AN ANALYTICAL STUDY OF A FAVOURABLE VELOCITY DISTRIBUTION FOR A LOW DRAG AEROFOIL

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Summary

Velocity distributions for a low drag aerofoil, which has little tendency to separate of turbulent boundary layer, are investigated. Stipulating the shape factor to be constant behind a proper situation on the aerofoil, an analytical method based on Garner's empirical formula is applied to calculate the turbulent boundary layer. The velocity distribution along the aerofoil obtained by the present method has a concave form, which is fairly well coincident with Wortmann's result.

Symbols

H : shape factor of turbulent boundary layer, *i.e.* the ratio of the displacement thickness to the momentum thickness

H': empirical value of shape factor due to Garner, *i.e.* 1.4.

H_c: a constant value of shape factor

R_e: free-stream Reynolds number $u_\infty c/\nu$

u_e : external velocity in turbulent boundary layer

u_∞: free-stream velocity

α : attack angle of aerofoil

θ : momentum thickness of turbulent boundary layer

ν : kinematic viscosity

ρ : density

1. Introduction

The form of wing sections of low drag has been determined by the velocity distribution preserving laminar boundary layer in the greater part of aerofoil surface. Further improvement can be expected by controlling the turbulent boundary layer to delay the separation. This can be done by using a suitable form of velocity distribution.

Walz[1] studied the effect of contour on the boundary layer flows and found that the turbulent boundary layer was thinner for the aerofoil with concave tail end than that of convex tail. Wortmann[2)3)] investigated the influence of velocity distributions on the development of a turbulent boundary layer with adverse pressure gradients, and presented an idea to control the turbulent separation by stipulating a parameter of boundary layer. This implies that the shape factor of turbulent boundary layer should be constant along the turbulent region of the aerofoil. Using the approximate method developed by Truckenbrodt,[4] Wortmann[2)3)] introduced several examples of concave velocity distributions by some trial and error processes.

An analytical calculation taking place of Wortmann's[2)3)] trial and error method, should be developed to improve the boundary layer characteristics. In this connection present authors tried to solve the turbulent boundary layer with constant shape factor and intended to obtain an analytical formula for the velocity distribution. Considering the small magnitude of pressure gradient Falkner's[5)] empirical formula is combined with the momentum integral equation for the first approximation. Integrating this equation for a constant value of shape factor, momentum thickness can be calculated in terms of velocity distribution. In order to solve the velocity distribution along a surface another equation of Garner's[6)] formula which governs the development of shape factor is employed. Two simultaneous equations for momentum thickness and external velocity are thus prepared. Solving these equations with suitable boundary conditions, the corresponding velocity distribution is obtained in a closed form. Several examples are calculated and numerical values of velocity distribution are shown in fair agreement with Wortmann's[2)3)] concave form of velocity distributions.

2. Fundamental Equations

The two-dimensional incompressible turbulent boundary layer is concerned. Taking coordinate x along the contour and y normal to it, the momentum integral equation can be written in the form:

$$\frac{\tau_0}{\rho u_e^2} = \frac{d\theta}{dx} + (H+2)\frac{\theta}{u_e}\frac{du_e}{dx} \tag{1}$$

where τ_0 is the frictional stress on the contour surface. Considering the small magnitude of pressure gradient Falkner's[5)] empirical formula for a flat plate is introduced as the first approximation.

$$\frac{\tau_0}{\rho u_e^2} = \frac{\alpha}{\left(\frac{u_e\theta}{\nu}\right)^{1/n}} \tag{2}$$

$$n = 6,\ \alpha = 0.006534$$

Substituting Eq. (2) into Eq. (1), the momentum integral equation can be rearranged in the following form:

$$\frac{d}{dx}\left[\theta\left(\frac{u_e\theta}{\nu}\right)^{1/n}\right] = \frac{n+1}{n}\alpha - \left\{\frac{n+1}{n}(H+2) - \frac{1}{n}\right\}\frac{\theta}{u_e}\left(\frac{u_e\theta}{\nu}\right)^{1/n}\frac{du_e}{dx} \tag{3}$$

For the flow of a constant shape factor Eq. (3) can be integrated in a closed form.

$$\theta\left(\frac{u_e\theta}{\nu}\right)^{1/n} = u_e^{-b}\left(C_1 + a\int_{x_t}^{x} u_e^b\,dx\right) \tag{4}$$

$$a = \frac{n+1}{n}\alpha,\ b = \frac{n+1}{n}(H+2) - \frac{1}{n}$$

where C_1 is the integration constant and x_t indicates the position of transition point. Eq. (4) gives a relation between momentum thickness and velocity distribution.

In order to solve the turbulent boundary layer another relation governing the change of shape factor along the contour is necessary. Von Doenhoff and Tetervin[7] presented such an equation and modifying their formula Garner[6] derived the following equation:

$$\theta\left(\frac{u_e\theta}{\nu}\right)^{1/n}\frac{dH}{dx} = e^{m(H-H')}\left[-\theta\left(\frac{u_e\theta}{\nu}\right)^{1/n}\frac{1}{u_e}\frac{du_e}{dx} - A(H-H')\right] \tag{5}$$

$$m=5,\quad H'=1.4,\quad A=0.0135$$

Putting the shape factor to be constant, an empirical relation θ and u_e is obtained.

$$\theta\left(\frac{u_e\theta}{\nu}\right)^{1/n}\frac{1}{u_e}\frac{du_e}{dx} = -A(H-H') \tag{6}$$

If Eqs. (4) and (6) are assumed to be simultaneous equations, the velocity distribution can be solved as a function of x. Eliminating θ from Eqs. (4) and (6) an equation for the velocity distribution is derived.

$$u_e^{-b}\left(C_1 + a\int_{x_t}^{x} u_e^b\, dx\right) = \frac{-A(H-H')}{\dfrac{1}{u_e}\dfrac{du_e}{dx}} \tag{7}$$

Differentiating Eq. (7) with x it is given:

$$\frac{d^2u_e}{dx^2} = \frac{k}{u_e}\left(\frac{du_e}{dx}\right)^2 \tag{8}$$

where

$$k = \frac{a}{A(H-H')} + b + 1$$

Eq. (8) is a differential equation for the velocity distribution when the shape factor is constant. A direct integration of Eq. (8) gives the following formula:

$$u_e = [q(C_2x + C_3)]^{1/q} \tag{9}$$

where C_2 and C_3 are the integration constants and $q=1-k$. These integration constants should be determined by the boundary conditions at the connecting point and at the trailing edge. Denoting the velocities at the connecting point and the trailing edge by u_{ec} and u_{et} respectively, C_2 and C_3 are determined by

$$C_2 = \frac{u_{et}^q - u_{ec}^q}{q(c-x_c)}, \quad C_3 = -\frac{u_{et}^q x_c - u_{ec}^q c}{q(c-x_c)} \tag{10}$$

where x_c and c denote the positions of the connecting point and of the trailing edge respectively. The velocity distribution in non-dimensional form is given by Eq. (9) in the following form:

$$\frac{u_e}{u_\infty} = \left[r\frac{x}{c} + s \right]^{1/q} \tag{11}$$

where

$$r = C_2 \frac{qc}{u_\infty^q} = \frac{\left(\dfrac{u_{et}}{u_\infty}\right)^q - \left(\dfrac{u_{ec}}{u_\infty}\right)^q}{1 - \dfrac{x_c}{c}}$$

$$s = C_3 \frac{q}{u_\infty^q} = -\frac{\left(\dfrac{u_{et}}{u_\infty}\right)^q \dfrac{x_c}{c} - \left(\dfrac{u_{ec}}{u_\infty}\right)^q}{1 - \dfrac{x_c}{c}}$$

When the values of x_c/c, u_{ec}/u_∞, u_{et}/u_∞ and H are indicated, the corresponding velocity distribution with a constant shape factor can be calculated by Eq. (11). In usual case the parameter q has a negative value and, therefore, it should be noticed that the velocity distribution with a constant shape factor has a concave form.

3. Numerical Examples

In order to compare the present formula with Wortmann's[2)3)] results, an example of NACA 64_2-015 aerofoil[8)] is employed. Assuming the point of transition to be $x_t/c=0.4$ at the free-stream Reynolds number $u_\infty c/\nu=10^6$, the development of turbulent boundary layer are calculated by either Garner's[6)] and Truckenbrodt's[4)] methods. Distribution of θ and H are shown in Fig. 1. For the sake of convenience Garner's[6)] method will be referred in the following calculations, since both method give similar results as shown in Fig. 1. The shape factor H in Fig. 1 exceeds the value of 1.8 at about $x/c=0.975$ and, therefore, there will be the possibility of separation of turbulent boundary layer.

Following Wortmann's[2)3)] idea the velocity distribution which has less tendency of separation will be calculated by keeping the shape factor at a constant value H_c less than 1.8. In the present examples the shape factor is assumed in the following form:

(a) $H_c = 1.45$ at $0.528 \leq x/c \leq 1$

(b) $H_c = 1.5$ at $0.673 \leq x/c \leq 1$

(c) $H_c = 1.55$ at $0.759 \leq x/c \leq 1$

(d) $H_c = 1.6$ at $0.820 \leq x/c \leq 1$

(e) $H_c = 1.6$ at $0.400 \leq x/c \leq 1$

Case (a) to (d) imply that the original form of the shape factor is kept up to the concerning point of x_c/c and that the value of H is remained constant thereafter.

The caluculated velocity distributions are shown in Fig. 2 together with the distributions of H. It is found that the favourable distributions of external velocity in turbulent boundary layer are concave along x axis, which show good agreement with Wortmann's[2)3)] prediction.

It is interesting to note that the slight modification of the velocity distribu-

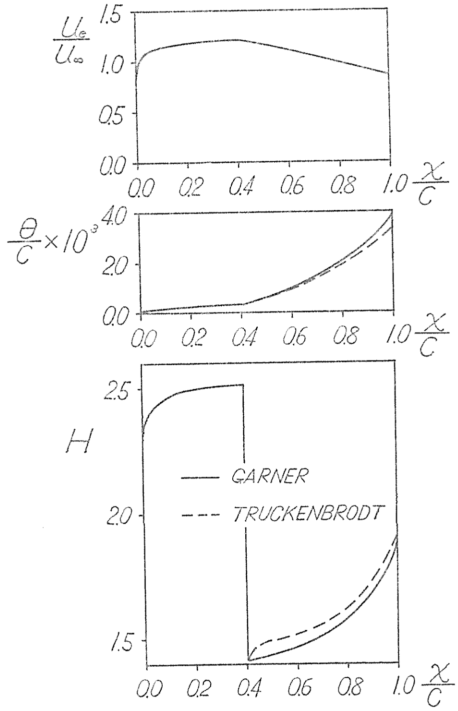

FIG. 1. The velocity distribution and the variations of momentum thickness, shape factor. NACA 64_2-015 $R_e=10^6$, $\alpha=0°$.

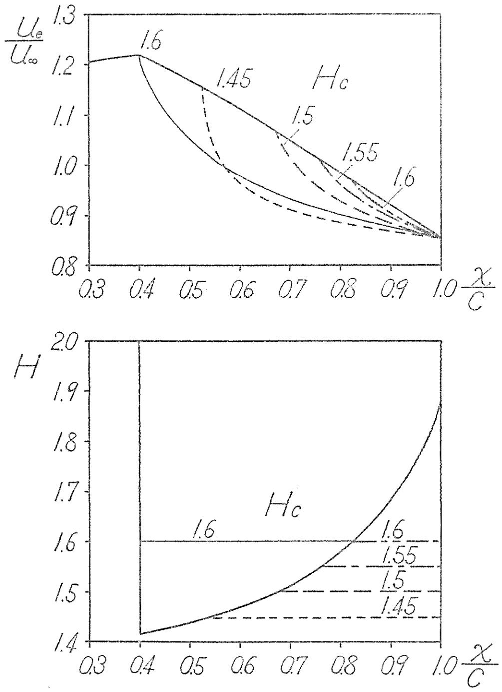

FIG. 2. The velocity distribution and the variations of shape factor.

tion in NACA laminar aerofoil makes the turbulent boundary layer hardly separate. In order to apply these result to practical designs the condition of connection of concave velocity distribution with the laminar one should be carefully investigated. In this connection Wortmann[2) 3)] has studied to find a proper condition of connecting point and has gained an idea to insert an instability range in between the laminar region and fully turbulent one.

The method to get a closed form of aerofoil contour is another problem, which will be attained in further studies.

4. Conclusion

An analytical form of the velocity distribution with constant shape factor is derived by the use of Garner's formula. When the values of the velocity at the connecting point and the trailing edge are given, the corresponding velocity distributions can be calculated by the present formula.

Applying this formula to NACA laminar aerofoil, the concave velocity distributions are obtained, which shows fair agreement with Wortmann's results.

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