11.11$. The equation of $Sh_m=1.58\cdot R_{er}^{0.4}$ is applicable to the disk in the cylinder with the limit of $R/r_0>1.9$. For $c=22$ mm, Sh_m was indicated to be parallel to the above equation, but the values are 10% lower than the above. $c=0$ means

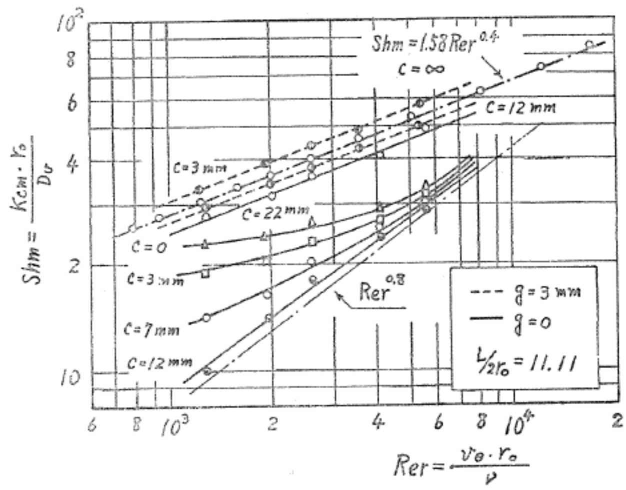

FIG. 13

the case of a disk in enclosed space. For $c=12$ mm Sh_m becomes the minimum value which corresponds to the state of $R/r_0=1.3$. For $c=0$, 3, 7, and 12 mm Sh_m increases with R_{er} and approaches to $R_{er}^{0.8}$.

It is said that the transition point from laminar to turbulent flows for the disk rotating in an open environment is at $R_{er}=2\times10^5$, but in authors' experiment the air flow in the cylinder was disturbed by vortexes of upward flow and downward flow and therefore in the region of $R_{er}=5\times10^3$, the relation of $Sh_m \propto R_{er}^{0.8}$ would be held. In the case of $g=3$ mm, Sh_m for $c=3$ and 12 mm were plotted with dotted lines and located near the equation of $Sh_m=1.58 \cdot R_{er}^{0.4}$. Generally the variation of Sh_m for the disk with annulus gap are not so different as shown in Figs. 9~11, Sh_m can be expressed as $Sh_m \propto R_{er}^{0.4}$.

Conclusion

(1) As Fig. 7 shows, the surface of naphthalene disk is not uniform in the direction of radius by influence of air flow on the surface of the disk. The local mass transfer coefficient near the center of the disk is different from that at outskirts of the disk.

(2) For the case of a rotating disk in a cylinder Sh_m decreases to the minimum value at $R/r_0=1.3$ and then turns to increase with decreases of R/r_0. For the case of $R/r_0>1.9$ Sh_m coincides with the value for a disk rotating in open air.

(3) For the case of a disk in a cylinder, the loss of naphthalene by sublimation is observed to be large in the region near outskirts of a disk by the vortexes caused by upward and downward flow in the cylinder.

(4) The similarity between heat and mass transfer was observed for this case, and k_{cm}/α_m was estimated to be about 1.35.

Acknowledgment

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Reference

1) J. W. Mitchell and D. E. Metzger: Trans. ASME., Ser. C, Vol. **87**, No. 4 (1965), p. 485.
2) C. Wagner: J. Appl. Phys., Vol. **19**, No. 2 (1948), p. 837.
3) K. Millsaps and K. Pohlhausen: J. Aeron. Sci., Vol. **19**, (1952), p. 120.
4) N. Riley: Quart. J. Mech. and Appl. Math., Vol. **17**, Part 3 (1964), p. 331.
5) D. R. Olander: Int. J. Heat and Mass transfer, Vol. **5**, No. 7 (1962), p. 825.
6) E. M. Sparrow and J. L. Gregg: Trans. ASME., Ser. C, Vol. **81**, No. 3 (1959), p. 249. and Trans. ASME., Ser. C, Vol. **81**, No. 2 (1959), p. 113.
7) C. L. Tien and D. T. Campbell: J. Fluid Mech., Vol. **17**, Part 1 (1963), p. 105.
8) J. P. Hartnett, Shing-Hwa Tsai and H. N. Jantscher: Trans. ASME., Ser. C, Vol. **87**, No. 3 (1965), p. 362.
9) R. L. Young: Trans. ASME, Vol. **78**, No. 6 (1956), p. 1163.
10) E. C. Cobb and O. A. Saunders: Proc. Roy. Soc., Vol. **236 A** (1959), p. 343.
11) R. Izumi: The bulletin of Yamagata Univ., Vol. **5**, No. 2 (1959), p. 269.
12) L. A. Dorfman: Hydrodynamic resistance and the heat loss of rotating solid (1963), Oliver and Boyd.
13) J. W. Daily and R. E. Nece: Trans. ASME., Ser. D, Vol. **82**, No. 1 (1960), p. 217.
14) F. Kreith, E. Doughman and H. Kozlowski: Trans. ASME., Ser. C Vol. **85**, No. 2 (1963), p. 153.
15) A. Iguchi and R. Izumi: Trans. of JSME., Vol. **33**, No. 247 (1967), p. 398.